Algorithmic Problems in Network Economics

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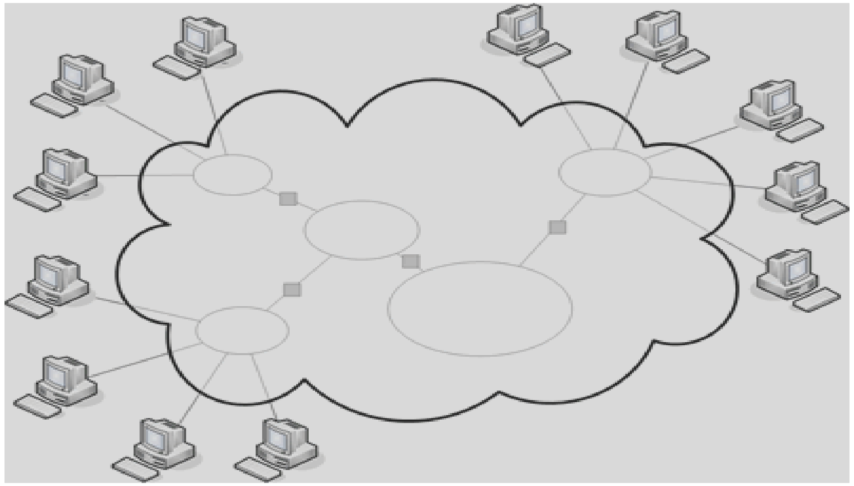
Networked World

- A classical view of the internet



- Open, evolutionary architecture
- Lacks central control and coordination
- Dynamically varying infrastructure and users
- Resource sharing
- Interesting mix of computational and strategic complexities

A Load Balancing Game

servers

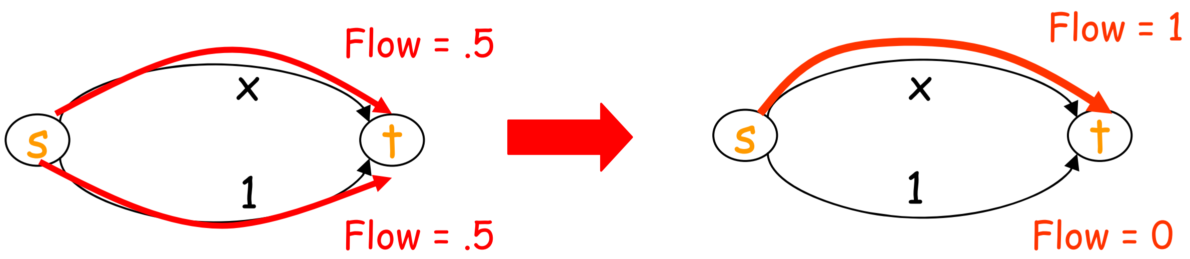
QuickTime?and a
decompressor
are needed to see this picture.

clients

- Matching n clients (users) to m servers (access points)
- A compatibility graph:
  - – edge (i,j) if client i can be served by j
- Identical servers with unit resource
- Latency as cost of matching:
  - – a server matched to k clients has latency = k
- Quality of matching in this uncoordinated world?

Price of Anarchy

- Selfish Routing [Roughgarden et al., Papadimitriou]



- (Social) optimum = 0.5 flow on each link
  - latency = 3/4
- Self-interested (Nash) optimum flow = 1 on top link
  - latency = 1
- Price of Anarchy = Ratio of Social to Nash Optimum
  - this example 4/3

Anarchy in Load Balancing

- What is the worst-case ratio between costs of optimum and Nash matching?

QuickTime?and a
decompressor
are needed to see this picture.

Input	Opt	Nash	Arbitrary
	Cost = 3	Cost = 5	Cost = 5

Anarchy in Load Balancing

- With identical servers, OPT is always NASH, but not vice versa.
- I.e. best case Nash = Opt
- Ratio between worst-case Nash and Opt?

QuickTime?and a
decompressor
are needed to see this picture.

Input

Opt

Cost = 3

Nash

Cost = 5

Arbitrary

Cost = 5

Bounds

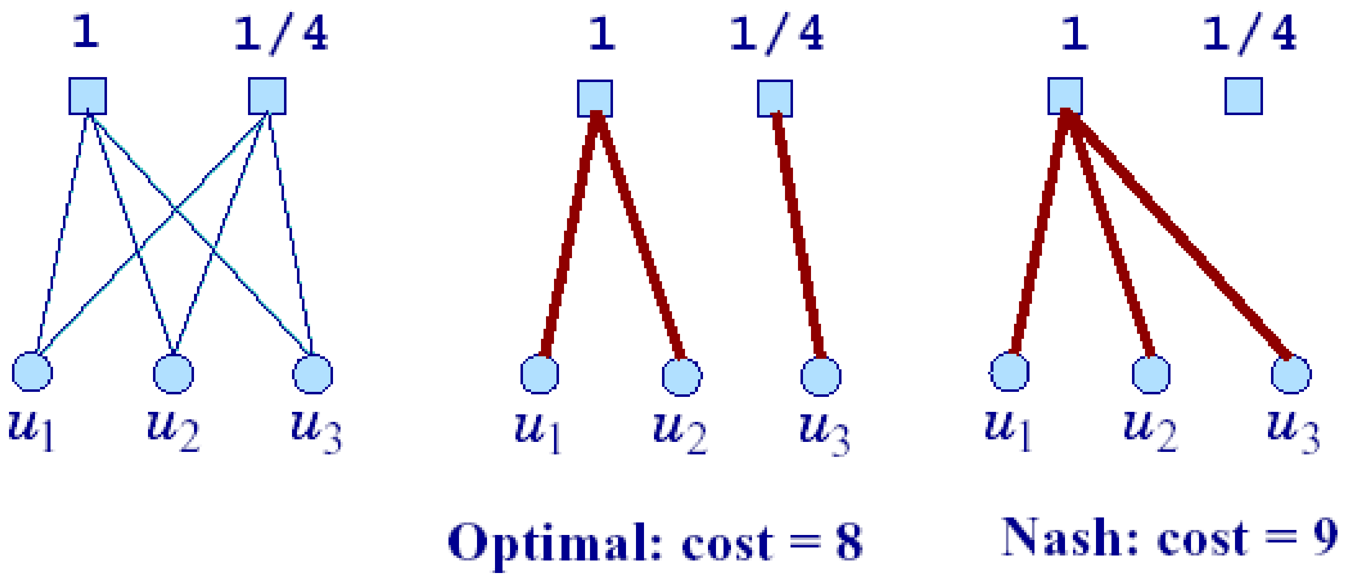
- Theorem 1: For identical servers, price of anarchy is atmost

$$(1 + 2/\sqrt{3}) = 2.155$$

- Theorem 2: Price of anarchy is at least 2.001

More Bounds

- For non-identical servers, social optimum no longer Nash Equilibrium.



- Theorem 3: PoA < 5/2.
- For Lp norm latency, PoA = O(p/logp)
- Selfish Load Balancing, S.-Toth-Zhou, Algorithmica ‘07.
- **Price of routing unsplittable flow**, Awerbuch**, Azar,** Epstein, **STOC ‘05**

Algorithms

- Nash matching by local swaps:
  - in each round, a user switches to better server.
  - Provably $O(n^2)$ rounds.
- Instead suppose clients arrive one by one and each chooses the best available server at that time.
  - Greedy Matching
  - Not necessarily a Nash matching
  - But can be shown to be O(1) factor optimal.

Mobility and Load Balancing

Mobility and Load Balancing

- Wireless access points (APs) at airport, malls, etc.
- User can select and use any AP
  - Selected AP need not be in range
  - User moves towards selected AP if necessary
- Strategic tradeoffs between cost of mobility and wireless service quality
  - Users are rational, selfish entities
    - Maximize personal benefit
    - No regard for system cost

Modeling the Game

- User arrive sequentially
- AP bandwidth shared equally among attached users
  - AP with fewer attached users preferable
- Distance of AP from user's location
  - Closer AP preferable (less mobility, better signal)

- Cost function (user I and AP j),

$$C_{ij} = \gamma * x_j + \beta * d_{i,j}$$

where x_j = number of users at AP *j*

$d_{i,j}$ = distance between user *i* and AP *j*

γ , β are constants (same for all users)

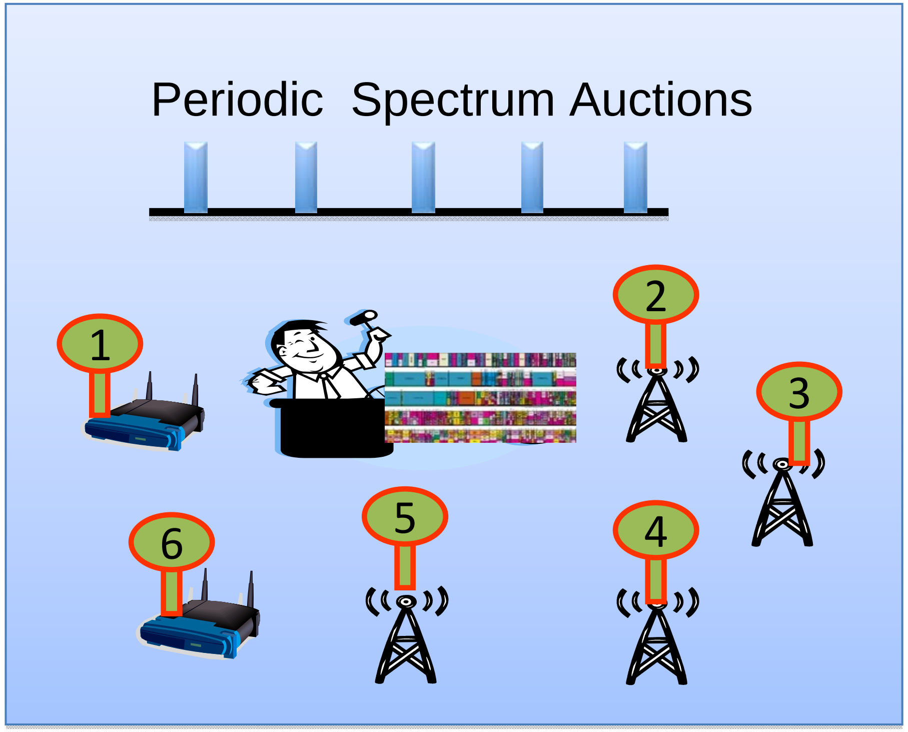
Simple Distributed Algorithm

- Greedy algorithm
  - Upon arrival, each user picks the AP with currently minimum cost
  - No future swaps done.
- Theorem: The greedy always produces a Nash equilibrium
- Social optimal always Nash.

Price of Anarchy

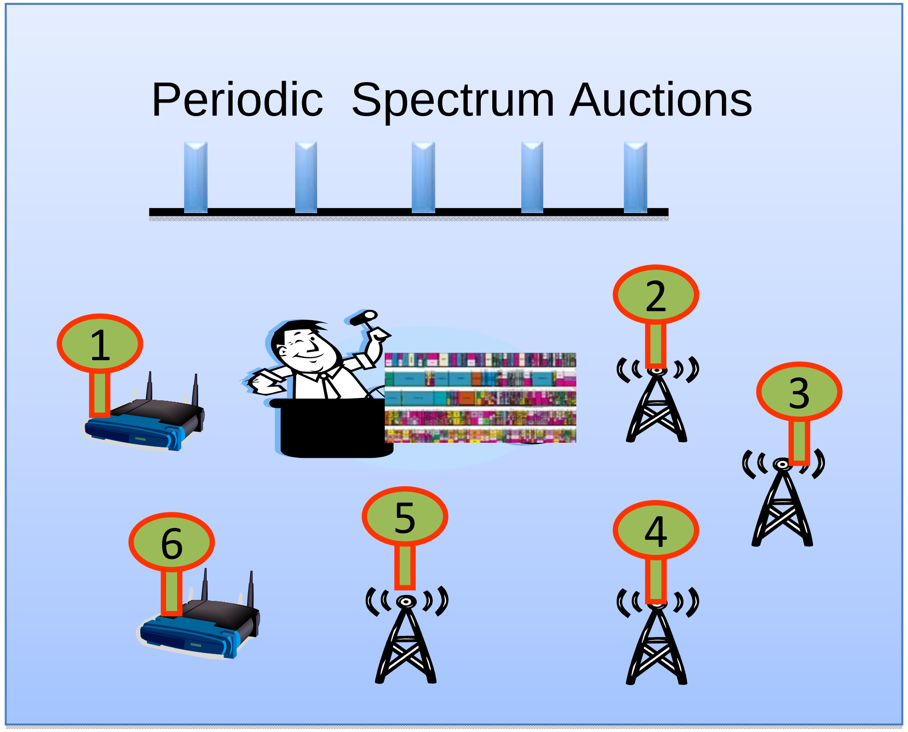
- $\beta = 0$ (Mobility cost zero)
  - Only Nash equilibriums are those that distribute users evenly
  - Pessimistic price of anarchy = 1
- $\gamma = 0$ (Users bandwidth-agnostic)
  - Unbounded price of anarchy
- General case (neither β nor γ zero):
  - Open

Spectrum Auctions



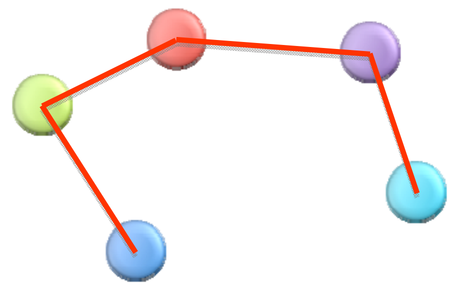
Spectrum Auctions

- **Auctions: efficient allocation of scarce resources**
- Auctioneer: d**ynamic price discovery based on demand**
- Users: **request and acquire spectrum when they need it**



Computational Complexity

- **Externality: interference**
  - **Spatial reuse possible**
  - **Nearby users cannot use same channel**



Interference constraints

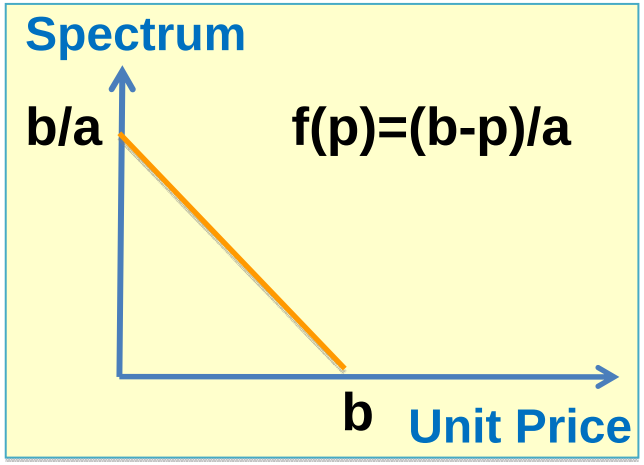
- **Combinatorial auctions NP-complete**
- **Hard even without expressive bidding due to graph coloring**

- **Focus on computational efficiency, without strategic considerations.**

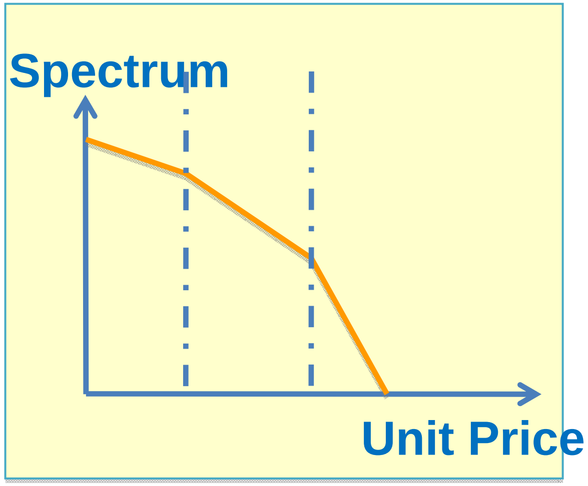
Piecewise Linear Price-Quantity Bids

- **Bids: the desired quantity of spectrum *f* at a per-unit price *p***

Linear bids



Piecewise linear bids



Compact

Approximate arbitrary bidding preferences

Bidding by Price-Quantity Curves

User

Bidding

Piecewise Linear Price Demand bids– compact yet expressive bidding format

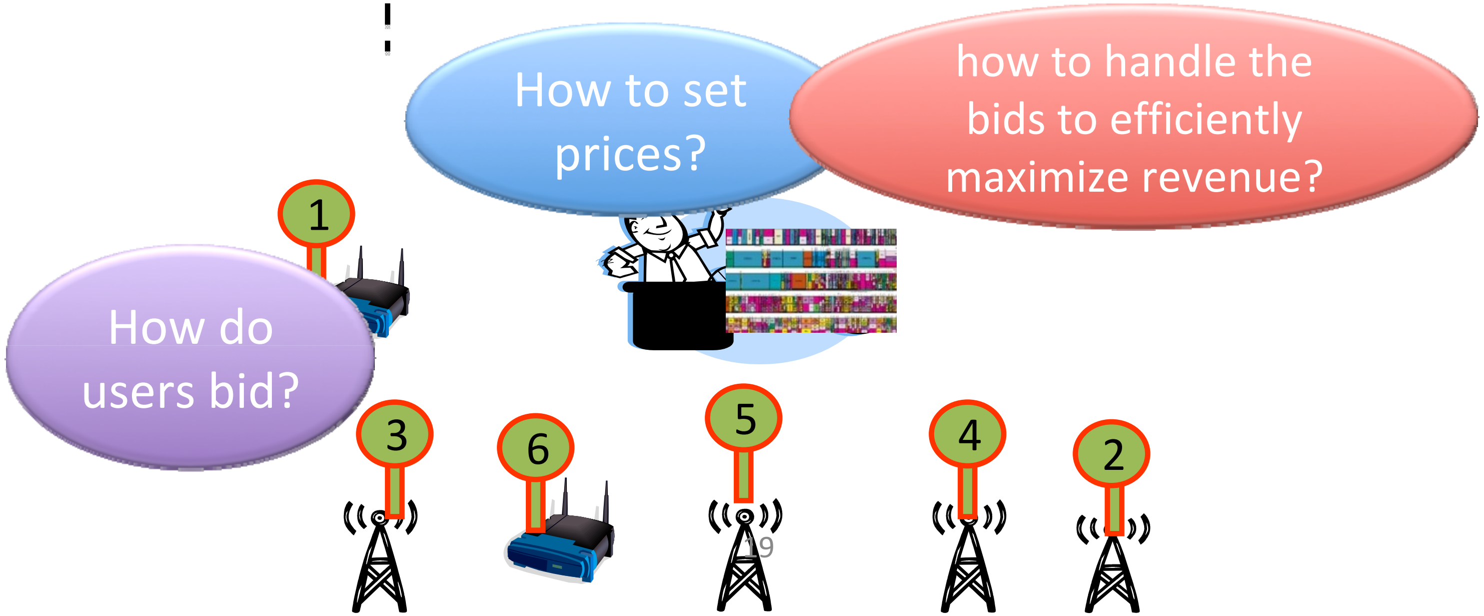
Auctioneer

Pricing Model

Uniform vs. Discriminatory– tradeoffs between efficiency and fairness

Allocation (clearing)

Fast *auction clearing* algorithms for both pricing models



Pricing Models

Uniform Pricing One per-unit price p* for everyone	Discriminatory Pricing Different prices for different bidders
Total Revenue $R(p^*) = \sum_{i, b_i > p^*} \frac{b_i p^* - p^{*2}}{a_i}$	Total Revenue $R(p_1, \ldots p_n) = \sum_i \frac{b_i p_i - p_i^2}{a_i}$

The Auction Clearing Problem

Allocate price(s) and spectrum to maximize the total revenue R(.) subject to **Interference Constraints**

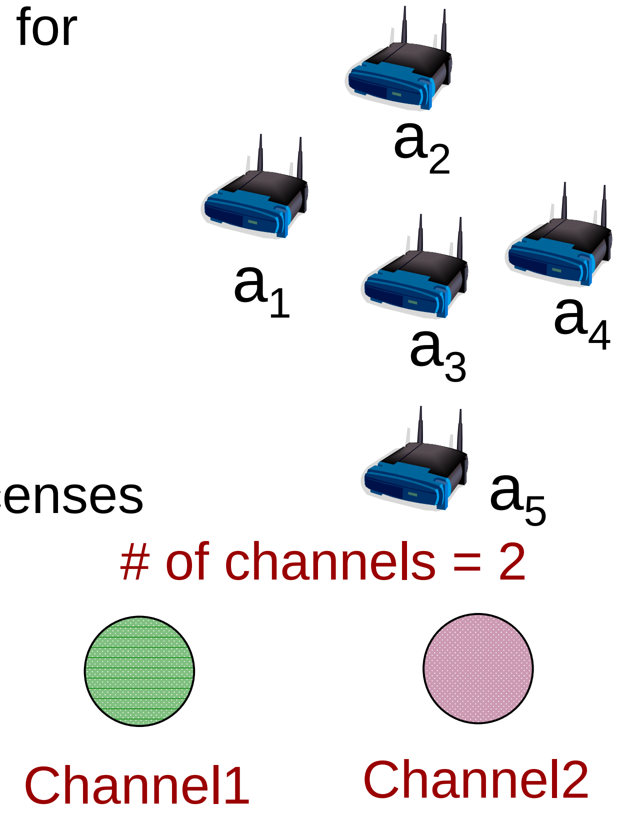
Analytical Bounds

Theoretical bounds	Clearing with Uniform Pricing	Clearing with Discriminatory Pricing
Revenue efficiency	$R \geq \frac{1}{3} R_{OPT}$	$R \geq \frac{n}{3(n+1)} R_{OPT}$
complexity	$O(n \log n + n \log U)$	*polynomial*
When the conflict graph is a tree	$R = R_{OPT}$	$R = R_{OPT}$

Strategy-proof Spectrum Auctions

Strategy-proof Spectrum Auctions

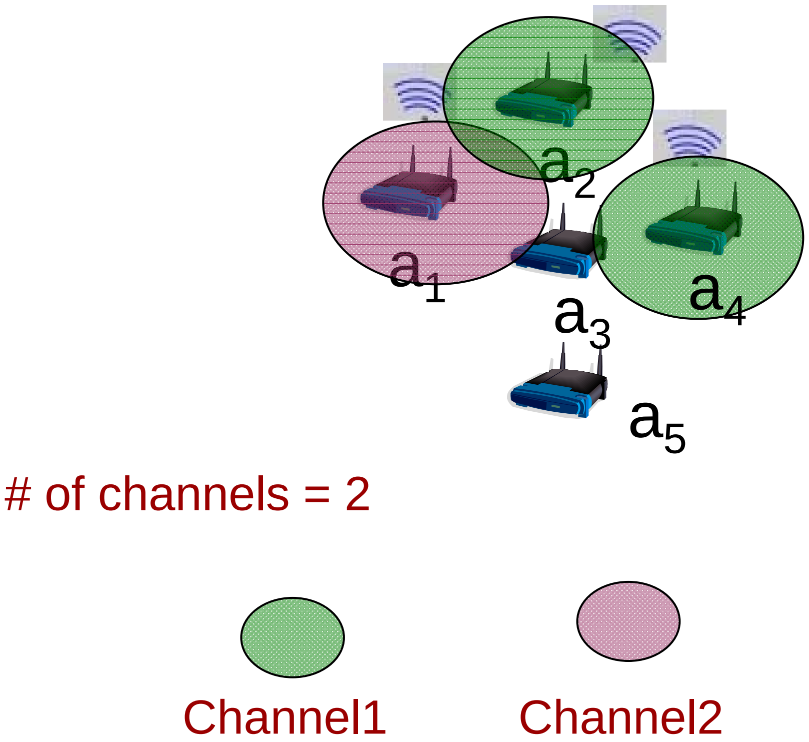
- Input:
  - Spectrum as k channels: 1, 2, …, k
  - A set of n bidders
- Output:
  - A polynomial time strategy-proof mechanism for spectrum allocation
  - Subject to interference constraints

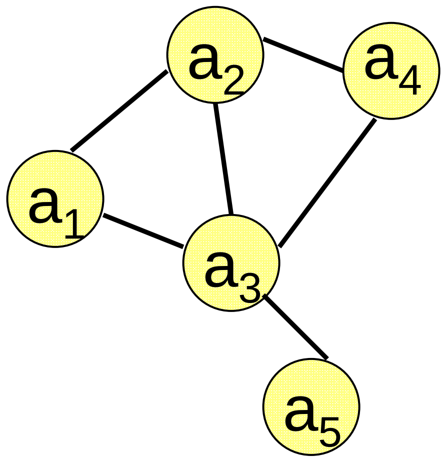


- Motivation:
  - Dynamic redistribution of FCC's long term licenses
  - Fair and open
  - Economic Efficiency

Graph Coloring

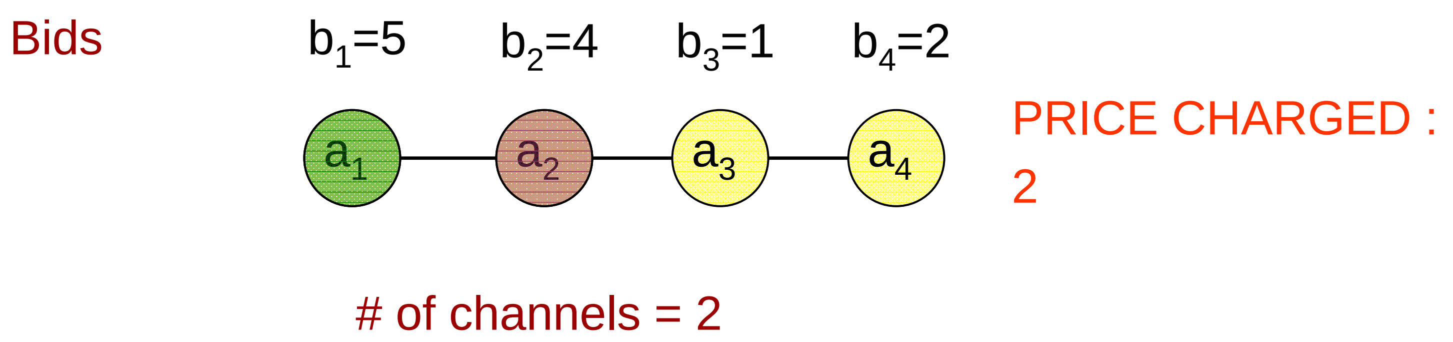
- **Conflict-free channel allocation = graph coloring**
- **Computationally, graph coloring intractable and in-approximable.**





Vickery Auction

- If all we care about is truthfulness, a trivial solution:
  - Allocate channels to k highest bidders
  - Price: Bid of (k+1)th highest bidder



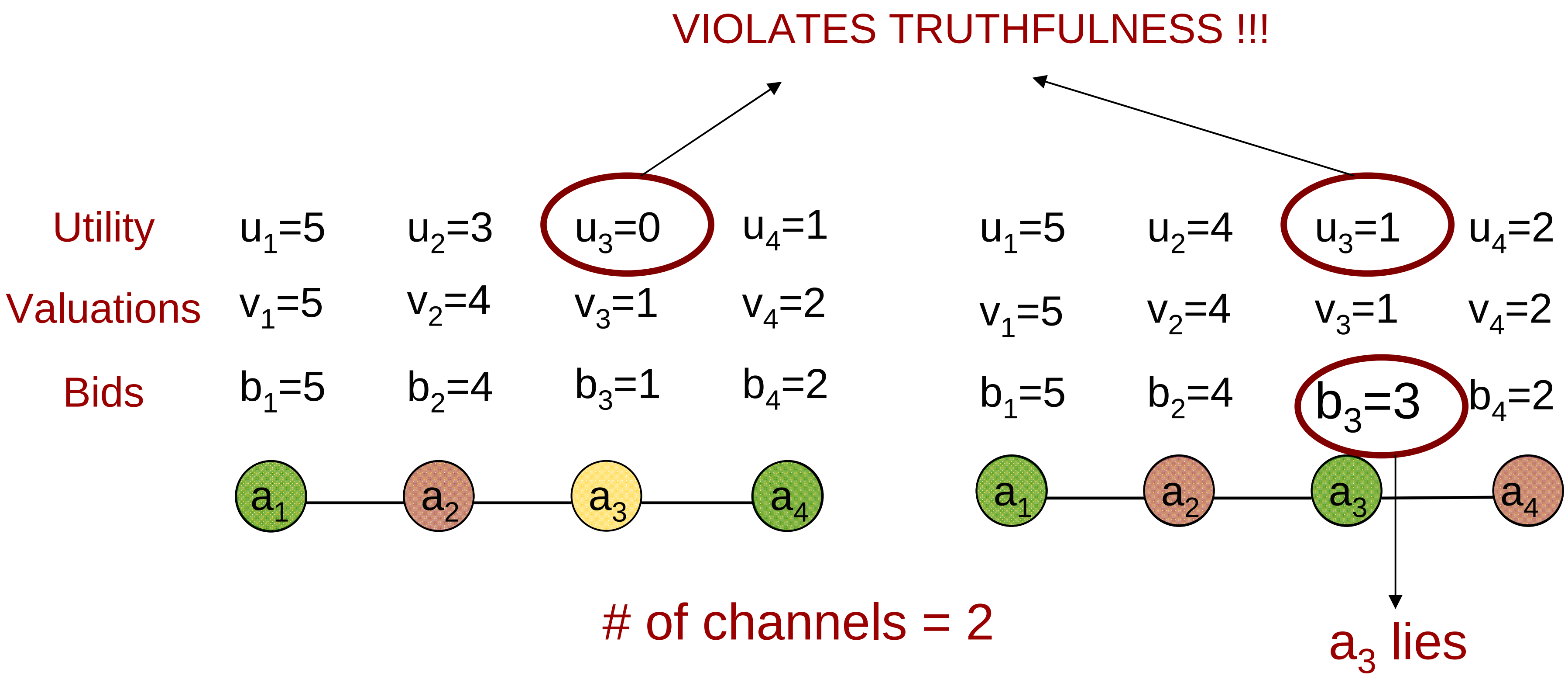
- Inefficient spectrum utilization: a_3 and a_4 left out

Truthfulness with Maximal Utilization

- Always allocate a channel unless doing so precludes another user
- Desiderata:
  - Truthfulness
  - Pareto optimality
  - Computational efficiency
- VCG doesn’t satisfy the computational efficiency requirement

First Attempt

- Sort and Greedily allocate channels
  - Allocate lowest available index
- Each winning bidder pays the bid of highest unallocated neighbor



Another Attempt

- Greedily allocate channels
- For each Winning bidder a_i determine neighbor a_j s.t.
  - – a_j loses when a_i is present, but
  - – a_j wins when a_i is absent
- Charge a_i the bid of a_j

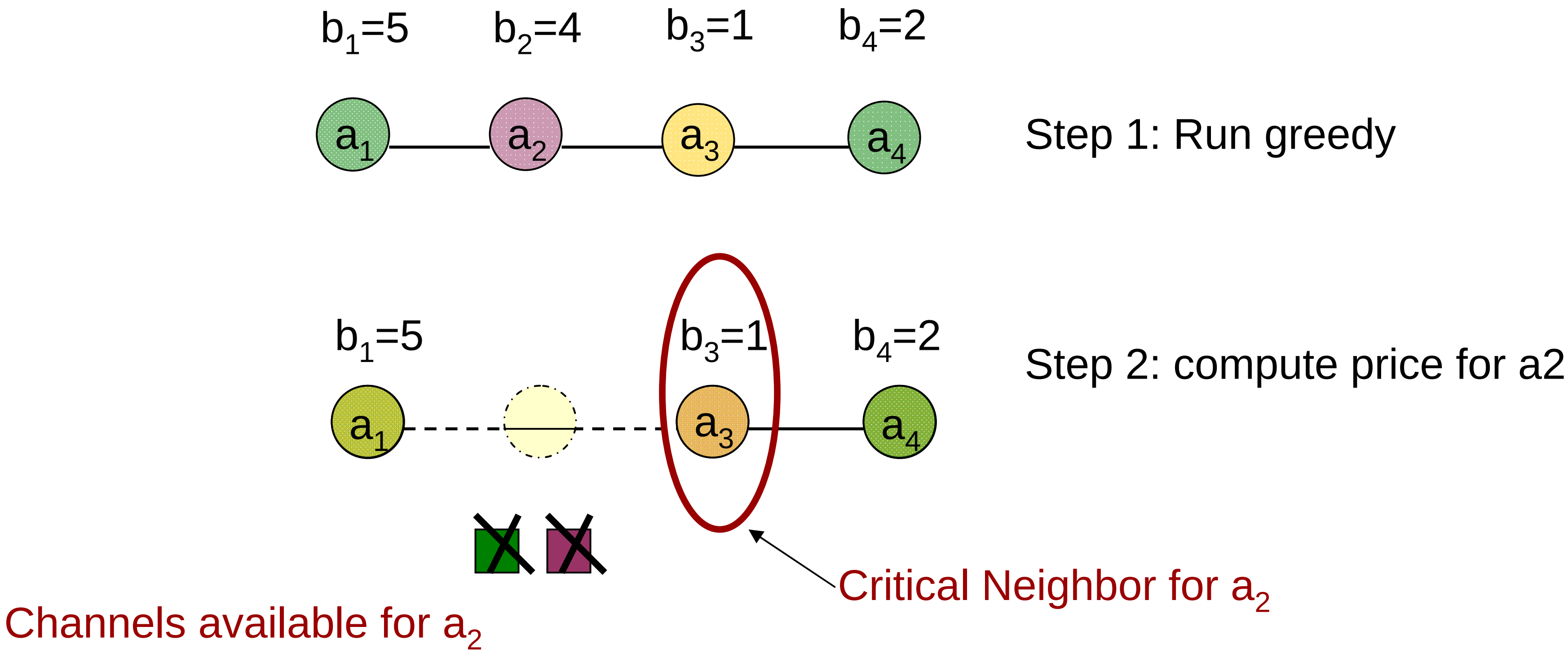
u_1=5 u_2=3 u_3=0 u_4=1 u_1=5 u_2=4 u_3=1 u_4=2

[redacted]

New Auction: Veritas

- Sort and Greedily allocate channels (lowest available first)
- Veritas-Pricing:
  - A winner i pays the bid of its critical neighbor C(i)
  - To determine Critical Neighbor for i
    - run greedy algorithm with B - b_i
    - Critical Neighbor of i is the first one to be denied a channel.

Veritas Example



# of channels = 2

Proof of Veritas

- Theorem: Veritas is truthful, achieves pareto optimality, and runs in $O(n^3k)$

- Proof sketch
  - Criticality: Unique critical value for each winning bidder.
  - Monotonicity: A bid above the critical value always wins.
  - Truthfulness: If we charge every bidder its critical value, no incentive to lie.

Bib and Collaborators

- Joint work with
  - Buragohain, Gandhi, Toth, Zheng, Zhou, Zhou
- Papers
  - Selfish Load Balancing, Algorithmica, 2007
  - A game-theoretic analysis of wireless access points selection by mobile users, Computer Communication '08
  - Towards real-time dynamic spectrum auctions, Computer Networks, '08
  - eBay in the sky: strategy-proof wireless spectrum auctions, Mobicom '08

Thank You!