## Integrating markets with control on electricity networks

John Ledyard Caltech For NEGT 11/20/14

With help from many others at Caltech and Southern California Edison Especially Desmond Cai

#### **A Work in Progress**

## Integrating markets with control on electricity networks

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### The Problem

- Increasing adoption of distributed energy resources (DER) implies difficulties for frequency and voltage control of electricity networks
  - Especially if economical storage is slow to come online
- Control of the wholesale side is not enough.
- Need to integrate distribution networks (retail) into the management of the grid.





#### How should we do this?

• Engineers – Give us control

– Implementable but not optimal

- Economists Let there be markets
  - Optimal but not implementable

There is a way that is both implementable and optimal – <u>Economic Control</u>

#### Outline

- Optimal Distribution Network Operation
- Direct control
- Markets
- Economic Control

#### We focus on Vol/Var control in 5 min increments — Principles would apply to frequency regulation, etc.

#### A Model of the DN: the network

There are N+1 buses. We identify i = 0 with the substation bus connecting the DN to the grid managed by the ISO.

Kirchoff's laws: 
$$\tilde{P}_i - P_i - \sum_j |V_i| |V_j| |Y_{ij}| \cos(\theta_{ij} + \delta_j - \delta_i) = 0, \forall i$$
  
 $\tilde{Q}_i - Q_i - \sum_j |V_i| |V_j| |Y_{ij}| \sin(\theta_{ij} + \delta_j - \delta_i) = 0, \forall i$ 

Voltage limits:  $V_i^{min} \leq V_i \leq V_i^{max}$ 

where

 $\tilde{P}_i$  = real power generation at i.  $\tilde{Q}_i$  = reactive power generation at i.  $P_i$  = real power demand of customer at bus i.  $Q_i$  = reactive power demand of customer at bus i.  $Y_{ij}$  = the  $ij^{th}$  term of the bus admittance matrix.  $V_i$  = the voltage at bus i.  $\theta_{ij}$  = the phase angle of the admittance  $Y_{ij}$ .  $\delta_i$  = the voltage angle at bus i.

#### A Model of the DN: the consumer

Utility function:  $u^i = u^i(c^i(t))$ 

Consumption requires power:

 $P^{i}(t) = F^{i}(c^{i}(t), V^{i}(t), e^{i}(t)) - \tilde{P}^{i}(t, e^{i}(t)).$ 

 $e^{i}(t)$  are temperature, work schedules, etc.

F captures thermal properties such as insulation, ... - Ignores dynamics. We will get to that later.

Faced with prices  $\pi(t)$ , the consumer will choose  $[c^i(1), \ldots, c^i(T)]$  to

$$\max_{(c,P)} \sum_{t} u^{i}(c^{i}(t)) - \pi(t) [F^{i}(c^{i}(t), V^{i}(t), e^{i}(t)) - \tilde{P}^{i}(t, e^{i}(t))].$$

We assume that  $Q^i$  and  $\tilde{Q}^i$  are fixed or proportional to P and  $\tilde{P}$ . NEGT - Caltech

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### The Goal is Socially Optimal Management

Maximize Consumers' welfare + Producer's welfare subject to Laws of Physics and Laws of Economics

For now we assume there is only one producer on the network – the Distribution Network Operator.

- Assumed to be regulated and willing to follow rules.
- Allows us to focus on consumer responses.

#### Socially Optimal Vol/Var control

Choose  $c(t), P^0(t), P(t), \tilde{P}(t), Q(t), \tilde{Q}(t), V(t), \theta(t)$  to solve

$$\max \sum_{t} \{\sum_{i} u^{i}(c^{i}(t))\} - \lambda(t)P^{0}(t) \qquad \text{subject to}$$

$$P^{i}(t) = F^{i}(c^{i}(t), V^{i}(t), e^{i}(t)) - \tilde{P}^{i}(t)$$
$$\tilde{P}^{i} - P^{i} - \sum_{j} |V^{i}||V^{j}||Y_{ij}|cos(\theta_{ij} + \delta_{j} - \delta_{i}) = 0$$
$$\tilde{Q}^{i} - Q^{i} - \sum_{j} |V^{i}||V^{j}||Y_{ij}|sin(\theta_{ij} + \delta_{j} - \delta_{i}) = 0$$
$$V^{i}_{min} \leq V^{i} \leq V^{i}_{max}$$

Socially Optimal Vol/Var control

Choose  $[c(t), P^0(t), P(t), Q(t), \tilde{Q}(t), V(t), \theta(t)]$  to solve

 $\max \sum_t \{\sum_i u^i(c(t))\} - \lambda(t) P^0(t)$  subject to

$$\frac{\text{Physics:}}{K(P^0(t), P(t), \tilde{P}(t), Q(t), \tilde{Q}(t), V(t), \theta(t))} \ge 0$$

$$P^i(t) = F^i(c^i(t), V^i(t), e^i(t)) - \tilde{P}^i(t).$$

• What about economics and computer science?

#### 3 difficulties in solving the optimal VVC

#### Computation

- NP hard (non-convexities)
- Time scale We do 5 minute intervals
- Time correlation (EV, pool pumps,...)

#### • Information

The DNO does not know the consumers' utility functions.

#### Incentive compatibility

- Getting the consumer to "do the right thing"

## Socially Optimal Vol/Var control

Choose  $[c(t), P^0(t), P(t), Q(t), \tilde{Q}(t), V(t), \theta(t)]$  to solve

 $\max \sum_t \{\sum_i u^i(c(t))\} - \lambda(t) P^0(t)$  subject to

$$\frac{\text{Physics:}}{K(P^0(t), P(t), \tilde{P}(t), Q(t), \tilde{Q}(t), V(t), \theta(t))} \geq 0$$

$$P^i(t) = F^i(c^i(t), V^i(t), e^i(t)) - \tilde{P}^i(t).$$

<u>Economics</u>: Information and Incentive constraints

 $\frac{\text{Computer Science:}}{\text{Computational constraints}}$ 

### The engineers' solution: Let me control everything

- Give the DNO access to the consumers' devices.
   Both controls and information.
- Minimize the cost of power (from ISO) subject to the laws of physics with bounds on what can be done to the consumers.

• Maintain standard regulatory average cost pricing policy with payments for access.

#### The engineers' solution

Given  $\bar{c}(t)$ , choose  $[P^0(t), P(t), Q(t), \tilde{Q}(t), V(t), \theta(t)]$  to solve

 $\max -\lambda(t)P^0(t)$  subject to

$$K(P^{0}(t), P(t), \tilde{P}(t), Q(t), \tilde{Q}(t), V(t), \theta(t)) \geq 0$$
  

$$P^{i}(t) = F^{i}(c^{i}(t), V^{i}(t), e^{i}(t)) - \tilde{P}^{i}(t)$$
  

$$\bar{c}^{i}(t) - \epsilon \leq c^{i}(t) \leq \bar{c}^{i}(t) + \epsilon.$$

The consumer chooses  $\bar{c}^i(t)$  to solve

$$\max u^{i}(c) - \bar{\pi}^{i}(t) [F^{i}(c, V^{i}(t), e^{i}(t)) - \tilde{P}^{i}(t)].$$

# The Problem with the engineers' solution

#### Not optimal

- Completely ignores consumer preferences
- Doesn't use the full range of potential consumer responses.
- Computation is not easy
  - Will assume it is computable so we can concentrate on the economics.
  - Open question: Can economics help the computation?

#### The economists' solution: Let there be markets

- A 5 minute market for power at each node.
   Centralized, bi-lateral, brokered, ... ?
- Given prices, consumers maximize utility.
- Given prices, the DNO maximizes net receipts from power.
- Prices are set (by whom) so that:
   Aggregate consumer demand = DNO supply.

#### The economists' solution: Let there be markets

Suppose there is a (locational) market for each  $P^{i}(t)$  with price  $\pi^{i}(t)$ .

Given  $\pi^{i}(t)$  and  $V^{i}(t)$  the rational consumer will solve

$$\max_{c,P^d} u^i(c(t)) - \pi^i(t)P^{di}(t)$$
  
subject to  
$$P^{di}(t) = [F^i(c^i(t), V^i(t), e^i(t)) - \tilde{P}^i(t, e^i(t))]$$

where  $P^{di}$  is i's demand for power.

The DNO (distribution network operator) will solve

$$\max_{P^s, \tilde{P}, Q, \tilde{Q}, V, \theta} \sum_i \pi^i(t) P^{si}(t) - \lambda(t) P^0(t)$$
  
subject to  
$$K(P^0, P^s(t), \tilde{P}(t), Q(t), \tilde{Q}(t), V(t), Y(t), \theta(t)) \ge 0.$$

<u>The market will choose</u>  $\pi^{i}(t)$  so that  $P^{di}(t) = P^{si}(t)$ .

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## The Problem with the economists' solution

- Optimal only if voltage is not a choice variable.
  - The equilibrium solves the optimal VVC only if dF/dV = 0.
- Markets do not equilibrate instantaneously.
  - Requires simultaneous solution by consumers and DNO and price setter, but no one has the information needed to equibrate in one shot.
  - If information and calculation are iterated, computational constraints require time to overcome. Not enough time.
- Temporary solution is not feasible during iteration.
  - Need to consider the dynamics

- The power demanded by the DN generates a substation LMCP from the ISO.
- Given c, the DNO minimizes the cost of power acquired from ISO. (This is just a CVR program.)
- The LMCP and the CVR problem determine a DMCP for each consumer.
- Given their DMCP, each consumer chooses c to maximize their utility minus their cost of power.

Given  $c_{t-1}, e_t, \tilde{P}_t$ , the DNO solves the CVR problem (conservation voltage reduction):

$$\min_{P^{0}, P, Q, \tilde{Q}, V, \theta} P^{0}$$
subject to
$$K(P^{0}, P, Q, \tilde{P}, \tilde{Q}, V, \theta) \geq 0$$

$$P_{t}^{i} = F^{i}(c_{t-1}^{i}, V_{t}^{i}, e_{t}^{i}) - \tilde{P}_{t}^{i}$$
(1)

This generates a DMCP price  $\beta_t^i$ , the multiplier on (1).

Given  $\pi_t^i = \lambda_t \beta_t^i, V_t^i, e_t^i, \tilde{P}_t^i$  the consumer solves:  $\max_{c_t^i} u^i(c_t^i) - \pi_t^i [F^i(c_t^i, V_t^i, e_t^i) - \tilde{P}_t^i].$ 

• <u>In equilibrium</u>, P2D is optimal.

• But there are lags in the dynamics which creates a potential for instability.

• The reason is found in the corn-hog cycle.

#### Prices to Devices is potentially unstable.

• The reason is the standard cobweb cycle.



• <u>In equilibrium</u>, P2D is optimal.

• But there are lags in the dynamics which creates a potential for instability.

• The network and DNO exacerbate the problem.

#### Prices to Devices dynamics simplified

Given  $\lambda_t, c_{t-1}, \tilde{P}_t$ , the DNO solves the CVR which implies  $V_t^i = V^i(c_{t-1}), \beta_t^i = \beta^i(c_{t-1})$ . Then

$$\pi_{t}^{i} = \lambda_{t}\beta^{i}(c_{t-1})$$

$$c_{t}^{i} = d^{i}(\pi_{t}^{i}, V^{i}(c_{t-1}))$$

$$P_{t}^{0} = P^{0}(c_{t}, V(c_{t-1}))$$

$$\lambda_{t+1} = H(P_{t}^{0}).$$

Therefore

$$c_t^i = d^i [H(P^0(c_{t-1}, V(c_{t-1}))\beta^i(c_{t-1}), V^i(c_{t-1})]$$

Locally this is unstable if

$$|d_{\pi}H_{P}P_{c^{i}}^{0}\beta^{i} + d_{\pi}H_{P}P_{V}^{0}V_{c^{i}}\beta + d_{\pi}H\beta_{c^{i}}^{i} + d_{V^{i}}V_{c^{i}}^{i}| > 1$$

#### **Stability of Prices to Devices**

Locally P2D is unstable if

 $|d_{\pi}H_{P}P_{c^{i}}^{0}\beta^{i} + d_{\pi}H_{P}P_{V}^{0}V_{c^{i}}\beta + d_{\pi}H\beta_{c^{i}}^{i} + d_{V^{i}}V_{c^{i}}^{i}| > 1$ 

 $H_P$  is the slope of the inverse supply function of the ISO. It is positive and grows towards  $\infty$  as  $P^0$  approaches capacity.

 $P_{c^i} d^i_{\pi}$  is how responsive  $P^0$  is to i's DMCP.

P2D is likely to be unstable(1) the more responsive consumers's are, and(2) the closer to capacity the system is.

## Stability of Prices to Devices

- P2D is more likely to be unstable
  - The more responsive consumers are to prices
  - The closer the ISO is to capacity.
- Paradox: We need consumers to be responsive when we are near capacity, but if they are too responsive this policy won't work.
- We can improve on P2D by reducing the lag.
  - Higher payoff
  - More stable

- The power demanded by the DN generates a substation LMCP from the ISO.
- Each consumer reports a utility function,  $\hat{u}$ , to the DNO.
- The DNO maximizes the sum of utilities less the cost of power acquired from ISO.
  - This is the optimal VVC program.
  - It determines the consumption of the consumer and the price they will pay.

• The consumer "reports"  $\hat{u}$  to the DNO.

• The DNO solves the Optimal VVC with that  $\hat{u}$ .

Choose  $[c(t), P^0(t), P(t), Q(t), \tilde{Q}(t), V(t), \theta(t)]$  to solve

$$\max\{\sum_{i} \hat{u}^{i}(c^{i})\} - \lambda(t)P^{0}(t) \text{ subject to}$$

$$\begin{split} &K(P^{0}(t), P(t), \tilde{P}(t), Q(t), \tilde{Q}(t), V(t), \theta(t)) \geq 0 \\ &P^{i}(t) = F^{i}(c^{i}(t), V^{i}(t), e^{i}(t)) - \tilde{P}^{i}(t) \end{split}$$

• The consumer "reports"  $\hat{u}$  to the DNO.

• The DNO solves the Optimal VVC with that  $\hat{u}$ .

Choose  $[c(t), P^0(t), P(t), Q(t), \tilde{Q}(t), V(t), \theta(t)]$  to solve

 $\max\{\sum_{i} \hat{u}^{i}(c^{i})\} - \lambda(t)P^{0}(t) \text{ subject to}$ 

$$\begin{split} &K(P^{0}(t), P(t), \tilde{P}(t), Q(t), \tilde{Q}(t), V(t), \theta(t)) \geq 0 \\ &P^{i}(t) = F^{i}(c^{i}(t), V^{i}(t), e^{i}(t)) - \tilde{P}^{i}(t) \end{split}$$

The consumer pays  $\pi^{i}(t)[F^{i}(c, V^{i}(t), e^{i}(t)) - \tilde{P}^{i}(t)].$ 

- Three issues
  - Communication: Will the consumer be able to describe their utility function?
  - Incentives: Will the consumer be willing to report their true utility function?
  - Computation: Will the DNO be able to solve the Optimal VVC with the utility functions in it?
- If the answer is yes to all three, then in equilibrium economic control will be optimal.

# Will the consumer be able to describe their utility function?

- Consider an approximation around the consumer's ideal setting c\*.
  - The ideal setting is what the consumer would choose if power were free.  $u_c(c^*) = 0$ .

$$u(c_t) \sim u(c^*) + u_c(c^*)(c_t - c^*) + \frac{1}{2}u_{cc}(c^*)(c_t - c^*)^2$$
  
$$\sim u(c^*) + \frac{1}{2}u_{cc}(c^*)(c_t - c^*)^2$$

- The consumer "reports" only two numbers
   A set-point, c\*
  - A strength of preference,  $u_{cc}(c^*)$

#### Today's simple thermostat



### Today's thermostat



#### Tomorrow's thermostat: Control



#### Tomorrow's thermostat: Economics



Will the consumer be able to describe their utility function? YES

• The technology is easy.

 Could also sign a contract with (c\*, σ) specified for the period of the contract.

 Could offer a menu of contracts with different values of (c\*, σ) and simulations of the possible payments required.

# Will the consumer be willing to report their true utility function?

 $\frac{\text{The DNO will solve the FOC}}{\hat{u}_c^i - \delta^i F_c^i = 0}$   $\frac{\delta^i + \beta K_{P^i} = 0}{-\delta^i F_{V^i}^i + \beta K_{V^i} = 0}$  $\lambda + \beta K_{P^0} = 0.$ 

The consumer wants the DNO to solve:  $u_c^i - \delta^i F_c^i = 0$ 

If the consumer believes she can not affect  $\delta^i$ , she will want to report the function  $\hat{u}_c^i(c) = u_c^i(c)$ .

If there are no network effects then  $\delta^i = \lambda$ , the ISO price. Behaviorally, the answer is yes.

# Will the consumer be willing to report their true utility function?

 $\frac{\text{The DNO will solve the FOC}}{\hat{u}_c^i - \delta^i F_c^i = 0}$   $\frac{\delta^i + \beta K_{P^i} = 0}{-\delta^i F_{V^i}^i + \beta K_{V^i} = 0}$  $\lambda + \beta K_{P^0} = 0.$ 

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If the consumer believes she can not affect  $\delta^i$ , she will want to report the function  $\hat{u}_c^i(c) = u_c^i(c)$ .

If there are no network effects then  $\delta^i = \lambda$ , the ISO price. Behaviorally, the answer is yes. Conjecture: use VCG or GL

## Can the DNO solve the Optimal VVC with the utility functions?

• Use the quadratic approximation with information from the consumer.

$$u(c_t) \approx u(c^*) + \frac{1}{2}\sigma(c_t - c^*)^2,$$

- Can ignore constant term in the optimization.
- We are only adding quadratic terms to something that is already quadratic.

#### • This problem is no harder than the CVR problem.

- Without a network, EC = P2D.
- With a network, in equilibrium, EC is optimal.
- With a network, <u>out of equilibrium</u>, EC eliminates one of the lags in P2D so that
  - EC is more stable that P2D.
  - EC is more efficient than P2D
    - With respect to sum  $u \lambda P$

#### Stability of EC

Given  $\lambda_t, \tilde{P}_t$ , the DNO solves the CVR which implies  $V_t^i = V^i(c_t), \beta_t^i = \beta^i(c_t)$  and

$$\pi_t^i = \lambda_t \beta^i(\mathbf{c_t})$$

$$c_t^i = d^i(\pi_t^i, V^i(\mathbf{c_t}))$$

$$P_t^0 = P^0(c_t, V(\mathbf{c_t}))$$

$$\lambda_{t+1} = H(P_t^0).$$

Therefore

$$c_t^i = d^i [H(P^0(c_{t-1}, V(c_{t-1}))\beta^i(c_t), V^i(c_t)]$$

Locally this is unstable if

$$|\frac{d_{\pi}H_{P}P_{c^{i}}\beta^{i} + d_{\pi}H_{P}P_{V}V_{c^{i}}\beta}{1 - d_{\pi}H\beta^{i}_{c^{i}} - d_{V^{i}}V^{i}_{c^{i}}}| > 1$$

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### Stability of EC vs P2D

Locally EC is unstable if  $\left|\frac{d_{\pi}H_{P}P_{c^{i}}\beta^{i}+d_{\pi}H_{P}P_{V}V_{c^{i}}\beta}{1-d_{\pi}H\beta^{i}_{c^{i}}-d_{V^{i}}V^{i}_{c^{i}}}\right| > 1$ 

Locally P2D is unstable if  $|d_{\pi}H_{P}P_{c^{i}}\beta^{i} + d_{\pi}H_{P}P_{V}V_{c^{i}}\beta + d_{\pi}H\beta^{i}_{c^{i}} + d_{V^{i}}V^{i}_{c^{i}}| > 1$ 

Since everything is negative these reduce to: EC: -1 < (a+b) - (c+d)P2D: -1 < (a+b) + (c+d)

#### EC is more stable than P2D.

#### Recap

- Increasing DER adoption requires integration of distribution networks grid management.
- The Engineers' solution is implementable but not efficient.
- The Economists' solution is efficient but not implementable.
- Efficient Control is nearly optimal and implementable.
- Preliminary simulations provide some support for this.



Change in Social Welfare for Summer Week **Under Optimal Approach** 

#### Change in Social Welfare for Summer Week Under Naïve Engineering Approach

### Open questions: or what I would like to know

- Optimal simple mechanism subject to feasible and stable implementation
- Understand the network effects for a DN – What is dP<sup>0</sup>/dP<sup>i</sup>?
- Does adding stochastics for solar or wind change the analysis at all?
  - Are expected values good enough?