Bargaining Strategies for Networked Multimedia Resource Management

Hyunggon Park and Mihaela van der Schaar

Abstract

Multi-user multimedia applications such as enterprise streaming, surveillance, and gaming are recently emerging, and they are often deployed over bandwidth-constrained network infrastructures. To ensure the Quality of Service required by the delay-sensitive and bandwidth intensive multimedia data for these applications, efficient resource (bandwidth) management becomes paramount. We propose to deploy the well-known game theoretic concept of bargaining to allocate the bandwidth fairly and optimally among multiple collaborative users. Specifically, we consider two bargaining solutions for our resource management problem: the Nash bargaining solution (NBS) and the Kalai-Smorodinsky bargaining solution (KSBS). We provide interpretations for the two investigated bargaining solutions for multi-user resource allocation: the NBS can be used to maximize the system utility, while the KSBS ensures that all users incur the same utility penalty relative to the maximum achievable utility. The bargaining strategies and solutions are implemented in the network using a resource manager, which explicitly considers the application-specific distortion for the bandwidth allocation. We show that the bargaining solutions exhibit important properties (axioms) that can be used for effective multimedia resource allocation. Moreover, we propose several criteria for determining bargaining powers for these solutions, which enable us to provide additional flexibility in choosing solution by taking into consideration the visual quality impact, the deployed spatio-temporal resolutions, etc. We also determine the complexity of these solutions for our application and quantify the performance of the proposed bargaining-based resource strategies for different scenarios.

Index Terms

Resource allocation, generalized Nash bargaining solution, Kalai-Smorodinsky bargaining solution, bargaining power, cooperative game, multimedia streaming.

This work was supported by the Korea Science and Engineering Foundation Grant funded by the Korean government (MOST) (No. M06-2004-000-10181).

The authors are with the Department of Electrical Engineering, University of California, Los Angeles (UCLA), Eng. IV, 420 Westwood Plaza, Los Angeles, CA 90095, USA. Tel. +1-310-825-5843, Fax: +1-310-206-4685, email: childman@ucla.edu (H. Park), mihaela@ee.ucla.edu (M. van der Schaar).

I. Introduction

A plethora of collaborative multimedia networking applications such as multi-camera surveillance and multi-user enterprise streaming are recently emerging. These applications are often deployed over time-varying and bandwidth-constrained infrastructures such as the Internet and wireless networks. These infrastructures do not provide support for the Quality of Service (QoS) required by the delay-sensitive and bandwidth-intensive multimedia data. To ensure the necessary QoS, recent research has focused on innovative solutions that provide efficient bandwidth allocation, rate-adaptation or joint source-channel coding to cope with the rapidly-varying resources [1]. However, these adaptation techniques have been performed in isolation, at each multimedia transmitter, and suffer from the important limitation of not considering their interactions (in terms of resource utilization) with other devices sharing the same network infrastructure. For example, over the Internet, ReSerVation Protocol (RSVP) [2], [3] was used to allocate bandwidth for a multimedia bitstream based on the traffic specification provided by the video application. In wireless LAN standards (e.g., IEEE 802.11e [4]), a similar resource allocation mechanism is deployed. The disadvantage of these static reservation-based solutions is that they do not scale to the number of users and time-varying network resources. Moreover, since the allocation is static and performed prior to transmission time, it does not consider the video sequence characteristics etc. Alternatively, the resource allocation can be determined dynamically based on the currently available resources, participating users and their video content characteristics. In this case, fairness policies are needed to allocate the available resources among the multiple multimedia users. Several resource allocation policies have been proposed in the literature. One of the simplest resource allocation policies is to equally allocate resources to the participating users. An important disadvantage of this policy is that it does not consider video content characteristics. Alternatively, the notion of *proportional fairness* was introduced in [5] to allocate resources based on the users' rate requirements. While this fairness criterion was successfully deployed in several applications [6], [7], it is not suitable for content-aware multimedia applications since it does not consider explicitly the resulting impact on video quality.

To address the above limitations, we propose a distributed resource management approach for multi-user multimedia transmission based on the well-suited game-theoretic concept from economics: the notion of bargaining [8]–[12]. Unlike inefficient conventional heuristic techniques for resource allocation, the proposed solution attempts to solve this problem directly in the multimedia utility domain and it leads to significantly better solutions for surveillance and enterprise streaming applications because it enables collaboration for the derivation of the global optimum operational point in terms of utility and it is robust to changes in the network infrastructure or the type and number of participating transmitters. This ensures that the resulting resource allocation can simultaneously consider the network constraints, video characteristics, and the number of participating users. Notice that bargaining is particularly suitable for such collaborative multimedia applications since each device can be considered as a trustworthy entity, which has incentives or is required to participate in the global system optimization. The resource exchanges are made possible by jointly adapting the compression and transmission strategies of the participating devices. Game theory was previously proposed to resolve resource allocation issues for various networks in a distributed and scalable manner [13]–[18]. However, prior research has not considered the dynamic bandwidth exchanges among collaborative devices and the resulting impact on the multimedia quality for various content-aware and delay-sensitive applications.

The main contribution of our paper is the use of bargaining solutions for collaborative multimedia streaming applications. We define an application-specific utility function and fairness criterion that enables an optimal allocation of resources among multimedia users. Unlike alternative resource allocation strategies that consider solely the network condition [19], we consider an application-specific utility which *explicitly* considers the content characteristics, resolutions, and delay constraints. Efficient resource allocation is especially important for multimedia applications as the necessary bandwidth for these applications is very huge and varies continuously based on the contents. We introduce the bargaining powers to fairly distribute the resources among users. We consider two bargaining solutions providing different fairness policies that can be used in various resource management problems: the generalized Nash bargaining solution and the Kalai-Smorodinsky bargaining solution. We show that these solutions exhibit important properties that can be used for effective resource allocation. Note that the bargaining solutions do not require global objective functions unlike conventional optimization methods such as Lagrangian or dynamic programming [20]. This is especially significant for video applications, where objective functions for the system utility cannot be easily defined as they need to jointly consider videos at different spatio-temporal resolutions. Alternatively, in our bargaining-based solution, independent utilities (with different parameters) can be deployed for the different users. We also determine the complexity of these solutions for our application and analyze the effect of different bargaining powers on the various users' performances.

This paper is organized as follows. In Section II, several basic concepts and definitions about cooperative game theory based on bargaining are reviewed. In Section III, we define the distortion-rate based utility function and prove the convexity of the feasible utility set, which enables us to deploy bargaining solutions. In Section IV, we analyze the Nash bargaining solution with a simple example and interpret the properties of this solution for our collaborative multimedia streaming problem. In Section V, we provide an alternative bargaining solution called Kalai-Smorodinsky bargaining solution and analyze a simple example for multimedia applications. In Section VI, we define a mechanism to implement the resource management and discuss possible strategies to assign bargaining powers. Simulation results to investigate the effect of the bargaining powers are presented. Conclusions are drawn in Section VII and several proofs used in the paper can be found in the Appendix.

II. BACKGROUND

In this section, we will briefly review several basic definitions and concepts related to bargaining solutions of cooperative game theory. Since there are many notations and abbreviations in this paper, we summarize them in Table I for reader's convenience.

Notation	Description	Notation	Description
$\mathbf{d}$	Disagreement Point, $(d_1, \ldots, d_n)$	R_{MAX}	Total Available Resource
$\mathbf{S}$	Feasible Utility Set	$\mathbf{X}^*$	Bargaining Solution
$\mathbf{B}$	Bargaining Set	X^i_{MAX}	Maximum Achievable Utility of user i
$\mathbf{U}_i(\cdot)$	Utility Function of user i	$\mathbf{X}_{MAX}$	Ideal Point, $(X^1_{MAX}, \ldots, X^n_{MAX})$
D	Distortion	α_i	Bargaining Power of user i
X_i	Utility of user i	$PSNR^*_i$	PSNR Achieved by Bargaining Solutions
$\mathbf{X}$	Joint Utility Point, $(X_1, \ldots, X_n)$	$PSNR^i_{MAX}$	Maximum Achievable PSNR for user i
x_i	Allocated Resource to user i		

Abbreviation	Full Name	Abbreviation	Full Name
PSNR	Peak Signal to Noise Ratio	NBS	Nash Bargaining Solution
KKT	Karush-Kuhn-Tucker	GNP	Generalized Nash Product
CIF	Common Interface Format	KSBS	Kalai-Smorodinsky Bargaining Solution
QCIF	Quarter CIF	TL	Temporal Level
		ERAS	Equal Resource Allocation Scenario

TABLE I

SUMMARY OF NOTATIONS AND ABBREVIATIONS

In cooperative games, players (in our case, multimedia transmitters or cameras) are assumed to try reaching an agreement that gives mutual advantage. Our resource management can be formulated as follows. There are n (video) users, which cooperate or "bargain" to divide the available network resource (bandwidth). Each user i has its own utility function ($U_i(x_i)$) that can be derived from the allocated resource (rate x_i) and it has also a minimum desired utility ($U_i(R_{0i})$), called the disagreement point. The disagreement point is the minimum utility that each user expects by joining the game without cooperation. Hence, we assume that the initial desired resource is at least guaranteed for each user in the cooperative game. Assume $\mathbf{S} = \{(U_1(x_1), \ldots, U_n(x_n))\} \subset \mathbb{R}^n$ is a joint utility set (or a feasible utility set) that is nonempty, convex, closed, and bounded and let $\mathbf{d} = (d_1, \ldots, d_n) = (U_1(R_{01}), \ldots, U_n(R_{0n})) \in \mathbb{R}^n$ be the disagreement point.

The pair $(\mathbf{S}, \mathbf{d})$ defines the bargaining problem. We define the Pareto optimal points/surface for a game among multiple users such that it is impossible to find another point that leads to a strictly superior advantage for all the users simultaneously [17].

Definition 1: Pareto Optimality

The resource allocation point $(X_1, \ldots, X_n) \in \mathbf{S}$ is *Pareto optimal* if for each $(X_1', \ldots, X_n') \in \mathbf{S}$ and $(X_1', \ldots, X_n') \geq (X_1, \ldots, X_n)$, then $(X_1', \ldots, X_n') = (X_1, \ldots, X_n)$.

Note that the inequality between two vectors in this paper represents component-wise inequality. In addition, it is known that there might exist an infinite number of Pareto optimal points in a game of multiple users [17]. Hence, we need criteria for deciding which Pareto optimal point is the best for the system, i.e., we need selection criteria for the bargaining solution. Multiple bargaining solutions that have different properties can be considered for our resource management problem (e.g. they provide consideration of optimality and fairness) [10], [21]. Specifically, we investigate two well-known solutions, which are the Nash Bargaining Solution (NBS) and the Kalai-Smorodinsky Bargaining Solution (KSBS).

A. Nash Bargaining Solution

NBS gives a unique and fair Pareto optimal solution that fulfills the following axioms [22]. Let F be a function $F : (\mathbf{S}, \mathbf{d}) \rightarrow \mathbb{R}^n$.

Definition 2: Bargaining Set [10]

The bargaining set $\mathbf{B}$ is the set of all individually rational, Pareto optimal payoff pairs in the cooperative payoff region $\mathbf{S}$.

Definition 3: Nash Bargaining Solution

$\mathbf{X}^* = F(\mathbf{S}, \mathbf{d})$ is said to be an NBS in $\mathbf{S}$ for the disagreement point $\mathbf{d}$, if the following axioms are satisfied.

1. *Individual Rationality*: $X_i^* \geq d_i$ for all i.
2. *Feasibility*: $\mathbf{X}^* \in \mathbf{S}$.
3. *Pareto Optimality*: $\mathbf{X}^*$ is Pareto optimal.
4. *Independence of Irrelevant Alternatives*: If $\mathbf{X}^* \in \mathbf{S}' \subset \mathbf{S}$ and $\mathbf{X}^* = F(\mathbf{S}, \mathbf{d})$, then $\mathbf{X}^* = F(\mathbf{S}', \mathbf{d})$.
5. *Independence of Linear Transformations*: For any linear scale transformation ψ, $\psi(F(\mathbf{S}, \mathbf{d})) = F(\psi(\mathbf{S}), \psi(\mathbf{d}))$.
6. *Symmetry*: If $\mathbf{S}$ is invariant under all exchanges of users, $F_i(\mathbf{S}, \mathbf{d}) = F_j(\mathbf{S}, \mathbf{d})$ for all possible users i, j.

The axioms 1, 2, and 3 define the bargaining set $\mathbf{B}$. Thus, the NBS is located in the bargaining set. The axioms 4, 5, and 6 are called axioms of fairness. The axiom 4 states that if the bargaining solution of the larger set is found on a smaller domain, then the solution is not affected by expanding the domain. This axiom provides a powerful property for our resource management problem when there are utility limits for each user. For example, if the utility is defined as the quality of multimedia content, it might be possible that in a specific application, a higher quality level (e.g., 40dB of Peak Signal to Noise Ratio) is not required. In this case, the NBS is invariant by limiting the maximum achievable utility if the axiom 4 is satisfied. The axiom 5 states that the bargaining solution is invariant if the utility function and disagreement point are scaled by a linear transformation. This axiom can also be used to obtain the NBS in a linearly transformed domain when the utility function and the disagreement point has a form of linear transformation. This is especially useful when the utility is a linear function of the rate like in e.g., MPEG-FGS video coders [23]. In this case, defining the bargaining solution in the utility or resource space is equivalent. The axiom 6 implies that if users have the same disagreement points and utility functions, they will have the same utility regardless of their indices. This represents an important fairness criteria for our problem that gives incentives to multimedia users to collaborate, as they can rely on the system to provide their equal treatment when their utility-resource tradeoffs vary over time.

B. Kalai-Smorodinsky Bargaining Solution

Another well-known bargaining solution is the KSBS [21]. The KSBS preserves all the axioms of the NBS except the independence of irrelevant alternatives that is replaced by the axiom of *individual monotonicity* [21]. This axiom states that increasing the bargaining set size in a direction favorable to user i always benefits user i. For example, let $(\mathbf{S}, \mathbf{d})$ and $(\mathbf{S}', \mathbf{d})$ be two bargaining problems, where $\mathbf{S} \subset \mathbf{S}'$ and the maximum achievable utilities of all users are the same except user i. Individual monotonicity states that the user i gains more utility in $(\mathbf{S}', \mathbf{d})$ than in $(\mathbf{S}, \mathbf{d})$. A simple example for this axiom is shown in Fig. 1. More details on the KSBS will be discussed in Section V. This axiom can be used to solve application specific problems. For instance, it might be necessary to improve the quality of some selected users (e.g. users transmitting more important content) by allocating them additional resources. In this example, the KSBS guarantees that this requirement keeps the optimality for all users.

III. Distortion-Rate based Utility and Convexity

In this section, we define the utility function based on the distortion-rate (DR) model. Since the requirement of the generalized NBS is a feasible utility set that is closed, convex, and bounded, we need to show that the feasible utility set of our problem is indeed convex.

A. Definition of Utility Function

Several distortion-rate (DR) models for wavelet video coders have been proposed [24]–[26]. Since the DR model proposed in [25] is well-suited for the average rate-distortion behavior [27] of the state-of-the-art

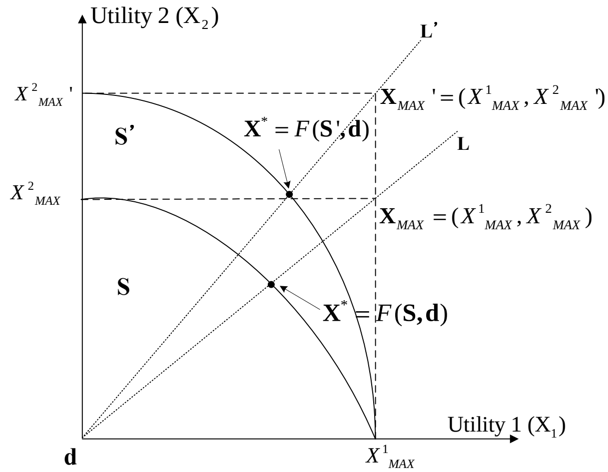


Fig. 1. A simple example to illustrate the axiom of *individual monotonicity* of the KSBS. In this example, there are two bargaining sets $\mathbf{S}$ and $\mathbf{S}'$ such that $\mathbf{S}' \supset \mathbf{S}$ and the maximum achievable utility of user 1 is fixed in both bargaining problems while the maximum achievable utility of user 2 is increased (i.e., $X_{MAX}^2{}' > X_{MAX}^2$). In this case, the KSBS always allocates more utility to user 2 due to the axiom of individual monotonicity of the KSBS.

video coders [28], we choose it as our DR model. The DR model in [25] is given by

$$D = \frac{\mu}{R - R_0} + D_0, \; R \geq R_0, \; D_0 \geq 0, \; \mu > 0, \tag{1}$$

where D is the distortion of the sequence, measured as the mean square error (MSE), and R is the rate for the video sequence. μ, R_0, and D_0 are the parameters for this DR model, which are dependent on video sequence characteristics, spatial and temporal resolutions, and delay. Note that the parameters μ is positive and D_0 is nonnegative. The corresponding Peak Signal to Noise Ratio (PSNR) is given by

$$PSNR = 10 \log_{10} \frac{255^2}{D}. \tag{2}$$

Correspondingly, we define the utility function that is from the definition of PSNR without considering the logarithm and constant multiplication as

$$U_i(x_i) \triangleq \frac{c}{D_i} = \frac{c \cdot (x_i - R_{0i})}{D_{0i}(x_i - R_{0i}) + \mu_i}, \tag{3}$$

where c is a nonnegative constant and subscript i represents user i (i.e., $U_i(x_i)$ represents the utility function for allocated rate x_i to user i). Note that $U_i(R_{0i}) = 0$ by the above definition of the utility function, thus the disagreement point $\mathbf{d}$ is the origin in our problem. Moreover, since each user expects a higher utility than the disagreement point, we assume that more than R_{0i} of resource is allocated to user i (i.e., $x_i > R_{0i}$). Thus, the utilities are positive (i.e., $U_i(x_i) > 0$). Note that the total available resource R_{MAX} is the constraint of this resource allocation problem.

In the next two sections, we will show that the utility function defined in (3) has two good properties: it leads to a convex feasible utility set and the physical meaning of the NBS and the KSBS corresponds to the desired resource optimization of our system.

B. Convexity of Feasible Utility Set

In this section, we show that the feasible utility set $\mathbf{S}$ of our problem is a convex set.

Theorem 1: The feasible utility set $\mathbf{S}$ is convex.

Proof: A set $\mathbf{C}$ is convex if for any $\mathbf{x_1}, \mathbf{x_2} \in \mathbf{C}$ and any θ with $0 \le \theta \le 1$, $\theta\mathbf{x_1} + (1-\theta)\mathbf{x_2} \in \mathbf{C}$. Let $\mathbf{X}$ and $\mathbf{Y}$ be two joint utility points in the feasible utility set $\mathbf{S}$. To show the convexity of the set $\mathbf{S}$, we must show that for any θ with $0 \le \theta \le 1$,

$$\begin{aligned} &\mathbf{X} = (X_1, \ldots, X_n) = (U_1(x_1), \ldots, U_n(x_n)) \in \mathbf{S},\ \mathbf{Y} = (Y_1, \ldots, Y_n) = (U_1(y_1), \ldots, U_n(y_n)) \in \mathbf{S} \\ &\quad \Rightarrow\ \theta\mathbf{X} + (1-\theta)\mathbf{Y} \in \mathbf{S}, \end{aligned} \tag{4}$$

where $(x_1, \ldots, x_n)$ and $(y_1, \ldots, y_n)$ are rate pairs which satisfy the rate constraints (i.e., for every i, $x_i > R_{0i}$, $y_i > R_{0i}$ and $\sum_{i=1}^n x_i \le R_{MAX}$, $\sum_{i=1}^n y_i \le R_{MAX}$). $X_i = U_i(x_i)$ and $Y_i = U_i(y_i)$ represent utilities when the rate x_i and y_i are allocated to user i, respectively. Since $x_i = \frac{\mu_i X_i}{c - D_{0i}X_i} + R_{0i}$ from the definition of the utility function and $\sum_{i=1}^n \frac{\mu_i X_i}{c-D_{0i}X_i} \le R_{MAX} - \sum_{i=1}^n R_{0i}$ from the rate constraint $\sum_{i=1}^n x_i \le R_{MAX}$, the feasible utility set $\mathbf{S}$ is expressed as

$$\mathbf{S} = \{\mathbf{X} \mid \sum_{i=1}^n \frac{\mu_i X_i}{c - D_{0i}X_i} \le R_{MAX} - \sum_{i=1}^n R_{0i},\ X_i > 0 \text{ for all } i\}. \tag{5}$$

To show the convexity of this set, we must show that

$$\sum_{i=1}^n \frac{\mu_i(\theta X_i + (1-\theta)Y_i)}{c - D_{0i}(\theta X_i + (1-\theta)Y_i)} \le R_{MAX} - \sum_{i=1}^n R_{0i} \text{ for } \mathbf{X}, \mathbf{Y} \in \mathbf{S}, \tag{6}$$

for any θ with $0 \le \theta \le 1$. From (6), the following inequalities hold:

$$\sum_{i=1}^n \frac{\mu_i(\theta X_i + (1-\theta)Y_i)}{c - D_{0i}(\theta X_i + (1-\theta)Y_i)} = \begin{cases} \sum_{i=1}^n \frac{\mu_i X_i}{c-D_{0i}X_i} \le R_{MAX} - \sum_{i=1}^n R_{0i} & \text{if} \quad \theta = 1 \\ \sum_{i=1}^n \frac{\mu_i Y_i}{c-D_{0i}Y_i} \le R_{MAX} - \sum_{i=1}^n R_{0i} & \text{if} \quad \theta = 0 \\ \le \max\{\sum_{i=1}^n \frac{\mu_i X_i}{c-D_{0i}X_i}, \sum_{i=1}^n \frac{\mu_i Y_i}{c-D_{0i}Y_i}\} & \text{otherwise.} \end{cases} \tag{7}$$

The last inequality is from the fact that the function $f(\theta) \triangleq \sum_{i=1}^n \frac{\mu_i(\theta X_i + (1-\theta)Y_i)}{c-D_{0i}(\theta X_i + (1-\theta)Y_i)}$ is convex for all $0 < \theta < 1$ (see Appendix I), and thus, $f(\theta) \le R_{MAX} - \sum_{i=1}^n R_{0i}$ for all $0 \le \theta \le 1$. Therefore, we showed that the conditions of convexity in (6) hold for any two joint utility points $\mathbf{X}$ and $\mathbf{Y}$, concluding that the feasible utility set $\mathbf{S}$ is convex. ■

IV. Nash Bargaining Solution

In this section, we analyze the NBS and verify its optimality for the investigated resource allocation problem. We also analyze the complexity of the NBS and provide a simple illustrative example of how the NBS-based resource allocation can be used in practice.

A. Analysis of the Generalized Nash Bargaining Solution

The function $F : \mathbf{S} \rightarrow \mathbb{R}^2$ is the generalized NBS for the two-user game corresponding to the bargaining powers α and β [10] if

$$F(\mathbf{S}, \mathbf{d}) \,=\, \{\mathbf{s} \,\in \mathbf{B} \,|\, \mathbf{s} = \,\alpha\mathbf{r}_1 + \beta\mathbf{r}_2,\ \alpha \geq 0,\ \beta \geq 0,\ \alpha + \beta = 1\}, \tag{8}$$

where the set $\mathbf{S}$ is the feasible utility set defined in (5) with $n = 2$ and $\mathbf{d}$ is the disagreement point. The point $\mathbf{s}$ is in the bargaining set $\mathbf{B}$ of the set $\mathbf{S}$ and the line through $\mathbf{r}_1$, $\mathbf{s}$, and $\mathbf{r}_2$ is a supporting line to $\mathbf{S}$ at $\mathbf{s}$. Note that $\mathbf{r}_1$ and $\mathbf{r}_2$ are the intersections of the supporting line and the horizontal and vertical lines through the disagreement point $\mathbf{d}$. The point $\mathbf{s}$ and the supporting line are chosen to ensure that $\mathbf{s} = \,\alpha\mathbf{r}_1 + \beta\mathbf{r}_2$. Then, the generalized NBS with bargaining powers α and β becomes the point $\mathbf{s}$, which is unique. Note that this satisfies all the axioms except the axiom 6 described in Section II-A. If the bargaining powers are the same (i.e., $\alpha = \beta = 1/2$), the axiom 6 is also satisfied. Moreover, the generalized NBS is the maximizer of the Generalized Nash Product (GNP) for the two-user case [10]. A simple example of the generalized NBS for two-user case is depicted in Fig. 2.

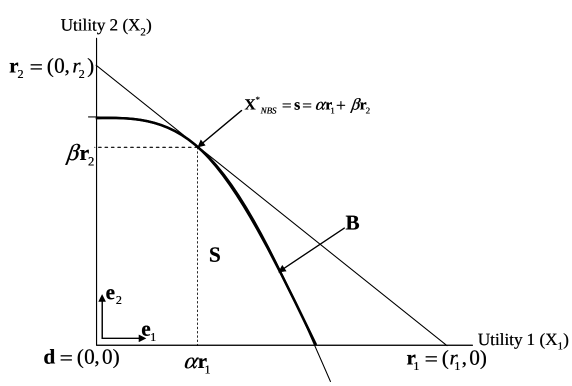


Fig. 2. A simple example of the NBS for two-user case with bargaining power α and β

The extension of the generalized NBS for the two-user game to the generalized NBS for the n user game can be found in [10]. The function $F : \mathbf{S} \rightarrow \mathbb{R}^n$ is the generalized NBS for n user game corresponding to the bargaining powers α_i for each user i, if

$$F(\mathbf{S},\mathbf{d}) \;=\; \{\mathbf{s} \in \mathbf{B} \,|\, \mathbf{s} = \sum_{i=1}^{n} \alpha_i \mathbf{r}_i,\; \sum_{i=1}^{n} \alpha_i = 1,\; \alpha_i \geq 0 \;\text{ for all } i\}, \tag{9}$$

where the set $\mathbf{S}$ is the feasible utility set defined in (5) and $\mathbf{d}$ is the disagreement point. $\mathbf{s}$ is a point in the bargaining set $\mathbf{B}$ of the set $\mathbf{S}$. The $(n-1)$ dimensional plane through $\mathbf{r}_1, \ldots, \mathbf{r}_n$ and $\mathbf{s}$ satisfying (9) is a supporting plane to $\mathbf{S}$ at $\mathbf{s}$. Then, this generalized NBS with bargaining powers α_i, $i = 1, \ldots, n$, is unique and is satisfying the axioms except the axiom 6 described in Section II-A. If the bargaining powers are the same (i.e., $\alpha_i = 1/n$ for all i), the axiom 6 is also satisfied. (See Appendix II). Moreover, the generalized NBS is the maximizer of the GNP for the n user case, which is shown in the next section. Recall that the disagreement point $\mathbf{d}$ is the origin in this problem. Thus, we can set $\mathbf{r}_i = r_i \mathbf{e}_i$ where $\mathbf{e}_i$ represents a unit vector of user i's utility axis X_i. Hence, the generalized NBS is the point $\mathbf{s}$ such that

$$\mathbf{s} = (\alpha_1 r_1,\, \ldots,\, \alpha_n r_n) \in \mathbf{B},\; \sum_{i=1}^{n} \alpha_i = 1,\; \alpha_i \geq 0 \text{ for all } i, \tag{10}$$

where the bargaining set $\mathbf{B}$ of the set $\mathbf{S}$ is given by

$$\mathbf{B} = \{\mathbf{X} = (X_1, \ldots, X_n) \,|\, \sum_{i=1}^{n} \frac{\mu_i X_i}{c - D_{0i} X_i} = R_{MAX} - \sum_{i=1}^{n} R_{0i},\; X_i > 0 \text{ for all } i\}. \tag{11}$$

Since the $(n-1)$ dimensional supporting plane at $\mathbf{s}$ is also determined by the normal vector at this point, the plane is perpendicular to the gradient at $\mathbf{s}$. The gradient at $\mathbf{s}$ is given by

$$\nabla \mathbf{B} \,|_{\mathbf{s}} = \left[\; \tfrac{c\mu_1}{(c - D_{01}\alpha_1 r_1)^2}, \ldots, \tfrac{c\mu_n}{(c - D_{0n}\alpha_n r_n)^2} \; \right]^T. \tag{12}$$

Since the supporting plane includes all the points of $\mathbf{r}_i$, the gradient at $\mathbf{s}$ is perpendicular to the vector $\mathbf{r}_m - \mathbf{r}_k$ for any $m, k \in \{1, \ldots, n\}$, $m \neq k$. This relationship gives

$$\{\nabla \mathbf{B} \,|_{\mathbf{s}}\}^T (\mathbf{r}_m - \mathbf{r}_k) = 0 \;\Rightarrow\; \frac{c\mu_m r_m}{(c - D_{0m}\alpha_m r_m)^2} = \frac{c\mu_k r_k}{(c - D_{0k}\alpha_k r_k)^2}. \tag{13}$$

By changing the variable $X_i^* = \alpha_i r_i$, we have

$$\frac{\alpha_m (c - D_{0m} X_m^*)^2}{\mu_m X_m^*} = \frac{\alpha_k (c - D_{0k} X_k^*)^2}{\mu_k X_k^*} \text{ for any } \; m, k \in \{1, \ldots, n\},\; m \neq k. \tag{14}$$

or, equivalently

$$\frac{\alpha_m \mu_m}{(x_m^* - R_{0m})(D_{0m}(x_m^* - R_{0m}) + \mu_m)} = \frac{\alpha_k \mu_k}{(x_k^* - R_{0k})(D_{0k}(x_k^* - R_{0k}) + \mu_k)}. \tag{15}$$

Recall that X_i represents the utility $U_i(x_i) = \frac{c(x_i - R_{0i})}{D_{0i}(x_i - R_{0i}) + \mu_i}$ and x_i is an allocated rate to user i. Moreover, since the generalized NBS $(X_1^*, \ldots, X_n^*)$ should be in the bargaining set $\mathbf{B}$ in (11), it must satisfy

$$\sum_{i=1}^{n} \frac{\mu_i X_i^*}{c - D_{0i} X_i^*} = R_{MAX} - \sum_{i=1}^{n} R_{0i},\; X_i^* > 0 \text{ for all } i. \tag{16}$$

or, equivalently

$$\sum_{i=1}^{n} x_i^* = R_{MAX},\ x_i^* > R_{0i} \text{ for all } i. \tag{17}$$

Using (14) or (15), we can express X_i^* based on X_1^* for $i = 2, \ldots, n$ or express x_i^* based on x_1^* for all $i = 2, \ldots, n$. Since (16) or (17) is generally an nth degree polynomial of X_1^* or x_1^*, there is no general solution for $n \geq 5$. The solution, however, can be obtained efficiently by simple numerical methods such as the Newton's method or the bisection method [29]. Since we know, for instance, the lower and upper bound of the utility or the rate, we can easily apply the bisection method to obtain the rate of user 1. To use this algorithm, we change the problem with one scalar variable. In this example, we focus on obtaining x_1^*. X_1^* can be obtained in a similar way. The relation between $x_i, i = 2, \ldots, n$ and x_1 from (15) is given by

$$x_k = \begin{cases} \frac{-\alpha_1\mu_1\mu_k + \sqrt{(\alpha_1\mu_1\mu_k)^2 + 4\alpha_1\alpha_k\mu_1\mu_k D_{0k} b_1}}{2\alpha_1\mu_1 D_{0k}} + R_{0k} & \text{if} \quad D_{0k} \neq 0 \\ \frac{\alpha_k b_1}{\alpha_1 \mu_1} + R_{0k} & \text{if} \quad D_{0k} = 0, \quad k = 2, \ldots, n, \end{cases} \tag{18}$$

where $b_1 = (x_1 - R_{01})(D_{01}(x_1 - R_{01}) + \mu_1)$. The set of x_i^* should satisfy the constraint $\sum_{i=1}^{n} x_i^* \leq R_{MAX}$. We apply the bisection method described in Algorithm 1 with this constraint to obtain x_1^* satisfying

$$x_1^* = \arg\max_{x_1} \{\sum_{i=1}^{n} x_i \leq R_{MAX}\}. \tag{19}$$

Since the generalized NBS is unique when the bargaining powers are given, x_1^* is the optimal solution. Based on the optimal solution x_1^*, we can obtain all other optimal solutions x_i^*, $i = 2, \ldots, n$ using (18). Note that the bisection method requires exactly $\lceil \log_2((u - l)/\epsilon) \rceil$ iterations.

Algorithm 1 Bisection Method

Require: lower bound $l := \min(R_{01}, \ldots, R_{0n})$, upper bound $u := R_{MAX}$, tolerance $\epsilon > 0$

repeat

1. set x_1 to be the mid-point of l and u; $x_1 := (l + u)/2$
2. obtain x_i $(i = 2, \ldots, n)$ based on x_1 using (18)
3. check the feasibility of x_i $(i = 1, \ldots, n)$ for the constraint $\sum_{i=1}^{n} x_i \leq R_{MAX}$
4. if feasible $l := x_1$, else $u := x_1$

until difference between l and u is less than the tolerance ϵ; $u - l \leq \epsilon$

Note that the tolerance ϵ represents the interval in which the optimal point x_1^* is located.

B. Optimality of the Generalized Nash Bargaining Solution

In this section, we verify the optimality of the generalized NBS that we obtained in Section IV-A. There exists only one generalized NBS $\mathbf{X}^* = (X_1^*, \ldots, X_n^*)$ that satisfies the six axioms and the generalized NBS is the solution of the following optimization problem [10], [17]:

$$\mathbf{X}^* = \arg\max_{\mathbf{X}\in\mathbf{S}} \quad G(\mathbf{X}) = \prod_{i=1}^{n}(X_i - d_i)^{\alpha_i}$$
$$\text{subject to} \quad (X_1, \ldots, X_n) \in \mathbf{S},\ X_i \geq d_i \quad \sum_{i=1}^{n}\alpha_i = 1,\ \alpha_i \geq 0 \text{ for all } i. \tag{20}$$

where $\mathbf{X} = (X_1, \ldots, X_n)$ is the joint utility point in the feasible utility set $\mathbf{S}$ defined in (5) with α_i, denoting bargaining power of user i. Recall that the disagreement point $\mathbf{d} = (d_1, \ldots, d_n)$ is the origin in our problem. We assume that the bargaining powers are given. We will verify that the generalized NBS is the optimal solution of the optimization problem in (20). For any optimization problem with differentiable objective and constraint functions for which strong duality holds, any pair of primal and dual optimal points must satisfy the Karush-Kuhn-Tucker (KKT) conditions. Moreover, if the primal problem is convex, the KKT conditions are sufficient for the points to be primal and dual optimal [29]. We verify that the generalized NBS is the solution of the optimization problem in (20) by showing that it satisfies the KKT condition. Since $G(\mathbf{X})$ is a concave function (see Appendix III) and $\mathbf{S}$ is a convex set, the primal problem is convex. Since the $G(\mathbf{X})$ is also nondecreasing function, the optimal solution should be in bargaining set $\mathbf{B}$. Thus, we can express the equivalent optimization problem:

$$\mathbf{X}^* = \arg\max_{\mathbf{X}\in\mathbf{S}} \quad G(\mathbf{X}) = \prod_{i=1}^{n} X_i^{\alpha_i}$$
$$\text{subject to} \quad \sum_{i=1}^{n}\frac{\mu_i X_i}{c - D_{0i}X_i} = R_{MAX} - \sum_{i=1}^{n} R_{0i},\ X_i > 0 \text{ for all } i, \tag{21}$$

where $\sum_{i=1}^{n}\alpha_i = 1,\ \alpha_i \geq 0$ for all i. Since the objective function and constraint functions are differentiable, we have the following KKT conditions:

1. *Primal Constraints*: $\sum_{i=1}^{n}\frac{\mu_i X_i}{c-D_{0i}X_i} = R_{MAX} - \sum_{i=1}^{n} R_{0i},\ X_i > 0$ for all i
2. *Dual Constraints*: $\boldsymbol{\lambda} \geq 0$
3. *Complementary Slackness*: $\lambda_i X_i = 0$ for all i
4. *Gradient of Lagrangian with respect to* $\mathbf{X}$ *vanishes*: $(\nabla G(\mathbf{X}))_i - \lambda_i + \nu(\frac{c\mu_i}{(c-D_{0i}X_i)^2}) = 0$ for all i,

where $(\nabla G(\mathbf{X}))_i$ represents ith element in the vector of $\nabla G(\mathbf{X})$ and λ_i and ν are Lagrange multipliers associated with ith inequality constraint and equality constraint, respectively. The gradient of the object function is:

$$\nabla G(\mathbf{X}) = \left[\ \alpha_1 \cdot X_1^{\alpha_1 - 1} \cdot \prod_{i=1, i\neq 1}^{n} X_i^{\alpha_i}, \ldots, \alpha_n \cdot X_n^{\alpha_n - 1} \cdot \prod_{i=1, i\neq n}^{n} X_i^{\alpha_i}\ \right]^T. \tag{22}$$

Next, we check that the generalized NBS satisfies all the KKT conditions. The condition 1 (primary constraints) is satisfied because the generalized NBS is a point in the bargaining set. The condition 3 (complementary slackness) is satisfied by setting $\lambda_i = 0$ because $X_i > 0$ for all i due to the assumption that a higher utility than the disagreement point is allocated to each user. Hence, condition 2 (dual constraints) is obviously satisfied. From the condition 4, we have $\nabla(G(\mathbf{X}))_i = -\nu \frac{c\mu_i}{(c-D_{0i}X_i)^2}$ for all i. Since ν is a constant, we can obtain the relation between X_m and X_k by setting mth and kth rows to be equal. Thus, we have

$$\frac{\alpha_m(c - D_{0m}X_m)^2}{\mu_m X_m} = \frac{\alpha_k(c - D_{0k}X_k)^2}{\mu_k X_k}. \tag{23}$$

This is exactly the same as the generalized NBS in (14) because X_i is located in the bargaining set by the condition 1. Therefore, the generalized NBS satisfies all the KKT conditions, being the optimal solution of the optimization problem for the GNP.

Let us now investigate the physical meaning of the generalized NBS and the extended GNP for our resource management problem. From (20) with $c = 255^2$ and the generalized NBS $(X_1^*, \ldots, X_n^*)$, we have

$$10\log_{10} G(\mathbf{X}^*) = 10\log_{10}\prod_{i=1}^{n}(X_i^*)^{\alpha_i} = \sum_{i=1}^{n}\alpha_i \cdot 10\log_{10}[\frac{255^2 \cdot (x_i^* - R_{0i})}{D_{0i}(x_i^* - R_{0i}) + \mu_i}] = \sum_{i=1}^{n}\alpha_i \cdot PSNR_i^*, \tag{24}$$

where $PSNR_i^*$ was achieved by the resource allocation provided by the generalized NBS X_i^* or x_i^* for the ith user (i.e., video sequence of user i). We can interpret (24) as the *weighted sum of PSNR* according to the bargaining powers (i.e., importance) of users or video sequences. In other words, the total resource R_{MAX} is divided into small bandwidth segments that are allocated to the user that has the highest increase of utility by gaining this resource. This allocation is repeated until the entire resource is used and it can be interpreted as assigning resources to the user with the largest rate-quality slope. This allocation also can be viewed as a fairness criterion, which maximizes the system utility represented by the weighted sum of PSNRs given the total rate R_{MAX} and the bargaining powers. Therefore, we conclude that the generalized NBS is the optimal and fair solution for resource allocation that leads to the maximum system utility.

C. Complexity of the Generalized Nash Bargaining Solution

In this section, we investigate the complexity for the generalized NBS. We use the ”flop” (floating-point operation) as a measure of complexity. Flop counts can give us a good estimate of the computation time of a numerical algorithm, and how the time grows with an increasing problem size [29]. We assume that the each operation of addition, substraction, multiplication and division counts one flop.

The generalized NBS is analyzed in Section IV-A. The generalized NBS is given in (18) with the constraint $\sum_{i=1}^{n} x_i \leq R_{MAX}$. As an example, we used the bisection method to obtain x_1^*. Since the number of iterations for the bisection method is exactly $\lceil \log_2((u-l)/\epsilon) \rceil$ and one iteration requires at most $(25+s_1)(n-1)+s_2$ flops in (18) (given x_1, computation of each x_i requires $(24+s_1)$ flops, the sum of them requires $(n-1)$

flops, and one comparison operation is required), where s_i, $i = 1, 2$ denotes some constant flops required for square root and comparison operation, respectively, the total number of required flops for obtaining x_i^* for all i is $\lceil \log_2((u-l)/\epsilon) \rceil \cdot ((25 + s_1)(n-1) + s_2)$. Therefore, we conclude that the complexity for the NBS is $O(n)$ if we use the bisection method.

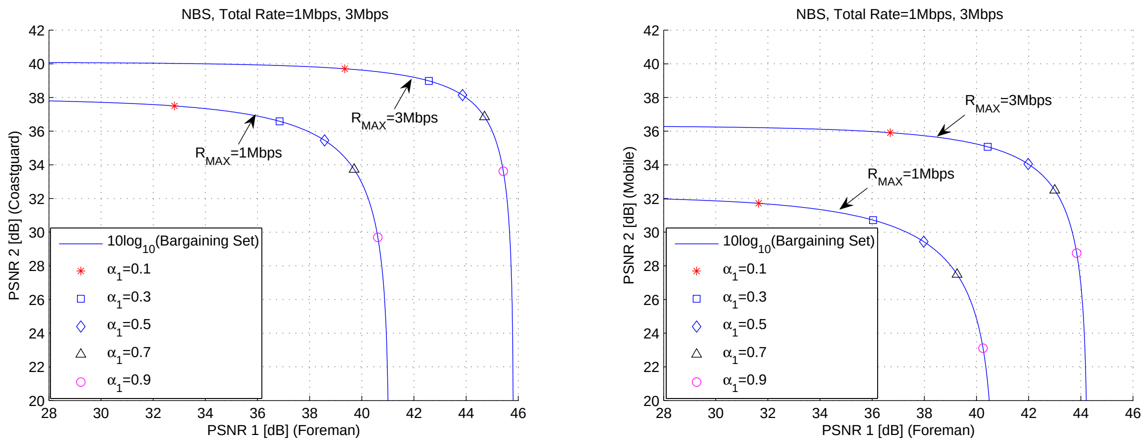
D. Example: Generalized Nash Bargaining Solution for the Two-User Case

In this section, we determine the generalized NBS for the two-user case, which is the simplest form of cooperative game. In this example, we focus on the rate allocation instead of the utility allocation due to the simplicity of calculation, since the rate and utility have a one-to-one mapping. For the two-user case, we use (15) to determine the rates allocated to user 1 and user 2. Let x_1^* and x_2^* be an optimal rate allocation by the generalized NBS for user 1 and user 2, respectively. Then, (15) gives,

$$\frac{\alpha_1 \mu_1}{(x_1^* - R_{01})(D_{01}(x_1^* - R_{01}) + \mu_1)} = \frac{\alpha_2 \mu_2}{(x_2^* - R_{02})(D_{02}(x_2^* - R_{02}) + \mu_2)}, \tag{25}$$

with rate constraint $x_1^* + x_2^* = R_{MAX}$, $x_1^* > R_{01}$, $x_2^* > R_{02}$ and $\alpha_1 + \alpha_2 = 1$. Since $x_2^* = R_{MAX} - x_1^*$, we can obtain x_1^* explicitly from (25). Note that since there are two solutions for x_1^*, we should choose the one of them which satisfies the rate constraint (i.e., $x_1^* > R_{01}$, $x_2^* > R_{02}$, $x_1^* + x_2^* = R_{MAX}$). The generalized NBS for some different bargaining powers and two different video sequences with $c = 255^2$ are in Fig. 3. Note that since we use the constant $c = 255^2$, we can interpret $10 \log_{10}(\text{utility})$ as PSNR. We observe that the resource (i.e., rate) is proportionally allocated by the bargaining powers such that the normalized sum of PSNRs is maximized. It implies that user i which has a higher bargaining power derives a higher utility.

It should be noted that the bargaining powers can be successfully used to enable different tradeoffs between the utilities of the various users based on the visual quality impact and not only the objective qualities (PSNR). For instance, when the total rate equals 1Mbps (i.e., R_{MAX}=1Mbps) in Fig. 3 (b), the achieved PSNRs of user 1 and user 2 with the bargaining powers $\alpha_1 = \alpha_2 = 0.5$ are approximately 38dB (no visual artifacts visible) and 29dB (poor video quality), respectively. However, to provide a fairer allocation of resources that considers the visual impact, the bargaining powers are set to $\alpha_1 = 0.3$ and $\alpha_2 = 0.7$, leading to a PSNR of approximately 36dB for user 1 and 31dB for user 2. By adjusting the bargaining powers, the visual quality of the two users becomes comparable. Moreover, in this case, the visual satisfaction increment for the user 2 due to the additional 2dB in PSNR is much higher than the visual satisfaction decrement experienced by the user 1. Therefore, the bargaining powers form an important tool to influence the visual satisfaction of the users. Other possible criteria for choosing the bargaining powers will be discussed in Section VI-B.



(a) (b)

Fig. 3. Plots of Bargaining Set and the NBS with different bargaining powers for *Foreman* and *Coastguard* sequences (a) and *Foreman* and *Mobile* sequences (b)

V. Alternative Bargaining Solution: Kalai-Smorodinsky Bargaining Solution

In this section, we present an alternative bargaining solution called the Kalai-Smorodinsky bargaining solution. The KSBS can be used when the feasible utility set is not convex. (Recall the NBS requires the convexity of the feasible utility set). Moreover, the KSBS provides different type of fairness as opposed to the NBS. We analyze the KSBS and the complexity of the KSBS.

A. *Analysis of the Generalized Kalai-Smorodinsky Bargaining Solution*

In this section, we analyze the KSBS. The KSBS for two users satisfies [21], [30]

$$\mathbf{X}^* = F(\mathbf{S}, \mathbf{d}) = \mathbf{d} + \lambda_{MAX}(\mathbf{X}_{MAX} - \mathbf{d}), \tag{26}$$

where $\mathbf{S}$ denotes the feasible utility set defined in (5) with $n = 2$ and $\mathbf{X}^* = (X_1^*, X_2^*)$ is the KSBS in the set $\mathbf{S}$ and $\mathbf{d} = (d_1, d_2)$ is the disagreement point. $\mathbf{X}_{MAX} = (X_{MAX}^1, X_{MAX}^2) \geq \mathbf{d}$ is called the ideal point [31], which is the point of the best achievable utility for user 1 and 2 in the set $\mathbf{S}$ and λ_{MAX} is the maximum value of λ such that $\mathbf{d} + \lambda(\mathbf{X}_{MAX} - \mathbf{d}) \in \mathbf{S}$. A simple example of the KSBS for the two-user case is depicted in Fig. 4. The extension of the two-user KSBS to the n-user KSBS satisfies

$$\mathbf{X}^* = F(\mathbf{S}, \mathbf{d}) = \mathbf{d} + \lambda_{MAX}(\mathbf{X}_{MAX} - \mathbf{d}), \tag{27}$$

where $\mathbf{S}$ is the feasible utility set defined in (5) with n and $\mathbf{X}^* = (X_1^*, \ldots, X_n^*)$ is the KSBS and $\mathbf{d} = (d_1, \ldots, d_n)$ is the disagreement point, which is the origin in our problem. $\mathbf{X}_{MAX} = (X_{MAX}^1, \ldots, X_{MAX}^n) \geq$

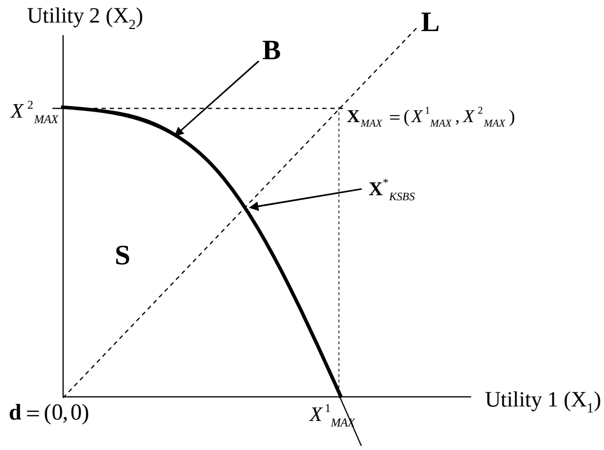


Fig. 4. A simple example of KSBS for the two-user case

$\mathbf{d}$ is the ideal point for n users and λ_{MAX} is the maximum value of λ such that $\mathbf{d} + \lambda(\mathbf{X}_{MAX} - \mathbf{d}) \in \mathbf{S}$. As we stated in Section IV-A, the bargaining set is defined as

$$\mathbf{B} = \{\mathbf{X} \,|\, \sum_{i=1}^{n} \frac{\mu_i X_i}{c - D_{0i} X_i} = R_{MAX} - \sum_{i=1}^{n} R_{0i},\ X_i > 0 \text{ for all } i\}. \tag{28}$$

The KSBS is the intersection between the bargaining set $\mathbf{B}$ and the line $\mathbf{L}$ defined by

$$\mathbf{L} = \{\,\mathbf{X} \,|\, \frac{X_1}{\alpha_1 X^1_{MAX}} = \ldots = \frac{X_n}{\alpha_n X^n_{MAX}},\ X_i > 0,\ \sum_{i=1}^{n} \alpha_i = 1,\ \alpha_i \geq 0,\ \text{for all } i\}, \tag{29}$$

where $X^i_{MAX} = U_i(R_{MAX})$ since the disagreement point is the origin. We can solve these equations by setting $\frac{X_1}{\alpha_1 X^1_{MAX}} = \ldots = \frac{X_n}{\alpha_n X^n_{MAX}} = k$ for a constant k and it makes possible to express each $X_i = k \cdot \alpha_i X^i_{MAX}$ for all i. Since this point should be in the bargaining set, we substitute these X_i's into equation $\sum_{i=1}^{n} \frac{\mu_i X_i}{c - D_{0i} X_i} = R_{MAX} - \sum_{i=1}^{n} R_{0i}$ and solve this equation. As we mentioned in Section IV-A, this equation is generally an nth degree polynomial of k. Thus, the same numerical method such as the bisection method can be applied to this problem:

$$k^* = \arg\max_{k}\{\sum_{i=1}^{n} \frac{k \cdot \alpha_i \mu_i X^i_{MAX}}{c - k \cdot \alpha_i D_{0i} X^i_{MAX}} \leq R_{MAX} - \sum_{i=1}^{n} R_{0i}\}. \tag{30}$$

With k^*, we can obtain all X_i^* using $X_i = k^* \cdot \alpha_i X^i_{MAX}$. Therefore, we obtain the KSBS.

Let us now investigate the physical meaning of the KSBS. Since the KSBS is located in the bargaining set as well as in the line in (29), the bargaining solution must satisfy

$$\frac{X_1^*}{\alpha_1 X^1_{MAX}} = \ldots = \frac{X_n^*}{\alpha_n X^n_{MAX}}, \tag{31}$$

where $(X_1^* \ldots, X_n^*) \in \mathbf{B},\ \sum_{i=1}^{n} \alpha_i = 1,\ \alpha_i \geq 0$ for all i. Taking $10\log_{10}$ in (31) with $c = 255^2$, we have

$$(PSNR^1_{MAX} - PSNR^*_1) + 10\log_{10}\alpha_1 = \ldots = (PSNR^n_{MAX} - PSNR^*_n) + 10\log_{10}\alpha_n, \tag{32}$$

and equivalently,

$$\triangle PSNR_1^{drop} + 10\log_{10}\alpha_1 = \ldots = \triangle PSNR_M^{drop} + 10\log_{10}\alpha_M, \tag{33}$$

where $PSNR_{MAX}^i = 10\log_{10} X_{MAX}^i$ is the maximum achievable PSNR for user i and $PSNR_i^*$ is achieved PSNR by the KSBS X_i^*. The PSNR drop denoted by $\triangle PSNR_i^{drop} \triangleq (PSNR_{MAX}^i - PSNR_i^*)$ represents the quality decrease (or drop) from user i's maximum achievable quality. If the same bargaining powers are used in (33), the KSBS allocates resources such that the quality drop for all users are the same. Importantly, note that the KSBS can be thus interpreted as an utility-based fair resource allocation, since all users incur the same utility penalty by participating in the resource management game. If different bargaining powers are used, the user with a higher bargaining power obtains a higher PSNR than the other users.

B. Complexity of the Generalized Kalai-Smorodinsky Bargaining Solution

The KSBS is analyzed in Section V-A. The KSBS is given in (27) with k^* in (30) as

$$\mathbf{X}^* = (k^* \cdot \alpha_1 X_{MAX}^1, \ldots, k^* \cdot \alpha_n X_{MAX}^n). \tag{34}$$

The same analysis in Section IV-C can be applied to the KSBS. The required flops for k^* are $\lceil \log_2((u - l)/\epsilon) \rceil \cdot (9n + s_2)$ in (30) and computation of each utility $X_i = k^* \cdot \alpha_i X_{MAX}^i$, $i = 1, \ldots, n$ requires $2n$ flops. Therefore, the total required flops is $\lceil \log_2((u-l)/\epsilon)\rceil \cdot (9n + s_2) + 2n$ and it also has a complexity of $O(n)$ if we use the bisection method for obtaining k^*.

Summarizing, since the complexity of the two bargaining solutions is $O(n)$, the two bargaining solutions provide efficient methods for the resource management that can be easily used in practice.

C. Example: Generalized Kalai-Smorodinsky Bargaining Solution for the Two-User Case

In this section, we present the optimal solution by the KSBS for the two-user case. We set the line given in (29) as

$$\mathbf{L} = \{(X_1, X_2) \,|\, \frac{X_1}{\alpha_1 X_{MAX}^1} = \frac{X_2}{\alpha_2 X_{MAX}^2},\ X_i > 0,\ i = 1, 2,\ \alpha_1 + \alpha_2 = 1\}, \tag{35}$$

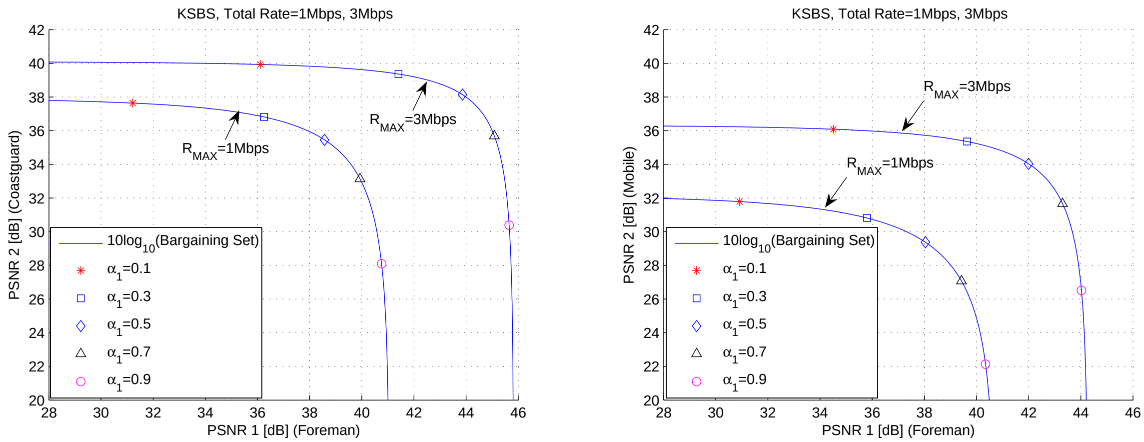
where $X_{MAX}^i = U_i(R_{MAX})$. Setting $\frac{X_1}{\alpha_1 X_{MAX}^1} = \frac{X_2}{\alpha_2 X_{MAX}^2} = k$ results in:

$$X_1 = k \cdot \alpha_1 X_{MAX}^1, \quad X_2 = k \cdot \alpha_2 X_{MAX}^2. \tag{36}$$

Since the KSBS (X_1, X_2) is in the bargaining set $\mathbf{B}$, the solution should satisfy

$$\begin{aligned}\frac{\mu_1 X_1}{c - D_{01} X_1} + \frac{\mu_2 X_2}{c - D_{02} X_2} &= \frac{k \cdot \mu_1 \alpha_1 X_{MAX}^1}{c - k \cdot D_{01}\alpha_1 X_{MAX}^1} + \frac{k \cdot \mu_2 \alpha_2 X_{MAX}^2}{c - k \cdot D_{02}\alpha_2 X_{MAX}^2} \\ &= R_{MAX} - R_{01} - R_{02}, \quad X_1 > 0,\ X_2 > 0.\end{aligned} \tag{37}$$

Hence, we can obtain k^* by solving the quadratic equation (37), and choose the solution k^* such that (X_1, X_2) is in the bargaining set $\mathbf{B}$. The KSBS for different bargaining powers and different video sequences is illustrated in Fig. 5. The KSBS gives proportional allocation of utility according to bargaining powers. Note that the KSBS is generally different from the NBS.



(a) (b)

Fig. 5. Plots of Bargaining Set and the KSBS with different bargaining powers for *Foreman* and *Coastguard* sequences (a) and *Foreman* and *Mobile* sequences (b)

VI. System Setup and Simulation Results

In this section, we define a mechanism or system to implement the previously analyzed bargaining solutions in a network infrastructure. Then we briefly discuss the possible criteria to determine the bargaining powers, provide several simulation results, and compare the achieved quality (i.e., PSNR) using the various bargaining solutions and resource allocation scenarios. In our simulations, we assume that there are two or three users and assume ”ideal” network conditions (i.e., no loss, the entire network resources (bandwidth) are allocated to the participating users). This scenario can be extended for wireless communications, congested networks, etc. The parameter values for the DR model are given in Table II determined based on a state-of-the-art wavelet video coder [28], which performs a number of temporal decomposition levels (TL) and adaptive motion compensated temporal filtering for each TL.

A. System Setup

The system configurations for the resource management are followings. There are multiple users, which try to transmit multimedia streams (e.g. video sequences) having different characteristics such as different resolutions, delay constraints, etc., over the same network. To allocate the available resources to users, there

Video Sequence	μ	D_0	R_0	Video Sequence	μ	D_0	R_0
Foreman (CIF, TL=4, 30Hz)	5232400	0	0	Foreman (QCIF, TL=4, 30Hz)	2653300	0	19614
Coastguard (CIF, TL=4, 30Hz)	6329700	4.3	0	Foreman (CIF, TL=4, 15Hz)	2760000	1	20720
Mobile (CIF, TL=4, 30Hz)	38230000	1	44040	Foreman (CIF, TL=2, 30Hz)	4610000	3	55080

TABLE II

MODEL PARAMETERS FOR VIDEO SEQUENCES. (VIDEO TYPE, TEMPORAL LEVEL (TL), FRAME RATE)

exists a resource manager, which decides the bargaining powers and distributes the entire available resources to users based on the bargaining powers. To do this, multiple users in a network send their information of parameters (μ, R_0, D_0) to the resource manager and the resource manager decides the bargaining powers for the bargaining solutions based on the information of the total available resource in a network, desired quality level for multimedia streams, and parameters that users sent. Subsequently, the resource manager computes the bargaining solutions and informs the users the rate at which they (transmitted videos) can operate.

B. Discussion about Assignment of Bargaining Powers

In this section, we present how to determine the bargaining powers for the various bargaining solutions. As we discussed in Section IV, the bargaining solutions are strongly dependent on the bargaining powers. Therefore, the bargaining powers should be determined appropriately based on applications and network constraints. Note that the bargaining solutions can be repeatedly applied every time a user's content characteristics change or the network constraints change. The following criteria can be considered for determining the bargaining powers.

1) Content Characteristics: Different motion and texture information can be extracted from the videos and higher bargaining powers can be assigned to the high motion or complex texture content. For example, the bargaining powers can be determined by explicitly considering the non-linear behavior of the Human Visual System [32]. Moreover, in our previous work [33], we have shown that this behavior can strongly depend on the specific users' Human Visual System. This can be easily exploited in the presented bargaining-based resource allocation by adjusting the bargaining powers depending on the instantaneous content and user characteristics.

2) Semantics of Videos: The semantic relevance of the various content can be extracted on the fly and a higher bargaining power can be assigned to the more important video sequence.

3) Spatio-Temporal Resolution of Videos: Different spatial resolution for video sequences such as the CIF and QCIF format and different temporal resolutions for the video sequences such as 30Hz and 10Hz can be deployed. The lower spatial and temporal resolutions lead to a higher PSNR benefit for the same allocation resource. However, PSNR does not account for the decreased resolution and this is compensated by increasing the bargaining power of the higher resolution sequence.

4) Channel Condition: Different users experience different channel conditions (e.g., different path loss, SNR, fading, etc.). We can allocate a higher bargaining power to users with a bad channel condition or a good channel condition.

5) Delay of Application Characteristics: Different delay constraints are required for multimedia applications. For example, interactive applications such as video conferencing or surveillance system require stringent delay constraint (less than 200ms) while for multimedia streaming applications delays of 1-5 second are tolerable [34]. A possible allocation of bargaining powers would be to allocate a higher power to low-delay video to facilitate its transmission.

Examples of the influence of bargaining powers on the bargaining solutions will be illustrated in the next sections.

C. Comparison of the generalized Nash Bargaining Solution and Equal Rate Allocation Scenario

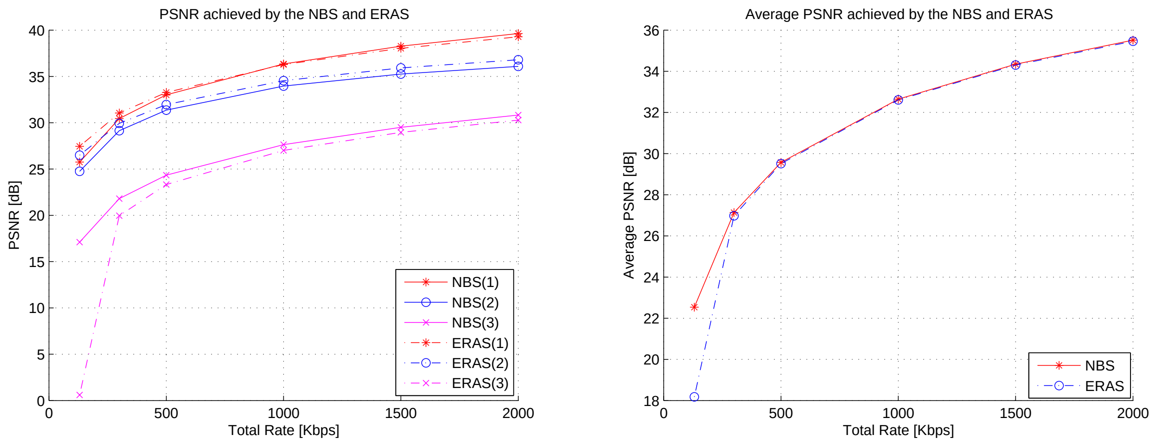
Based on (18), the generalized NBS allocates resources according to the video characteristics and bargaining powers. We compare this solution with the same bargaining powers to Equal Resource Allocation Scenario (ERAS) [35] that allocates the same amount of resource to each user without considering the video characteristics or their importance. Assume that there are three users and set the same bargaining powers, i.e., $\alpha_1 = \alpha_2 = \alpha_3 = \frac{1}{3}$ for the generalized NBS and compare the system performance with ERAS. For this, we determined the average PSNR for the generalized NBS and ERAS solutions as

$$\text{NBS}: \frac{1}{3}\left[PSNR_1(x_1^*) + PSNR_2(x_2^*) + PSNR_3(x_3^*)\right] \tag{38}$$

$$\text{ERAS}: \frac{1}{3}\left[PSNR_1(R_{MAX}/3) + PSNR_2(R_{MAX}/3) + PSNR_3(R_{MAX}/3)\right], \tag{39}$$

where x_1^*, x_2^*, and x_3^* are determined by the generalized NBS. Recall that the generalized NBS is the maximizer of the $\sum_{i=1}^{n} \alpha_i PSNR_i$. Thus, (38) leads to the maximum value at the generalized NBS for the same bargaining powers. Note that the average PSNR for the generalized NBS is always higher than that of ERAS since the generalized NBS always achieves the maximum value of $\frac{1}{n}\sum_{i=1}^{n} PSNR_i$.

Fig. 6 shows the average PSNR for the generalized NBS with the same bargaining powers and ERAS. User 1, user 2, and user 3 trasmit the *Foreman*, *Coastguard*, and *Mobile* sequences at CIF resolution 30Hz, respectively. Table III shows the allocated rates to each user by the generalized NBS and ERAS for different total available rates R_{MAX}. We observe that the generalized NBS gives always a higher average PSNR value than that of ERAS. This is especially true at low rates (when the resources are scarce), where a judicious use of resource is essential. Therefore, we conclude that the generalized NBS always provides better average



(a) (b)

Fig. 6. Plots of each user's achieved PSNR and Average PSNR by the generalized NBS and ERAS. User 1: Foreman (CIF, TL=4, 30Hz), User 2: Coastguard (CIF, TL=4, 30Hz), User 3: Mobile (CIF, TL=4, 30Hz)

	NBS			**ERAS**		
R_{MAX} (Mbps)	x_1^* (bps)	x_2^* (bps)	x_3^* (bps)	x_1^* (bps)	x_2^* (bps)	x_3^* (bps)
0.13	30241 (22.5%)	29644 (22.1%)	74257 (55.4%)	44700 (33.3%)	44700 (33.3%)	44700 (33.3%)
0.3	89408 (29.1%)	84551 (27.5%)	133240 (43.4%)	102400 (33.3%)	102400 (33.3%)	102400 (33.3%)
0.5	161070 (31.5%)	146491 (28.6%)	204437 (39.9%)	170667 (33.3%)	170667 (33.3%)	170667 (33.3%)
1.0	346684 (33.8%)	289678 (28.3%)	387636 (37.9%)	341333 (33.3%)	341333 (33.3%)	341334 (33.3%)
1.5	539721 (35.1%)	419927 (27.4%)	576350 (37.5%)	512000 (33.3%)	512000 (33.3%)	512000 (33.3%)
2.0	738683 (36.1%)	540339 (26.4%)	768976 (37.5%)	682666 (33.3%)	682667 (33.3%)	682667 (33.3%)

TABLE III

ALLOCATED RATES BY THE GENERALIZED NBS AND ERAS. FOREMAN, COASTGUARD, AND MOBILE (CIF, TL=4, 30HZ)

PSNR than ERAS even though we are not considering the bargaining powers. (i.e., the same bargaining powers are used).

D. Comparison of the Nash Bargaining Solutions with Different Bargaining Powers

In the previous section, we compared the average PSNR for the generalized NBS with the same bargaining powers and ERAS. In this section, we compare the weighted PSNR for the generalized NBS with different bargaining powers to examine the effect of bargaining powers. By the definition of the generalized NBS, more resource is allocated to the user having higher bargaining powers. Different bargaining powers are essential when we consider *fairness* for the cooperative game theory. For example, consider the two-user case with different spatial resolution video sequences. User 1 transmits *Foreman* sequence at CIF resolution at 30Hz and user 2 transmits *Foreman* sequence at QCIF resolution at 30Hz. Definitely user 2 always achieves higher PSNR than user 1 if the same rate is allocated. In this case, the resource manager could allocate more rates to user 1 to achieve a similar level of quality between users. We consider another example for different temporal resolution video sequences. User 1 transmits *Foreman* sequence at CIF resolution at 30Hz while user 2 transmits *Foreman* sequence at CIF resolution at 15Hz. Obviously, user 2 always achieves higher PSNR that user 1 if the same rate is allocated. In this case, the resource manager could allocate more rate to user 1 to achieve similar level of quality between users. These arbitrations are performed by changing bargaining powers to achieve a similar level of quality between users. In this paper, we present the simulation results that show that the change of bargaining powers plays an important role in performance and fairness. We provide a simple algorithm in Algorithm 2, as an illustration, to obtain bargaining powers to allocate resources such that each user achieves similar quality. Our future study will investigate the optimal and fair decision of bargaining powers.

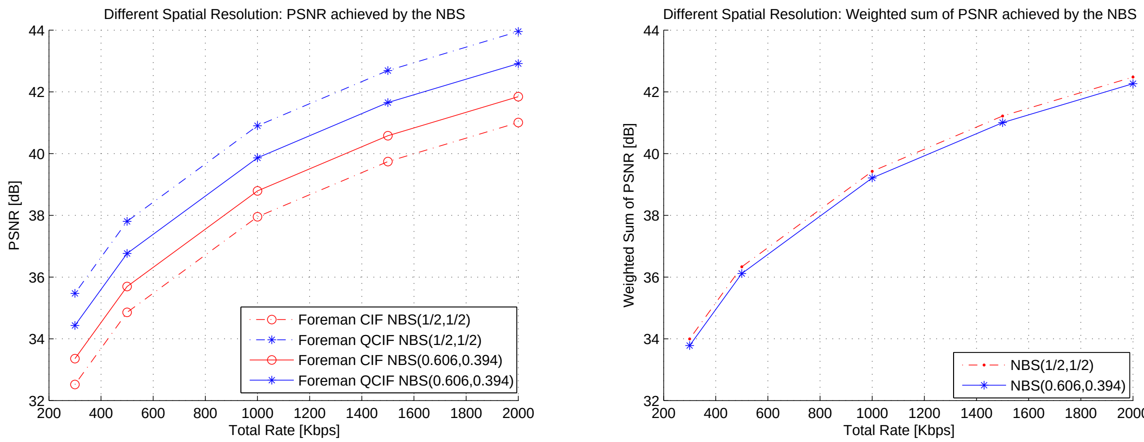
Algorithm 2 Determine Bargaining Powers to Achieve a Similar Quality Level

Require: Desired Quality: Q, DR Model Parameters: μ_i, R_{0i}, D_{0i}

1. Calculate rates x_i corresponding to the desired quality Q for each user from the DR model.
2. Determine bargaining powers α_i such that $\alpha_i = \frac{x_i}{\sum_{i=1}^{n} x_i}$ for all i.

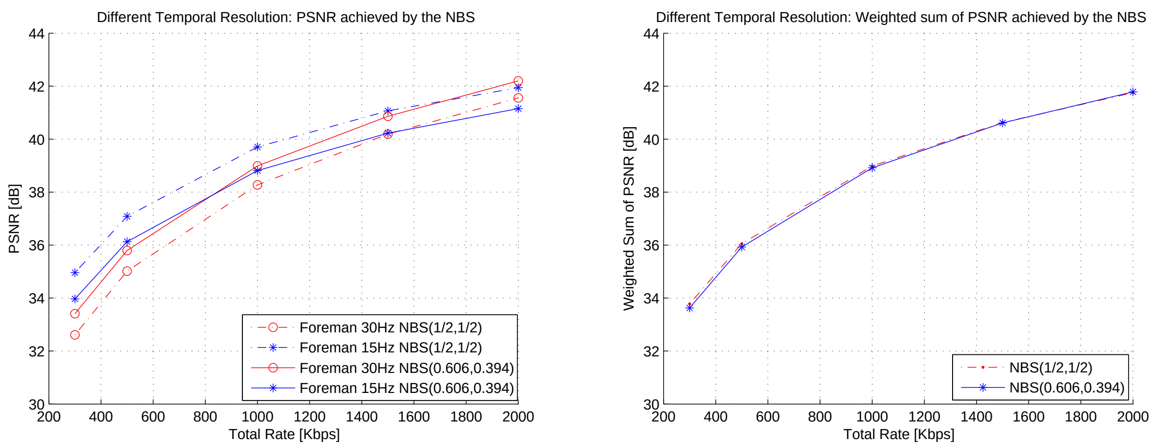
Fig. 7, Fig. 8, Fig. 9, and Fig. 10 show the weighted sum of PSNR and individual PSNR of each user achieved by the generalized NBS for the same and different bargaining powers for sequences that are different *spatial resolution*, *temporal resolution*, *application delays* and *video*, respectively. The bargaining powers are obtained using Algorithm 2 such that the each user's achieved PSNR is similar quality. Note that $NBS(a, b)$ in the figures represents the generalized NBS for the bargaining powers a and b for user 1 and user 2, respectively,

Fig. 7 shows individual PSNRs and the weighted sum of PSNRs, which are achieved by the generalized NBS with the same and different bargaining powers, for the two users that transmit different *spatial* resolution sequences. User 1 transmits *Foreman* sequence at CIF resolution 30Hz and user 2 transmits *Foreman* sequence at QCIF resolution 30Hz. In this case, user 2 can achieve higher PSNR than that of user 1 if the same bargaining powers are used. To achieve a similar level of quality, the bargaining powers can be adapted by the Algorithm 2 with the desire quality level $Q = 35$dB. For these sequences, the determined bargaining powers are $\alpha_1 = 0.606$ and $\alpha_2 = 0.394$. Since the bargaining power for user 1 is decreased than the equally divided bargaining power while the bargaining power for user 2 is increased than the equally divided bargaining power, we expect that user 1 achieves less PSNR and user 2 achieves more PSNR compared with the same bargaining power case. This argument is verified from the individual PSNR and the weighted



(a) (b)

Fig. 7. Different Spatial Resolution: Plots of individual PSNRs and weighted sum of PSNRs for the same and different bargaining powers. User 1: Foreman (CIF, TL=4, 30Hz), User 2: Foreman (QCIF, TL=4, 30Hz)



(a) (b)

Fig. 8. Different Temporal Resolution: Plots of individual PSNRs and weighted sum of PSNRs for the same and different bargaining powers. User 1: Foreman (CIF, TL=4, 30Hz), User 2: Foreman (CIF, TL=4, 15Hz)

sum of PSNR in Fig. 7. This can be viewed as a tradeoff between *fairness* and *system performance*. We can focus on the fairness at the expense of the overall system performance. (i.e., the weighted sum of PSNR). The same arguments can be applicable to other simulation results.

Fig. 8 shows individual PSNRs and weighted sum of PSNRs for the same and different bargaining powers for the sequences that are different *temporal* resolution. User 1 transmits *Foreman* sequence at CIF resolution 30Hz and user 2 transmits *Foreman* sequence at CIF resolution 15Hz. Since two sequences are different from the temporal resolution, they can achieve different PSNRs at the same rate. (i.e., the sequence of lower temporal resolution can achieve higher PSNR than that of higher temporal resolution if the same rates are

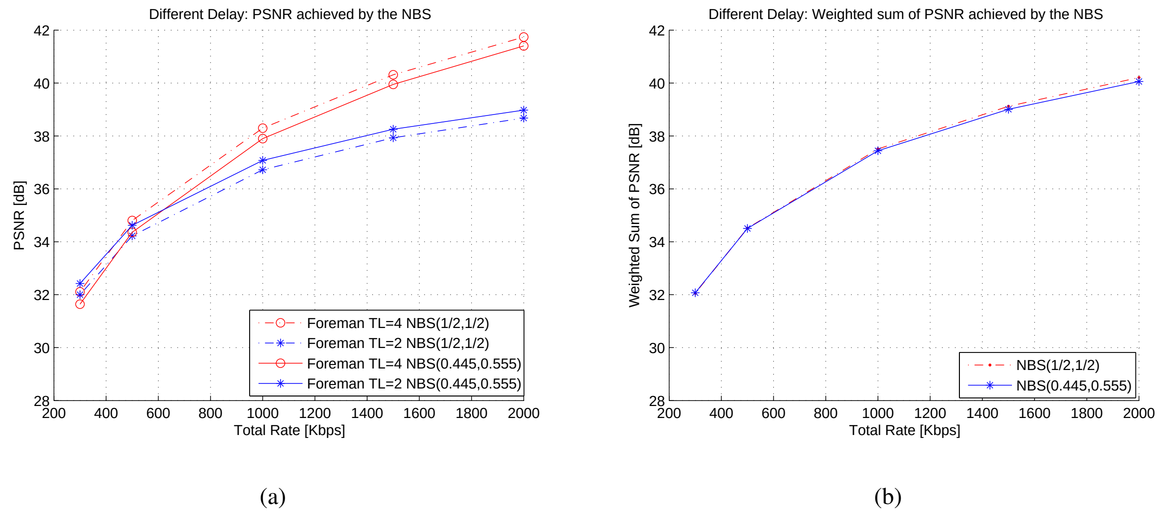


Fig. 9. Different Delay: Plots of individual PSNRs and weighted sum of PSNRs for the same and different bargaining powers. User 1: Foreman (CIF, TL=4, 30Hz), User 2: Foreman (CIF, TL=2, 30Hz)

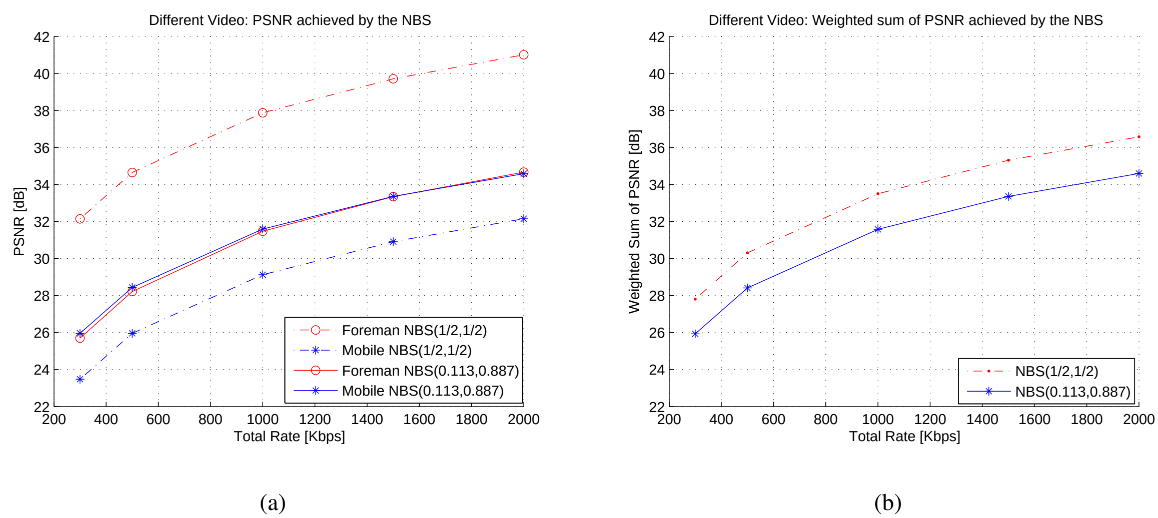


Fig. 10. Different Video Sequences: Plots of individual PSNRs and weighted sum of PSNRs for the same and different bargaining powers. User 1: Foreman (CIF, TL=4, 30Hz), User 2: Mobile (CIF, TL=4, 30Hz)

allocated). By changing the bargaining powers using the Algorithm 2 with the desired quality level Q=35dB, the individual PSNRs and the weighted sum of PSNRs with the bargaining powers are in Fig. 8.

Fig. 9 and Fig. 10 show the individual PSNRs and weighted sum of PSNRs for the sequences that are different delay and different video. In Fig. 9, higher delay is due to the larger number of temporal level in a state-of-the-art wavelet video coder [28]. User 1 transmits 1 transmits *Foreman* sequence at CIF resolution 30Hz with 4 temporal levels and user 2 transmits *Foreman* sequence at CIF resolution 30Hz with 2 temporal level. Since the sequence of the user 2 has lower temporal level, user 2 can achieve lower PSNR with lower delay. Thus, the resource manager allocates more rate to user 2 to achieve a similar level of quality between users. In Fig. 10, the different PSNRs come from the different video characteristics. User 1 and user 2 transmit the *Foreman* and *Mobile* sequences at CIF resolution 30Hz. Due to the own characteristics of each sequence, the user 1 can achieve higher PSNR than user 2 if the same rates are allocated. Hence, the resource manager would allocate more rates to user 2 to achieve a similar level of quality.

From above examples, therefore, we conclude that bargaining powers of the generalized NBS play an important role for the tradeoff between *fairness* and *performance* and they need to be chosen appropriately depending on the application requirements.

E. Comparison of the Kalai-Smorodinsky Bargaining Solution with Different Bargaining Powers

In this section, we investigate an alternative bargaining solution and compare the KSBS with the same and different bargaining powers. In this simulation, we assume that there are three users that transmit three different video sequences. The achieved PSNRs for the same and different bargaining powers are listed in Table IV. The different bargaining powers are also determined by the Algorithm 2 to achieve a similar level of quality, and they are $\alpha_1 = 0.0832$, $\alpha_2 = 0.1543$ and $\alpha_3 = 0.7625$. Compared with the same bargaining powers (i.e., $\alpha_1 = \alpha_2 = \alpha_3 = \frac{1}{3}$), only the user 3 has higher bargaining powers after adapting bargaining powers. Hence, we expect that the user 3 obtains higher PSNRs in different bargaining power case. In Table IV, we observe that the user 3 achieves higher PSNR after adapting bargaining powers. Moreover, the achieved PSNR for each user is a similar level of quality after changing bargaining powers even though the average PSNRs are lowered compared with the same bargaining case. Therefore, the KSBS also has a tradeoff between fairness and performance.

KSBS	R_{MAX} (Mbps)	$PSNR_1$ (dB)	$PSNR_2$ (dB)	$PSNR_3$ (dB)	$\overline{PSNR}$ (dB)
same bargaining powers	0.5	33.3017	31.1786	24.2212	29.5672
	1.0	36.6487	33.5285	27.7108	32.6293
	2.0	39.9999	35.3868	31.0466	35.4778
KSBS	R_{MAX} (Mbps)	$PSNR_1$ (dB)	$PSNR_2$ (dB)	$PSNR_3$ (dB)	$\overline{PSNR}$ (dB)
different bargaining powers	0.5	27.3190	27.8771	27.8568	27.6843
	1.0	30.4845	30.0456	31.1649	30.5650
	2.0	33.6509	31.7191	34.3159	33.2286

TABLE IV

ALLOCATED PSNR BY KSBS (SAME BARGAINING POWERS $\alpha_1 = \alpha_2 = \alpha_3 = \frac{1}{3}$ AND DIFFERENT BARGAINING POWERS $\alpha_1 = 0.0832$, $\alpha_2 = 0.1543$, $\alpha_3 = 0.7625$. USER 1: FOREMAN (CIF), USER 2: COASTGUARD (CIF), USER 3: MOBILE (CIF)

VII. Conclusion

In this paper we propose an alternative and novel solution to the problem of rate allocation for collaborative video users, based on the bargaining methodology from game theory. As shown in this paper, in axiomatic bargaining theory, a solution is selected out of the set of possible choices that satisfies a set of rational and desirable axioms. Hence, unlike the conventional rate allocation methods, which require a global objective function, the purpose is *not* to maximize a system utility, but rather select a solution from the Pareto optimal surface and satisfy several rational properties in making the choice. We provided physical interpretations for the two investigated bargaining solutions. We found that the NBS and the KSBS can be interpreted as two different fairness criteria from which a resource manager can choose depending on the application scenario. The NBS can be used to maximize the system utility (i.e., weighted sum of PSNRs of the users), while the KSBS ensures that all users incur the same utility penalty relative to the maximum achievable utility. We also showed that the complexities for the bargaining solutions are both linear to the number of users. In addition, the bargaining powers can be used to provide additional flexibility in choosing solution by taking into consideration the visual quality impact, the deployed spatio-temporal resolutions, etc. Summarizing, the proposed bargaining solutions can provide a good solution for fair and optimal resource allocation for multi-user multimedia transmission with reasonable complexity, robustness, and flexibility.

Appendix I
Proof of the convexity of the function $f(\theta)$

To show that the function $f(\theta) \triangleq \sum_{i=1}^{n} \frac{\mu_i(\theta X_i + (1-\theta)Y_i)}{c - D_{0i}(\theta X_i + (1-\theta)Y_i)}$ is convex, the second derivative of $f(\theta)$ must be nonnegative for all $0 < \theta < 1$. Let $f_i(\theta)$ be ith term of $f(\theta)$. The second derivatives of $f_i(\theta)$ is given by

$$\frac{d^2 f_i(\theta)}{d\theta^2} = \frac{2c\mu_i D_{0i}(X_i - Y_i)^2}{(c - D_{0i}(\theta X_i + (1-\theta)Y_i))^3}. \tag{40}$$

Since μ_i is positive, D_{0i} is nonnegative, and c is a positive constant, $2c\mu_i D_{0i}(X_i - Y_i)^2$ is obviously nonnegative. From the constraint $x_i > R_{0i}$, we have $\frac{\mu_i X_i}{c - D_{0i} X_i} > 0$, and thus $c - D_{0i} X_i > 0$ since μ_i and X_i are positive. With the same argument, we have $c - D_{0i} Y_i > 0$. Hence, for $0 < \theta < 1$, we have

$$\theta \cdot (c - D_{0i} X_i) > 0, \text{ and } (1-\theta) \cdot (c - D_{0i} Y_i) > 0. \tag{41}$$

Sum of two inequality in (41) gives $c - D_{0i}(\theta X_i + (1-\theta)Y_i) > 0$. Thus, the second derivative of the function $f_i(\theta)$ is positive, and it means that the function $f_i(\theta)$ is convex. Since the sum of convex function is also convex, $f(\theta)$ is convex. ■

APPENDIX II

PROOF OF THE UNIQUENESS OF THE GENERALIZED NBS FOR THE n-USER CASE

In this section, we will prove that the generalized NBS for n-user case is unique and satisfies the axioms described in the Section II-A. Note that this proof is an extension for n-user case from the proof in [10] for the two-user case.

A bargaining solution is a function $F : \mathbf{S} \to \mathbb{R}^n$ with the property that $F(\mathbf{S}, \mathbf{d})$ is in the set $\mathbf{S}$, where $\mathbf{S}$ is the feasible utility set and $\mathbf{d}$ is the disagreement point. A generalized NBS is a function $G : \mathbf{S} \to \mathbb{R}^n$ such that $\mathbf{s} = G(\mathbf{S}, \mathbf{d}) = \sum_{i=1}^{n} \alpha_i \mathbf{r}_i$, $\sum_{i=1}^{n} \alpha_i = 1$, $\alpha_i \geq 0$ for all i as in (9). We can rewrite the axioms.

1. *Independence of Linear Transformations*: For any linear scale transformation ψ, $\psi(F(\mathbf{S}, \mathbf{d})) = F(\psi(\mathbf{S}), \psi(\mathbf{d}))$.
2. *Pareto Optimality*: (i) $F(\mathbf{S}, \mathbf{d}) \geq \mathbf{d}$. (ii) $\mathbf{y} > F(\mathbf{S}, \mathbf{d}) \Rightarrow \mathbf{y} \notin \mathbf{S}$.
3. *Independence of Irrelevant Alternatives*: If $\mathbf{d} \in \mathbf{Y} \subseteq \mathbf{S}$, then $F(\mathbf{S}, \mathbf{d}) \in \mathbf{Y} \Rightarrow F(\mathbf{Y}, \mathbf{d}) = F(\mathbf{S}, \mathbf{d})$.
4. *Symmetry*: If $\mathbf{S}$ is invariant under all exchanges of users, $F_i(\mathbf{S}, \mathbf{d}) = F_j(\mathbf{S}, \mathbf{d})$ for all possible user i, j.

Note that any generalized NBS $G : \mathbf{S} \to \mathbb{R}^n$ satisfies the axioms 1, 2, and 3 [10]. We will show that there are the only bargaining solutions that satisfy the axioms. That is, if $G : \mathbf{S} \to \mathbb{R}^n$ satisfies the axioms 1, 2, and 3, then F is a generalized NBS for bargaining powers α_i for all i.

First, consider the simple bargaining problem $(\mathbf{Z}, 0)$ in which the disagreement point is the origin and the feasible utility set $\mathbf{Z}$ consists of all payoff pairs $\mathbf{X}$ that satisfy $\sum_{i=1}^{n} X_i \leq 1$. By the axiom 2, the solution $\mathbf{s}' = F(\mathbf{Z}, 0)$ for the bargaining problem $(\mathbf{Z}, 0)$ lies somewhere on the $(n-1)$ dimensional plane joining $\mathbf{r}'_i = \mathbf{e}_i$ for all i, where $\mathbf{e}_i$ represents a unit vector of user i's utility axis X_i. Choose α_i satisfying $\sum_{i=1}^{n} \alpha_i = 1$, $\alpha_i \geq 0$ for all i so that $\mathbf{s}' = \sum_{i=1}^{n} \alpha_i \mathbf{r}'_i$.

Next consider any bargaining problem $(\mathbf{S}, \mathbf{d})$. Let $\mathbf{s} = G(\mathbf{S}, \mathbf{d})$, where G is the generalized NBS corresponding to the bargaining powers α_i, $i = 1, \ldots, n$. Then, $\mathbf{s} = \sum_{i=1}^{n} \alpha_i \mathbf{r}_i$, where $\mathbf{r}_i$ is defined in (9). The aim is to prove that $F(\mathbf{S}, \mathbf{d}) = G(\mathbf{S}, \mathbf{d})$. By choosing linear transformation functions $\psi_i : \mathbb{R} \to \mathbb{R}$ such that $\psi_i(d_i) = 0$ and $\psi_i(r_i) = 1$ for all i, the linear transformation function $\psi : \mathbb{R}^n \to \mathbb{R}^n$ has the property that $\psi(\mathbf{d}) = 0$, $\psi(\mathbf{r}_i) = \mathbf{r}'_i = \mathbf{e}_i$ for all i. Since linear transformation functions preserve convexity, the image of the $(n-1)$ dimensional plane through $\mathbf{s}$ and $\mathbf{r}_i$, $i = 1, \ldots, n$ remains a supporting plane to the image of the set $\mathbf{S}$. That is, the $(n-1)$ dimensional plane $\sum_{i=1}^{n} X_i = 1$ through $\mathbf{r}'_i = \mathbf{e}_i$ and $\mathbf{s}'$ is a supporting plane to the convex set $\mathbf{S}' = \psi(\mathbf{S})$. Thus, by the axioms 1,

$$F(\mathbf{Z}, 0) = \psi(G(\mathbf{S}, \mathbf{d})). \tag{42}$$

Since $\mathbf{S}' \subseteq \mathbf{Z}$, $F(\mathbf{S}', 0) = F(\mathbf{Z}, 0)$ by the axiom 3. Since $\mathbf{S}' = \psi(\mathbf{S})$ and $0 = \psi(\mathbf{d})$, it follows from (42) that $F(\psi(\mathbf{S}), \psi(\mathbf{d})) = \psi(G(\mathbf{S}, \mathbf{d}))$. Thus,

$$G(\mathbf{S}, \mathbf{d}) = \psi^{-1}(F(\psi(\mathbf{S}), \psi(\mathbf{d})), \tag{43}$$

where ψ^{-1} is the inverse function to ψ. Applying the axiom 1, we have

$$G(\mathbf{S}, \mathbf{d}) = F(\psi^{-1}(\psi(\mathbf{S})), \psi^{-1}(\psi(\mathbf{d}))) = F(\mathbf{S}, \mathbf{d}). \tag{44}$$

This completes the proof for the uniqueness of the generalized NBS.

If the generalized NBS satisfies the axiom 4, it requires that the solutions be symmetric since the bargaining problem $(\mathbf{Z}, 0)$ is symmetric. Therefore, the bargaining powers should be $\alpha_i = 1/n$ for $i = 1, \ldots, n$. ■

APPENDIX III
PROOF OF CONCAVITY OF THE GENERALIZED NASH PRODUCT

In this section, we show that the GNP is concave function. We prove that the GNP is concave by showing that the Hessian of the GNP is negative semidefinite on the convex set $\mathbf{S}$. The GNP is given by $G(\mathbf{X}) = \prod_{i=1}^{n} X_i^{\alpha_i}$ with $\sum_{i=1}^{n} \alpha_i = 1$, $\alpha_i \geq 0$, and $X_i > 0$ for all i. The gradient of the GNP is given in (22) and the Hessian of the GNP is given by

$$[\nabla^2 G(\mathbf{X})]_{k,l} = \begin{cases} \alpha_k(\alpha_k - 1)X_k^{\alpha_k - 2}\prod_{i\neq k}^{n} X_i^{\alpha_i} & \text{if} \quad k = l \\ \alpha_k \alpha_l X_k^{\alpha_k - 1} X_l^{\alpha_l - 1}\prod_{i\neq k,l}^{n} X_i^{\alpha_i} & \text{if} \quad k \neq l, \quad k, l = 1, \ldots, n. \end{cases} \tag{45}$$

The function $G(\mathbf{X})$ is concave if and only if $\nabla^2 G(\mathbf{X})$ is negative semidefinite. (i.e., $\mathbf{y}^T \nabla^2 G(\mathbf{X})\mathbf{y} \leq 0$ for all nonzero vectors $\mathbf{y}$ since $\nabla^2 G(\mathbf{X})$ is real symmetric matrix). Direct computation of $\mathbf{y}^T \nabla^2 G(\mathbf{X})\mathbf{y}$ for nonzero vector $\mathbf{y} = (y_1, \ldots, y_n)$ gives

$$\mathbf{y}^T \nabla^2 G(\mathbf{X})\mathbf{y} = \sum_{i=1}^{n}\sum_{j=1}^{n} y_i y_j [\nabla^2 G(\mathbf{X})]_{ij} \tag{46}$$

$$= [\sum_{k=1}^{n}(y_k \alpha_k X_k^{\frac{\alpha_k - 2}{2}} \prod_{i=1, i\neq k}^{n} X_i^{\frac{\alpha_i}{2}})]^2 - \sum_{k=1}^{n}(y_k^2 \alpha_k X_k^{\alpha_k - 2} \prod_{i=1, i\neq k}^{n} X_i^{\alpha_i}) \tag{47}$$

$$\leq \sum_{k=1}^{n} \alpha_k (y_k^2 X_k^{\alpha_k - 2} \prod_{i=1, i\neq k}^{n} X_i^{\alpha_i}) - \sum_{k=1}^{n}(y_k^2 \alpha_k X_k^{\alpha_k - 2} \prod_{i=1, i\neq k}^{n} X_i^{\alpha_i}) = 0. \tag{48}$$

The inequality from (47) to (48) is from the Jensen's inequality since the function $f(x) = x^2$ is convex, $\sum_{i=1}^{n} \alpha_i = 1$, and $\alpha_i \geq 0$ for all i. Therefore, $\nabla^2 G(\mathbf{X})$ is negative semidefinite, and thus, the function $G(\mathbf{X})$ is concave. ■

REFERENCES

[1] B. Girod and N. Färber, *"Wireless Video" in A. Reibman, M.-T. Sun (eds.), Compressed Video over Networks*. Marcel Dekker, 2000.

[2] L. Zhang, S. Deering, D. Estrin, S. Shenker, and D. Zappala, "RSVP: A new resource reservation protocol," *IEEE Commun. Mag.*, vol. 31, pp. 8–18, Sept. 1993.

[3] S. Berson, “Aggregation of internet integrated services state,” in *6th International Workshop on Quality of Service (IWQoS 98)*, Napa, May 1998, pp. 26–28.

[4] “IEEE 802.11e/D5.0, Wireless Medium Access Control (MAC) and Physical Layer (PHY) Specifications: Medium Access Control (MAC) Enhancements for Quality of Service (QoS), draft supplement,” June 2003.

[5] F. Kelly, “Charging and rate control for elastic traffic,” *Eur. Trans. Telecommun.-”Focus on Elastic Services Over ATM Networks”*, vol. 8, no. 1, pp. 33–37, 1997.

[6] F. Kelly, A. Maulloo, and D. Tan, “Rate control for communication networks: Shadow prices, proportional fairness and stability,” *Journal of the Operational Research Society*, vol. 49, no. 3, pp. 237–252, Mar. 1998.

[7] L. Massoulie and J. Roberts, “Bandwidth sharing: objectives and algorithms,” in *Proc. 18th Annu. Joint Conf. IEEE Computer and Communications Socieities (INFOCOM 99)*, vol. 3, Mar. 1999, pp. 1395–1403.

[8] J. Nash, “The bargaining problem,” *Econometrica*, vol. 18, pp. 155–162, 1950.

[9] J. P. Conley and S. Wilkie, “Implementing the Nash extension bargaining solution for non-convex problems,” *Economic Design*, vol. 1, no. 2, pp. 205–216, 1995.

[10] K. Binmore, *Fun and Games: A Text on Game Theory*. Lexington, MA: D.C. Health, 1992.

[11] G. Owen, *Game Theory*, 3rd ed. New York, NY: Academic Press, 1995.

[12] D. Fudenberg and J. Tirole, *Game Theory*. Cambridge, MA: MIT Press, 1991.

[13] C. Douligeris and R. Mazumdar, “A game theoretic perspective to flow control in telecommunication networks,” *Journal Franklin Inst.*, vol. 329, no. 2, pp. 383–402, Mar. 1992.

[14] Z. Dziong and L. G. Mason, “Fair-efficient call admission control policies for broadband networks - a game theoretic framework,” *IEEE/ACM Trans. Networking*, vol. 4, pp. 123–136, Dec. 1991.

[15] R. La and V. Anantharam, “Optimal routing control: Repeated game approach,” *IEEE Trans. Automat. Contr.*, vol. 47, no. 3, pp. 437–450, Mar. 2002.

[16] R. Mazumdar, L. G. Mason, and C. Douligeris, “Fairness in network optimal flow control: Optimality of product forms,” *IEEE Trans. Commun.*, vol. 39, no. 5, pp. 775–782, May 1991.

[17] H. Yaïche, R. R. Mazumdar, and C. Rosenberg, “A game theoretic framework for bandwidth allocation and pricing in broadband networks,” *IEEE/ACM Trans. Networking*, vol. 8, no. 5, pp. 667–678, Oct. 2000.

[18] T. Corinne, A. Eitan, and G. Jerome, “Generalized Nash bargaining solution for bandwidth allocation,” *Computer Networks*, available at http://www.sciencedirect.com/science/article/B6VRG-4J2VXY8-1/2/1aaa152af37eb17798c17a66f8c16239.

[19] H. Zheng and C. Peng, “Collaboration and fairness in opportunistic spectrum access,” in *Proc. 40th annual IEEE International Conference on Communications*, vol. 5, May 2005, pp. 3132–3136.

[20] A. Ortega and K. Ramchandran, “Rate-distortion techniques in image and video compression,” *IEEE Signal Processing Magazine*, vol. 15, no. 6, pp. 23–50, Nov. 1998.

[21] G. Rocheteau and C. Waller, “Bargaining and the value of money,” Federal Reserve Bank of Cleveland, Working Paper 0501, 2005, available at http://ideas.repec.org/p/fip/fedcwp/0501.html.

[22] Z. Han, Z. Ji, and K. J. R. Liu, “Fair multiuser channel allocation for OFDMA networks using Nash bargaining solutions and coalition,” *IEEE Trans. Commun.*, vol. 53, no. 8, pp. 1366–1376, Aug. 2005.

[23] H. M. Radha, M. van der Schaar, and Y. Chen, “The MPEG-4 Fine-Grained Scalable video coding method for multimedia streaming over IP,” *IEEE Trans. Multimedia*, vol. 3, no. 1, pp. 53–68, Mar. 2001.

[24] M. Wang and M. van der Schaar, “Rate-distortion modeling for wavelet video coders,” pp. 281–284, Mar. 2005.

[25] K. Stuhlmüller, N. Färber, M. Link, and B. Girod, “Analysis of video transmission over lossy channels,” *IEEE J. Select. Areas Commun.*, vol. 18, no. 6, June 2000.

[26] M. Dai, D. Loguinov, and H. Radha, “Rate-distortion modeling of scalable video coders,” *IEEE International Conference on Image Processing (ICIP ’04)*, Oct. 2004.

[27] X. Lu, Y. Wang, and E. Erkip, “Power efficient H.263 video transmission over wireless channels,” in *Proc. Int. Conf. Image Processing*, vol. 1, Sept. 2002, pp. 533–536.

[28] Y. Andreopoulos, A. Munteanu, J. Barbarien, M. van der Schaar, J. Cornelis, and P. Schelkens, “In-band motion compensated temporal filtering,” *Signal Processing: Image Communication(special issue on ”Subband/Wavelet Interframe Video Coding”)*, vol. 19, no. 7, pp. 653–673, Aug. 2004.

[29] S. Boyd and L. Vandenberghe, *Convex Optimization*. New York, NY: Cambridge University Press, 2004.

[30] H. Houba, X. Tieman, and R. Brinksma, “The Nash- and Kalai-Smorodinsky bargaining solution for decision weight utility functions,” Game Theory and Information 9611001, Economics Working Paper Archive EconWPA,” Working Paper, Nov. 1996.

[31] B. Chakravorti and J. Conley, “Bargaining efficiency and the repeated prisoner’s dilemma,” *Economics Bulletin*, Feb. 2004.

[32] A. C. Bovik, Ed., *The Handbook of Image and Video Processing*. San Diego: Academic, 2000.

[33] Y. Wang, M. van der Schaar, S. F. Chang, and A. Loui, “Content-based optimal MDA operation prediction for scalable video coding systems using subjective quality evaluation,” *IEEE Trans. Circuits Syst. Video Technol.*, vol. 15, pp. 1270–1279, Oct. 2005.

[34] A. Scaglione and M. van der Schaar, “Cross-layer resource allocation for delay-constrainted wireless video transmission,” in *Proc. IEEE International Conference on Acoustics, Speech, and Signal Processing*, Philadelphia, PA, Mar. 2005.

[35] D. Tse and S. Hanly, “Multi-access fading channels: Part I: Polymatroid structure, optimal resource allocation and throughput capacities,” *IEEE Trans. Inform. Theory*, vol. 44, no. 7, pp. 2796–2815, 1998.