Informationally Efficient Multi-user communication

Yi Su

Advisor: Professor Mihaela van der Schaar

Electrical Engineering, UCLA
Outline

• Motivation and existing approaches
• Informationally efficient multi-user communication
  – Vector cases
    • Convergence conditions with decentralized information
    • Improve efficiency with decentralized information
  – Scalar cases
    • Achieve Pareto efficiency with decentralized information
• Conclusions
Multi-user communication networks

Power control

Distributed routing

Peer-to-peer system etc...
Constraints in communication networks

- **Resources**
  - Bandwidth, power, spectrum, etc.

- **Information**
  - Real-time
    - Local observation
Constraints in communication networks

- Resources
  - Bandwidth, power, spectrum, etc.
- Information
  - Real-time
    - Local observation
    - Exchanged message
Constraints in communication networks

• Resources
  – Bandwidth, power, spectrum, etc.

• Information
  – Real-time
    • Local observation
    • Exchanged message
  – Non-real-time
    • A-priori information about inter-user coupling, protocols, etc.
Constraints in communication networks

- Resources
  - Bandwidth, power, spectrum, etc.

- Information
  - Real-time
    - Local observation
    - Exchanged message
  - Non-real-time
    - A-priori information about inter-user coupling, protocols, etc.

Goal: multi-user communication without information exchange
A standard strategic game formulation

- Consider a tuple

\[ \Gamma = \langle \mathcal{N}, \mathcal{A}, u \rangle \]

- The set of players: \( \mathcal{N} = \{1, 2, \ldots, N\} \)
- The set of actions: \( \mathcal{A} = \times_{n \in \mathcal{N}} \mathcal{A}_n \) and \( \mathcal{A}_n \subseteq \mathcal{R}^K \)
- Utility function: \( u = \times_{n \in \mathcal{N}} u_n \) and \( u_n : \mathcal{A} \rightarrow \mathcal{R} \)
- Utility region: \( \mathcal{U} = \{(u_1(a), \ldots, u_N(a)) | \exists a \in \mathcal{A}\} \)

In communication networks, different operating points in \( \mathcal{U} \) can be chosen based on the information availability.
Existing approaches

- Local observation

\[ \max_{a_n \in A_n} u_n(a_n, a_{-n}) \]

Nash equilibrium
Existing approaches

- Local observation
  \[ \max_{a_n \in A_n} u_n(a_n, a_{-n}) \]
  Nash equilibrium

- Exchanged messages
  \[ \max_{a \in A} \sum_{n=1}^{N} u_n(a) \]
  \[ \Rightarrow \frac{\partial u_n(a)}{\partial a_n^k} + \sum_{m \neq n} \frac{\partial u_m(a)}{\partial a_n^k} \]
  Pareto optimality

Price!
Existing approaches

- Local observation
  \[
  \max_{a_n \in A_n} u_n(a_n, a_{-n})
  \]
  Nash equilibrium

- Exchanged messages
  \[
  \max_{a \in A} \sum_{n=1}^{N} u_n(a)
  \Rightarrow \frac{\partial u_n(a)}{\partial a_n^k} + \sum_{m \neq n} \frac{\partial u_m(a)}{\partial a_n^k}
  \]
  Pareto optimality

Existing results usually assume some specific action and utility structures!
Existing approaches (cont’d)

- Results with specific action and utility structures
  - Pure Nash equilibrium
    - Concave games
      - $A_n$: convex and compact; $u_n(a_n, a_{-n})$: quasi-concave in $a_n$
      - Use gradient play to find NE
Existing approaches (cont’d)

- Results with specific action and utility structures
  - Pure Nash equilibrium
    - Concave games
      i) $\mathcal{A}_n$: convex and compact; ii) $u_n(a_n, a_{-n})$: quasi-concave in $a_n$
    - Potential games [Shapley]
      $$\forall (m, n) \in \mathcal{N}^2, m \neq n, \quad \frac{\partial^2(u_n - u_m)}{\partial a_n \partial a_m} = 0$$

- Use gradient play to find NE
- Use best response to find NE
Existing approaches (cont’d)

• Results with specific action and utility structures
  – Pure Nash equilibrium
    • Concave games
      i) $\mathcal{A}_n$: convex and compact; ii) $u_n(a_n, a_{\neg n})$: quasi-concave in $a_n$
    • Potential games [Shapley]
      $$\forall (m, n) \in \mathcal{N}^2, m \neq n, \quad \frac{\partial^2 (u_n - u_m)}{\partial a_n \partial a_m} = 0$$
    • Super-modular games [Topkis]
      i) $\mathcal{A}_n$ is a lattice; ii) $\forall (m, n) \in \mathcal{N}^2, m \neq n, \quad \frac{\partial^2 u_n}{\partial a_n \partial a_m} \geq 0$
Existing approaches (cont’d)

• Results with specific action and utility structures
  – Pure Nash equilibrium
    • Concave games Use gradient play to find NE
      i) \( \mathcal{A}_n \) is convex and compact; ii) \( u_n(a_n, a_{-n}) \): quasi-concave in \( a_n \)
    • Potential games [Shapley] Use best response to find NE
      \[ \forall (m, n) \in \mathcal{N}^2, m \neq n, \quad \frac{\partial^2(u_n - u_m)}{\partial a_n \partial a_m} = 0 \]
    • Super-modular games [Topkis] Use best response to find NE
      i) \( \mathcal{A}_n \) is a lattice; ii) \[ \forall (m, n) \in \mathcal{N}^2, m \neq n, \quad \frac{\partial^2 u_n}{\partial a_n \partial a_m} \geq 0 \]
  – Pareto optimality
    • Network utility maximization [Kelly]
      • Convexity is the watershed
## Existing approaches (cont’d)

<table>
<thead>
<tr>
<th>Researchers</th>
<th>Applications</th>
<th>Tools</th>
</tr>
</thead>
<tbody>
<tr>
<td>Altman</td>
<td>CDMA uplink power control</td>
<td>S-modular games</td>
</tr>
<tr>
<td>Berry</td>
<td>Distributed interference compensation</td>
<td>S-modular games</td>
</tr>
<tr>
<td>Barbarossa</td>
<td>Power control</td>
<td>Potential games</td>
</tr>
<tr>
<td>Tse</td>
<td>Spectrum sharing</td>
<td>Repeated games</td>
</tr>
<tr>
<td>Kelly</td>
<td>End-to-end congestion control</td>
<td>Pricing</td>
</tr>
<tr>
<td>Goodman</td>
<td>CDMA uplink power control</td>
<td>Pricing</td>
</tr>
<tr>
<td>Low</td>
<td>End-to-end flow control</td>
<td>Pricing</td>
</tr>
<tr>
<td>Chiang</td>
<td>Joint congestion and power control</td>
<td>Pricing</td>
</tr>
<tr>
<td>Poor</td>
<td>Energy efficient power and rate control</td>
<td>Equilibrium analysis</td>
</tr>
<tr>
<td>Cioffi</td>
<td>Power control in DSL systems</td>
<td>Equilibrium analysis</td>
</tr>
<tr>
<td>Yates</td>
<td>Uplink power control for cellular radio</td>
<td>Equilibrium analysis</td>
</tr>
<tr>
<td>Wicker</td>
<td>Selfish users in Aloha</td>
<td>Equilibrium analysis</td>
</tr>
<tr>
<td>Lazar</td>
<td>Non-cooperative optimal flow control</td>
<td>Equilibrium analysis</td>
</tr>
</tbody>
</table>
Existing approaches (cont’d)

• Game theory
  – Equilibrium characterization
  – Incentive design

• Optimization theory
  – Computational complexity
  – Distributed algorithms

• Information theory
  – Fundamental limits
  – Encoding and decoding schemes

The focus is on strategic interactions among users

Information is usually costless

Decentralization is not the focus
Existing approaches (cont’d)

General models
- e.g. concave/potential/supermodular games

Specific multi-user communication applications

But in many communication systems, information is constrained and no message passing is allowed!
Our goals

General models
e.g. concave/potential/supermodular games

New classes of communication games

Specific multi-user communication applications

If information is constrained and no message passing is allowed...

Pareto boundary
Global (exchanged) information

Nash equilibrium
Decentralized (limited) information
Our goals

If information is constrained and no message passing is allowed...

- Nash equilibrium
- Decentralized (limited) information
- Global (exchanged) information
- Pareto boundary

When will it converge to a NE? And how fast?

General models
- e.g. concave/potential/supermodular games

New classes of communication games

Specific multi-user communication applications

If information is constrained and no message passing is allowed...

- Nash equilibrium
- Decentralized (limited) information
- Global (exchanged) information
- Pareto boundary

When will it converge to a NE? And how fast?
Our goals

If information is constrained and no message passing is allowed...

- Pareto boundary
  - Global (exchanged) information
- Nash equilibrium
  - Decentralized (limited) information

- General models
  - e.g. concave/potential/supermodular games

- New classes of communication games

- Specific multi-user communication applications

- How to improve an inefficient NE without message passing?
Our goals

If information is constrained and no message passing is allowed...

General models
e.g. concave/potential/supermodular games

New classes of communication games

Specific multi-user communication applications

Pareto boundary
Global (exchanged) information

And can we still achieve Pareto optimality?

Nash equilibrium
Decentralized (limited) information

Our goals

When will it converge to a NE? And how fast?

How to improve an inefficient NE without message passing?

And can we still achieve Pareto optimality?
Outline

• Motivation and existing approaches

• Informationally efficient multi-user communication
  – Vector cases
    • Convergence conditions with decentralized information
    • Improve efficiency with decentralized information
  – Scalar cases
    • Achieve Pareto efficiency with decentralized information

• Conclusions
A reformulation of multi-user interactions

- Consider a tuple

\[ \Gamma = (\mathcal{N}, \mathcal{A}, \mathcal{S}, s, u) \]

  - The set of players: \( \mathcal{N} = \{1, 2, \ldots, N\} \)
  - The set of actions: \( \mathcal{A} = \times_{n \in \mathcal{N}} \mathcal{A}_n \)
  - State space: \( \mathcal{S} = \times_{n \in \mathcal{N}} \mathcal{S}_n \)
  - State determination function:
    \[ s = \times_{n \in \mathcal{N}} s_n \text{ and } s_n : \mathcal{A}_{-n} \rightarrow \mathcal{S}_n \]
  - Utility function:
    \[ u = \times_{n \in \mathcal{N}} u_n \text{ and } u_n : \mathcal{S}_n \times \mathcal{A}_n \rightarrow \mathcal{R} \]

In standard strategic game, \( S_n = A_{-n} \)

It captures the structure of the coupling between action and state
A reformulation of multi-user interactions

- Consider a tuple

\[ \Gamma = \langle \mathcal{N}, \mathcal{A}, \mathcal{S}, s, u \rangle \]

- The set of players: \( \mathcal{N} = \{1, 2, \ldots, N\} \)
- The set of actions: \( \mathcal{A} = \times_{n \in \mathcal{N}} \mathcal{A}_n \)
- **State space:** \( \mathcal{S} = \times_{n \in \mathcal{N}} \mathcal{S}_n \)
- **State determination function:**
  \[ s = \times_{n \in \mathcal{N}} s_n \text{ and } s_n : \mathcal{A}_{-n} \to \mathcal{S}_n \]
- **Utility function:**
  \[ u = \times_{n \in \mathcal{N}} u_n \text{ and } u_n : \mathcal{S}_n \times \mathcal{A}_n \to \mathcal{R} \]

Many communication networking applications have simple \( \mathcal{S}_n \), which captures the aggregate effects of \( \mathcal{A}_{-n} \)

In standard strategic game, \( \mathcal{S}_n = \mathcal{A}_{-n} \)

It captures the structure of the coupling between action and state
Communication games with simple states

- Power control
  
  aggregate interference

\[ s_n = \sum_{m \neq n} h_{mn} P_m, \quad u_n = \log_2 \left( 1 + \frac{h_{nn} P_n}{\sigma_n + s_n} \right). \]
Communication games with simple states

- **Power control**
  \[ s_n = \sum_{m \neq n} h_{mn} P_m, \quad u_n = \log_2 \left( 1 + \frac{h_{nn} P_n}{\sigma_n + s_n} \right). \]

- **Flow control**
  \[ s_n = \sum_{m \neq n} \psi_m, \quad u_n = \frac{\psi_n}{\mu - \psi_n - s_n}. \]
Communication games with simple states

- Power control
  \[ s_n = \sum_{m \neq n} h_{mn} P_m, \quad u_n = \log_2 \left( 1 + \frac{h_{nn} P_n}{\sigma_n + s_n} \right). \]

- Flow control
  \[ s_n = \sum_{m \neq n} \psi_m, \quad u_n = \frac{\psi_n}{\mu - \psi_n - s_n}. \]

- Random access
  \[ s_n = \prod_{m \neq n} (1 - P_m), \quad u_n = P_n \cdot s_n. \]
Outline

• Motivation and existing approaches
• Informationally efficient multi-user communication
  – Vector cases
    • Convergence conditions with decentralized information
    • Improve efficiency with decentralized information
  – Scalar cases
    • Achieve Pareto efficiency with decentralized information
• Conclusions
Additively Coupled Sum Constrained Games

• Definition

  – A multi-user interaction $\Gamma = \langle \mathcal{N}, \mathcal{A}, u \rangle$ in which

  \textbf{A1:} $\forall n \in \mathcal{N}$, action set $\mathcal{A}_n$ is defined to be

  $\mathcal{A}_n = \left\{ (a_n^1, \ldots, a_n^K) \mid a_n^k \in [a_{n,k}^{\min}, a_{n,k}^{\max}] \text{ and } \sum_{k=1}^K a_n^k \leq M_n \right\}$
Additively Coupled Sum Constrained Games

• Definition
  – A multi-user interaction $\Gamma = \langle \mathcal{N}, \mathcal{A}, \mathcal{U} \rangle$ in which

  **A1:** $\forall n \in \mathcal{N}$, action set $\mathcal{A}_n$ is defined to be

  $\mathcal{A}_n = \{ (a_1^n, \ldots, a^K_n) | a^K_n \in [a_{n,k}^{\min}, a_{n,k}^{\max}]$ and $\sum_{k=1}^{K} a^K_n \leq M_n \}.$

  Structure of the action set: resource is constrained
Additively Coupled Sum Constrained Games

- Definition
  - A multi-user interaction $\Gamma = \langle \mathcal{N}, \mathcal{A}, u \rangle$ in which

**A2:** The utility function satisfies

$$u_n(a) = \sum_{k=1}^{K} \left[ h_n^k(a_n^k + f_n^k(a_{-n})) - g_n^k(a_{-n}) \right],$$

in which $h_n^k(\cdot) : \mathcal{R} \rightarrow \mathcal{R}$ is an increasing and strictly concave function. Both $f_n^k(\cdot) : \mathcal{A}_{-n} \rightarrow \mathcal{R}$ and $g_n^k(\cdot) : \mathcal{A}_{-n} \rightarrow \mathcal{R}$ are twice differentiable.
Additively Coupled Sum Constrained Games

• Definition
  – A multi-user interaction $\Gamma = \langle \mathcal{N}, \mathcal{A}, \mathcal{U} \rangle$ in which

**A2:** The utility function satisfies

$$u_n(a) = \sum_{k=1}^{K} \left[ h_n^k(a_n^k + f_n^k(a_{-n})) - g_n^k(a_{-n}) \right],$$

in which $h_n^k(\cdot) : \mathcal{R} \to \mathcal{R}$ is an increasing and strictly concave function. Both $f_n^k(\cdot) : \mathcal{A}_{-n} \to \mathcal{R}$ and $g_n^k(\cdot) : \mathcal{A}_{-n} \to \mathcal{R}$ are twice differentiable.
Additively Coupled Sum Constrained Games

- Definition
  - A multi-user interaction \( \Gamma = \langle \mathcal{N}, \mathcal{A}, u \rangle \) in which

  \( u_n(a) = \sum_{k=1}^{K} \left[ h_n^k(a_n^k + f_n^k(a_{-n})) - g_n^k(a_{-n}) \right] \),

  in which \( h_n^k(\cdot) : \mathcal{R} \rightarrow \mathcal{R} \) is an increasing and strictly concave function. Both \( f_n^k(\cdot) : \mathcal{A}_{-n} \rightarrow \mathcal{R} \) and \( g_n^k(\cdot) : \mathcal{A}_{-n} \rightarrow \mathcal{R} \) are twice differentiable.
Additively Coupled Sum Constrained Games

- Definition
  - A multi-user interaction $\Gamma = \langle \mathcal{N}, \mathcal{A}, \mathcal{U} \rangle$ in which

  **A2:** The utility function satisfies

  $$u_n(a) = \sum_{k=1}^{K} \left[ h_n^k(a_n^k + f_n^k(a_{-n})) - g_n^k(a_{-n}) \right],$$

  in which $h_n^k(\cdot) : \mathcal{R} \rightarrow \mathcal{R}$ is an increasing and strictly concave function. Both $f_n^k(\cdot) : \mathcal{A}_{-n} \rightarrow \mathcal{R}$ and $g_n^k(\cdot) : \mathcal{A}_{-n} \rightarrow \mathcal{R}$ are twice differentiable.
Examples of ACSCG

- Power control in interference channels
Examples of ACSCG

- Power control in interference channels
Examples of ACSCG

• Power control in interference channels

\[ \mathcal{A}_n = \{ \mathbf{P} = (P^1_n, \ldots, P^K_n) | P^k_n \geq 0 \text{ and } \sum_{k=1}^{K} P^k_n \leq P^{\text{max}}_n \} \]

\[ r_n(\mathbf{P}) = \sum_{k=1}^{K} \log_2 \left( 1 + \frac{H^k_{nn} P^k_n}{\sigma^k_n + \sum_{m \neq n} H^k_{mn} P^k_m} \right) \]

\[ = \sum_{k=1}^{K} \left( \log_2 (\sigma^k_n + \sum_{m=1}^{N} H^k_{mn} P^k_m) - \log_2 (\sigma^k_n + \sum_{m \neq n} H^k_{mn} P^k_m) \right). \]

<table>
<thead>
<tr>
<th>\quad</th>
<th>\quad</th>
<th>\quad</th>
</tr>
</thead>
<tbody>
<tr>
<td>\quad</td>
<td>\quad</td>
<td>\quad</td>
</tr>
<tr>
<td>\quad</td>
<td>\quad</td>
<td>\quad</td>
</tr>
</tbody>
</table>

\[ f^k_n(a_{-n}) \quad h^k_n(x) \quad g^k_n(a_{-n}) \]

| $\sum_{m \neq n} \frac{H^k_{mn}}{H^k_{nn}} P^k_m$ | $\log_2 (\sigma^k_n + H^k_{nn} x)$ | $\log_2 (\sigma^k_n + \sum_{m \neq n} H^k_{mn} P^k_m)$ |
Examples of ACSCG (cont’d)

- Delay minimization in Jackson networks
Examples of ACSCG (cont’d)

- Delay minimization in Jackson networks

\[ \mathcal{A}_n = \left\{ \Psi = (\psi_1^1, \ldots, \psi^K_n) \mid \psi_n^k \geq 0 \right\} \]

and

\[ \sum_{k=1}^{K} \psi_n^k \geq \Psi_n^{\text{min}} \]

\[ d_n(\Psi) = \sum_{k=1}^{K} \frac{1}{\mu_n^k - \sum_{m=1}^{N} \nu_{mn}^k \psi_n^k} \]

<table>
<thead>
<tr>
<th>( f_n^k(a_{-n}) )</th>
<th>( h_n^k(x) )</th>
<th>( g_n^k(a_{-n}) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \sum_{m \neq n} \frac{\nu_{mn}^k \psi_m^k}{\nu_{nn}^k} )</td>
<td>( -\frac{1}{\mu_n^k - \nu_{nn}^k x} )</td>
<td>0</td>
</tr>
</tbody>
</table>
Examples of ACSCG (cont’d)

- Delay minimization in Jackson networks

$$\mathcal{A}_n = \{ \Psi = (\psi_1^n, \ldots, \psi^K_n) | \psi_n^k \geq 0 \}
\text{ and } \sum_{k=1}^{K} \psi_n^k \geq \Psi_n^{\text{min}} \}$$

$$d_n(\Psi) = \sum_{k=1}^{K} \frac{1}{\mu_n^k - \sum_{m=1}^{N} \nu_{mn}^k \psi_m^k}$$

$$[R^k]_{mn} = r_{nm}^k, \ U^k = (I - R^k)^{-1}, \ v_m^k = [U^k]_{mn}.$$
Nash equilibrium in ACSCG

• Existence of pure NE
  – A subclass of concave games

• When is the NE unique? When does best response converges to such a NE?
  – Existing literatures are not immediately applicable
    • Diagonal strict convexity condition [Rosen]
      • Use gradient play and stepsizes need to be carefully chosen
    • Super-modular games [Topkis]
      • Action space is not a lattice
    • Sufficient conditions for specific $h_n^k(\cdot)$ and $f_n^k(\cdot)$ [Yu]
Best response dynamics

- Best response iteration

\[
\begin{align*}
&f_n^1(a_{t-n}^{t-1}) & f_n^2(a_{t-n}^{t-1}) & \ldots & f_n^K(a_{t-n}^{t-1}) \\
g_n^1(a_{t-n}^{t-1}) & g_n^2(a_{t-n}^{t-1}) & & g_n^K(a_{t-n}^{t-1})
\end{align*}
\]

\[
u_n(a) = \sum_{k=1}^{K} \left[ h_n^k(a_n^k + f_n^k(a_{-n})) - g_n^k(a_{-n}) \right]
\]

\[A_n = \left\{ (a_1^n, a_2^n, \ldots, a^K_n) \mid a_n^k \in [a_{n,k}^{\text{min}}, a_{n,k}^{\text{max}}] \text{ and } \sum_{k=1}^{K} a_n^k \leq M_n \right\}.
\]
Best response dynamics

- Best response iteration

\[
a_{n}^{k,t} \triangleq \left[ \left\{ \frac{\partial h_{n}^{k}(x)}{\partial x} \right\}^{-1}(\lambda) - f_{n}^{k}(a_{-n}^{t-1}) \right]_{a_{n,k}^{\min}}^{a_{n,k}^{\max}}
\]

in which \( \lambda \) is chosen such that

\[
\sum_{k=1}^{K} a_{n}^{k,t} = M_{n}.
\]
Best response dynamics

- Best response iteration

$$a^{k,t}_n \triangleq \left[ \left\{ \frac{\partial h^n_k(x)}{\partial x} \right\}^{-1}(\lambda) - f_n^k(a^{t-1}_{-n}) \right]a_{n,k}^{\max} a_{n,k}^{\min}$$

in which $\lambda$ is chosen such that

$$\sum_{k=1}^{K} a^{k,t}_n = M_n.$$  

A competition scenario in which every user aggressively uses up all his resources.
Best response dynamics

- Best response iteration

\[ a_{n,t}^k \triangleq \left( \left\{ \frac{\partial h_n^k(x)}{\partial x} \right\}^{-1} (\lambda) - f_n^k(a_{t-1}^{-n}) \right) \text{max}_{a_{n,k}} \text{min}_{a_{n,k}}^a \]

in which \( \lambda \) is chosen such that

\[ \sum_{k=1}^{K} a_{n,t}^k = M_n. \]

- When does it converges?

  - By intuition, the weaker the mutual coupling is, the more likely it converges
  - How to measure and quantify this coupling strength?
A measure of the mutual coupling

Define

\[ [T^{\text{max}}]_{mn} \triangleq \begin{cases} \max_{a \in A, k'} \sum_{k=1}^{K} \left| \frac{\partial f_n^k(a_m - a_n^k)}{\partial a_m^{k'}} \right|, & \text{if } m \neq n \\ 0, & \text{otherwise}. \end{cases} \]

\[
\max_{a \in A, k'} \sum_{k=1}^{K} \left| \frac{\partial f_n^k(a_m - a_n^k)}{\partial a_m^{k'}} \right|
\]
represents the maximum impact that user m’s action can make over user n’s state.
Convergence conditions

**Theorem 1:** If

\[ \rho(T_{\text{max}}) < \frac{1}{2}, \]

then best response dynamics converges linearly to a unique pure NE for any set of initial conditions.
Convergence conditions

**Theorem 1:** If

\[ \rho(T_{\text{max}}) < \frac{1}{2}, \]

then best response dynamics converges linearly to a unique pure NE for any set of initial conditions.

Contraction mapping

\[ a_{n}^{k,t} = \left( \left\{ \frac{\partial h_{n}^{k}(x)}{\partial x} \right\}^{-1} (\lambda) - f_{n}^{k}(a_{n-1}^{t-1}) \right] a_{n,k}^{\text{max}} a_{n,k}^{\text{min}} \]
Convergence conditions

**Theorem 1:** If
\[ \rho(\tilde{T}^{\max}) < \frac{1}{2}, \]
then best response dynamics converges linearly to a unique pure NE for any set of initial conditions.

**Contraction mapping**
\[ a^{k,t}_{n} \triangleq \left[ \left\{ \frac{\partial h^{k}_{n}(x)}{\partial x} \right\}^{-1} (\lambda) - f^{k}_{n}(a^{t-1}_{n}) \right] a^{\max}_{n,k} a^{\min}_{n,k} \]

- The contraction factor \( \rho(\tilde{T}^{\max}) \) is a measure of the overall coupling strength.
- If \( f^{k}_{n}(a^{t}_{n}) \) is affine, the condition in Theorem 1 is not impacted by \( M_{n} \); otherwise it may depend on \( M_{n} \).

\[ \max_{a \in A,k'} \sum_{k=1}^{K} \left| \frac{\partial f^{k}_{n}(a^{t}_{n})}{\partial a^{t}_{m}} \right| \text{ is a constant for affine } f^{k}_{n}(a^{t}_{n}) \]
Convergence conditions

- If \( \frac{\partial f_n^k(a_{-n})}{\partial a_{m}^{k'}} \) have the same sign, \( \forall m \neq n, k, k', a \in \mathcal{A} \), the condition in Theorem 1 can be relaxed to

\[
\rho \left( \overline{T}^{\max} \right) < 1
\]

- This is true in many communication scenarios
  - Increasing power causes stronger interference
  - Increasing input rate congests the server
Convergence conditions

- If \( \frac{\partial f_n^k(a_{-n})}{\partial a_m^{k'}} \) have the same sign, \( \forall m \neq n, k, k', a \in \mathcal{A} \), the condition in Theorem 1 can be relaxed to

\[
\rho(\bar{T}^{\text{max}}) < 1
\]

- This is true in many communication scenarios
  - Increasing power causes stronger interference
  - Increasing input rate congests the server

Strategic complements (or strategic substitutes)

\[
\frac{\partial^2 u_n}{\partial a_n \partial a_m} \geq 0 \quad \text{(or)} \quad \frac{\partial^2 u_n}{\partial a_n \partial a_m} \leq 0
\]
A special class of $h_n^k (\cdot)$

For $\alpha_n^k \in \mathcal{R}$ and $F_{nn}^k > 0$, define [Walrand]

$$h_n^k (x) = \begin{cases} 
\frac{(\alpha_n^k + F_{nn}^k x)^{\theta+1}}{\theta+1}, & \text{if } -1 < \theta < 0, \\
\log(\alpha_n^k + F_{nn}^k x), & \text{if } \theta = -1.
\end{cases}$$

$\theta = -1$: proportional fairness;

$\theta = -2$, harmonic mean fairness;

$\theta = -\infty$, max-min fairness.
For \( \alpha_n^k \in \mathcal{R} \) and \( F_{nn}^k > 0 \), define [Walrand]

\[
h_n^k(x) = \begin{cases} 
\frac{(\alpha_n^k + F_{nn}^k x)^{\theta+1}}{\theta+1}, & \text{if } -1 < \theta < 0, \\
\log(\alpha_n^k + F_{nn}^k x), & \text{if } \theta = -1.
\end{cases}
\]

Define

\[
[S_{\text{max}}^m]_{mn} \triangleq \begin{cases} 
\frac{\sum_{k=1}^{K} (F_{nn}^k)^{1+\frac{1}{\theta}}}{\sum_{k=1}^{K} (F_{nn}^k)^{1+\frac{1}{\theta}}} \max_{a \in A,k'} \left\{ \sum_{k=1}^{K} \left| \frac{\partial f_n^k(a_{-n})}{\partial a_{m}^k} \right| \left( \frac{F_{nn}^k}{F_{mm}^k} \right)^{1+\frac{1}{\theta}} \right\}, & \text{if } m \neq n \\
0, & \text{otherwise.}
\end{cases}
\]

A measure of the similarity between users’ parameters \( F_{nn}^k \)

\[
[S_{\text{max}}^m]_{mn} \leq \zeta_{mn} \cdot [T_{\text{max}}^m]_{mn}, \quad \zeta_{mn} \in \left[ 1, \frac{\max_k (F_{nn}^k/F_{mm}^k)^{1+\frac{1}{\theta}}}{\min_k (F_{nn}^k/F_{mm}^k)^{1+\frac{1}{\theta}}} \right].
\]
Convergence conditions

Theorem 2: If

$$\rho(\bar{S}^{\text{max}}) < 1,$$

then best response dynamics converges linearly to a unique pure NE for any set of initial conditions.
Convergence conditions

**Theorem 2:** If

\[ \rho(\bar{S}^{\text{max}}) < 1, \]

then best response dynamics converges linearly to a unique pure NE for any set of initial conditions.

In general

\[ \zeta_{mn} < 2, \forall m \neq n \]
Conclusion so far...

Concave games
ACSCG
Power control, Flow control

Pareto boundary

If Information is constrained and no message passing is available...

When will it converge to a NE? And how fast?

Nash equilibrium
Conclusion so far...

Concave games

ACSCG

Power control, Flow control

If Information is constrained and no message passing is available...

When will it converge to a NE? And how fast?

Sufficient conditions that guarantee linear convergence

Nash equilibrium

Pareto boundary

Multimedia Communications and Systems Laboratory
Power control as an ACSCG

- Power control in interference channels

\[ r_n(P) = \sum_{k=1}^{K} \log_2 \left( 1 + \frac{H_{nn}^k P_n^k}{\sigma_n^k + \sum_{m \neq n} H_{mn}^k P_m^k} \right) \]
Performance comparison

• Solutions without information exchange
  – Iterative water-filling algorithm [Yu]

\[
P_{1}^{(0)} \rightarrow P_{2}^{(0)} \rightarrow P_{1}^{(1)} \rightarrow P_{2}^{(1)} \rightarrow \ldots
\]

• Solutions with information exchange

\[
\max \sum_{k} \omega_{k} R_{k}
\]
Performance comparison

• Solutions without information exchange
  – Iterative water-filling algorithm [Yu]

• Solutions with information exchange

\[
\sigma_k m_n^k \neq \sum \text{user} n' \text{\'s spectrum max } k \omega_k R_k
\]

OSB = Optimal Spectrum Balancing
ASB = Autonomous Spectrum Balancing

Best Available Today (IW)

UCLA Multimedia Communications and Systems Laboratory
Outline

- Motivation and existing approaches
- Informationally efficient multi-user communication
  - Vector cases
    - Convergence conditions with decentralized information
    - Improve efficiency with decentralized information
  - Scalar cases
    - Achieve Pareto efficiency with decentralized information
- Conclusions
How to model the mutual coupling

- A reformulation of the coupling
  - State space \( S = \times_{n \in \mathcal{N}} S_n \)
  - Utility function \( u_n : S_n \times \mathcal{A}_n \rightarrow \mathcal{R} \)
  - State determination function \( s_n : \mathcal{A}_{-n} \rightarrow S_n \)
  - Belief function \( \tilde{s}_n : \mathcal{A}_n \rightarrow S_n \)
  - Conjectural Equilibrium (CE): a configuration of belief functions \((\tilde{s}_1^*, \ldots, \tilde{s}_N^*)\) and joint action \( a^* = (a_1^*, \ldots, a_N^*) \) satisfying
    \[
    \tilde{s}_n^*(a_n^*) = s_n(a_{-n}^*) \quad \text{and} \quad a_n^* = \arg \max_{a_n \in \mathcal{A}_n} u_n(\tilde{s}_n^*(a_n), a_n)
    \]
How to model the mutual coupling

- **A reformulation of the coupling**
  - **State space** \( \mathcal{S} = \times_{n \in \mathcal{N}} \mathcal{S}_n \)
  - **Utility function** \( u_n : \mathcal{S}_n \times \mathcal{A}_n \rightarrow \mathcal{R} \)
  - **State determination function** \( s_n : \mathcal{A}_{-n} \rightarrow \mathcal{S}_n \)
  - **Belief function** \( \tilde{s}_n : \mathcal{A}_n \rightarrow \mathcal{S}_n \)
  - **Conjectural Equilibrium (CE)**: a configuration of belief functions \( (\tilde{s}_1^*, \ldots, \tilde{s}_N^*) \) and joint action \( a^* = (a_1^*, \ldots, a_N^*) \) satisfying
    \[
    \tilde{s}_n^*(a_n^*) = s_n(a_{-n}^*) \quad \text{and} \quad a_n^* = \arg \max_{a_n \in \mathcal{A}_n} u_n(\tilde{s}_n^*(a_n^*), a_n)
    \]

  - It captures the aggregate effect of the other users’ actions
  - It models the aggregate effect of the other users’ actions

UCLA Multimedia Communications and Systems Laboratory
How to model the mutual coupling

- A reformulation of the coupling
  - State space $\mathcal{S} = \times_{n \in \mathcal{N}} \mathcal{S}_n$
  - Utility function $u_n : \mathcal{S}_n \times \mathcal{A}_n \rightarrow \mathcal{R}$
  - State determination function $s_n : \mathcal{A}_{-n} \rightarrow \mathcal{S}_n$
  - Belief function $\tilde{s}_n : \mathcal{A}_n \rightarrow \mathcal{S}_n$
  - Conjectural Equilibrium (CE): a configuration of belief functions $(\tilde{s}_1^*, \ldots, \tilde{s}_N^*)$ and joint action $a^* = (a_1^*, \ldots, a_N^*)$ satisfying

\[
\tilde{s}_n^*(a_n^*) = s_n(a_{-n}^*) \quad \text{and} \quad a_n^* = \arg \max_{a_n \in \mathcal{A}_n} u_n(\tilde{s}_n^*(a_n), a_n)
\]

beliefs are realized

it captures the aggregate effect of the other users’ actions

it models the aggregate effect of the other users’ actions

each user behaves optimally according to its expectation
CE in power control games [SuTSP’09]

- One leader and multiple followers
- State space
  - \( I_n^k \): the interference caused to user \( n \) in channel \( k \)
- Utility function
  \[
  R_n = \sum_{k=1}^{K} \log_2 \left( 1 + \frac{P_n^k}{\sigma_n^k + I_n^k} \right)
  \]
- State determination function
  \[
  I_n^k = \sum_{i=1, i \neq n}^{N} \alpha_{in}^k P_i^k
  \]
- Belief function (linear form)
  \[
  \tilde{I}_1^k = \beta^k - \gamma^k P_1^k
  \]
Why Linear belief?

\[
\frac{\partial I_1^k}{\partial P_1^k} \text{ is piece-wise linear; } \frac{\partial I_1^k}{\partial P_1^j} = 0, j \neq k, \text{ if the number of frequency bins is sufficiently large.}
\]

\[\Rightarrow \text{Linear belief is sufficient to capture the interference coupling!}\]
Why Linear belief?

\[ \frac{\partial I_1^k}{\partial P_1^k} \text{ is piece-wise linear; } \frac{\partial I_1^k}{\partial P_1^j} = 0, \; j \neq k, \text{ if the number of frequency bins is sufficiently large.} \]

→ **Linear belief is sufficient** to capture the interference coupling!
Why Linear belief?

\[ \frac{\partial I_1^k}{\partial P_1^k} \text{ is piece-wise linear; } \frac{\partial I_1^k}{\partial P_1^j} = 0, j \neq k, \text{ if the number of frequency bins is sufficiently large.} \]

→ **Linear belief is sufficient** to capture the interference coupling!
Why Linear belief?

\[ \frac{\partial I_1^k}{\partial P_1^k} \] is piece-wise linear; \[ \frac{\partial I_1^k}{\partial P_1^j} = 0, \ j \neq k \], if the number of frequency bins is sufficiently large.

→ Linear belief is sufficient to capture the interference coupling!
Why Linear belief?

\[
\frac{\partial I_k^1}{\partial P_k^1} \text{ is piece-wise linear; } \frac{\partial I_k^1}{\partial P_j^1} = 0, \ j \neq k, \text{ if the number of frequency bins is sufficiently large.}
\]

→ **Linear belief is sufficient** to capture the interference coupling!

\[
P_n^k(P_t^{-1}) = \left[ \frac{1}{\lambda} - \frac{\sigma_n^k}{H_{nn}^k} - \sum_{m \neq n} \frac{H_{mn}^k}{H_{nn}^k} P_{m}^{k,t-1} \right]^{+\infty}_{0}
\]
Main results

- Stackelberg equilibrium
  - Strategy profile \( (a_1^*, NE(a_1^*)) \) that satisfies
    \[
    u_1(a_1^*, NE(a_1^*)) \geq u_1(a_1, NE(a_1)), \forall a_1 \in \mathcal{A}_1
    \]

- NE and SE are special CE
  - NE: \( \beta^k = \sum_{i=2}^{N} \alpha_i^k P_i^k, \gamma^k = 0 \)
  - SE: \( \beta^k = I_1^k - P_1^k \cdot \frac{\partial I_1^k}{\partial P_1^k}, \gamma^k = -\frac{\partial I_1^k}{\partial P_1^k} \).

- Infinite set of CE
  - Open sets of CE that contain NE and SE may exist
Achieving the desired CE

- Conjecture-based rate maximization (CRM)

<table>
<thead>
<tr>
<th>State $I^n_t$</th>
<th>User 1 leader</th>
<th>User 2, ..., $N$ followers</th>
</tr>
</thead>
<tbody>
<tr>
<td>$I^{k,t}<em>n = \sum</em>{i=1, i \neq n}^N \alpha^{k}_{in} P^{k,t}_i$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Belief function $\tilde{s}_n : A_n \rightarrow S_n$</th>
<th>$\beta^{k}<em>{t}, \gamma^{k}</em>{t} \leftarrow \text{Update}_1\left(I^{k,t}_1, P^{k,t}_1\right)$</th>
<th>$\tilde{I}^{k,t}<em>n = I^{k,t}<em>n = \sum</em>{i=1, i \neq n}^N \alpha^{k}</em>{in} P^{k,t}_i$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\tilde{I}^{k,t}<em>t = \beta^{k}</em>{t} - \gamma^{k}_{t} P^{k,t}_1$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Action $a^t_1, \ldots, a^t_K$</th>
<th>$P^{t+1}_1 \leftarrow \text{Update}_2\left(P^t_1, \tilde{I}^t_1\right)$</th>
<th>$P^t_k = \arg \max_{P^t_k \in A_n} \sum_{k=1}^K \log_2 \left(1 + \frac{P^{\prime k}_n}{\sigma^k_n + \tilde{I}^{k,t}_n}\right)$</th>
</tr>
</thead>
</table>

Dynamic updates of the play

$$\beta^k_t = \left.I^k_t - P^k_1 \cdot \frac{\partial I^k_t}{\partial P^k_1}\right|_{P_1 = P^t_1}$$

$$\gamma^k_t = - \left.I^k_t \cdot \frac{\partial I^k_t}{\partial P^k_1}\right|_{P_1 = P^t_1}$$

$$\max_{\left\{P^t_k\right\}} \sum_{k=1}^K \log_2 \left(1 + \frac{P^{k}_1}{\sigma^k_1 + \beta^k_t - \gamma^k_t P^{k}_1}\right)$$

solvable using dual method
Discussion about CRM

• Essence of CRM
  – local approximation of the computation of SE

• Advantages
  – the structure of the utility function is explored
  – only local information is required
  – it can be applied in the cases where N>2
  – if it converges, the outcome is a CE
Simulation results

Average rate improvements:
2-user case: 24.4% for user 1; 33.6% for user 2
3-user case: 26.3% for user 1; 9.7% for user 2&3

\[
\sum_k |\alpha_{ij}^k|^2 = 0.5, \ i \neq j \\
\sum_k |\alpha_{ij}^k|^2 = 0.33, \ i \neq j
\]
Conclusions so far...

If Information is constrained and no message passing is allowed

How to improve an inefficient NE without message passing?

Nash equilibrium

Pareto boundary

Concave games

ACSCG

Power control

\[ u_2 \]

\[ u_1 \]
Conclusions so far...

- Concave games
- ACSCG
- Power control

Overall efficiency may be improved!

If information is constrained and no message passing is allowed

How to improve an inefficient NE without message passing?

Build belief, learn, and adapt

Nash equilibrium

UCLA Multimedia Communications and Systems Laboratory
Outline

• Motivation and existing approaches
• Informationally efficient multi-user communication
  – Vector cases
    • Convergence conditions with decentralized information
    • Improve efficiency with decentralized information
  – Scalar cases
    • Achieve Pareto efficiency with decentralized information
• Conclusions
Linearly coupled games

- A non-cooperative game model
- Users’ states are linearly impacted by their competitor’s actions
- Contributions
  - Characterize the structures of the utility functions
  - Explicitly compute Nash equilibrium and Pareto boundary
  - A conjectural equilibrium approach to achieve Pareto boundary without real-time information exchange
Definition

A multi-user interaction is considered a *linearly coupled game* if the action set \( \mathcal{A}_n \subseteq \mathcal{R}_+ \) is convex and the utility function \( u_n \) satisfies

\[
    u_n(a) = a_n^{\beta_n} \cdot s_n(a),
\]

in which \( \beta_n > 0 \). In particular, the basic assumptions about \( s_n(a) \) include:

**A1:** \( s_n(a) \) is non-negative;

**A2:** \( s_n(a) \) is strictly linearly decreasing in \( a_m, \forall m \neq n \);

\( s_n(a) \) is non-increasing and linear in \( a_n \).
Definition (cont’d)

Denote $s'_{nm}(a) = \frac{\partial s_n(a)}{\partial a_m}$.

A3: $\frac{s_n(a)}{s'_{nm}(a)}$ is an affine function, $\forall n \in \mathcal{N} \setminus \{m\}$.

A4: $\frac{s'_{nm}(a)}{s_n(a)} = \frac{s'_{km}(a)}{s_k(a)}$, $\forall n, k \in \mathcal{N} \setminus \{m\}$;

$\frac{s'_{mm}(a)}{s_m(a)} = 0$ or $\frac{s'_{nm}(a)}{s_n(a)}$, $\forall n \neq m$.

Actions are linearly coupled at NE and PB
Two basic types

- For the games satisfying A1-A4, the utility functions can take two types of form:
  - Type I [SuJSAC’10]

\[ u_n(a) = a_n^{\beta_n} \cdot \prod_{m \neq n} (\mu_m - \tau_m a_m) \]

- e.g. random access

\[ u_n(p) = p_n \prod_{m \neq n} (1 - p_m) \]
Two basic types

- For the games satisfying A1-A4, the utility functions can take two types of form:
  - Type I [SuJSAC’10]
    \[ u_n(a) = a_n^{\beta_n} \cdot \prod_{m \neq n} (\mu_m - \tau_m a_m) \]
    - e.g. random access
    \[ u_n(p) = p_n \prod_{m \neq n} (1 - p_m) \]
  - Type II [SuTR’09]
    \[ u_n(a) = a_n^{\beta_n} \cdot (\mu - \sum_{m=1}^{N} \tau_m a_m) \]
    - e.g. rate control
    \[ u_n(\psi) = \psi_n^{\beta_n} (\mu - \sum_{m=1}^{N} \psi_m) \]
Type I games: wireless random access

- **Player set:**
  - nodes in a single cell

- **Action set:**
  - transmission probability

- **Payoff:**
  - throughput $u_k(p) = p_k \prod_{i \in \mathcal{K} \setminus \{k\}} (1 - p_i)$.

- **Key issues**
  - stability, convergence, throughput, and fairness
Conjecture-based Random Access

- Individual conjectures
  - state: \( s_k = \prod_{i \in \mathcal{K} \setminus \{k\}} (1 - p_i) \)
  - linear belief: \( u_k^t(\tilde{s}_k^t(p_k), p_k) = p_k \left[ \prod_{i \in \mathcal{K} \setminus \{k\}} (1 - p_i^{t-1}) - a_k(p_k - p_k^{t-1}) \right] \)

- Two update mechanisms
  - Best response
  \[ p_k^t = \arg \max_{p_k \in P_k} u_k^t(\tilde{s}_k^t(p_k), p_k) = \min \left\{ \frac{p_k^{t-1}}{2} + \prod_{i \in \mathcal{K} \setminus \{k\}} (1 - p_i^{t-1}), 1 \right\} \]
  - Gradient play
  \[ p_k^t = p_k^{t-1} + \gamma_k \frac{\partial u_k^t(\tilde{s}_k^t(p_k))}{\partial p_k} \bigg|_{p_k=p_k^{t-1}}. \]
Main results

- **Existence of CE**
  - all operating points in action space are CE

- **Stability and convergence**
  - sufficient conditions

- **Throughput performance**
  - the entire throughput region can be achieved with stable CE

- **Fairness issue**
  - conjecture-based approaches attain weighted fairness

Protocol design: how to achieve efficient outcomes?
How to select suitable $a_k$?

- Adaptively alter $a_k$ when the network size changes
- Adopt aggregated throughput or “idle interval” as the indicator of the system efficiency
- Advantages
  - No need of a centralized solver
  - Throughput efficient with fairness guarantee
  - Stable equilibrium
  - Autonomously adapt to traffic fluctuation
Engineering interpretation

• DCF vs. the best response update
  – re-design the random access protocol

\[
p_k^t = \arg \max_{p_k \in P_k} u_k^t(s_k^t(p_k), p_k) = \min \left\{ \frac{p_k^{t-1}}{2} + \frac{\prod_{i \in \mathcal{K} \setminus \{k\}} (1 - p_i^{t-1})}{2a_k}, 1 \right\}
\]

\[
p_k^t = \frac{1}{2} \mathbb{E}\{p_k^{t-1} \mathbf{1}\{T_{-k}^{t-1}=1\}|p^{t-1}\} + \frac{1}{2a_k} \mathbb{E}\{\mathbf{1}\{T_{-k}^t=0\} \mathbf{1}\{T_k^t-1=0\}|p^{t-1}\}
\]

\[
+ \frac{1}{2} \left(1 + \frac{1}{a_k}\right) \mathbb{E}\{\mathbf{1}\{T_{-k}^t=0\} \mathbf{1}\{T_k^t-1=1\}|p^{t-1}\}
\]
Engineering interpretation

- DCF vs. the best response update
  - re-design the random access protocol

\[ p_k^t = \arg \max_{p_k \in P_k} u_k^t(s_k^t(p_k), p_k) = \min \left\{ \frac{p_k^{t-1}}{2} + \frac{\prod_{i \in \mathcal{K} \setminus \{k\}} (1 - p_i^{t-1})}{2a_k}, 1 \right\} \]

\[ p_k^t = \frac{1}{2} \mathbb{E}\{p_k^{t-1}\} \mathbf{1}_{\{T_{-k}^{t-1} = 1\}} | p_k^{t-1} \} + \frac{1}{2a_k} \mathbb{E}\{\mathbf{1}_{\{T_{-k}^{t-1} = 0\}} \mathbf{1}_{\{T_k^{t-1} = 0\}} | p_k^{t-1} \} \]

\[ + \frac{1}{2} (1 + \frac{1}{a_k}) \mathbb{E}\{\mathbf{1}_{\{T_{-k}^{t-1} = 0\}} \mathbf{1}_{\{T_k^{t-1} = 1\}} | p_k^{t-1} \} \]

\[ \text{similar} \]

<table>
<thead>
<tr>
<th></th>
<th>( T_{-k}^{t-1} = 1 )</th>
<th>( T_{-k}^{t-1} = 0 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>CBRA</td>
<td>( p_k^t = p_k^{t-1} / 2 )</td>
<td>( p_k^t = p_k^{t-1} / 2 )</td>
</tr>
<tr>
<td>DCF</td>
<td>( p_k^t = 1/2 + 1/2a_k )</td>
<td>( p_k^t = 1/2a_k )</td>
</tr>
</tbody>
</table>

\[ \text{different} \]
Engineering interpretation

- DCF vs. the best response update
  - re-design the random access protocol

\[ p_k^t = \arg \max_{p_k \in P_k} u_k^t(s_k(p_k), p_k) = \min \left\{ \frac{p_k^{t-1}}{2} + \frac{\prod_{i \in K \setminus \{k\}} (1 - p_i^{t-1})}{2a_k}, 1 \right\} \]

\[ p_k^t = \frac{1}{2} \mathbb{E}\{p_k^{t-1}\} \mathbb{1}_{\{T_{-k}^{t-1} = 1\}} |p_k^{t-1}| + \frac{1}{2a_k} \mathbb{E}\{\mathbb{1}_{\{T_{-k}^{t-1} = 0\}} \mathbb{1}_{\{T_k^{t-1} = 0\}} |p_k^{t-1}|}\]

\[ + \frac{1}{2} \left( 1 + \frac{1}{a_k} \right) \mathbb{E}\{\mathbb{1}_{\{T_{-k}^{t-1} = 0\}} \mathbb{1}_{\{T_k^{t-1} = 1\}} |p_k^{t-1}|\} \]

Similar to CBRA, uses 4-bit information, while DCF only uses 2 bits.
Simulation results

- Throughput
- Stability and convergence

DCF: low throughput;
P-MAC: needs to know the number of nodes

P-MAC: instability due to the online estimation
Conventional solutions in Type II games

- Utility function
  \[ u_n(a) = a_n^{\beta_n} \cdot (\mu - \sum_{m=1}^{N} \tau_m a_m). \]

- Nash equilibrium
  \[ a_n^{NE} = \frac{\beta_n \mu}{\tau_n (1 + \sum_{m=1}^{N} \beta_m)}, \quad \forall n \in \mathcal{N} \]

- Pareto boundary
  \[ \max_{a} \sum_{n=1}^{N} \omega_n \log[u_n(a)] \rightarrow a_n^{PB} = \frac{\omega_n \beta_n \mu}{\tau_n (1 + \sum_{m=1}^{N} \omega_m \beta_m)}, \quad \forall n \in \mathcal{N} \]

- Efficiency loss
  \[ (1 + \sum_{n=1}^{N} \omega_n \beta_n) \cdot \log \frac{(1 + \sum_{n=1}^{N} \omega_n \beta_n)^2}{(1 + \sum_{n=1}^{N} \omega_n^2 \beta_n)(1 + \sum_{n=1}^{N} \beta_n)} < \sum_{n=1}^{N} \omega_n \log \frac{u_n(a_n^{NE})}{u_n(a_n^{PB})} < 0. \]
Best response dynamics in Type II games

• At stage $t$,

$$u_n^t(s_n^t(a_n), a_n) = a_n^{\beta_n} \cdot \left[ \mu - \sum_{m=1}^{N} \tau_m a_m^{t-1} - \lambda_n (a_n - a_n^{t-1}) \right]$$

• Theorem 5: A necessary and sufficient condition for the best response dynamics to converge is

$$\sum_{n=1}^{N} \frac{\tau_n \beta_n}{\lambda_n (1 + 2\beta_n)} < 1.$$
Stability of the Pareto boundary

- **Theorem 6**: All the operating points on the Pareto boundary are globally convergent CE under the best response dynamics. The belief configurations \( \{ \lambda_n \}_{n=1}^{N} \) lead to Pareto-optimal operating points if and only if

\[
\sum_{n=1}^{N} \frac{T_n}{\lambda_n} = 1
\]

- \( \frac{T_n}{\lambda_n} \): the ratio between the immediate performance degradation and the conjectured long-term effect

Theorem 5 and expressions of Pareto boundary and CE
Pricing vs. conjectural equilibrium

• Pricing mechanism in communication networks [Kelly][Chiang]
  – Users repeatedly exchange coordination signals

• Conjectural equilibrium for linearly coupled games
  – Coordination is implicitly implemented when the participating users initialize their belief parameters
  – Pareto-optimality can be achieved solely based on local observations on the states
  – No message passing is needed during the convergence process
  – The key problem is how to design belief functions
Conclusions so far...

The optimal way of designing the beliefs and updating the actions based on conjectural equilibrium is addressed.

Concave games

LCG

Random Access, Rate control

Can we still achieve Pareto optimality?

Pareto boundary
Global (exchanged) information

If Information is constrained and no message passing is available...

Decentralized (insufficient) information

Nash equilibrium
Decentralized (limited) information
Conclusions so far...

Can we still achieve Pareto optimality?

Conjectural equilibrium
Decentralized (insufficient) information
Nash equilibrium
Decentralized (limited) information

Pareto boundary
Global (exchanged) information

If Information is constrained and no message passing is available...

The optimal way of designing the beliefs and updating the actions based on conjectural equilibrium is addressed.
Conclusions

• We define new classes of games emerging in multi-user communication networks and investigate the information and efficiency trade-off
  – Provide sufficient convergence conditions to NE
  – Suggest a conjectural equilibrium based approach to improve efficiency
  – Quantify the performance improvement
References

References (cont’d)


References (cont’d)

Linear convergence

- A sequence $x^{(k)}$ with limit $x^*$ is linearly convergent if there exists a constant $c \in (0, 1)$ such that

\[ |x^{(k)} - x^*| \leq c |x^{(k-1)} - x^*| \]

for $k$ sufficiently large.
Solutions with information exchange

• Users aim to solve

\[ \max_{\mathbf{a} \in \mathcal{A}} \sum_{n=1}^{N} u_n(\mathbf{a}) \]

user n’s impact over user m’s utility

• They can pass coordination messages

\[ \pi_{mn}^k(a_m, a_{-m}) = -\frac{\partial u_m(\mathbf{a})}{\partial a_n^k} \]

and user n behaves according to

\[ \max_{a_n \in \mathcal{A}_n} u_n(\mathbf{a}) - \sum_{k=1}^{K} a_n^k \left( \sum_{m \neq n} \pi_{mn}^k \right) \]
Solutions with information exchange

- **Gradient play**

\[
\pi_{mn}^{k,t-1} = -\frac{\partial u_n(a_n^{t-1})}{\partial a_n^k}
\]

\[
a_n^{k,t} = a_n^{k,t-1} + \kappa\left(\frac{\partial u_n(a_n, a_{-n}^{t-1})}{\partial a_n^k} - \sum_{m \neq n} \pi_{mn}^{k,t-1}\right).
\]

\[
a_n^t = [a_n^{1,t}, a_n^{2,t}, \ldots, a_n^{K,t}] = [a_n^{1,t}, a_n^{2,t}, \ldots, a_n^{K,t}]A_n
\]

**Theorem 3:** If \( \forall n, k, x, y \in A_n \),

\[
\inf_{x} \frac{\partial^2 h_n^k(x)}{\partial^2 x} > -\infty, \text{ and } \left\| \nabla g_n^k(x) - \nabla g_n^k(y) \right\| \leq L' \|x - y\|,
\]

gradient play converges for a small enough stepsize.

Lipschitz continuity and gradient projection algorithm
Solutions with information exchange

- Jacobi update

\[
\pi_{mn}^{k,t-1} = -\frac{\partial u_m(a_n^{t-1})}{\partial a_n^k}
\]

\[
B(a_{-n}^{t-1}) = \arg \max_{a_n \in \mathcal{A}_n} u_n(a, a_{-n}^{t-1}) - \sum_{k=1}^{K} a_n^k \left( \sum_{m \neq n} \pi_{mn}^{k,t-1} \right)
\]

\[
a_n^{k,t} = a_n^{k,t-1} + \kappa \left\{ \left[ B(a_{-n}^{t-1}) \right]_n^k - a_n^{k,t-1} \right\}
\]

**Theorem 4:** If

\[
\inf_x \frac{\partial^2 h_n^k(x)}{\partial^2 x} > -\infty, \quad \sup_x \frac{\partial^2 h_n^k(x)}{\partial^2 x} < 0,
\]

and \( \left\| \nabla g_n^k(x) - \nabla g_n^k(y) \right\| \leq L' \left\| x - y \right\| , \)

Jacobi update converges for a small enough stepsize.

Lipschitz continuity, descent lemma, and mean value theorem
Solutions with information exchange

- Convergence to an operating point that satisfies the KKT conditions is guaranteed.

- Total utility is monotonically increasing.

- Global optimality is guaranteed if the original problem is convex, otherwise not.

- Developed for general non-convex problem in which convex NUM solutions may not apply in general.
Stackelberg equilibrium

• Definition
  – Leader (foresighted): only one
  – Follower (myopic): the remaining ones
  – Strategy profile \( (a_n^*, NE(a_n^*)) \) that satisfies
    \[
    u_n(a_n^*, NE(a_n^*)) \geq u_n(a_n, NE(a_n)), \forall a_n \in A_n
    \]

• Existence and computation of SE in the power control games [SuTWC’09]
A two-user formulation

- Bi-level Programming

\[
\begin{align*}
\text{upper level} & \quad \max_{P_1} \sum_{k=1}^{K} \ln \left( 1 + \frac{P_1^k}{N_1^k + \alpha_2^k P_2^k} \right) & (a) \\
\text{problem} & \quad \sum_{k=1}^{K} P_1^k \leq P_1^{\text{max}}, \quad P_1^k \geq 0, & (b) \\
\text{lower level} & \quad P_2 = \arg \max_{P_2'} \sum_{k=1}^{K} \ln \left( 1 + \frac{P_2'^k}{N_2^k + \alpha_1^k P_1^k} \right) & (c) \\
\text{problem} & \quad \sum_{k=1}^{K} P_2'^k \leq P_2^{\text{max}}, \quad P_2'^k \geq 0. & (d)
\end{align*}
\]

where
\[
\begin{align*}
N_1^k & = \sigma_1^k \left/ \left| H_{11}^k \right|^2 \right., \alpha_1^k = \left| H_{12}^k \right|^2 \left/ \left| H_{22}^k \right|^2 \right., \quad N_2^k = \sigma_2^k \left/ \left| H_{22}^k \right|^2 \right., \alpha_2^k = \left| H_{21}^k \right|^2 \left/ \left| H_{11}^k \right|^2 \right.
\end{align*}
\]
Problems with the SE formulation

• Computational complexity
  – intrinsically hard to compute

• Information required for playing SE
  – Global information
    \[ \left\{ \alpha_{ij}^k \right\}, \left\{ \sigma_i^k \right\}, \left\{ P_{i}^{\text{max}} \right\} \]

• Realistic assumption
  – Local information
    \[ \sum_{n=2}^{N} \alpha_n^k P_n^k + \sigma_1^k, P_1^{\text{max}} \]
  – Any appropriate solutions other than SE and NE?
Weighted Fairness

• Priority-based fair medium access control
  – Traffic classes with positive weights

\[
SU_i = p_i \cdot (1 - p_i)^{|f_i| - 1} \cdot \prod_{j \neq i} (1 - p_j)^{|f_j|}
\]

\[\forall i, j \in \{1, \cdots n\}, \quad \frac{SU_i}{\phi_i} = \frac{SU_j}{\phi_j}\]

\[\iff \forall i, j \in \{1, \cdots n\}, \quad \frac{p_i(1 - p_j)}{\phi_i} = \frac{p_j(1 - p_i)}{\phi_j}\]

• Conjecture-based protocol

\[\forall i, j \in \{1, 2, \cdots, N\}, \phi_i p_i (1 - p_i) = \phi_j p_j (1 - p_j) \Rightarrow \frac{\phi_i p_i}{1 - p_i} \approx \frac{\phi_j p_j}{1 - p_j}\]
Some distributed iterative algorithms

- Best response

\[ a_{n}^{BR,t} = \arg \max_{a_n \in A_n} u_n(a_n, a_{-n}^{t-1}) \]

- Jacobi update

\[ a_{n}^{JU,t} = (1 - \kappa) a_n^{t-1} + \kappa a_{n}^{BR,t} \]

- Gradient play

\[ a_{n}^{GP,t} = a_n^{t-1} + \kappa \nabla_n u_n(a_n, a_{-n}^{t-1}) \]