Endogenous Matching: Adverse Selection & Moral Hazard

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Motivation

Familiar problem: two-sided matching with transfers

- $w \in W$, $t \in T$
  - buyers, sellers
  - workers, firms
  - men, women
  - workers, tasks
- Output/Value $Y(w, t)$
- Objective: match W to T

$$\mu : W \to T$$

to maximize total output

$$\int_W Y(w, \mu(w))dw$$

Some background

- Shapley & Shubik
    - Housing market
    - Buyers, Sellers characterized by valuations for houses
    - Assignment game
    - Core = Stable matches = Walrasian allocations
- Becker
    - Marriage market with transfers
    - Women, Men ordered by "quality"
    - Supermodularity $\rightarrow$ *assortative matching*
      better women matched to better men

What is left out?

Worker types not observable
$\rightarrow$ adverse selection

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Output must be produced
Production requires effort
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Interaction ongoing
$\rightarrow$ repeated game

Literature

Not-so-closely related

- Search (Smith & Sorensen)
- Directed search (Shimer)
- Holdup (Cole, Mailath & Postlewaite; Makowski)
- Signaling (Hopkins)
- Repeated games
    - Indirect reciprocity (Kandori)
    - Random matching

Example

Firm (Capgemini)

- Set of workers fixed
- Flow of tasks arrive each period
- Firm matches tasks to workers, arranges payment (piece-rate)

Who knows/observes what?

Firm knows/observes

- Ordering of task (types)
- History of output

Firm does not know/observe

- Output function
- Worker type
- Effort

Who knows/observes what? (cont)

Workers know/observe

- Own output function
- Own cost function
- Own history
- Output distribution

Workers do not know/observe

- Others' output functions
- Others' cost functions

Essence of the Problem

Adverse selection

- Firm does not observe worker characteristics

Moral hazard

- Firm does not observe worker effort

Repeated interaction

- Each period: Firm matches workers to tasks
- Each period: Workers choose effort

Repeated interaction

Endogenous matching: match better producers to better tasks

- assortative matching $\rightarrow$ optimal matching
  eliminate adverse selection
- mitigate moral hazard
- comparisons
  - random matching
  - myopic optimum with assortative matching
  - incentive optimum with assortative matching

Model

Workers

- $W = [0, 1]$
- productivity $= F(w),\ F(0) \geq 0,\ F' > 0$
- Cost $= C(e),\ C(0) = 0,\ C' > 0,\ C'' > 0$
- distribution $=$ uniform
  workers identified by location in the distribution

(Paper is more general)

Model (cont)

Tasks

- $T = [0, 1]$
- quality $= H(t)$, $H(0) \geq 0$, $H' > 0$
- distribution $=$ uniform
  tasks identified by location in the distribution

(Paper is more general)

Formalism (cont)

- Output
$$Y(e, w, t) = e\,F(w)H(t)$$
separable: tractability
linear in effort: normalization
- Payment rule
$$P : [0, \infty) \to [0, \infty)$$
$$0 \leq P(y) \leq y, P' > 0, P'' \leq 0$$
- Worker utility
$$U(e, w, t) = P[Y(e, w, t)] - C(e)$$

(Paper is more general)

Formalism (cont)

Canonical Example

- Output

$$Y(e, w, t) = ewt$$

- Payment rule

$$P(y) = \lambda y$$

$$\lambda \in (0, 1]$$

- Cost

$$C(e) = e^2$$

- Worker utility

$$U(e, w, t) = \lambda ewt - e^2$$

Firm Objective

Maximize

total output - total payment to workers

Steady-State Assortative Equilibrium

Equilibrium notion:

- Worker strategy: effort

$$g : \text{history} \times [0,1] \to [0,\infty)$$

More convenient: output

$$G(w) = g(w)F(w)H(w)$$

- Workers discount future utility at constant rate $\delta \in (0,1]$
- Workers optimize
  (given that others play equilibrium)
- Steady state
- Assortative: match better workers to better tasks

Worst worker/Worst task

Two cases: value of matching worst worker to worst task

- worthless $F(0)H(0) = 0$
- not worthless $F(0)H(0) > 0$

Main Results

Theorem 1 Steady-state Assortative Equilibrium exists
(Note that adverse selection has disappeared completely)

Theorem 2 $F(0)H(0) > 0 \Rightarrow$ SSA Equilibrium unique

Theorem 3 Output net of payments dominates output net of payments under

- random matching
- myopic effort choice

(Comparison with incentive optimum)

Proof ideas

Steps in proof of Theorem 1

- **Step 1** Equilibrium $\rightarrow$ no profitable deviations
  $\rightarrow$ no profitable *stationary deviations*
- **Step 2** No profitable stationary deviations
  FOC $\rightarrow$ *ODE*
  $$G' = \Phi(w, G)$$
- **Step 3** *'Good' solution* G_* to ODE
  no profitable infinitesimal stationary deviations
- **Step 4** *Monotone* deviations enough
- **Step 5** Monotonicity $\rightarrow$ *stationary* deviations enough
- **Step 6** Stationarity $\rightarrow$ G_* is *SSA Equilibrium*
- **Step 7** $F(0)H(0) > 0 \rightarrow$ uniqueness

How much rationality is necessary?

- Very little knowledge necessary on either side
- Workers do not have to solve the Bellman problem
  – no profitable one-step deviations
  $\rightarrow$ no profitable deviations at all

Canonical example: deriving ODE

Equilibrium

$\rightarrow$ worker w prefers constant $G(w)$ to constant $G(\hat{w})$

$$U(e, w, t) = \lambda ewt - e^2$$

$$\begin{aligned} U(\hat{w}|w) &= \lambda G(\hat{w}) - [G(\hat{w})/w^2]^2 \\ &\quad + \delta/(1-\delta)\left\{\lambda G(\hat{w}) - [G(\hat{w})/w\hat{w}]^2\right\} \end{aligned}$$

FOC: $dU/d\hat{w} = 0$ when $\hat{w} = w$

$$G' = \frac{2\delta G^2}{w[2G - \lambda w^4]}$$

Canonical example: solving ODE

Guess $G(w) = Aw^4$; plug in and equate

$$\begin{aligned} 4Aw^3 &= \frac{2\delta A^2 w^8}{w[2Aw^4 - \lambda w^4]} \\ 4A &= \frac{2\delta A^2}{[2A - \lambda]} \end{aligned}$$

$$A = \frac{2\lambda}{4-\delta} \; ; \; A = 0$$

Two solutions; both satisfy the initial condition $G(0) = 0$

Canonical example: OOPS

Why not inconsistent with existence/uniqueness for ODE's?

- $G' = \Phi(w, G)$
  Φ is not well-behaved: denominator can be 0
- numerical methods $\rightarrow$ other solutions (?)
- For ODE's of this type:
  - uniqueness does not obtain
  - existence is in doubt

Moreover

- some solutions definitely not equilibria
- we *construct* solution G_* that is a candidate equilibrium
  $G'_* > 0$, derived worker utility > 0

Comparisons in the Canonical Example

Random matching, worker optimization

Worker w matched with task t exerts effort to maximize

$$\lambda ewt - e^2$$

$\rightarrow$ effort $e = \lambda wt/2$, output $= \lambda w^2 t^2/2$
Total net of payments

$$\int_0^1 \int_0^1 (1-\lambda)[\lambda w^2 t^2/2] dt dw = [(1-\lambda)\lambda][1/18]$$

Comparisons in the Canonical Example (cont)

Assortative matching, myopic worker optimization

Worker w matched with task w exerts effort to maximize

$$\lambda eww - e^2$$

$\rightarrow$ effort $e = \lambda w^2/2$, output $= \lambda w^4/2$
Total net of payments

$$\int_0^1 (1-\lambda)[\lambda w^4/2]dw = [(1-\lambda)\lambda][1/10]$$

Comparisons in the Canonical Example (cont)

Steady-state Assortative Equilibrium

Worker w matched with task w produces

$$G(w) = [2\lambda/(4-\delta)]w^4$$

Total net of payments

$$\int_0^1 (1-\lambda)\big(2\lambda/(4-\delta)]w^4\big)\,dw = [(1-\lambda)\lambda][2/5(4-\delta)]$$

Comparisons in the Canonical Example

2nd best: assortative + incentive-optimal payment

Worker w matched with task w is paid p to maximize

$$ew^2 - p \quad \text{subject to} \quad p \geq e^2$$

$\rightarrow$ effort $e = w^2/2$, output $= w^4/2$, net $= w^4/4$
Total net of payments

$$\int_0^1 [w^4/4]dw = 1/20$$

Comparisons in the Canonical Example (cont)

Optimal $\lambda = 1/2$

$$1/72 < 1/40 < 1/[10(4-\delta)] < 1/20$$

Random $<$ Myopic Assortative $<$ SSA Equilibrium $<$ 2nd best