Efficient Bilateral Trade

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Buyer and seller privately know their values for an indivisible good

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- no Bayesian incentive-compatible, interim individually-rational, budget-balanced mechanism is ex-post Pareto efficient.

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This negative answer presumes quasilinear utilities.

Beyond Quasilinearity

- Risk aversion
- Wealth effects
- Risk of bankruptcy

Beyond Quasilinearity

Paper:

- Normal goods: each agent's reservation price for the good increases with the agent's money holding (Cook and Graham, 1977).
- The semi-elasticities of the marginal utilities of money and good with respect to private information are well-behaved.

Talk:

- Separable example:

$$u(x, m; v) = vx + U(m)$$

where $U' > 0 > U''$.

Contribution

- Efficient trade is possible if agents' utilities are not too responsive to their private information.

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- Away from quasilinearity, efficient trade is possible even if utilities are highly responsive to private information.

- Technology for mechanism design with normal goods.

Literature on Efficient Trade

- Disjoint domains of types (Myerson and Sattherwaite 1983)
- Infinite risk-aversion (Chatterjee and Samuelson 1983)
- Correlated types (McAfee and Reny 1992)
- Many agents (Wilson 1985, Makowski and Ostroy 1989, Makowski and Mezzetti 1994, Rustichini, Satterthwaite, and Williams 1994, Reny and Perry 2006, Cripps and Swinkels 2006)
- Budget-breakers and large ex ante gains from trade (McAfee 1991, Riley 2012)
- Divisible good initially co-owned by both agents (Cramton, Gibbons, and Klemperer 1987)
- Many goods (Jackson and Sonnenschein 2007, Jackson, Sonnenschein, and Xing 2014)

Literature on Mechanisms and Efficiency Beyond Quasilinearity

Deterministic Mechanisms

- Holt (1980), Matthews (1983), Maskin and Riley (1984), Morimoto and Serizawa (2014)

Pareto frontier for normal goods

- Garratt (1999)
- Baisa (2014)

Example 1: Log Utility

Utility $u(x, m; v) = vx + \log m$ where

$x = 1$ if the agent has the good, or $x = 0$ otherwise;

$m \geq 0$ money holdings of the agent;

initial money holdings m_S, m_B such that the median types of buyer and seller have the same utility;

$v \geq 0$ agent's privately known type; v^S and v^B distributed iid uniformly on $[2, 100]$

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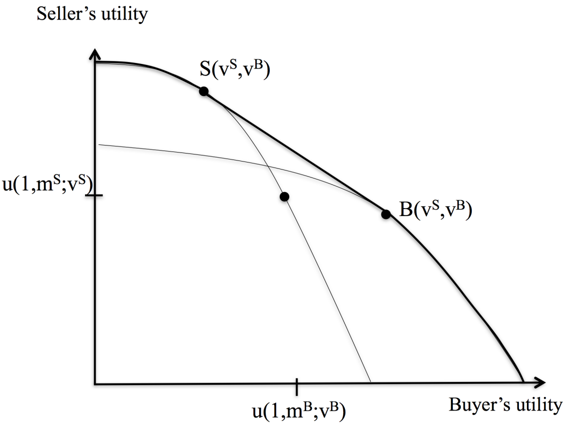
Efficient trade is then possible!

Example 1: Pareto Frontier



Example 1: Log Utility

- at point S the seller's money holdings are

$$m^S(v^s, v^b) = \frac{v^b}{v^s + v^b} M$$

- at point B the buyer's money holdings are

$$m^B(v^s, v^b) = \frac{v^s}{v^s + v^b} M$$

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Efficient money holdings do not depend on who has the good:
$M - m^S(v^s, v^b) = m^B(v^s, v^b)$.

Example 1: Mechanism

- The probability that the seller gets the item if the seller reports v^s and the buyer reports v^b is

$$\begin{aligned}\pi(v^s, v^b) &= \frac{1}{2} + \frac{1}{98}\int_{51}^{v^s} \frac{\log(100+x) - \log(2+x)}{x} dx \\ &\quad + \frac{1}{98}\int_{51}^{v^b} \frac{-\log(100+x) + \log(2+x)}{x} dx.\end{aligned}$$

- This mechanism is Bayesian incentive compatible, individually rational, and Pareto efficient

Example 2: Shifted Cobb-Douglas

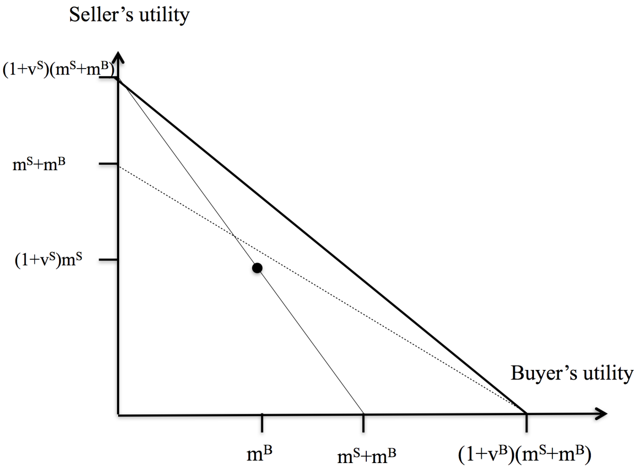
Utility $U(x, m; v) = (1 + vx)\, m$ where

$x = 1$ if the agent has the good, or $x = 0$ otherwise;

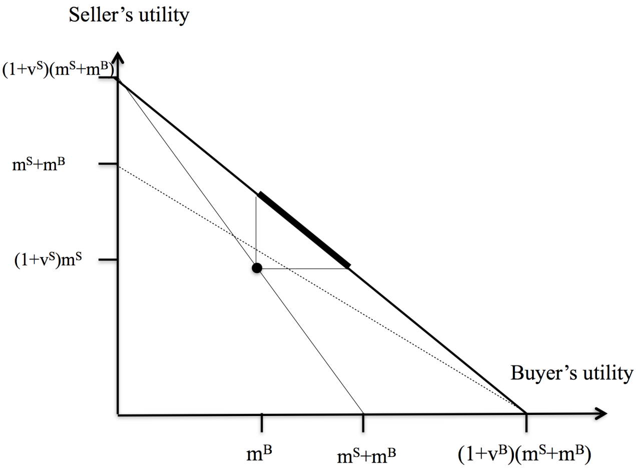
$m \geq 0$ money holdings of the agent; m_S, m_B initial money holdings;

$v \geq 0$ agent's privately known type; v^S and v^B distributed arbitrarily (correlation allowed but not needed).

Example 2: Pareto Frontier



Example 2: IR Set on the Pareto Frontier



Example 2: Efficient Mechanism

Give the good and all money to the seller with probability $\frac{m_S}{m_S+m_B}$,
Give the good and all money to the buyer with probability $\frac{m_B}{m_S+m_B}$.

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- Incentive compatible and efficient.

- Individually rational for the seller:

$$\frac{m_S}{m_S+m_B}\left(1+v^S\right)(m_S+m_B) \geq \left(1+v^S\right)m_S.$$

- Individually rational for the buyer:

$$\frac{m_B}{m_S+m_B}\left(1+v^B\right)(m_S+m_B) \geq m_B.$$

An Aside: Do People Use Such Lottery Mechanisms?

- Betting the farm.
- Poker tournaments.

Model

Endowments (total amount of money M fixed):

- seller's: the indivisible good and money m^S.
- buyer's: money $m^B = M - m^S$.

Utility $u(x, m; v)$

- strictly increasing in x, m, and v,
- strictly concave in m, and
- twice differentiable in m and v.

Privately known types v^S, v^B; arbitrary continuous distribution.

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Separable example: $u(x, m; v) = vx + U(m)$

Normality

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The indivisible good is normal for v if for any $m, p, \epsilon > 0$:

$$u(0, m; v) = u(1, m - p; v) \implies u(0, m + \epsilon; v) < u(1, m + \epsilon - p; v).$$

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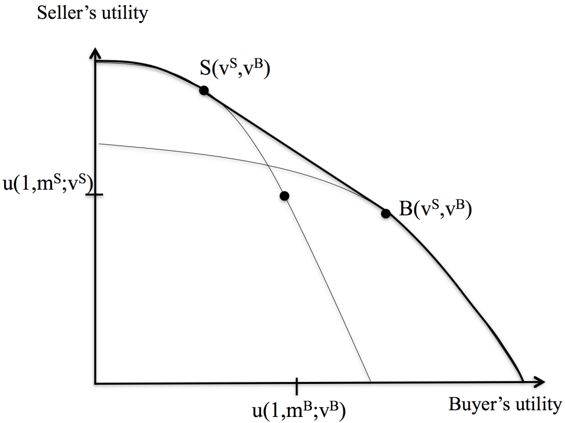

A Condition on Semi-Elasticities

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$$\frac{\partial}{\partial v} \log \left(u \left(1, m, v \right) - u \left(0, m, v \right) \right) > \frac{\partial}{\partial v} \log \left(\frac{\partial}{\partial m} u \left(x, m, v \right) \right) = \text{constant}$$

Main Theorem

Fix $u^s(\cdot,\cdot;v^*)$, $u^b(\cdot,\cdot;v^*)$, *and any initial money endowments but one. There is* $\Delta > 0$ *such that if*

$$\max_{x\in\{0,1\},\, m\in[0,M],\, v} \left| u^\theta(x,m,v) - u^\theta(x,m,v^*) \right| < \Delta,$$

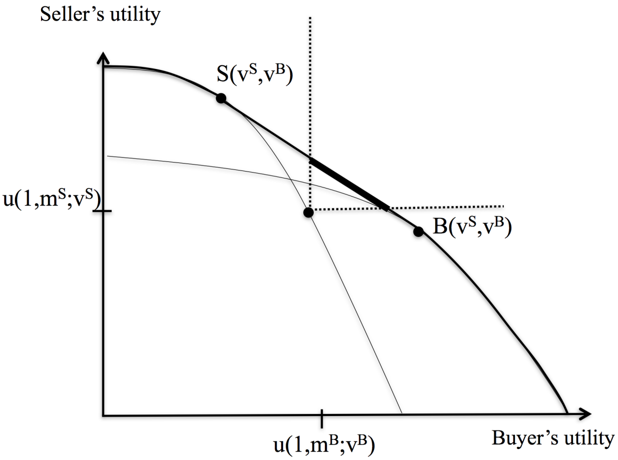
then there is an incentive-compatible and individually-rational mechanism that generates efficient trade.

Alternative Formulation

Fix any $u^s(\cdot,\cdot;\cdot)$, $u^b(\cdot,\cdot;\cdot)$, *any* v^*, *and any initial money endowments but one. There is an interval* $(\underline{v},\overline{v}) \ni v^*$ *such that:*

- *for any distribution of agents' types on* $[\underline{v},\overline{v}] \times [\underline{v},\overline{v}]$, *there is an incentive-compatible, individually-rational mechanism that generates efficient trade.*

Pareto Frontier



CRRA Example $u(x, m; v) = vx + m^r$

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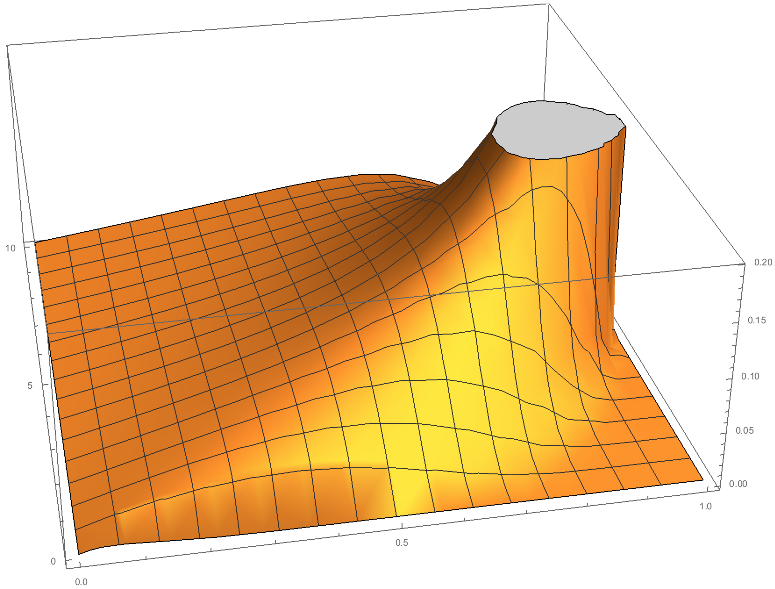

Separable Example $u(x, m; v) = vx + U(m)$

$$\frac{\partial \pi \left(v^s, v^b\right)}{\partial v^s} = -\frac{\partial U\left(m^S\left(v^s, v^b\right)\right)}{\partial m} \frac{\partial m^S\left(v^s, v^b\right)}{\partial v^s}$$

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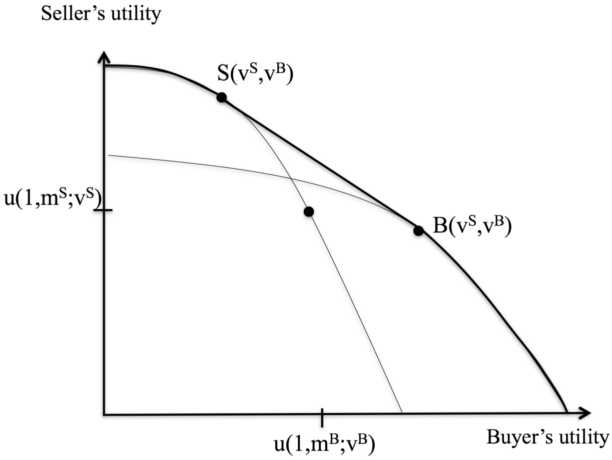
CRRA utility of money $U(m) = m^r$

$$m^S(v^s, v^b) = \frac{\left(v^b\right)^{\frac{1}{1-r}}}{\left(v^s\right)^{\frac{1}{1-r}} + \left(v^b\right)^{\frac{1}{1-r}}} M$$

Proof of the Main Theorem



Proof: How to Elicit Types?

Mechanism: agents obtain allocation $S\left(v^s, v^b\right)$ with probability $\pi\left(v^s, v^b\right)$ and allocation $B\left(v^s, v^b\right)$ with probability $1 - \pi\left(v^s, v^b\right)$.

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Challenge: find function $\pi\left(v^s, v^b\right)$ such that agents report their true types in a Bayesian Nash equilibrium.

Step 1: we solve the agents' first order condition to find π

Step 2: we verify the agents' second order condition.

First Order Conditions

$$\begin{aligned}
\mathbb{E}_{v^b|v^s}\left[S_1\left(v^s,v^b\right)\frac{\partial}{\partial v^s}\pi\left(v^s,v^b\right)+S_2\left(v^s,v^b\right)\pi\left(v^s,v^b\right)\right] &= \phi\left(v^s\right) \\
\mathbb{E}_{v^s|v^b}\left[B_1\left(v^s,v^b\right)\frac{\partial}{\partial v^b}\pi\left(v^s,v^b\right)+B_2\left(v^s,v^b\right)\pi\left(v^s,v^b\right)\right] &= \psi\left(v^b\right)
\end{aligned}$$

First Order Conditions

$$S_1\left(v^s, v^b\right) = u\left(1, m^S\left(v^s, v^b\right), v^s\right) - u\left(0, M - m^B\left(v^s, v^b\right), v^s\right)$$
$$B_1\left(v^s, v^b\right) = u\left(1, m^B\left(v^s, v^b\right), v^b\right) - u\left(0, M - m^S\left(v^s, v^b\right), v^b\right)$$

$$\begin{aligned}
S_2\left(v^s, v^b\right) &= \left[\frac{\partial}{\partial m} u\left(1, m^S\left(v^s, v^b\right), v^s\right)\right]\left[\frac{\partial}{\partial v^s} m^S\left(v^s, v^b\right)\right] \\
&\quad + \left[\frac{\partial}{\partial m} u\left(0, M - m^B\left(v^s, v^b\right), v^s\right)\right]\left[\frac{\partial}{\partial v^s} m^B\left(v^s, v^b\right)\right] \\
B_2\left(v^s, v^b\right) &= \left[\frac{\partial}{\partial m} u\left(1, m^B\left(v^s, v^b\right), v^b\right)\right]\left[\frac{\partial}{\partial v^b} m^B\left(v^s, v^b\right)\right] \\
&\quad + \left[\frac{\partial}{\partial m} u\left(0, M - m^S\left(v^s, v^b\right), v^b\right)\right]\left[\frac{\partial}{\partial v^b} m^S\left(v^s, v^b\right)\right]
\end{aligned}$$

$$\begin{aligned}
\phi\left(v^s\right) &= E_{v^b}\left\{\left[\frac{\partial}{\partial m} u\left(0, M - m^B\left(v^s, v^b\right), v^s\right)\right]\left[\frac{\partial}{\partial v^s} m^B\left(v^s, v^b\right)\right]\right\} \\
\psi\left(v^b\right) &= E_{v^s}\left\{\left[\frac{\partial}{\partial m} u\left(1, m^S\left(v^s, v^b\right), v^b\right)\right]\left[\frac{\partial}{\partial v^b} m^S\left(v^s, v^b\right)\right]\right\}
\end{aligned}$$

First Order Conditions

$$\begin{aligned}
\mathbb{E}_{v^b|v^s}\left[S_1\left(v^s, v^b\right)\frac{\partial}{\partial v^s}\pi\left(v^s, v^b\right) + S_2\left(v^s, v^b\right)\pi\left(v^s, v^b\right)\right] &= \phi\left(v^s\right) \\
\mathbb{E}_{v^s|v^b}\left[B_1\left(v^s, v^b\right)\frac{\partial}{\partial v^b}\pi\left(v^s, v^b\right) + B_2\left(v^s, v^b\right)\pi\left(v^s, v^b\right)\right] &= \psi\left(v^b\right)
\end{aligned}$$

First Order Conditions

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$$\mathbb{E}_{v^s|v^b}\left[B_1\left(v^s,v^b\right)\frac{\partial}{\partial v^b}\pi\left(v^s,v^b\right)+B_2\left(v^s,v^b\right)\pi\left(v^s,v^b\right)\right] = \psi\left(v^b\right)$$

Solution

$$\pi\left(v^s,v^b\right)=b\left(v^b\right)\pi^B\left(v^s,v^b\right)+s\left(v^s\right)\pi^S\left(v^s,v^b\right)$$

where π^B and π^S solve

$$S_1\left(v^s,v^b\right)\frac{\partial}{\partial v^s}\pi^B\left(v^s,v^b\right)+S_2\left(v^s,v^b\right)\pi^B\left(v^s,v^b\right) = 0$$
$$B_1\left(v^s,v^b\right)\frac{\partial}{\partial c}\pi^S\left(v^s,v^b\right)+B_2\left(v^s,v^b\right)\pi^S\left(v^s,v^b\right) = 0$$

Second Order Condition

The second order condition is implied by our assumption on the semi-elasticities:

$$\frac{\partial}{\partial v} \log\left(u\left(1, m, v\right) - u\left(0, m, v\right)\right) > \frac{\partial}{\partial v} \log\left(\frac{\partial}{\partial m} u\left(x, m, v\right)\right) = \text{constant}$$

Impossibility of Ex Post Implementation

When m^S, m^B are interior and efficiency requires randomization, then generically no mechanism is ex-post incentive compatible, individually rational, and implements efficient trade.

Conclusion

Eliciting money holdings.

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Public good provision.

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Efficient trade is possible in a natural class of environments.