

# Network Dynamics With Incomplete Information and Learning

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# Networks have Malicious Users!

## Wireless sensor networks

Sensors that send useless information drain other sensors' batteries

## Organizations/companies

Workers who have a low level of expertise consume other workers' time and effort

## Social networks

People who constantly share a lot of random ads annoy their connected friends



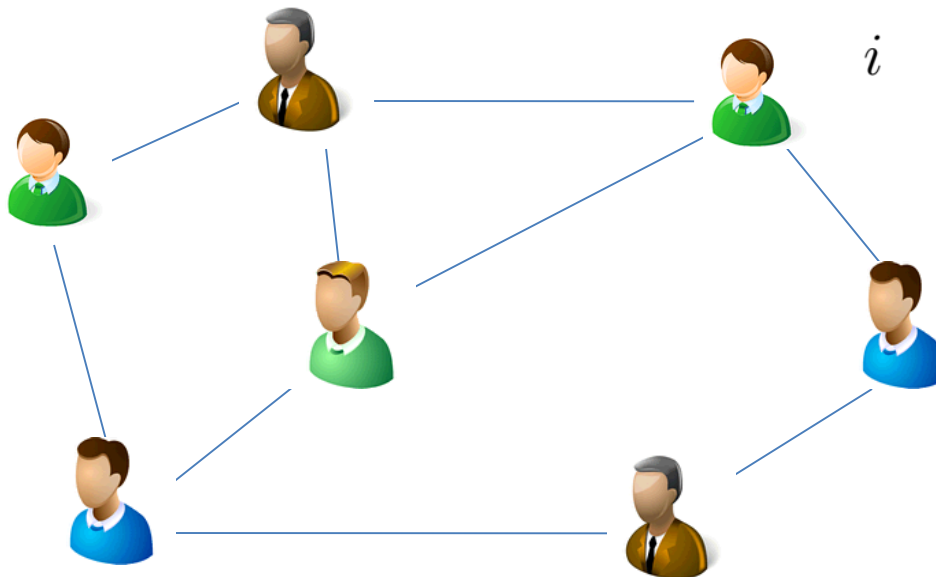
## Networks, Agents, Quality

Agents that are of low “quality” should be ostracized from the network...  
How can networks best learn about and weed out such agents?

**The network evolves over time as agents learn**

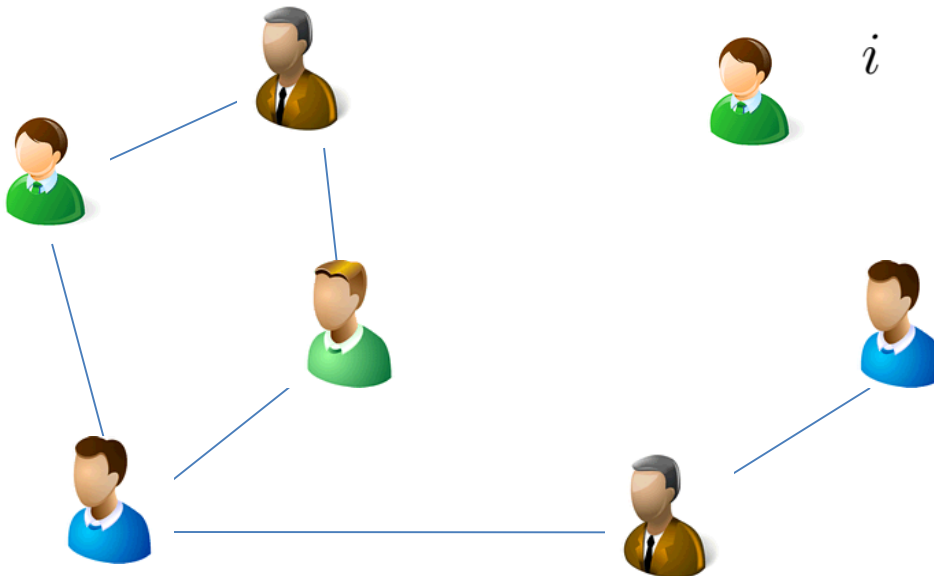
# Network Model

- Infinite horizon continuous time
  - Interactions are on-going and dynamic
- A number of  $N$  agents, initially linked according to  $G^0$ 
  - Physical/geographical/communication connection constraints
  - Can also be planned
- Network evolves  $G^t$ 
  - Links of each agent changes over time as learning occurs



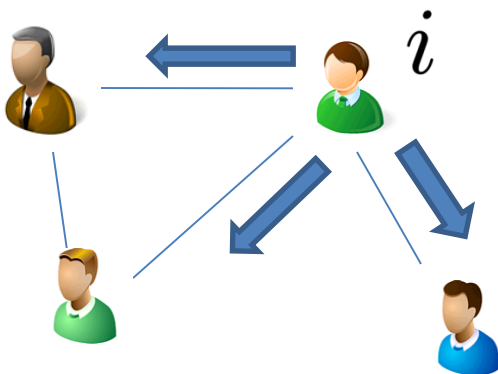
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# Agent Quality

- Each agent has a quality  $q_i$ 
  - Unknown a priori to all agents
  - Prior belief is drawn from a normal distribution  $\mathcal{N}(\mu_i, \sigma_i^2)$ 
    - We assume  $\mu_i > 0$  for all agents
- An agent sends a (flow) benefit to his neighbors
  - Benefit **reflects the quality** of that agent with **noise**
  - Modeled using Brownian motion diffusion

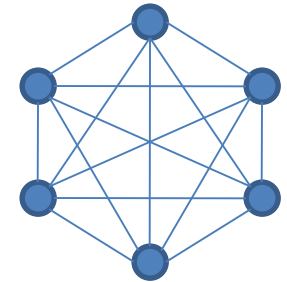


$$dB_i(t) = q_i dt + (k_i^t \tau_i)^{-1/2} dZ(t)$$

Total benefit sent by agent  $i$  up to time  $t$

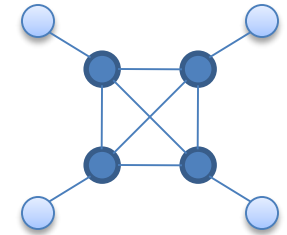
# Optimal Initial Networks

- Fully connected network (**the world is flat**)



**Theorem.** A fully connected initial network is optimal if the prior mean quality is sufficiently high

- Core-periphery network (**the world is not flat**)
  - Heterogeneous agents: two levels  $\mu_H$   $\mu_L$

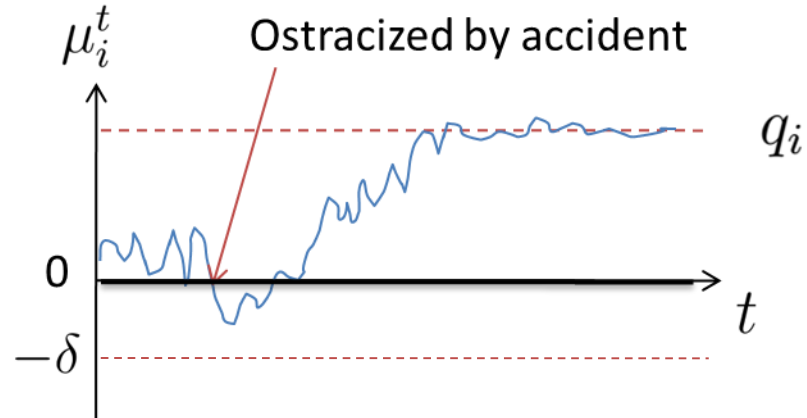


**Theorem.** Core-periphery is optimal if  $\mu_H$  is sufficiently higher than  $\mu_L$

- Why?
  - Connected at the core  $\rightarrow$  learned more quickly
  - Connected at the periphery  $\rightarrow$  less harm **Dominant**

# When to cut-off?

Suppose the network designer can choose a threshold  $\delta$  at which agents cut off links with each other



**Theorem. (1)**  $W(\delta) > W(0)$

(2) There exists  $\delta^* < \infty$ , such that  $\delta^* = \arg \max_{\delta} W(\delta)$

- Implication
  - Encouraging more experimentation is good for the network
  - Cannot be too tolerant of bad behaviors
  - The exact value depends on the specific networks and we can compute!

# Conclusion

- Current Results
  - A first model for analyzing **endogenously** evolving network formation under incomplete information
    - Rigorous characterization of learning and network co-evolution
    - Understanding emergent behaviors of strategic agents
  - Guidelines for building safer networks
    - Planning for initial connection
    - When to cut-off the links with malicious agents
- Please see the poster for more details and results!