Multimedia Communications and Systems Laboratory

Informationally Efficient Multi-user communication

Yi Su

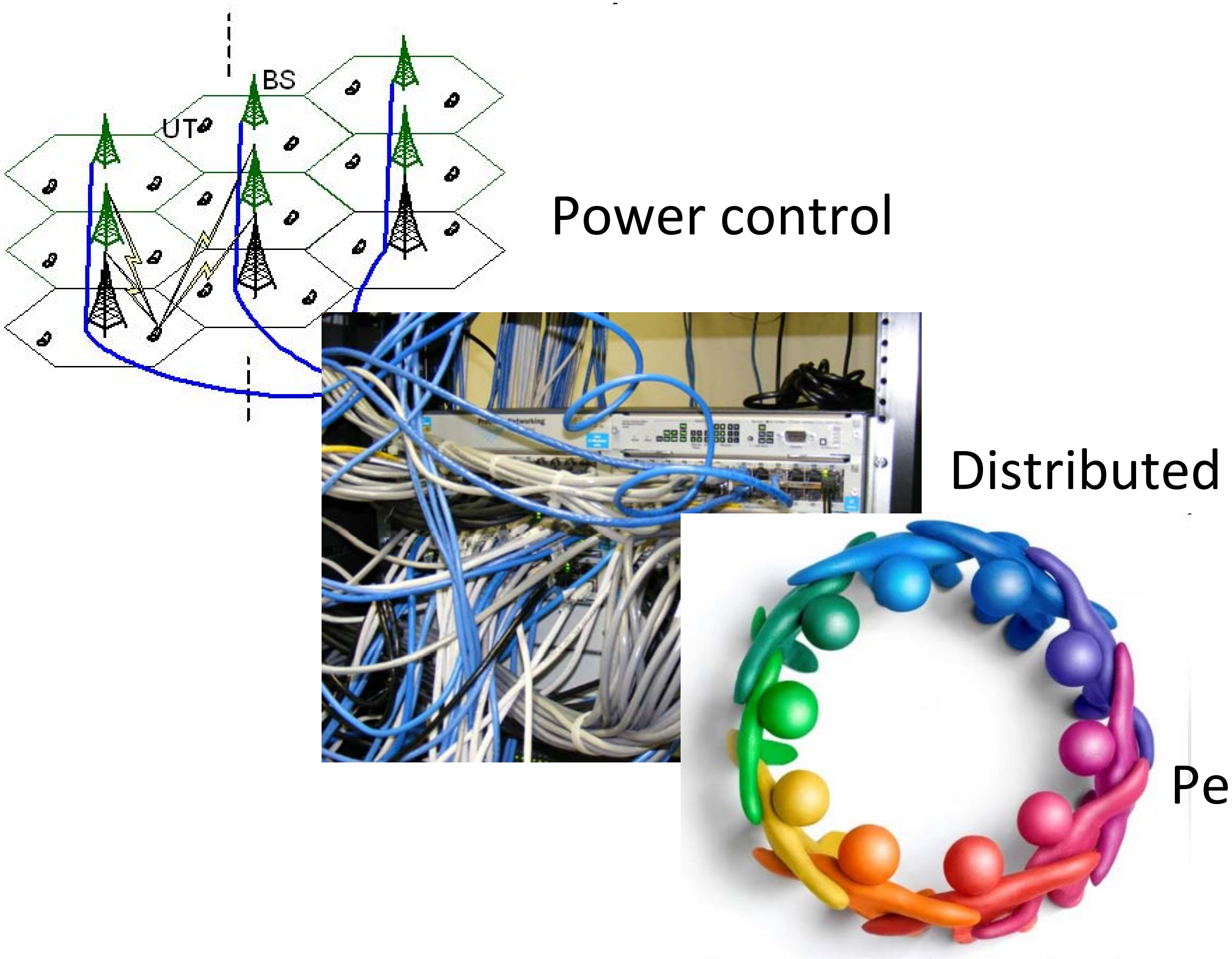
Advisor: Professor Mihaela van der Schaar

Electrical Engineering, UCLA

Outline

- **Motivation and existing approaches**
- Informationally efficient multi-user communication
  - Vector cases
    - Convergence conditions with decentralized information
    - Improve efficiency with decentralized information
  - Scalar cases
    - Achieve Pareto efficiency with decentralized information
- Conclusions

Multi-user communication networks



Power control

Distributed routing

Peer-to-peer system

etc…

Constraints in communication networks

- Resources
  - Bandwidth, power, spectrum, etc.
- Information
  - Real-time
    - Local observation

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Goal: multi-user communication without information exchange

A standard strategic game formulation

- Consider a tuple

$$\Gamma = \langle \mathcal{N}, \mathcal{A}, u \rangle$$

  - The set of players : $\mathcal{N} = \{1, 2, \ldots, N\}$
  - The set of actions: $\mathcal{A} = \times_{n \in \mathcal{N}} \mathcal{A}_n$ and $\mathcal{A}_n \subseteq \mathcal{R}^K$
  - Utility function: $u = \times_{n \in \mathcal{N}} u_n$ and $u_n : \mathcal{A} \rightarrow \mathcal{R}$
  - Utility region: $\mathcal{U} = \{(u_1(\mathbf{a}), \ldots, u_N(\mathbf{a}))| \ \exists \ \mathbf{a} \in \mathcal{A}\}$

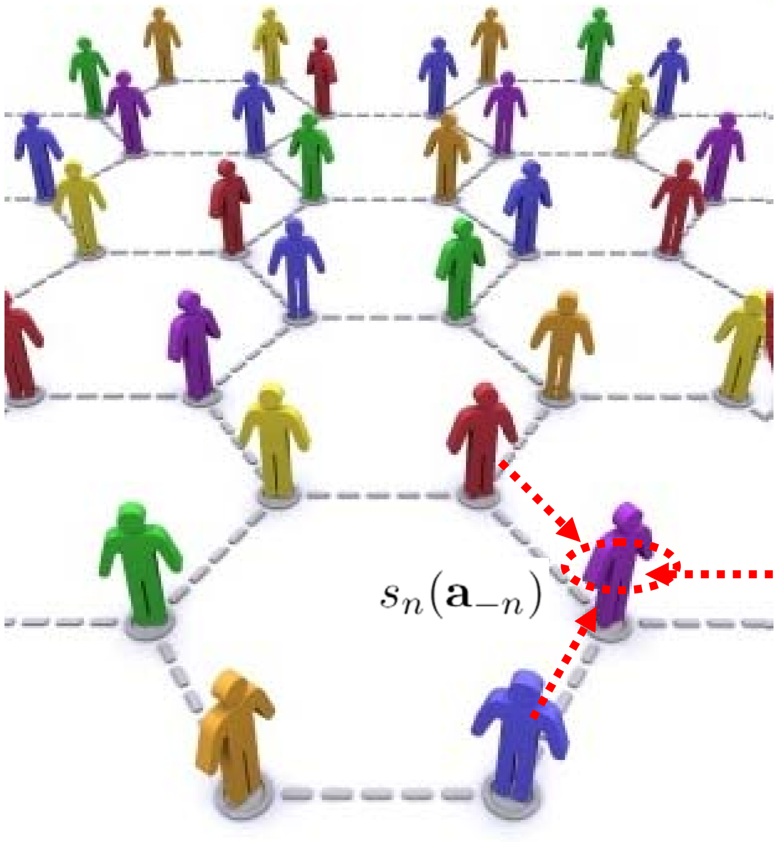
  In communication networks, different operating points in $\mathcal{U}$ can be chosen based on the information availability

Existing approaches

- Local observation

$$\max_{\mathbf{a}_n \in \mathcal{A}_n} u_n(\mathbf{a}_n, \mathbf{a}_{-n})$$

Nash equilibrium



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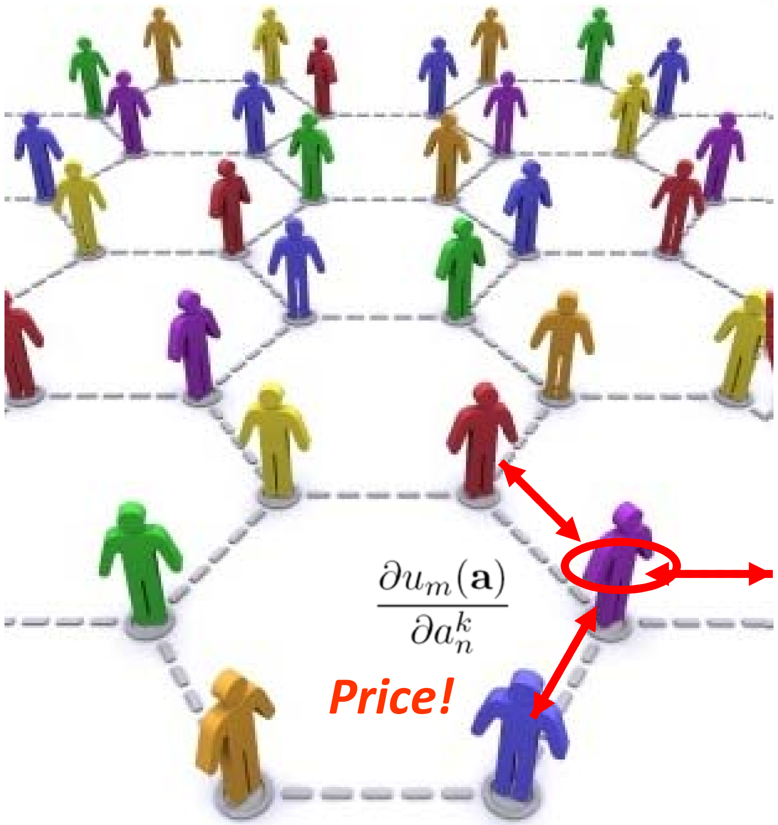
$$\max_{\mathbf{a} \in \mathcal{A}} \sum_{n=1}^{N} u_n(\mathbf{a})$$

$$\Rightarrow \frac{\partial u_n(\mathbf{a})}{\partial a_n^k} + \sum_{m \neq n} \frac{\partial u_m(\mathbf{a})}{\partial a_n^k}$$

Pareto optimality



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Pareto optimality

Existing results usually assume some specific action and utility structures!

Existing approaches (cont'd)

- Results with specific action and utility structures
  - Pure Nash equilibrium
    - Concave games

Use gradient play to find NE

i) $\mathcal{A}_n$: convex and compact; ii) $u_n(\mathbf{a}_n, \mathbf{a}_{-n})$: quasi-concave in $\mathbf{a}_n$

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    - Potential games [Shapley]

      Use best response to find NE

$$\forall (m,n) \in \mathcal{N}^2, m \neq n, \ \frac{\partial^2 (u_n - u_m)}{\partial \mathbf{a}_n \partial \mathbf{a}_m} = 0$$

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    - Super-modular games [Topkis]

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  - Pareto optimality
    - Network utility maximization [Kelly]
      - Convexity is the watershed

Existing approaches (cont'd)

Researchers	**Applications**	**Tools**
Altman	CDMA uplink power control	S-modular games
Berry	Distributed interference compensation	S-modular games
Barbarossa	Power control	Potential games
Tse	Spectrum sharing	Repeated games
Kelly	End-to-end congestion control	Pricing
Goodman	CDMA uplink power control	Pricing
Low	End-to-end flow control	Pricing
Chiang	Joint congestion and power control	Pricing
Poor	Energy efficient power and rate control	Equilibrium analysis
Cioffi	Power control in DSL systems	Equilibrium analysis
Yates	Uplink power control for cellular radio	Equilibrium analysis
Wicker	Selfish users in Aloha	Equilibrium analysis
Lazar	Non-cooperative optimal flow control	Equilibrium analysis

Special utility

Existing approaches (cont'd)

- Game theory
  - Equilibrium characterization
  - Incentive design

The focus is on strategic interactions among users

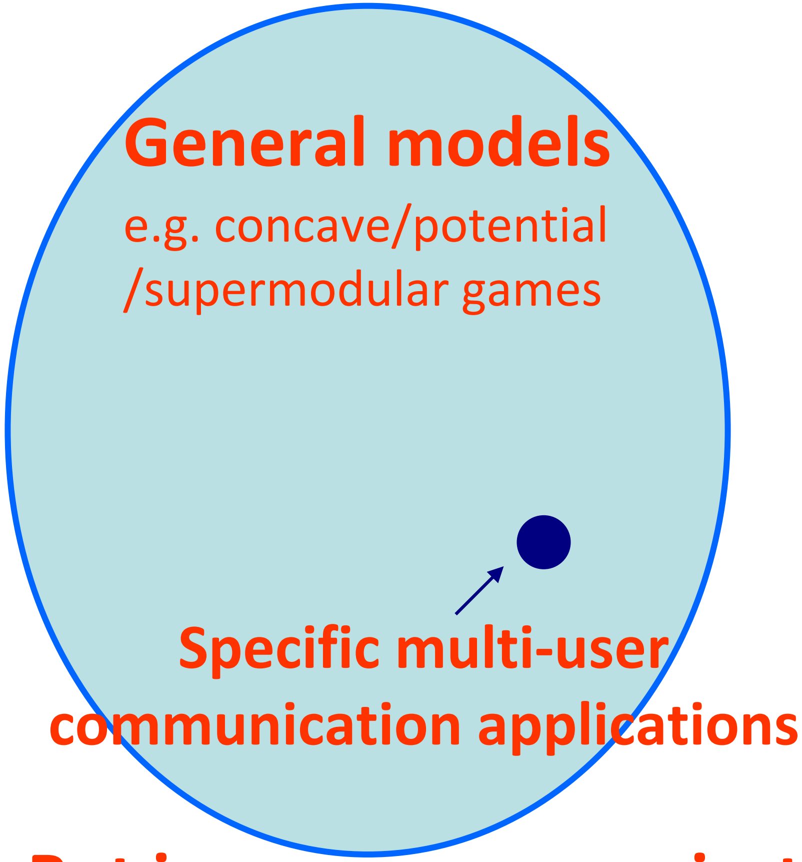
- Optimization theory
  - Computational complexity
  - Distributed algorithms

Information is usually costless

- Information theory
  - Fundamental limits
  - Encoding and decoding schemes

Decentralization is not the focus

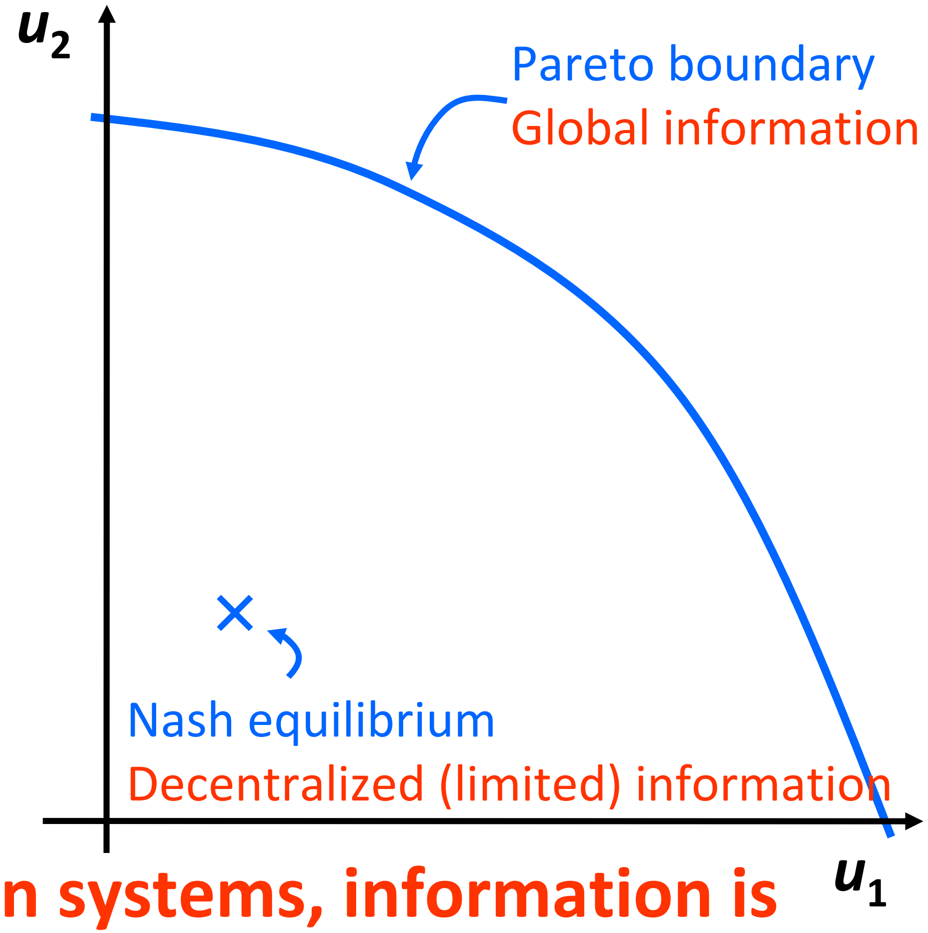
Existing approaches (cont'd)



General models

e.g. concave/potential /supermodular games

Specific multi-user communication applications



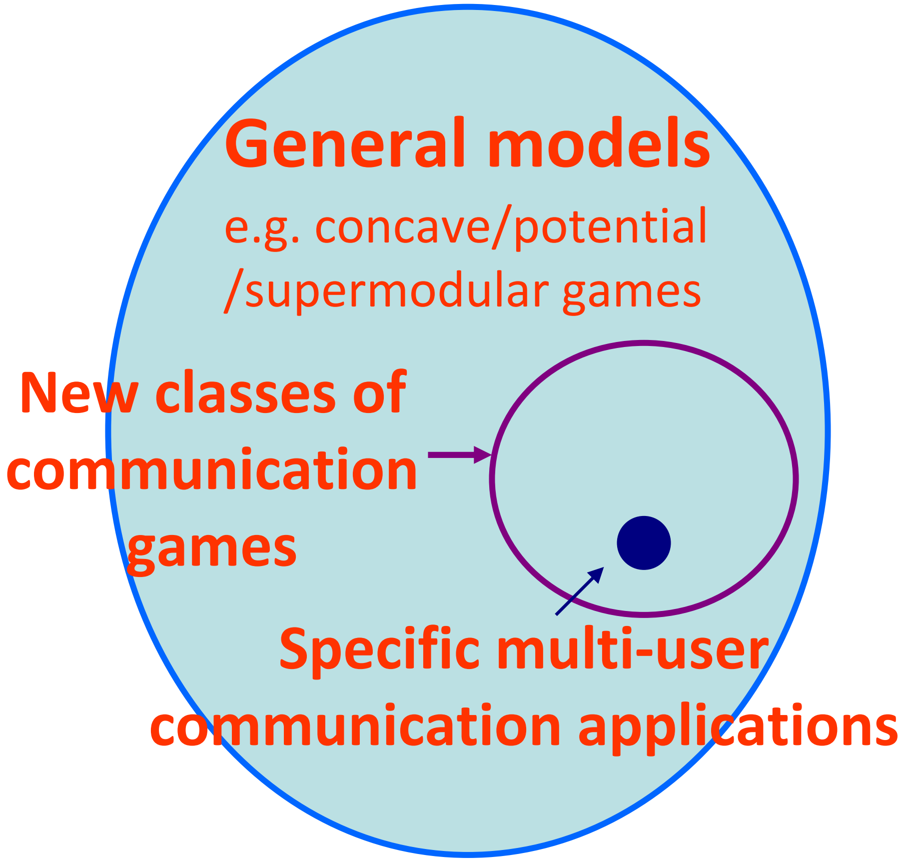
Pareto boundary
Global information

Nash equilibrium
Decentralized (limited) information

But in many communication systems, information is constrained and no message passing is allowed!

Our goals

If information is constrained and no message passing is allowed…



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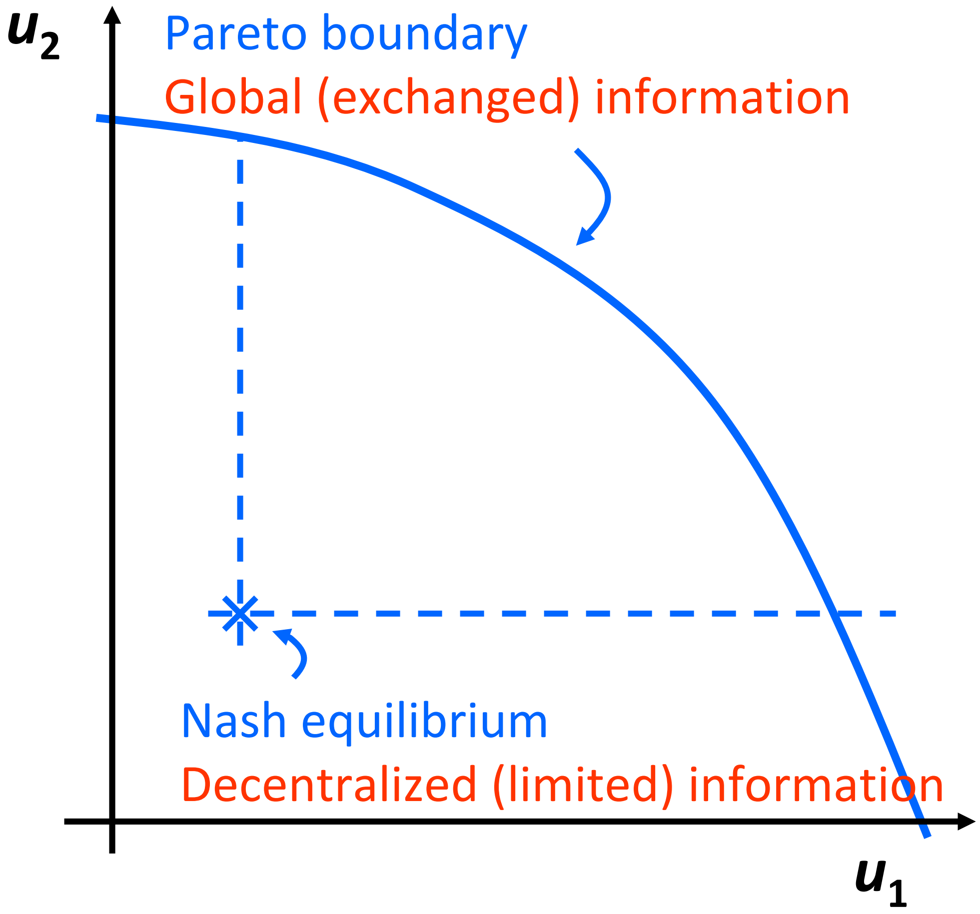
New classes of communication games

Specific multi-user communication applications



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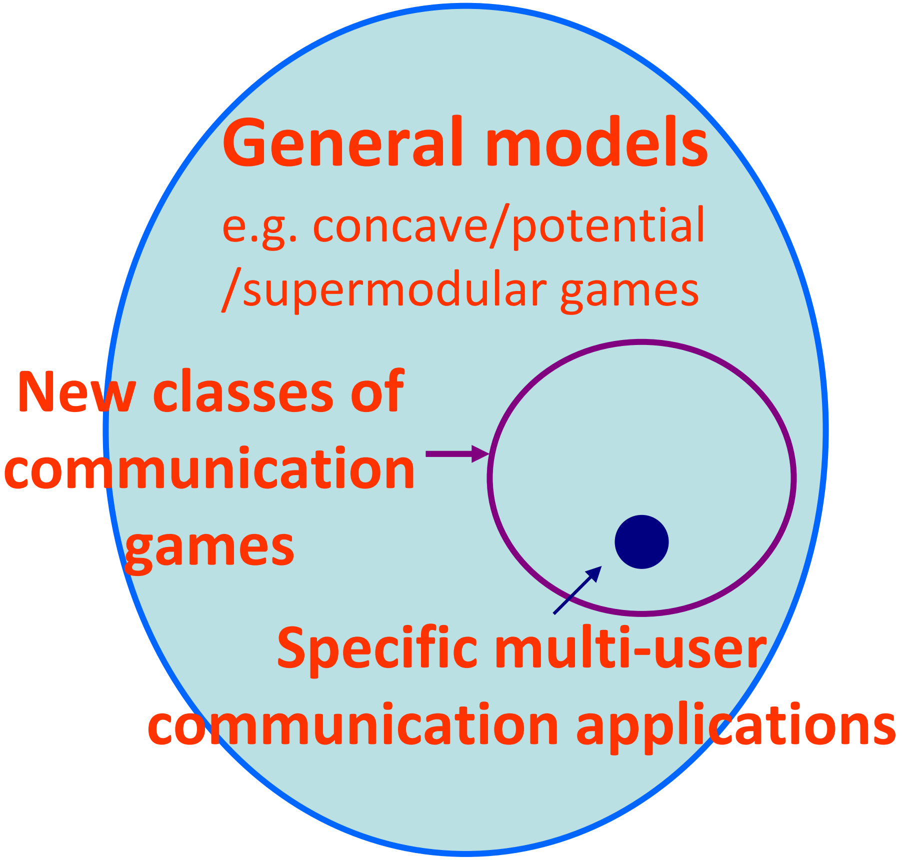

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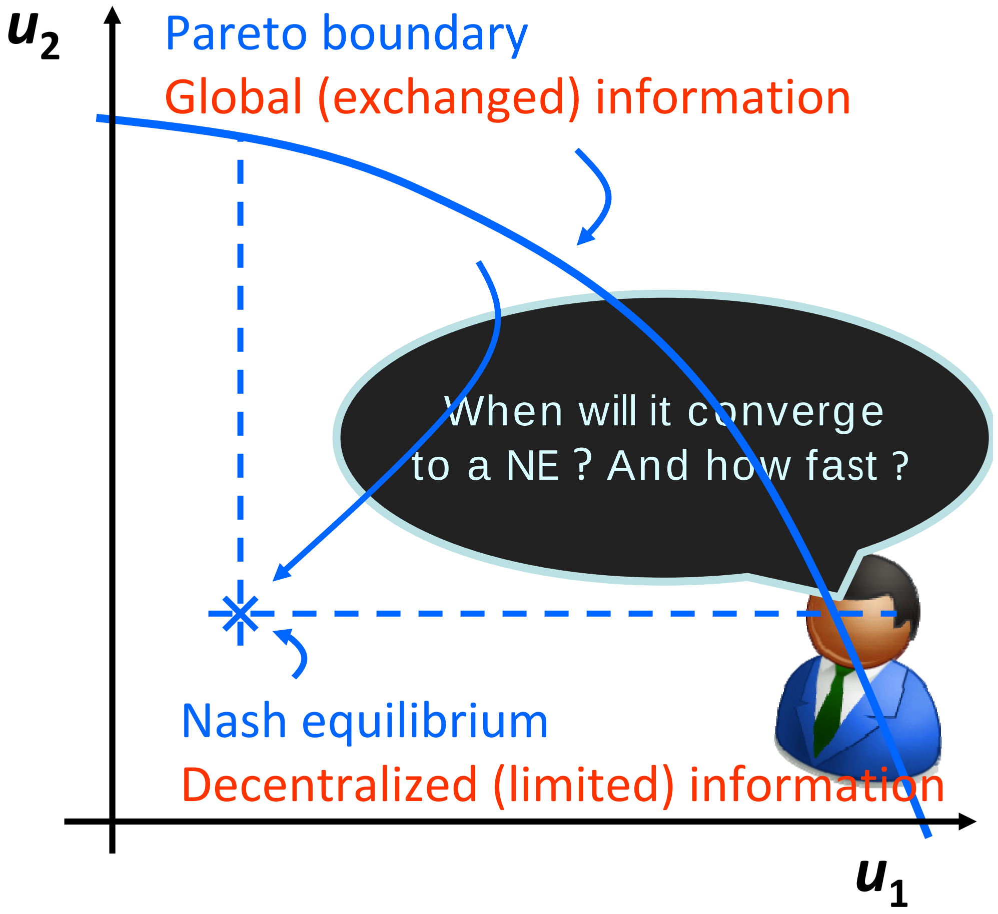

Pareto boundary

Global (exchanged) information

When will it converge to a NE ? And how fast ?

Nash equilibrium

Decentralized (limited) information

Our goals

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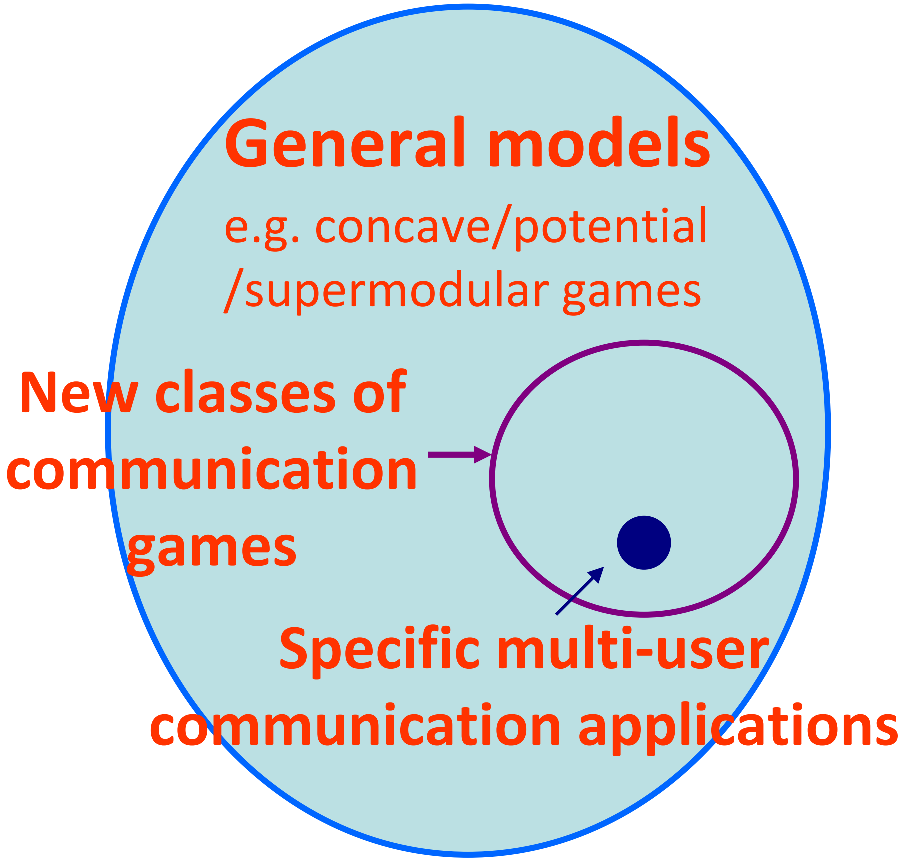

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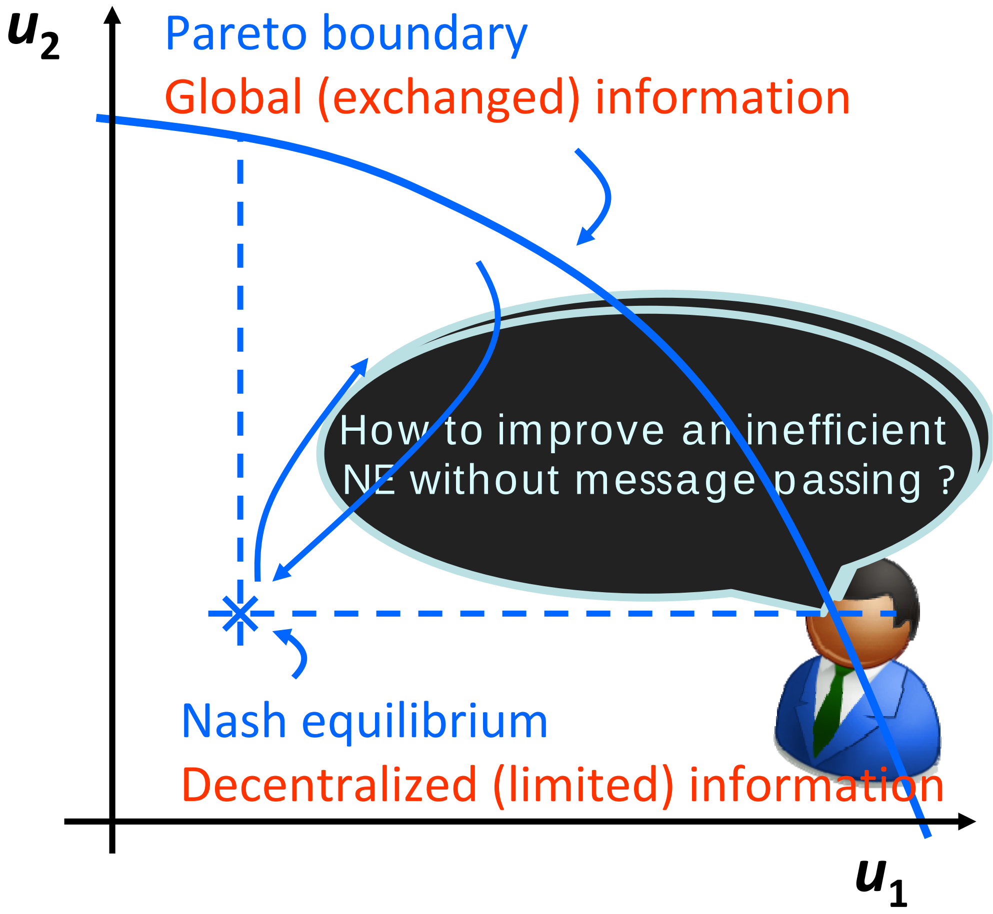

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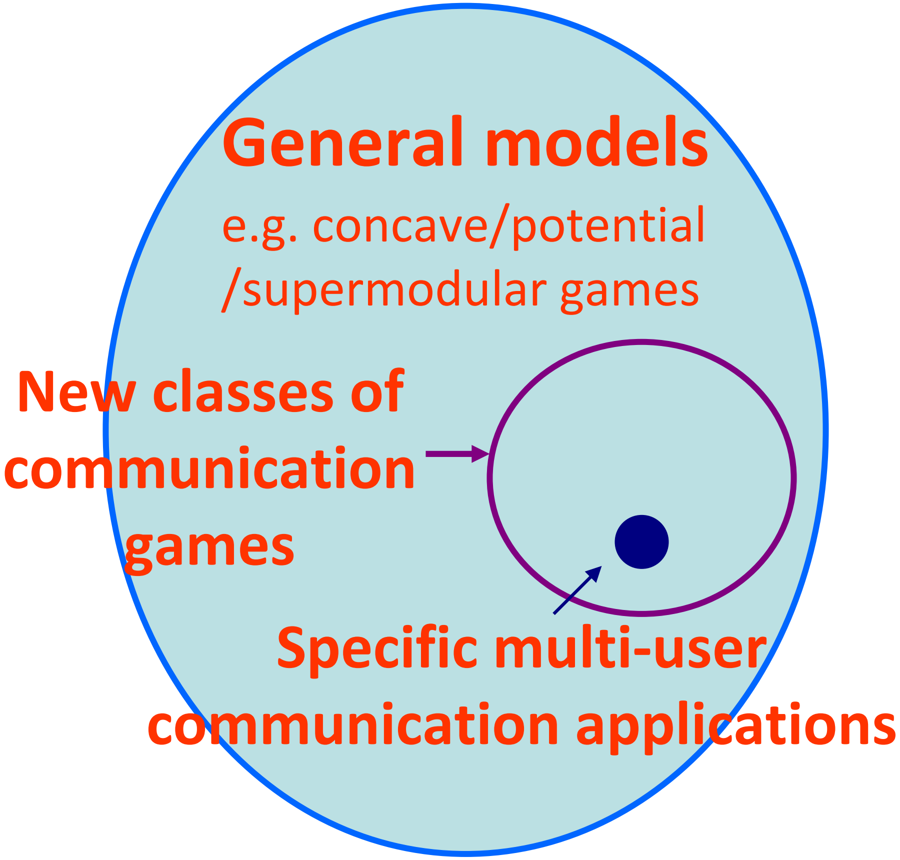

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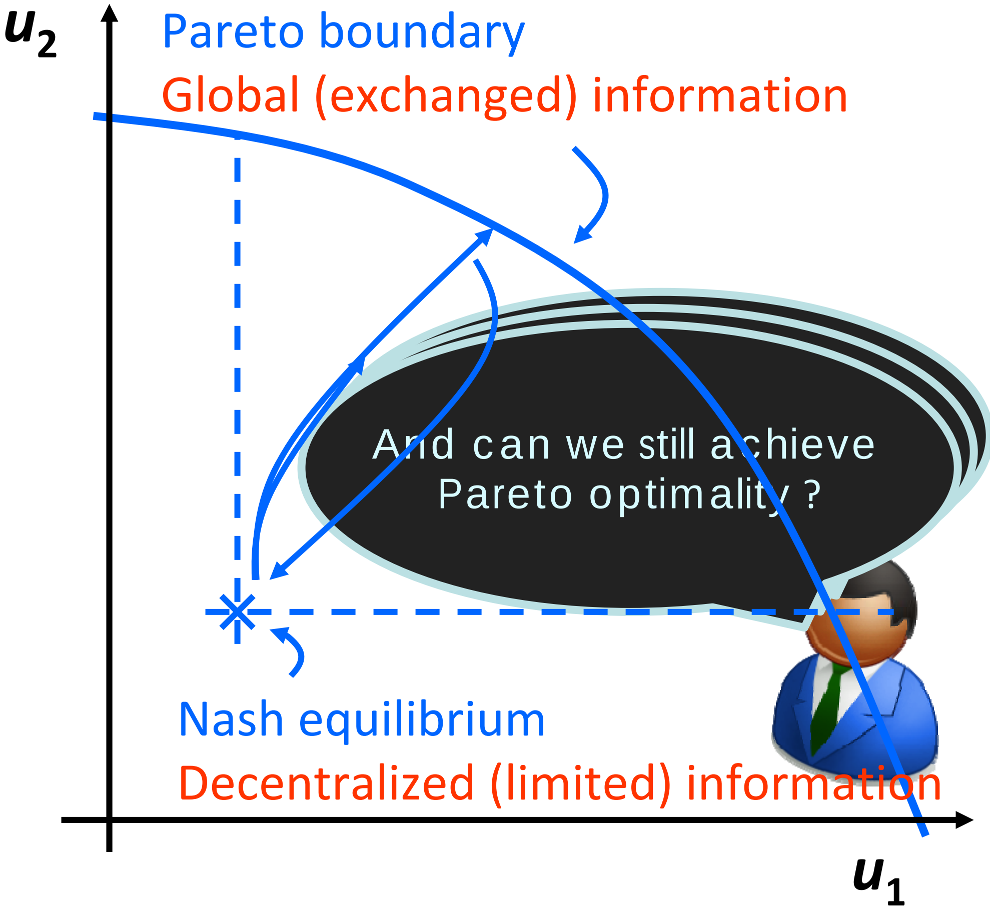

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A reformulation of multi-user interactions

- Consider a tuple

$$\Gamma = \langle \mathcal{N}, \mathcal{A}, \mathcal{S}, s, u \rangle$$

  - The set of players: $\mathcal{N} = \{1, 2, \ldots, N\}$
  - The set of actions: $\mathcal{A} = \times_{n \in \mathcal{N}} \mathcal{A}_n$
  - **State space:** $\mathcal{S} = \times_{n \in \mathcal{N}} \mathcal{S}_n$
  - **State determination function:**

$$s = \times_{n \in \mathcal{N}} s_n \text{ and } s_n : \mathcal{A}_{-n} \rightarrow \mathcal{S}_n$$

  - Utility function:

In standard strategic game, $\mathcal{S}_n = \mathcal{A}_{-n}$

$$u = \times_{n \in \mathcal{N}} u_n \text{ and } u_n : \boxed{\mathcal{S}_n \times \mathcal{A}_n \rightarrow \mathcal{R}}$$

It captures the structure of the coupling between action and state

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Many communication networking applications have simple $\mathcal{S}_n$, which captures the aggregate effects of $\mathcal{A}_{-n}$

In standard strategic game,

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  - Utility function:

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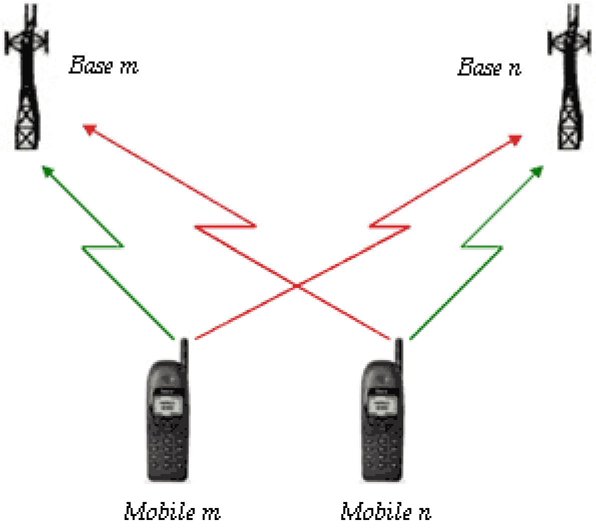
It captures the structure of the coupling between action and state

Communication games with simple states

- Power control

aggregate interference

$$s_n = \sum_{m \neq n} h_{mn} P_m, \quad u_n = \log_2\left(1 + \frac{h_{nn} P_n}{\sigma_n + s_n}\right).$$



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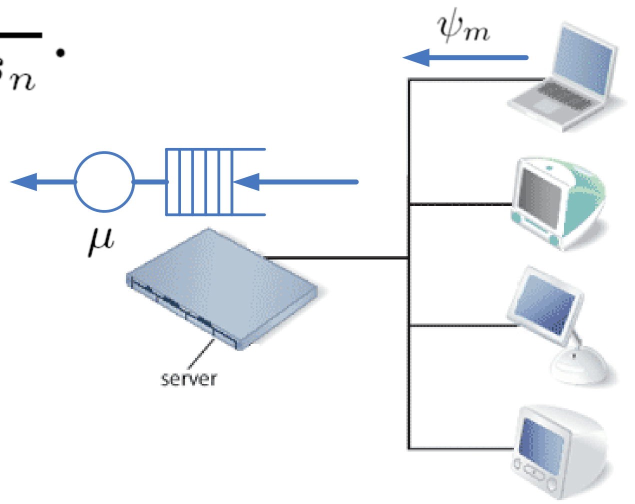
- Flow control

remaining capacity

$$s_n = \sum_{m \neq n} \psi_m, \quad u_n = \frac{\psi_n}{\mu - \psi_n - s_n}.$$



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$$s_n = \sum_{m\neq n} \psi_m, \quad u_n = \frac{\psi_n}{\mu - \psi_n - s_n}.$$

- Random access

idle probability

$$s_n = \prod_{m\neq n}(1-p_m), \quad u_n = p_n \cdot s_n.$$

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Additively Coupled Sum Constrained Games

- Definition
  - A multi-user interaction $\Gamma = \langle \mathcal{N}, \mathcal{A}, u \rangle$ in which

A1: $\forall n \in \mathcal{N},$ action set $\mathcal{A}_n$ is defined to be

$$\mathcal{A}_n = \{(a_n^1, \ldots, a_n^K) |\ a_n^k \in [a_{n,k}^{\min}, a_{n,k}^{\max}] \text{ and } \sum_{k=1}^K a_n^k \leq M_n.\}$$

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↓

Structure of the action set: resource is constrained

Additively Coupled Sum Constrained Games

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A2: The utility function satisfies

$$u_n(\mathbf{a}) = \sum_{k=1}^{K} \Big[h_n^k\big(a_n^k + f_n^k(\mathbf{a}_{-n})\big) - g_n^k(\mathbf{a}_{-n}) \Big],$$

in which $h_n^k(\cdot) : \mathcal{R} \rightarrow \mathcal{R}$ is an increasing and strictly concave function. Both $f_n^k(\cdot) : \mathcal{A}_{-n} \rightarrow \mathcal{R}$ and $g_n^k(\cdot) : \mathcal{A}_{-n} \rightarrow \mathcal{R}$ are twice differentiable.

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Structure of the utility:
additive coupling between
action and state

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diminishing return
per invested action

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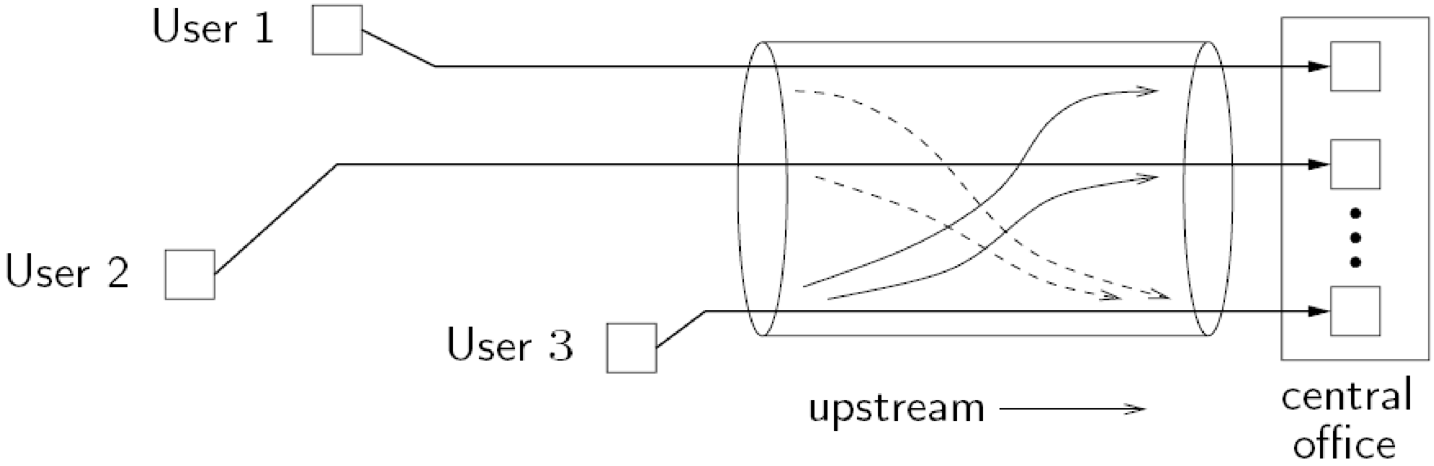
Examples of ACSCG

- Power control in interference channels



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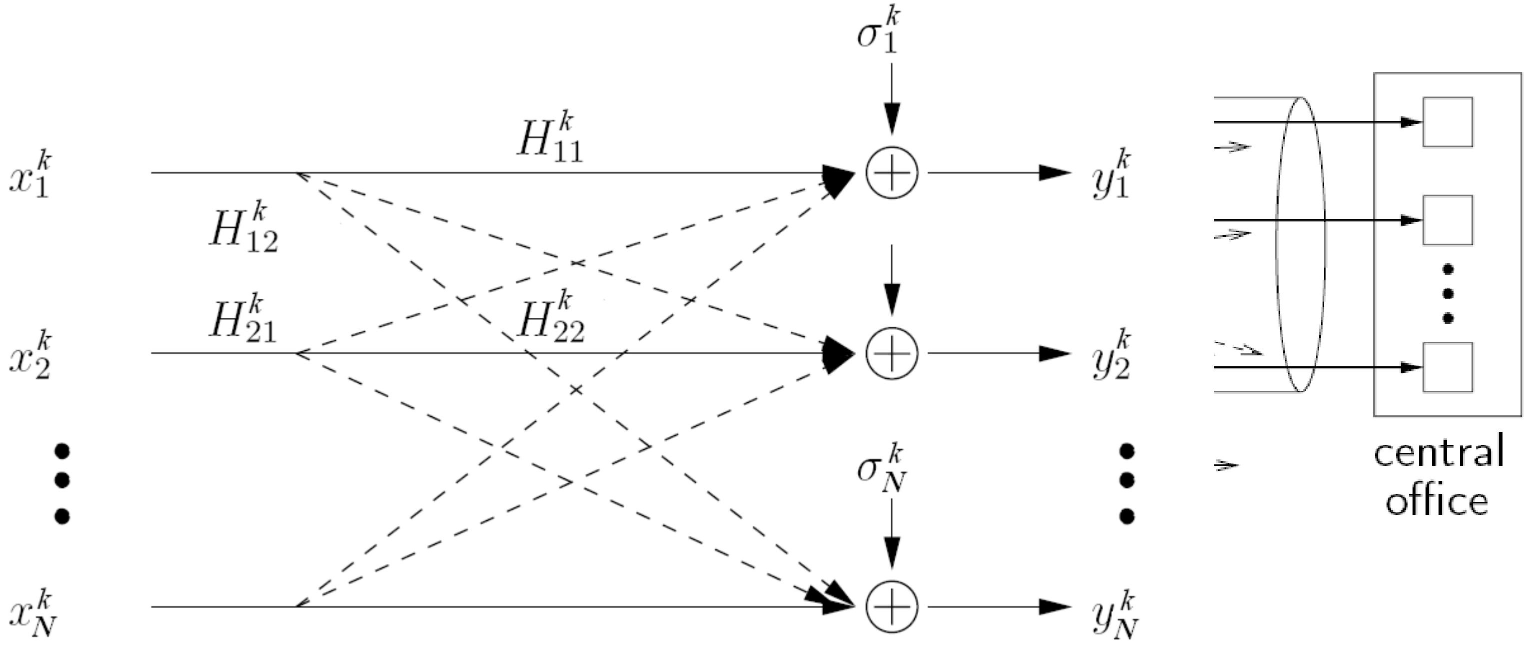

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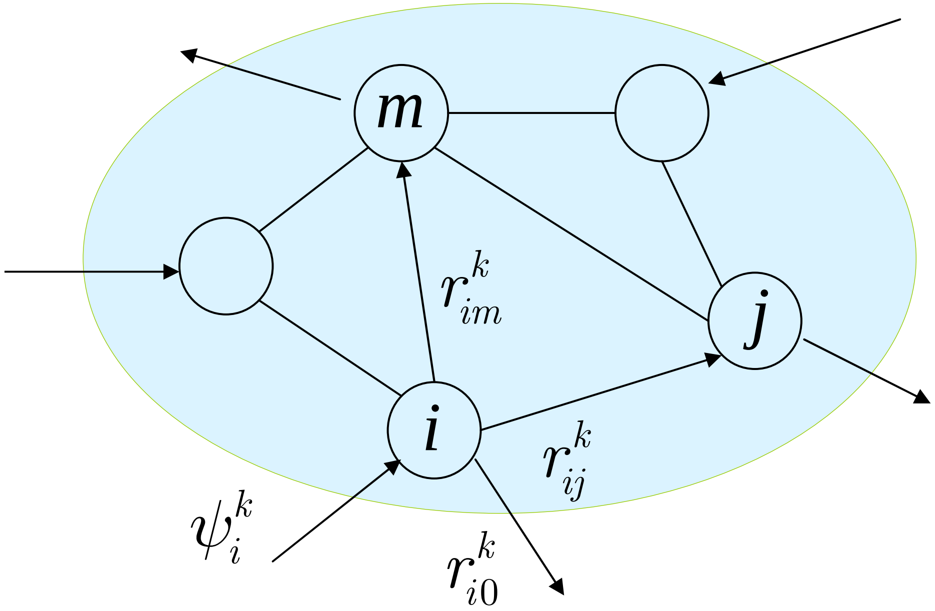
$$\mathcal{A}_n = \{\mathbf{P} = (P_n^1, \ldots, P_n^K) |\ P_n^k \geq 0 \text{ and } \textstyle\sum_{k=1}^K P_n^k \leq \mathbf{P}_\mathbf{n}^\mathbf{max}\}$$

$$\begin{aligned} r_n(\mathbf{P}) &= \sum_{k=1}^K \log_2 \left(1 + \frac{H_{nn}^k P_n^k}{\sigma_n^k + \sum_{m \neq n} H_{mn}^k P_m^k}\right) \\ &= \sum_{k=1}^K \left(\log_2(\sigma_n^k + \sum_{m=1}^N H_{mn}^k P_m^k) - \log_2(\sigma_n^k + \sum_{m \neq n} H_{mn}^k P_m^k)\right). \end{aligned}$$

$f_n^k(\mathbf{a}_{-n})$	$h_n^k(x)$	$g_n^k(\mathbf{a}_{-n})$
$\sum_{m \neq n} \frac{H_{mn}^k}{H_{nn}^k} P_m^k$	$\log_2(\sigma_n^k + H_{nn}^k x)$	$\log_2(\sigma_n^k + \sum_{m \neq n} H_{mn}^k P_m^k)$

Examples of ACSCG (cont'd)

- Delay minimization in Jackson networks



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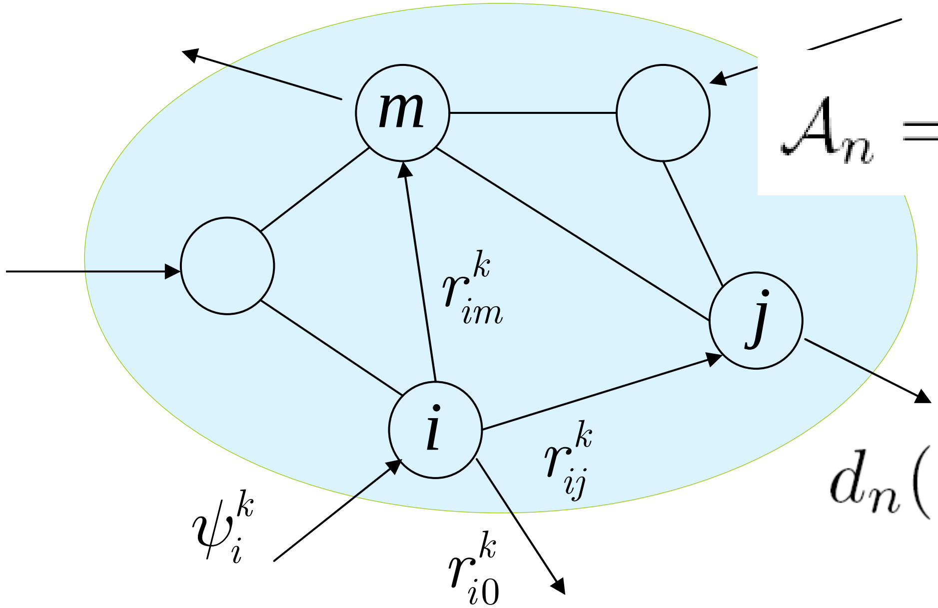

$$\mathcal{A}_n = \{\boldsymbol{\Psi} = (\psi_n^1,\ldots,\psi_n^K) \mid \psi_n^k \ge 0 \text{ and } \sum_{k=1}^K \psi_n^k \ge \boldsymbol{\Psi}_{\mathbf{n}}^{\mathbf{min}}\}$$

$$d_n(\boldsymbol{\Psi}) = \sum_{k=1}^{K} \frac{1}{\mu_n^k - \sum_{m=1}^{N} \upsilon_{mn}^k \psi_m^k}$$

$f_n^k(\mathbf{a}_{-n})$	$h_n^k(x)$	$g_n^k(\mathbf{a}_{-n})$
$\sum_{m \neq n} \frac{\upsilon_{mn}^k}{\upsilon_{nn}^k} \psi_m^k$	$-\frac{1}{\mu_n^k - \upsilon_{nn}^k x}$	0

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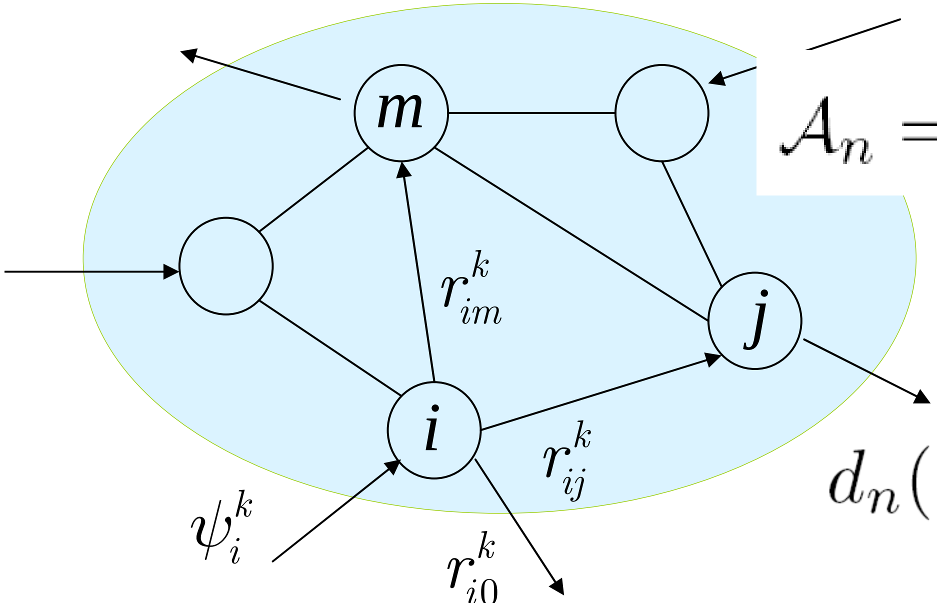

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$$[\mathrm{R}^k]_{mn} = r^k_{nm},\ \Upsilon^k = (\mathrm{I} - \mathrm{R}^k)^{-1},\ \upsilon^k_{mn} = [\Upsilon^k]_{nm}.$$

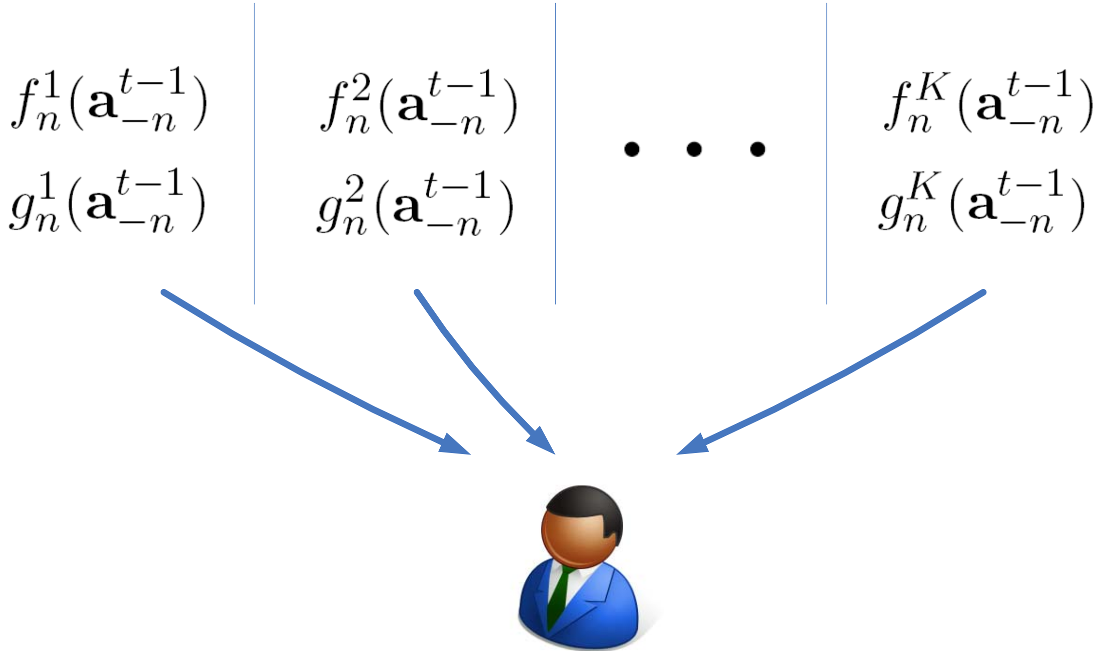
$\sum_{m \neq n} \frac{\upsilon^k_{mn}}{\upsilon^k_{nn}} \psi^k_m$	$-\frac{1}{\mu^k_n - \upsilon^k_{nn} x}$	0

Nash equilibrium in ACSCG

- Existence of pure NE
  - A subclass of concave games
- When is the NE unique? When does best response converges to such a NE?
  - Existing literatures are not immediately applicable
    - Diagonal strict convexity condition [Rosen]
      - Use gradient play and stepsizes need to be carefully chosen
    - Super-modular games [Topkis]
      - Action space is not a lattice
    - Sufficient conditions for specific $h_n^k(\cdot)$ and $f_n^k(\cdot)$ [Yu]

Best response dynamics

- Best response iteration



$f_n^1(\mathbf{a}_{-n}^{t-1})$ $f_n^2(\mathbf{a}_{-n}^{t-1})$ $\cdots$ $f_n^K(\mathbf{a}_{-n}^{t-1})$

$g_n^1(\mathbf{a}_{-n}^{t-1})$ $g_n^2(\mathbf{a}_{-n}^{t-1})$ $g_n^K(\mathbf{a}_{-n}^{t-1})$

$$u_n(\mathbf{a}) = \sum_{k=1}^{K} \left[h_n^k(a_n^k + f_n^k(\mathbf{a}_{-n})) - g_n^k(\mathbf{a}_{-n}) \right]$$

$$\mathcal{A}_n = \left\{ (a_n^1, a_n^2, \cdots, a_n^K) \mid a_n^k \in [a_{n,k}^{\min}, a_{n,k}^{\max}] \text{ and } \sum_{k=1}^{K} a_n^k \le M_n \right\}.$$

Best response dynamics

- Best response iteration

$$a_n^{k,t} \triangleq \left[\left\{ \frac{\partial h_n^k(x)}{\partial x} \right\}^{-1}(\lambda) - f_n^k(\mathbf{a}_{-n}^{t-1}) \right]_{a_{n,k}^{\min}}^{a_{n,k}^{\max}}$$

in which λ is chosen such that

$$\sum_{k=1}^{K} a_n^{k,t} = M_n.$$

Best response dynamics

- Best response iteration

sum constraint

additive coupling

$$a_n^{k,t} \triangleq \left[\left\{ \frac{\partial h_n^k(x)}{\partial x} \right\}^{-1} (\lambda) - f_n^k(\mathbf{a}_{-n}^{t-1}) \right]_{a_{n,k}^{\min}}^{a_{n,k}^{\max}}$$

state

in which λ is chosen such that

$$\sum_{k=1}^{K} a_n^{k,t} = M_n.$$

A competition scenario in which every user aggressively uses up all his resources

Best response dynamics

- Best response iteration

sum constraint

additive coupling

$$a_n^{k,t} \triangleq \left[\left\{ \frac{\partial h_n^k(x)}{\partial x} \right\}^{-1} (\lambda) \; \boxed{-} \; \boxed{f_n^k(\mathbf{a}_{-n}^{t-1})} \right]_{a_{n,k}^{\min}}^{a_{n,k}^{\max}}$$

in which λ is chosen such that

$$\sum_{k=1}^{K} a_n^{k,t} = M_n.$$

- When does it converges?
  - By intuition, the weaker the mutual coupling is, the more likely it converges
  - How to measure and quantify this coupling strength?

A measure of the mutual coupling

Define

$$[\bar{\mathbf{T}}^{\max}]_{mn} \triangleq \begin{cases} \max_{\mathbf{a}\in\mathcal{A},k'} \sum_{k=1}^{K} \left| \frac{\partial f_n^k(\mathbf{a}_{-n})}{\partial a_m^{k'}} \right|, & \text{if } m \neq n \\ 0, & \text{otherwise.} \end{cases}$$

$\max_{\mathbf{a}\in\mathcal{A},k'} \sum_{k=1}^{K} \left| \frac{\partial f_n^k(\mathbf{a}_{-n})}{\partial a_m^{k'}} \right|$ represents the maximum impact that user m’s action can make over user n’s state

Convergence conditions

Theorem 1: If

$$\rho(\bar{\mathbf{T}}^{\max}) < \frac{1}{2},$$

then best response dynamics converges linearly to a unique pure NE for any set of initial conditions.

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Contraction mapping

$$a_n^{k,t} \triangleq \left[\left\{\frac{\partial h_n^k(x)}{\partial x}\right\}^{-1}(\lambda) \boxed{- f_n^k(\mathbf{a}_{-n}^{t-1})}\right]_{a_{n,k}^{\min}}^{a_{n,k}^{\max}}$$

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- The contraction factor $\rho(\bar{\mathbf{T}}^{\max})$ is a measure of the overall coupling strength
- If $f_n^k(\mathbf{a}_{-n})$ is affine, the condition in Theorem 1 is not impacted by M_n; otherwise it may depend on M_n .

$\max_{\mathbf{a}\in\mathcal{A},k'} \sum_{k=1}^{K} \left| \frac{\partial f_n^k(\mathbf{a}_{-n})}{\partial a_m^{k'}} \right|$ is a constant for affine $f_n^k(\mathbf{a}_{-n})$

Convergence conditions

- If $\frac{\partial f_n^k(\mathbf{a}_{-n})}{\partial a_m^{k'}}$ have the same sign, $\forall m \neq n, k, k', \mathbf{a} \in \mathcal{A}$, the condition in Theorem 1 can be relaxed to

$$\rho(\bar{\mathbf{T}}^{\max}) < 1$$

- **This is true in many communication scenarios**
  - Increasing power causes stronger interference
  - Increasing input rate congests the server

Convergence conditions

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$$\rho(\bar{\mathbf{T}}^{\max}) < 1$$

- **This is true in many communication scenarios**
  - Increasing power causes stronger interference
  - Increasing input rate congests the server

Strategic complements (or strategic substitutes)

$$\frac{\partial^2 u_n}{\partial a_n \partial a_m} \geq 0 \text{ (or } \frac{\partial^2 u_n}{\partial a_n \partial a_m} \leq 0)$$

A special class of $h_n^k(\cdot)$

For $\alpha_n^k \in \mathcal{R}$ and $F_{nn}^k > 0$, define [Walrand]

$$h_n^k(x) = \begin{cases} \frac{(\alpha_n^k + F_{nn}^k x)^{\theta+1}}{\theta+1}, & \text{if } -1 < \theta < 0, \\ \log(\alpha_n^k + F_{nn}^k x), & \text{if } \theta = -1. \end{cases}$$

$\theta = -1$: proportional fairness;

$\theta = -2$, harmonic mean fairness;

$\theta = -\infty$, max-min fairness.

A special class of $h_n^k(\cdot)$

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Define

$$[\bar{\mathbf{S}}^{\max}]_{mn} \triangleq \begin{cases} \frac{\sum_{k=1}^{K} (F_{mm}^k)^{1+\frac{1}{\theta}}}{\sum_{k=1}^{K} (F_{nn}^k)^{1+\frac{1}{\theta}}} \max_{\mathbf{a} \in \mathcal{A}, k'} \left\{ \sum_{k=1}^{K} \left| \frac{\partial f_n^k(\mathbf{a}_{-n})}{\partial a_m^{k'}} \right| \left(\frac{F_{nn}^{k'}}{F_{mm}^{k'}} \right)^{1+\frac{1}{\theta}} \right\}, & \text{if } m \neq n \\ 0, & \text{otherwise.} \end{cases}$$

A measure of the similarity between users' parameters F_{nn}^k

$$[\bar{\mathbf{S}}^{\max}]_{mn} \le \zeta_{mn} \cdot [\bar{\mathbf{T}}^{\max}]_{mn}, \quad \zeta_{mn} \in \left[1, \frac{\max_k (F_{nn}^k / F_{mm}^k)^{1+\frac{1}{\theta}}}{\min_k (F_{nn}^k / F_{mm}^k)^{1+\frac{1}{\theta}}} \right].$$

Convergence conditions

Theorem 2: If

$$\rho(\bar{\mathbf{S}}^{\max}) < 1,$$

then best response dynamics converges linearly to a unique pure NE for any set of initial conditions.

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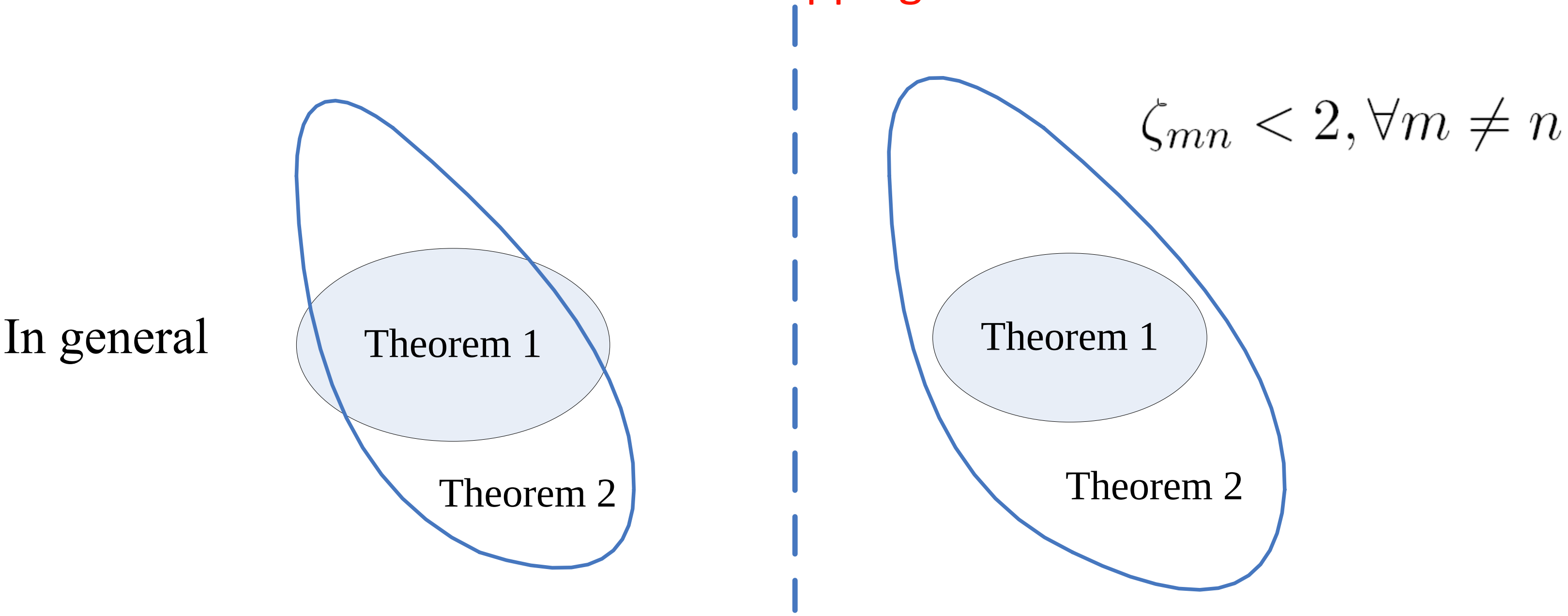
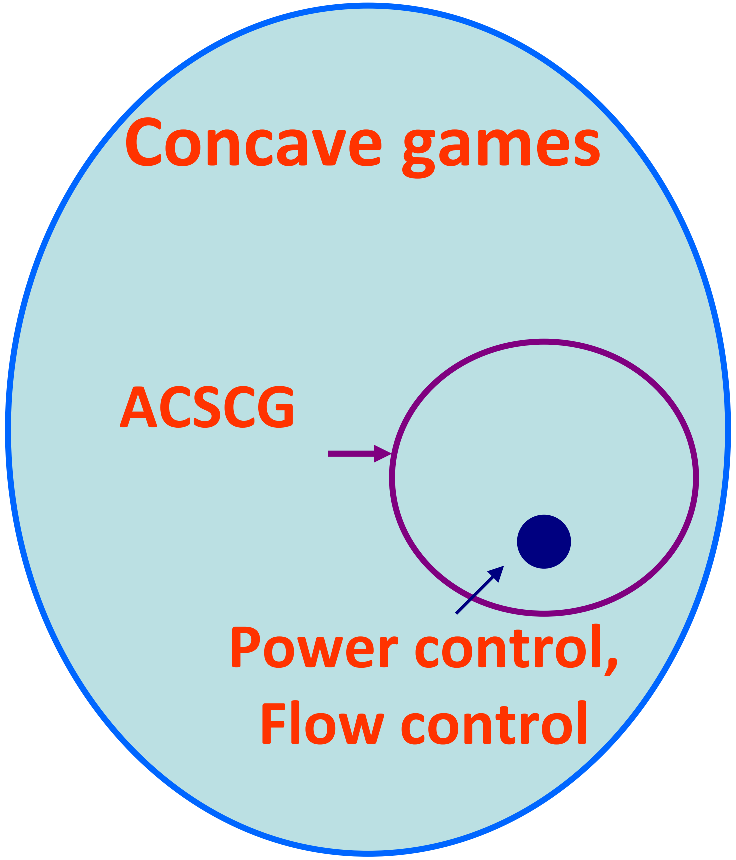
Contraction mapping

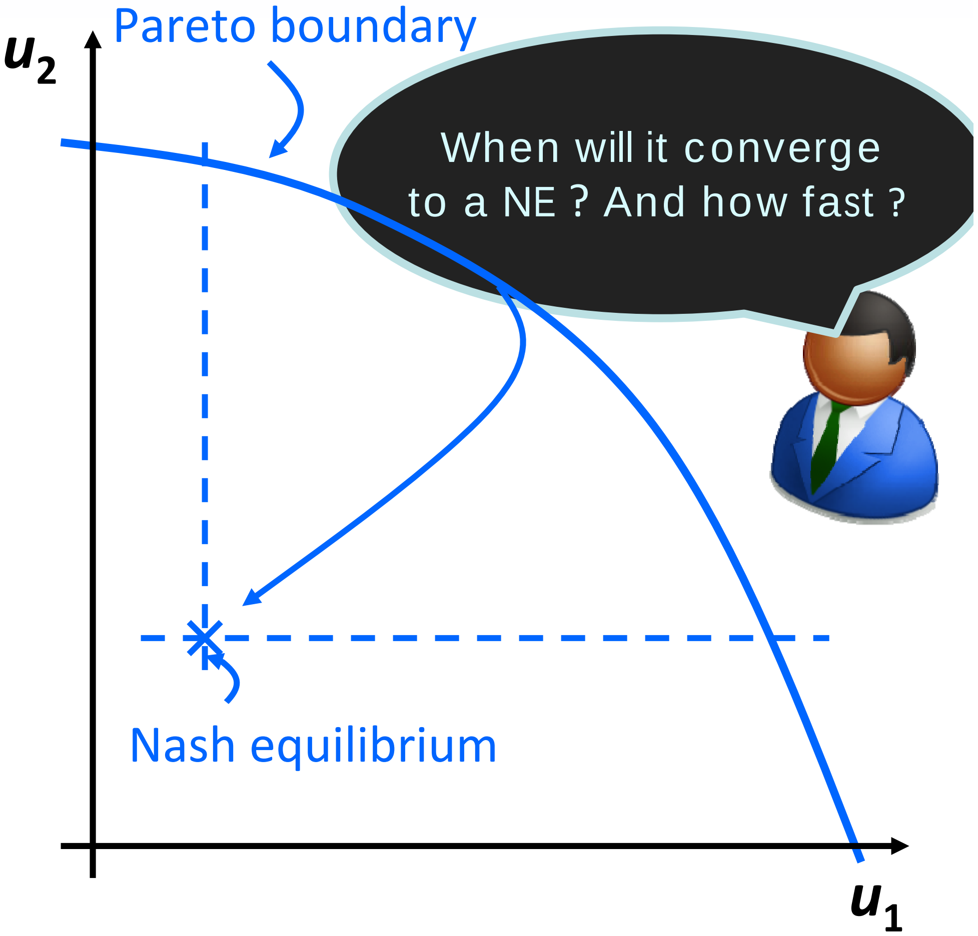


$\zeta_{mn} < 2, \forall m \neq n$

Conclusion so far…

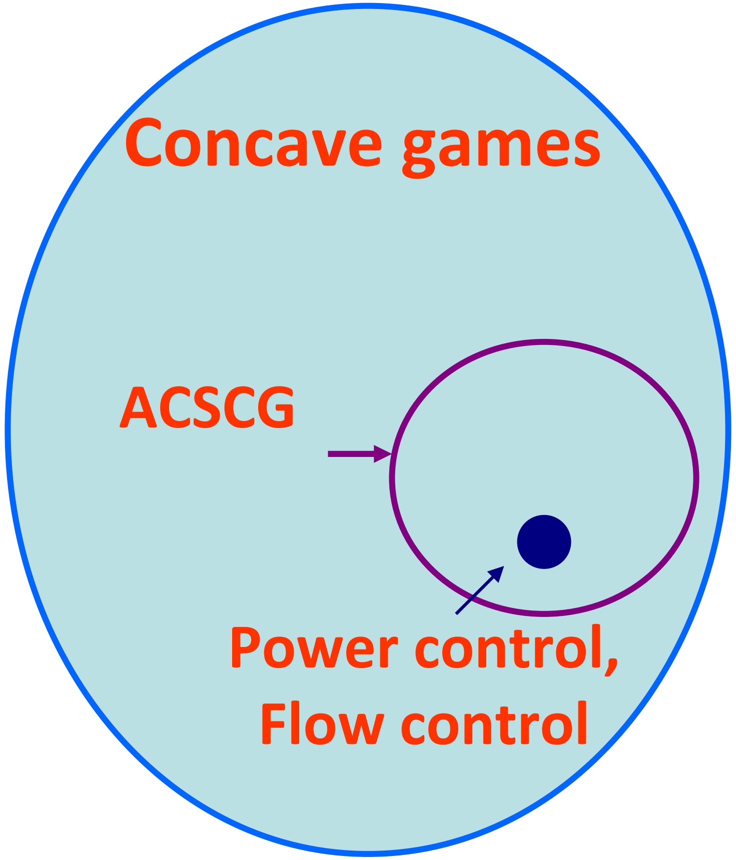
If Information is constrained and no message passing is available…

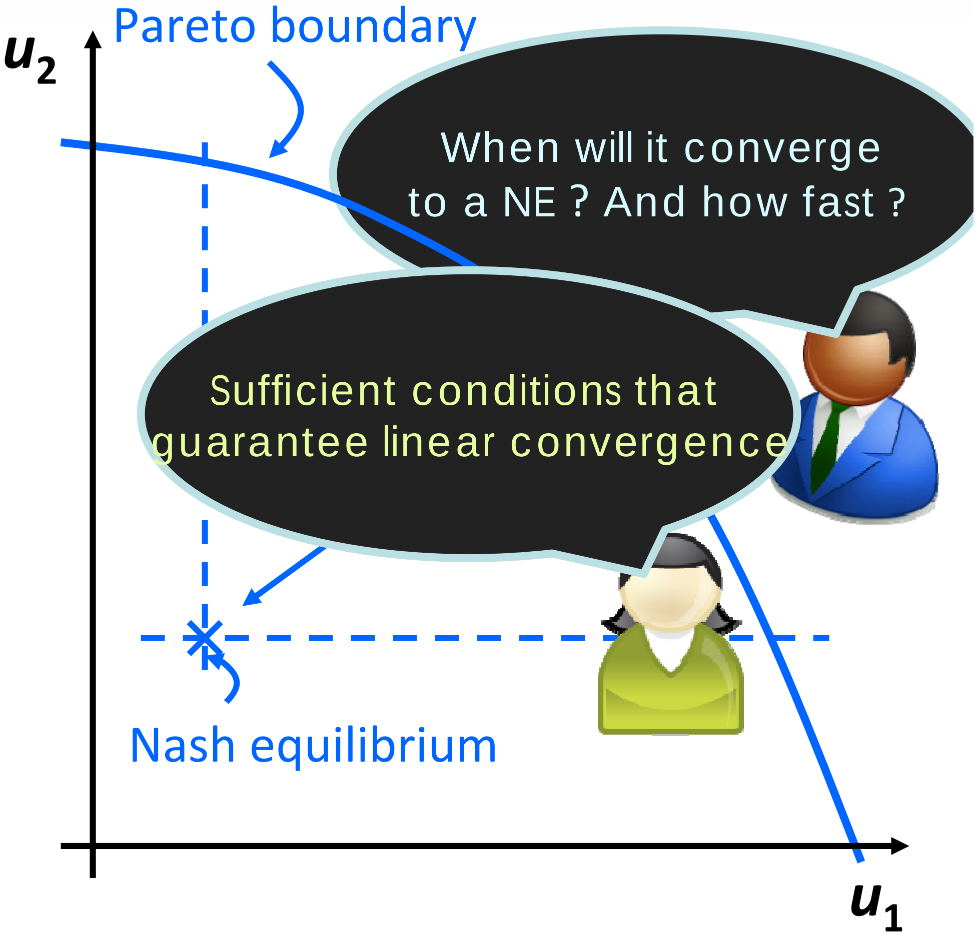




Conclusion so far…

If Information is constrained and no message passing is available…

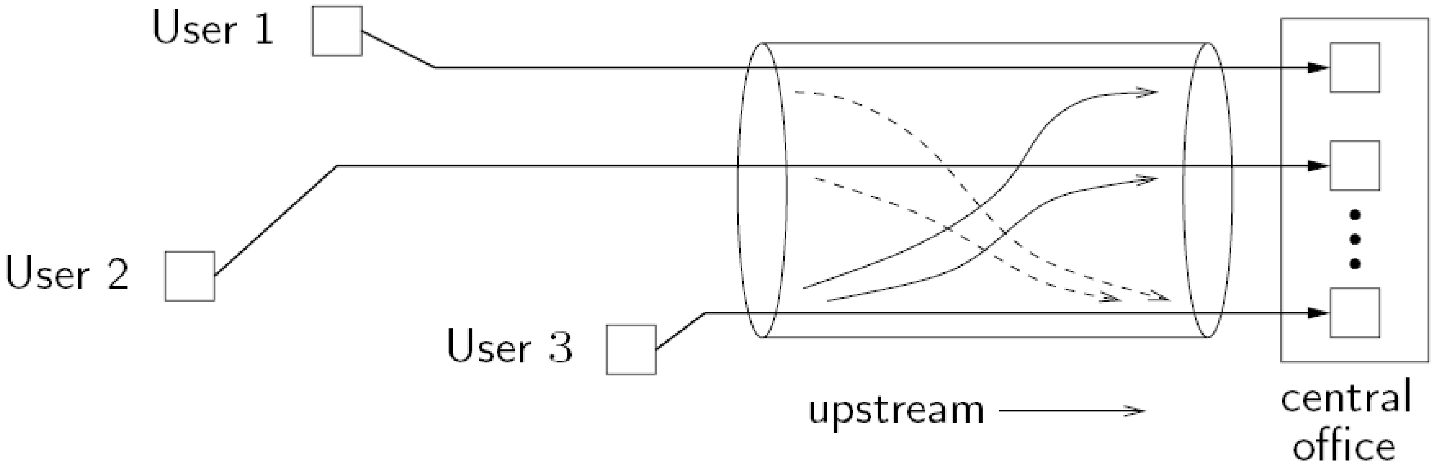




Power control as an ACSCG

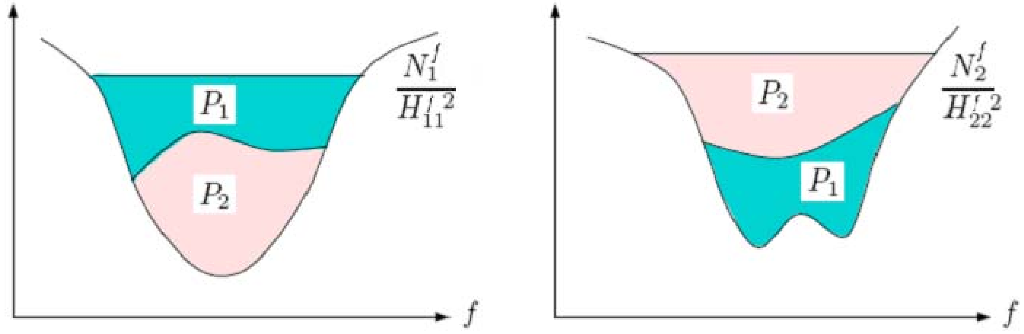
- Power control in interference channels

$$r_n(\mathbf{P}) = \sum_{k=1}^{K} \log_2 \left(1 + \frac{H_{nn}^k P_n^k}{\sigma_n^k + \sum_{m \neq n} H_{mn}^k P_m^k}\right)$$



Performance comparison

- Solutions without information exchange
  - Iterative water-filling algorithm [Yu]



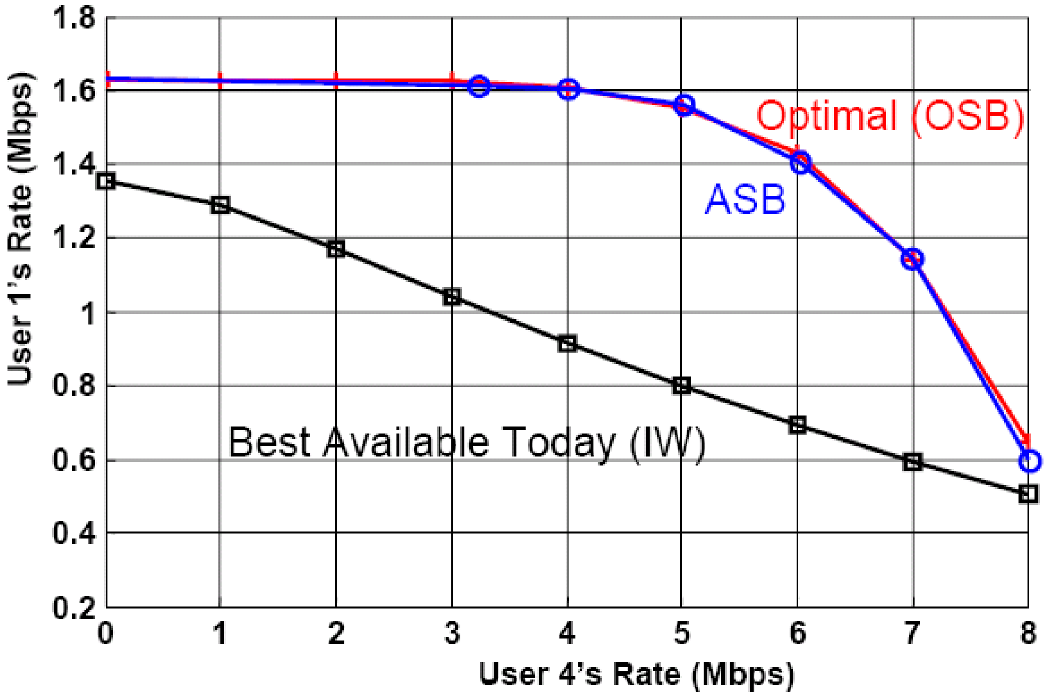
$$P_1^{f(0)} \rightarrow P_2^{f\,(0)} \rightarrow P_1^{f(1)} \rightarrow P_2^{f(1)} \rightarrow \cdots$$

- Solutions with information exchange

$$\max \sum\nolimits_k \omega_k R_k$$

Performance comparison

- Solutions without information exchange
  - Itera



OSB = Optimal Spectrum Balancing

ASB = Autonomous Spectrum Balancing

- Solutic

$$\max \sum_k \omega_k R_k$$

Outline

- Motivation and existing approaches
- Informationally efficient multi-user communication
  - Vector cases
    - Convergence conditions with decentralized information
    - **Improve efficiency with decentralized information**
  - Scalar cases
    - Achieve Pareto efficiency with decentralized information
- Conclusions

How to model the mutual coupling

- A reformulation of the coupling
  - State space $\mathcal{S} = \times_{n \in \mathcal{N}} \mathcal{S}_n$
  - Utility function $u_n : \mathcal{S}_n \times \mathcal{A}_n \to \mathcal{R}$
  - State determination function $s_n : \mathcal{A}_{-n} \to \mathcal{S}_n$
  - **Belief function** $\tilde{s}_n : \mathcal{A}_n \to \mathcal{S}_n$
  - **Conjectural Equilibrium (CE)** : a configuration of belief functions $(\tilde{s}_1^*, \cdots, \tilde{s}_N^*)$ and joint action $a^* = (a_1^*, \cdots, a_N^*)$ satisfying

$$\tilde{s}_n^*(a_n^*) = s_n(\mathbf{a}_{-n}^*) \qquad \text{and } a_n^* = \arg\max_{a_n \in \mathcal{A}_n} u_n\left(\tilde{s}_n^*(a_n), a_n\right)$$

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beliefs are realized

each user behaves optimally according to its expectation

CE in power control games [SuTSP'09]

- One leader and multiple followers
- State space
  - I_n^k : the interference caused to user *n* in channel *k*
- Utility function

$$R_n = \sum_{k=1}^{K} \log_2 \left(1 + \frac{P_n^k}{\sigma_n^k + I_n^k} \right)$$

- State determination function

$$I_n^k = \sum\nolimits_{i=1, i \neq n}^{N} \alpha_{in}^k P_i^k$$

actual play

- Belief function (linear form)

$$\tilde{I}_1^k = \beta^k - \gamma^k P_1^k$$

conceived play

Why Linear belief?

$\frac{\partial I_1^k}{\partial P_1^k}$ is piece-wise linear; $\frac{\partial I_1^k}{\partial P_1^j} = 0,\ j \neq k$, if the number of frequency bins is sufficiently large.

→ Linear belief is sufficient to capture the interference coupling!

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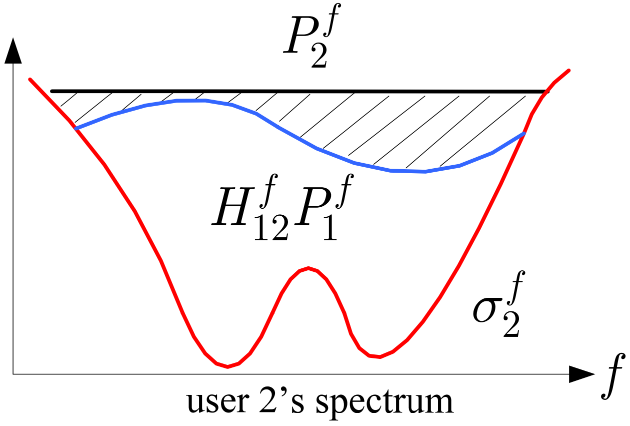

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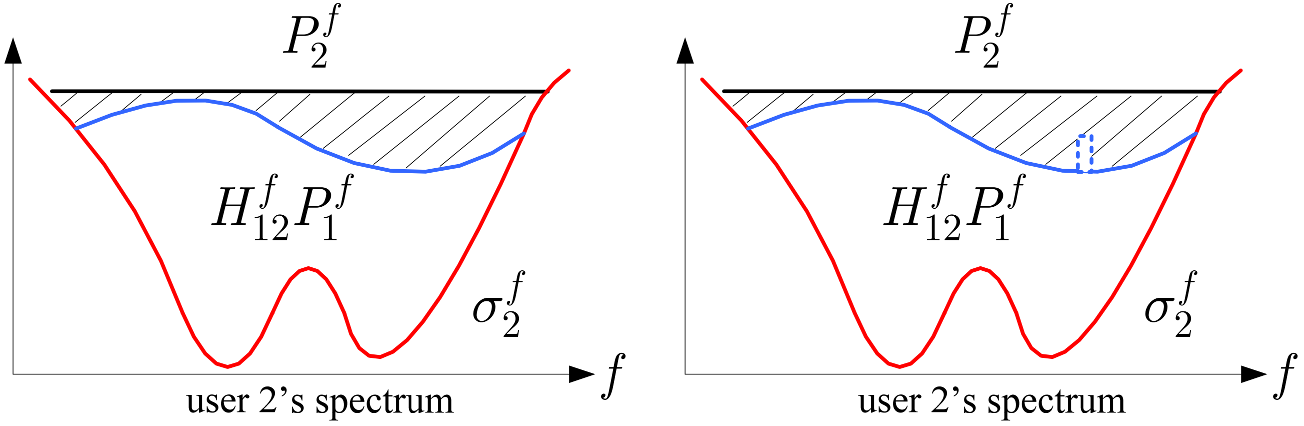

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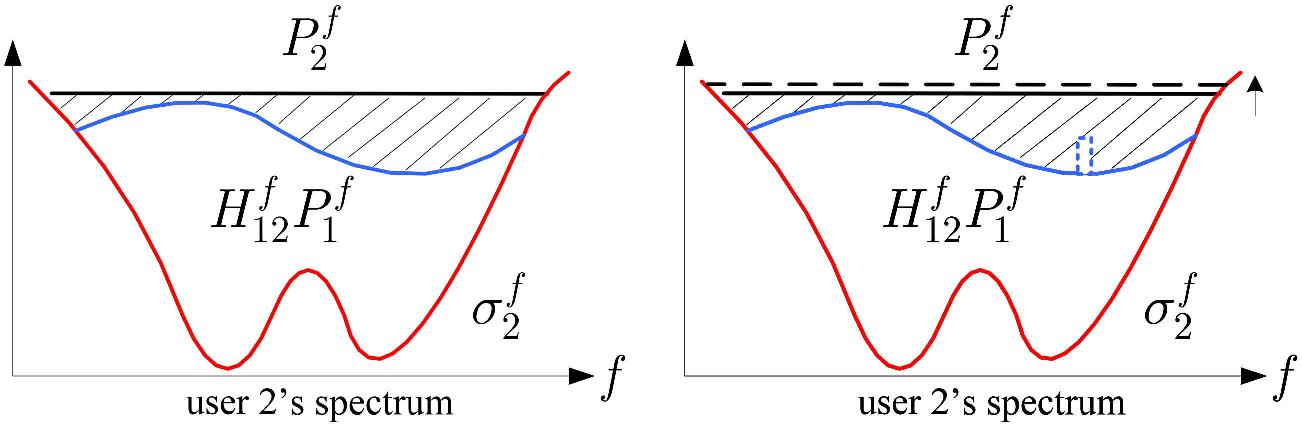

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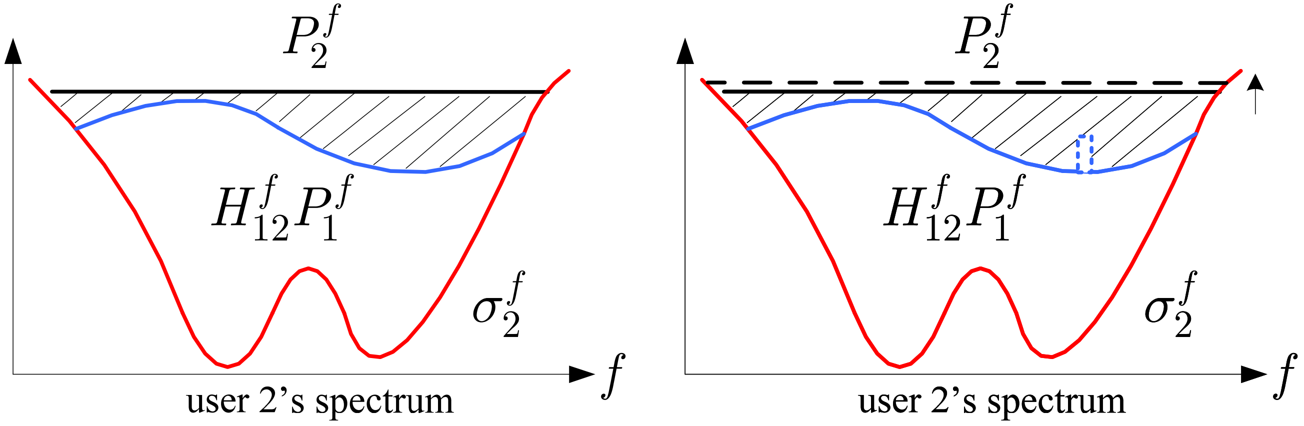


$$P_n^k(\mathbf{P}_{-n}^{t-1}) = \left[\frac{1}{\lambda} - \frac{\sigma_n^k}{H_{nn}^k} - \sum_{m \neq n} \frac{H_{mn}^k}{H_{nn}^k} P_m^{k,t-1}\right]_0^{+\infty}$$

Main results

- Stackelberg equilibrium
  - Strategy profile $\left(a_1^*, NE\left(a_1^*\right)\right)$ that satisfies

$$u_1\left(a_1^*, NE\left(a_1^*\right)\right) \geq u_1\left(a_1, NE\left(a_1\right)\right), \forall a_1 \in \mathcal{A}_1$$

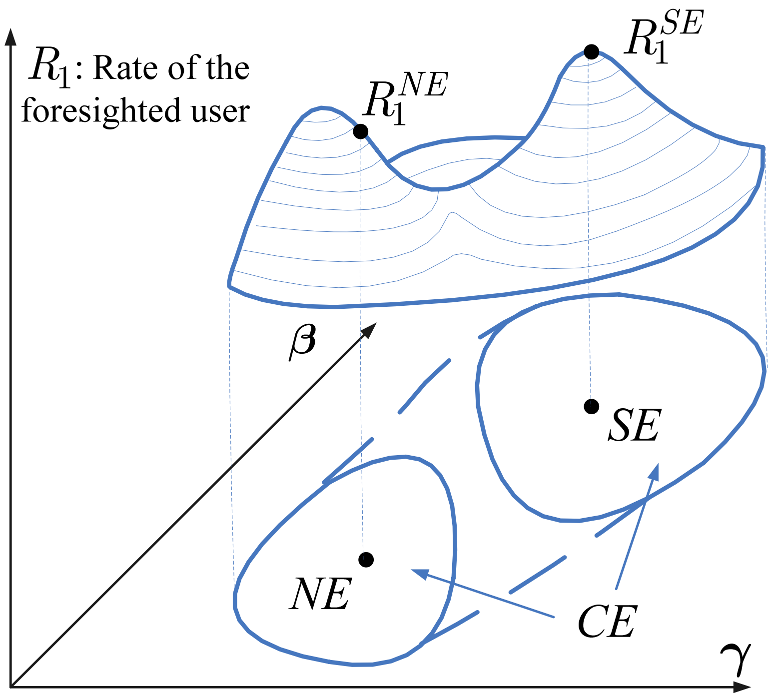
- NE and SE are special CE

NE: $\beta^k = \sum_{i=2}^{N} \alpha_{i1}^k P_i^k, \gamma^k = 0$

SE: $\beta^k = I_1^k - P_1^k \cdot \frac{\partial I_1^k}{\partial P_1^k}, \gamma^k = -\frac{\partial I_1^k}{\partial P_1^k}.$

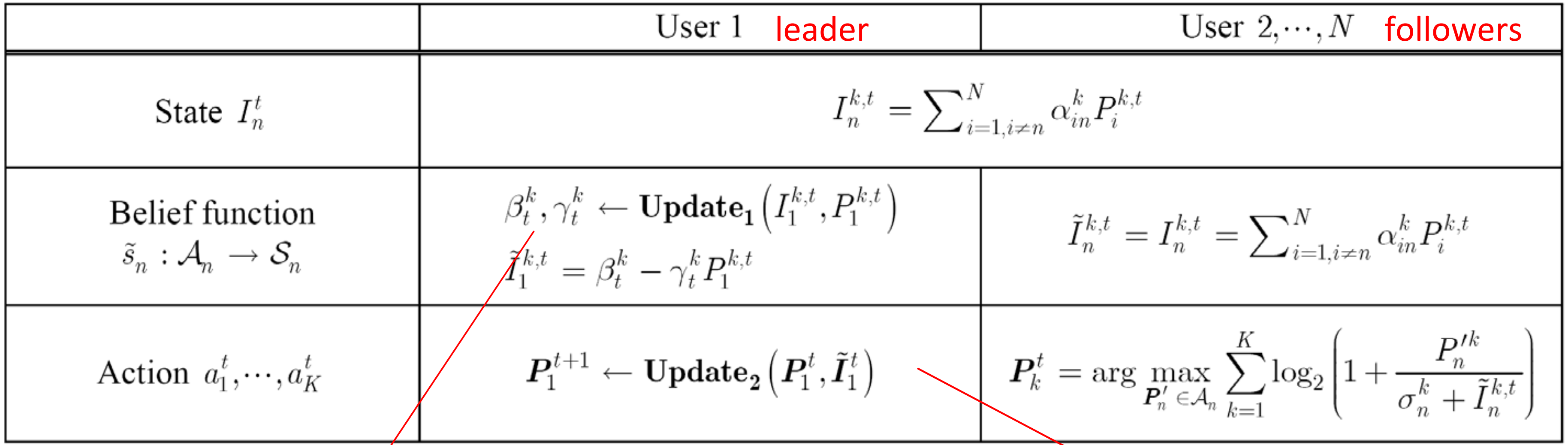
- Infinite set of CE

Open sets of CE that contain NE and SE may exist



Achieving the desired CE

- Conjecture-based rate maximization (CRM)



	User 1 leader	User $2,\cdots,N$ followers
State I_n^t	$I_n^{k,t} = \sum_{i=1,i\neq n}^{N} \alpha_{in}^k P_i^{k,t}$	
Belief function $\tilde{s}_n : \mathcal{A}_n \rightarrow \mathcal{S}_n$	$\beta_t^k, \gamma_t^k \leftarrow \mathbf{Update_1}\left(I_1^{k,t}, P_1^{k,t}\right)$ $\tilde{I}_1^{k,t} = \beta_t^k - \gamma_t^k P_1^{k,t}$	$\tilde{I}_n^{k,t} = I_n^{k,t} = \sum_{i=1,i\neq n}^{N} \alpha_{in}^k P_i^{k,t}$
Action $a_1^t,\cdots,a_K^t$	$\boldsymbol{P}_1^{t+1} \leftarrow \mathbf{Update_2}\left(\boldsymbol{P}_1^t, \tilde{\boldsymbol{I}}_1^t\right)$	$\boldsymbol{P}_k^t = \arg\max_{\boldsymbol{P}_n' \in \mathcal{A}_n} \sum_{k=1}^{K} \log_2\left(1 + \frac{P_n'^k}{\sigma_n^k + \tilde{I}_n^{k,t}}\right)$

Dynamic updates of the play

$$\beta_t^k = \left(I_1^k - P_1^k \cdot \frac{\partial I_1^k}{\partial P_1^k}\right)\Bigg|_{\boldsymbol{P}_1=\boldsymbol{P}_1^t} \text{ and } \gamma_t^k = -\frac{\partial I_1^k}{\partial P_1^k}\Bigg|_{\boldsymbol{P}_1=\boldsymbol{P}_1^t}$$

$$\max_{\{P_1^k\}} \sum_{k=1}^{K} \log_2\left(1 + \frac{P_1^k}{\sigma_1^k + \beta_t^k - \gamma_t^k P_1^k}\right)$$

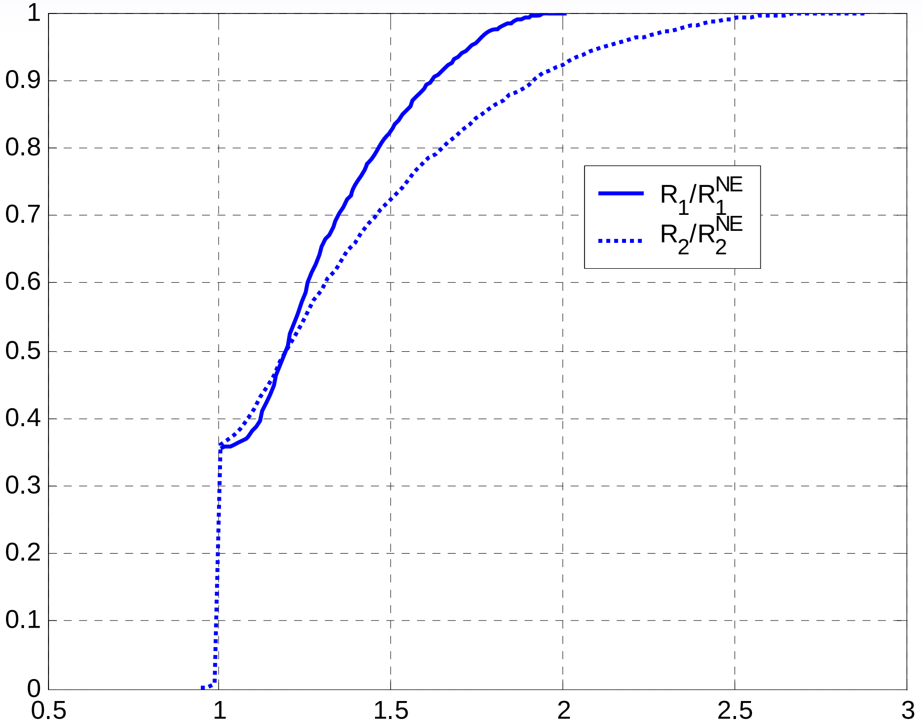
↓

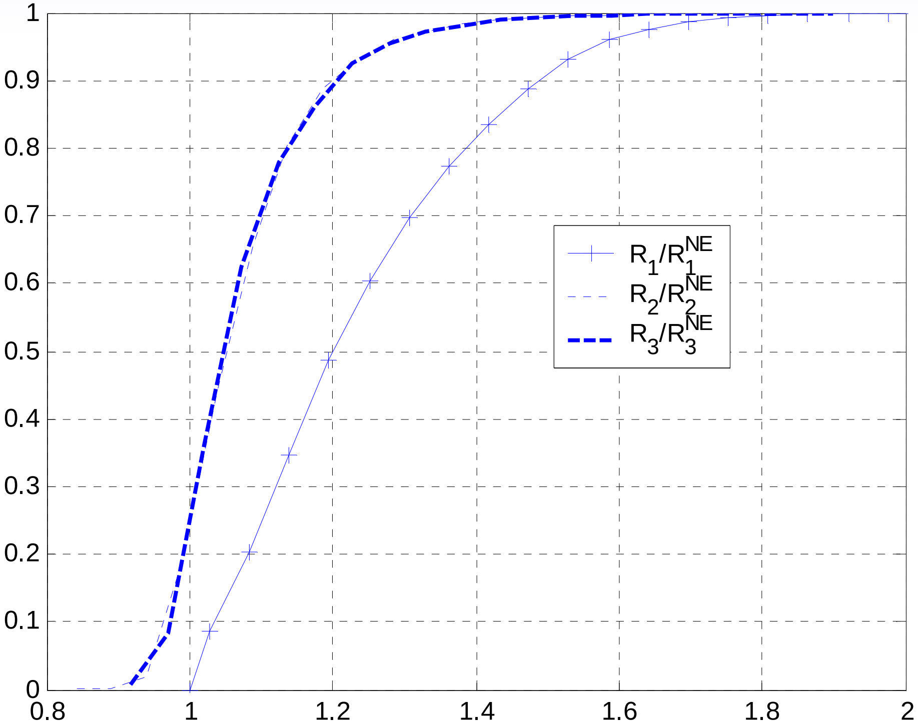
solvable using dual method

Discussion about CRM

- Essence of CRM
  - local approximation of the computation of SE
- Advantages
  - the structure of the utility function is explored
  - only local information is required
  - it can be applied in the cases where N>2
  - if it converges, the outcome is a CE

Simulation results





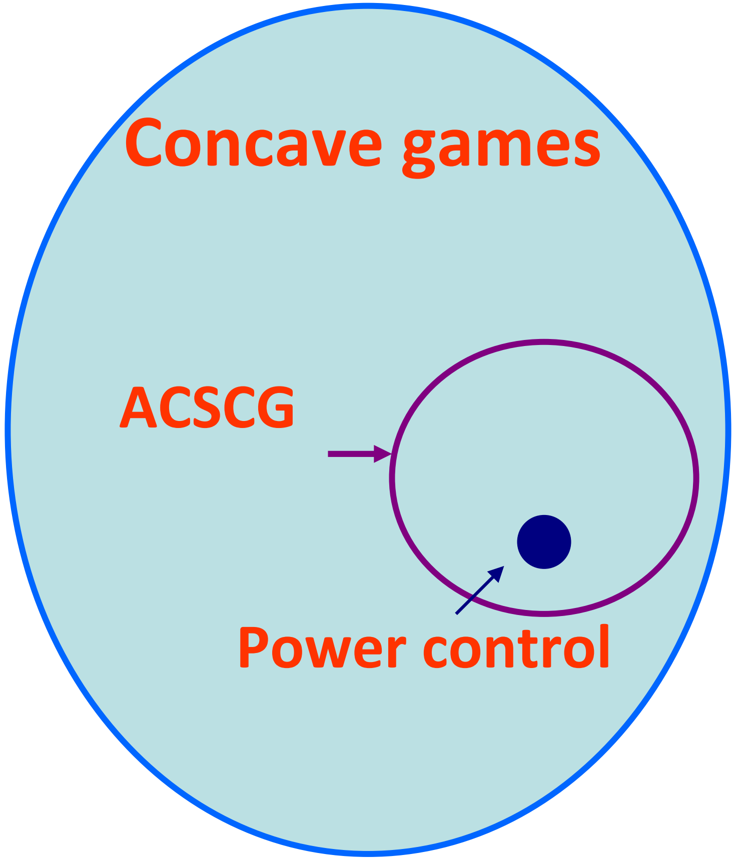
Average rate improvements:

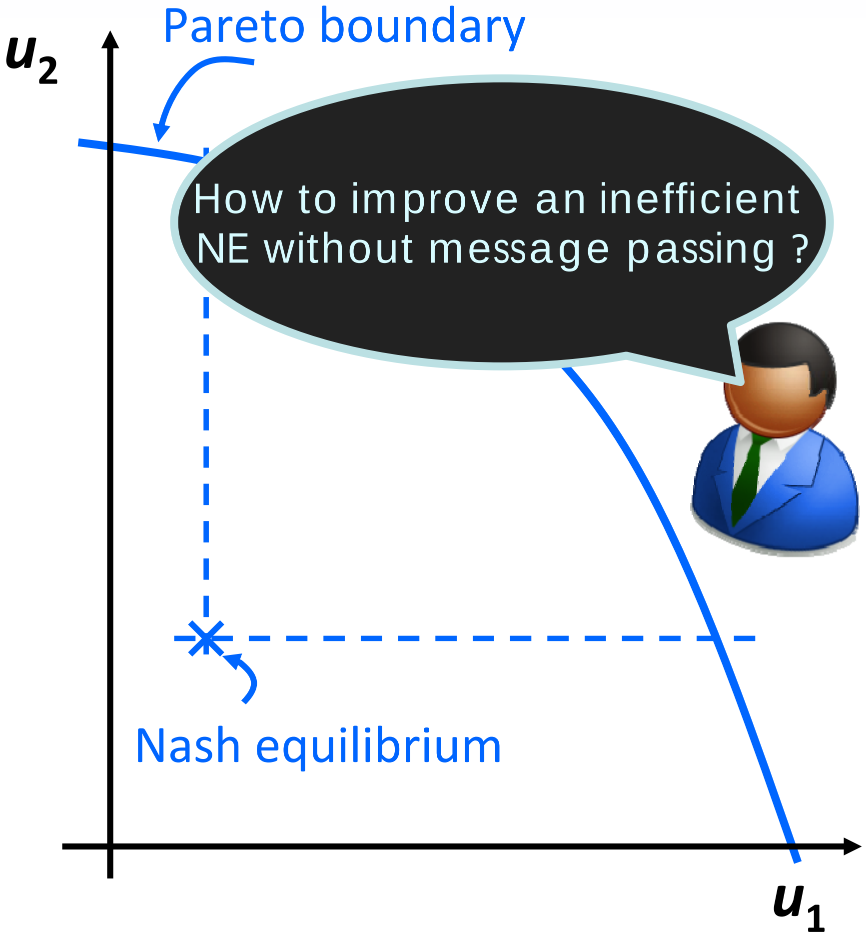
2-user case: 24.4% for user 1; 33.6% for user 2 $\left(\sum_k \left|\alpha_{ij}^k\right|^2 = 0.5,\ i \neq j\right)$

3-user case: 26.3% for user 1; 9.7% for user 2&3 $\left(\sum_k \left|\alpha_{ij}^k\right|^2 = 0.33,\ i \neq j\right)$

Conclusions so far…

If Information is constrained and no message passing is allowed





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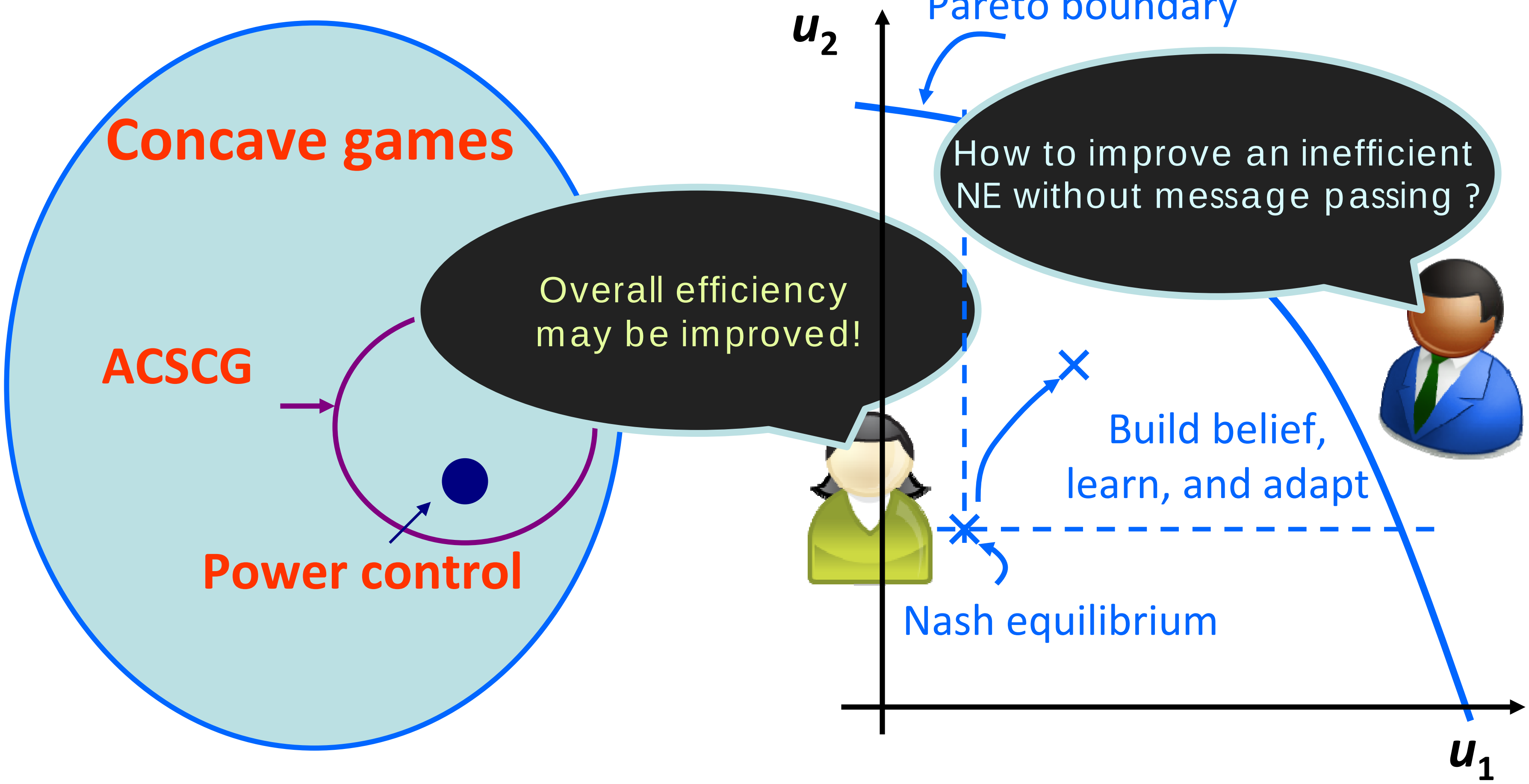

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Linearly coupled games

- A non-cooperative game model
- Users' states are linearly impacted by their competitor's actions
- Contributions
  - Characterize the structures of the utility functions
  - Explicitly compute Nash equilibrium and Pareto boundary
  - A conjectural equilibrium approach to achieve Pareto boundary without real-time information exchange

Definition

A multi-user interaction is considered a *linearly coupled game* if the action set $\mathcal{A}_n \subseteq \mathcal{R}_+$ is convex and the utility function u_n satisfies

$$u_n(\mathbf{a}) = a_n^{\beta_n} \cdot s_n(\mathbf{a}),$$

in which $\beta_n > 0$. In particular, the basic assumptions about $s_n(\mathbf{a})$ include:

States are linearly impacted by actions

A1: $s_n(\mathbf{a})$ is non-negative;

A2: $s_n(\mathbf{a})$ is strictly linearly decreasing in $a_m, \forall m \neq n$;

$s_n(\mathbf{a})$ is non-increasing and linear in a_n.

Definition (cont'd)

Denote $s'_{nm}(\mathbf{a}) = \frac{\partial s_n(\mathbf{a})}{\partial a_m}$.

A3: $\frac{s_n(\mathbf{a})}{s'_{nm}(\mathbf{a})}$ is an affine function, $\forall n \in \mathcal{N} \setminus \{m\}$.

A4: $$\frac{s'_{nm}(\mathbf{a})}{s_n(\mathbf{a})} = \frac{s'_{km}(\mathbf{a})}{s_k(\mathbf{a})}, \forall n, k \in \mathcal{N} \setminus \{m\};$$

$$\frac{s'_{mm}(\mathbf{a})}{s_m(\mathbf{a})} = 0 \text{ or } \frac{s'_{nm}(\mathbf{a})}{s_n(\mathbf{a})}, \ \forall n \neq m.$$

↓

Actions are linearly coupled at NE and PB

Two basic types

- For the games satisfying A1-A4, the utility functions can take two types of form:
  - Type I [SuJSAC'10]

$$u_n(\mathbf{a}) = a_n^{\beta_n} \cdot \prod_{m \neq n} (\mu_m - \tau_m a_m)$$

- e.g. random access $u_n(\mathbf{p}) = p_n \prod_{m \neq n}(1 - p_m)$

Two basic types

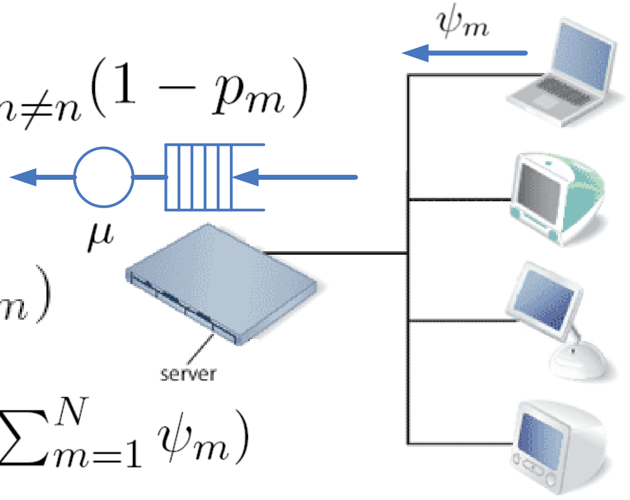
- For the games satisfying A1-A4, the utility functions can take two types of form:
  - Type I [SuJSAC'10]

$$u_n(\mathbf{a}) = a_n^{\beta_n} \cdot \prod_{m \neq n} (\mu_m - \tau_m a_m)$$

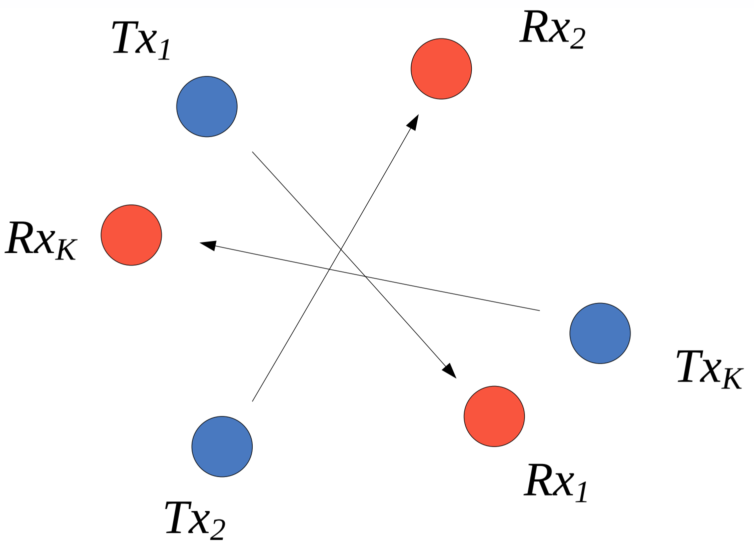
    - e.g. random access $u_n(\mathbf{p}) = p_n \prod_{m \neq n}(1 - p_m)$
  - Type II [SuTR'09]

$$u_n(\mathbf{a}) = a_n^{\beta_n} \cdot (\mu - \sum_{m=1}^{N} \tau_m a_m)$$

    - e.g. rate control $u_n(\psi) = \psi_n^{\beta_n}(\mu - \sum_{m=1}^{N} \psi_m)$



Type I games: wireless random access



- Player set:
  - nodes in a single cell
- Action set:
  - transmission probability
- Payoff:
  - throughput $u_k(\mathbf{p}) = p_k \prod_{i \in \mathcal{K} \setminus \{k\}} (1 - p_i).$
- Key issues
  - stability, convergence, throughput, and fairness

Conjecture-based Random Access

- Individual conjectures

  actual play

  - state: $s_k = \prod_{i\in\mathcal{K}\setminus\{k\}}(1-p_i)$

  conceived play

  - linear belief: $u_k^t(\tilde{s}_k^t(p_k),p_k) = p_k\Big[\prod_{i\in\mathcal{K}\setminus\{k\}}(1-p_i^{t-1}) - a_k(p_k - p_k^{t-1})\Big]$
- Two update mechanisms
  - Best response

$$p_k^t = \arg\max_{p_k\in P_k} u_k^t(\tilde{s}_k^t(p_k),p_k) = \min\left\{\frac{p_k^{t-1}}{2} + \frac{\prod_{i\in\mathcal{K}\setminus\{k\}}(1-p_i^{t-1})}{2a_k}, 1\right\}$$

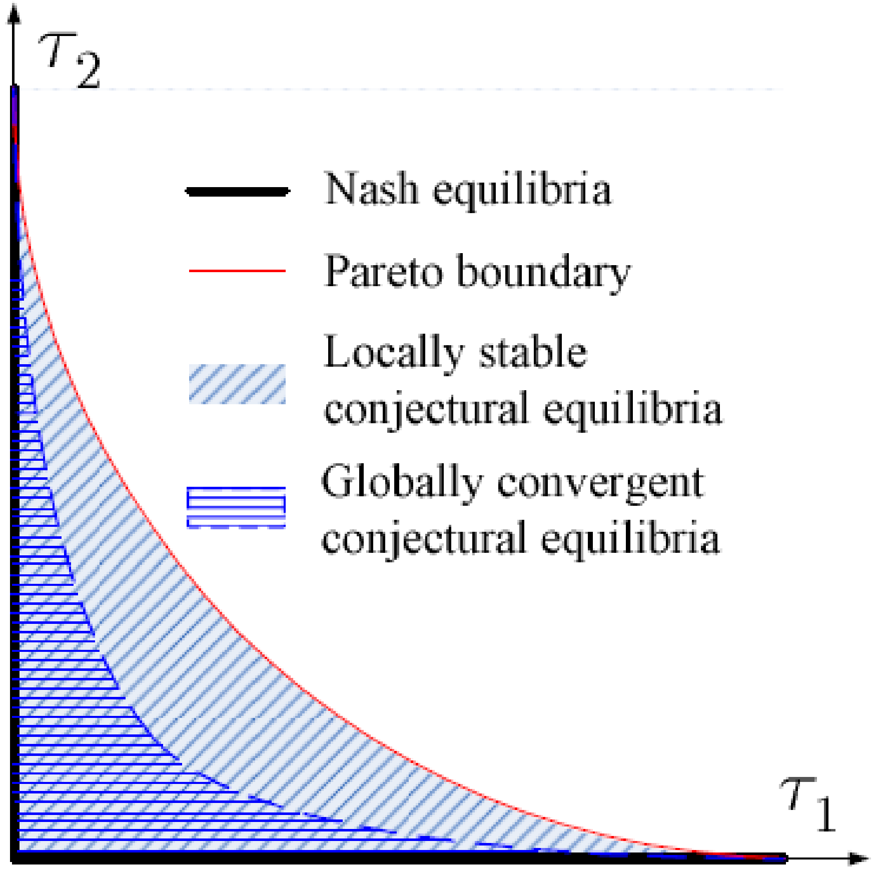
  - Gradient play

$$p_k^t = p_k^{t-1} + \gamma_k \frac{\partial u_k^t(\tilde{s}_k^t(p_k))}{\partial p_k}\bigg|_{p_k=p_k^{t-1}}.$$

Main results

Protocol design: how to achieve efficient outcomes?

- Existence of CE
  - all operating points in action space are CE
- Stability and convergence
  - sufficient conditions
- Throughput performance
  - the entire throughput region can be achieved with stable CE
- Fairness issue
  - conjecture-based approaches attain weighted fairness



How to select suitable a_k?

- Adaptively alter a_k when the network size changes
- Adopt aggregated throughput or “idle interval” as the indicator of the system efficiency
- Advantages
  - No need of a centralized solver
  - Throughput efficient with fairness guarantee
  - Stable equilibrium
  - Autonomously adapt to traffic fluctuation

Engineering interpretation

- DCF vs. the best response update
  - re-design the random access protocol

$$p_k^t = \arg\max_{p_k \in P_k} u_k^t(\tilde{s}_k^t(p_k), p_k) = \min\left\{\frac{p_k^{t-1}}{2} + \frac{\prod_{i\in\mathcal{K}\setminus\{k\}}(1-p_i^{t-1})}{2a_k}, 1\right\}$$

$$\big\downarrow$$

$$\begin{aligned} p_k^t = {} & \frac{1}{2}\mathrm{E}\{p_k^{t-1}\mathbf{1}_{\{T_{-k}^{t-1}=1\}}|\mathbf{p}^{t-1}\} + \frac{1}{2a_k}\mathrm{E}\{\mathbf{1}_{\{T_{-k}^{t-1}=0\}}\mathbf{1}_{\{T_k^{t-1}=0\}}|\mathbf{p}^{t-1}\} \\ & + \frac{1}{2}(1+\frac{1}{a_k})\mathrm{E}\{\mathbf{1}_{\{T_{-k}^{t-1}=0\}}\mathbf{1}_{\{T_k^{t-1}=1\}}|\mathbf{p}^{t-1}\} \end{aligned}$$

Engineering interpretation

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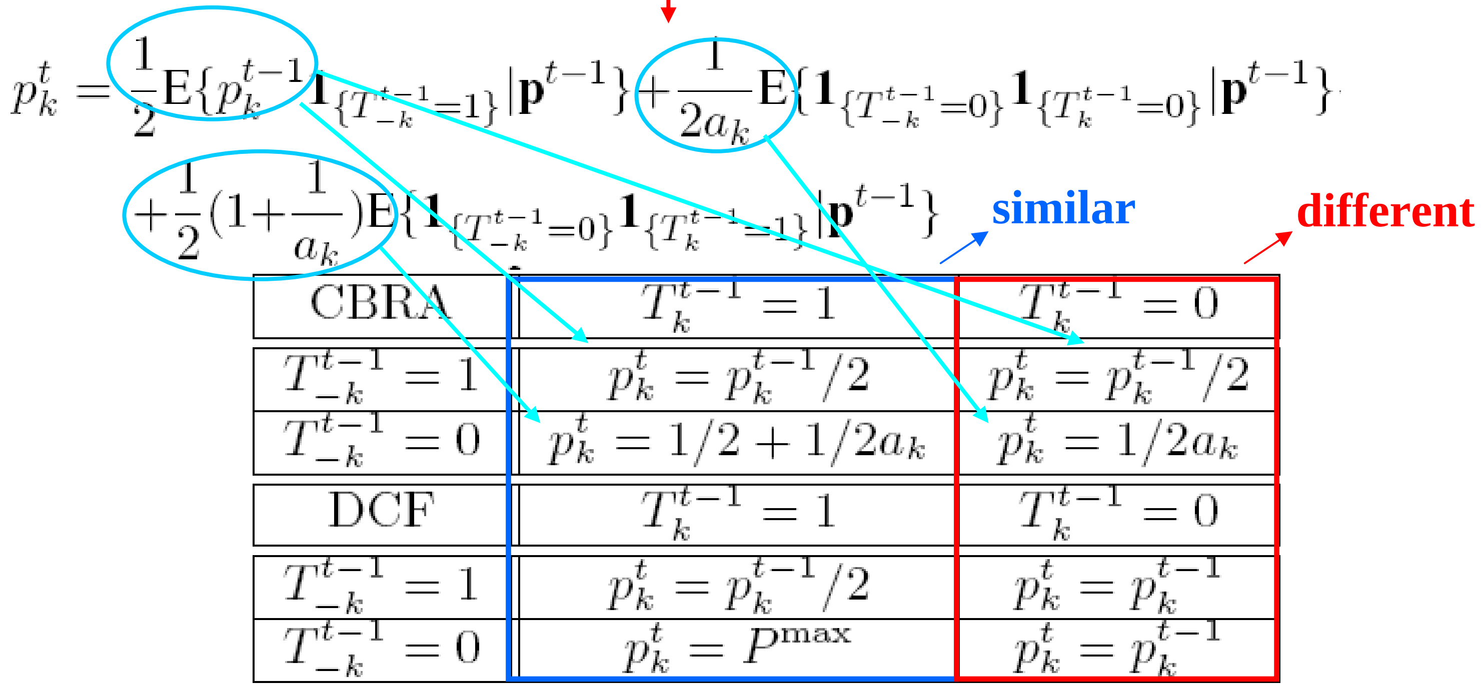
$$p_k^t = \arg\max_{p_k\in P_k} u_k^t(\tilde{s}_k^t(p_k),p_k) = \min\left\{\frac{p_k^{t-1}}{2}+\frac{\prod_{i\in\mathcal{K}\setminus\{k\}}(1-p_i^{t-1})}{2a_k},1\right\}$$



$$p_k^t = \frac{1}{2}\mathrm{E}\{p_k^{t-1}\mathbf{1}_{\{T_{-k}^{t-1}=1\}}|\mathbf{p}^{t-1}\}+\frac{1}{2a_k}\mathrm{E}\{\mathbf{1}_{\{T_{-k}^{t-1}=0\}}\mathbf{1}_{\{T_k^{t-1}=0\}}|\mathbf{p}^{t-1}\}$$
$$+\frac{1}{2}(1+\frac{1}{a_k})\mathrm{E}\{\mathbf{1}_{\{T_{-k}^{t-1}=0\}}\mathbf{1}_{\{T_k^{t-1}=1\}}|\mathbf{p}^{t-1}\}$$

similar

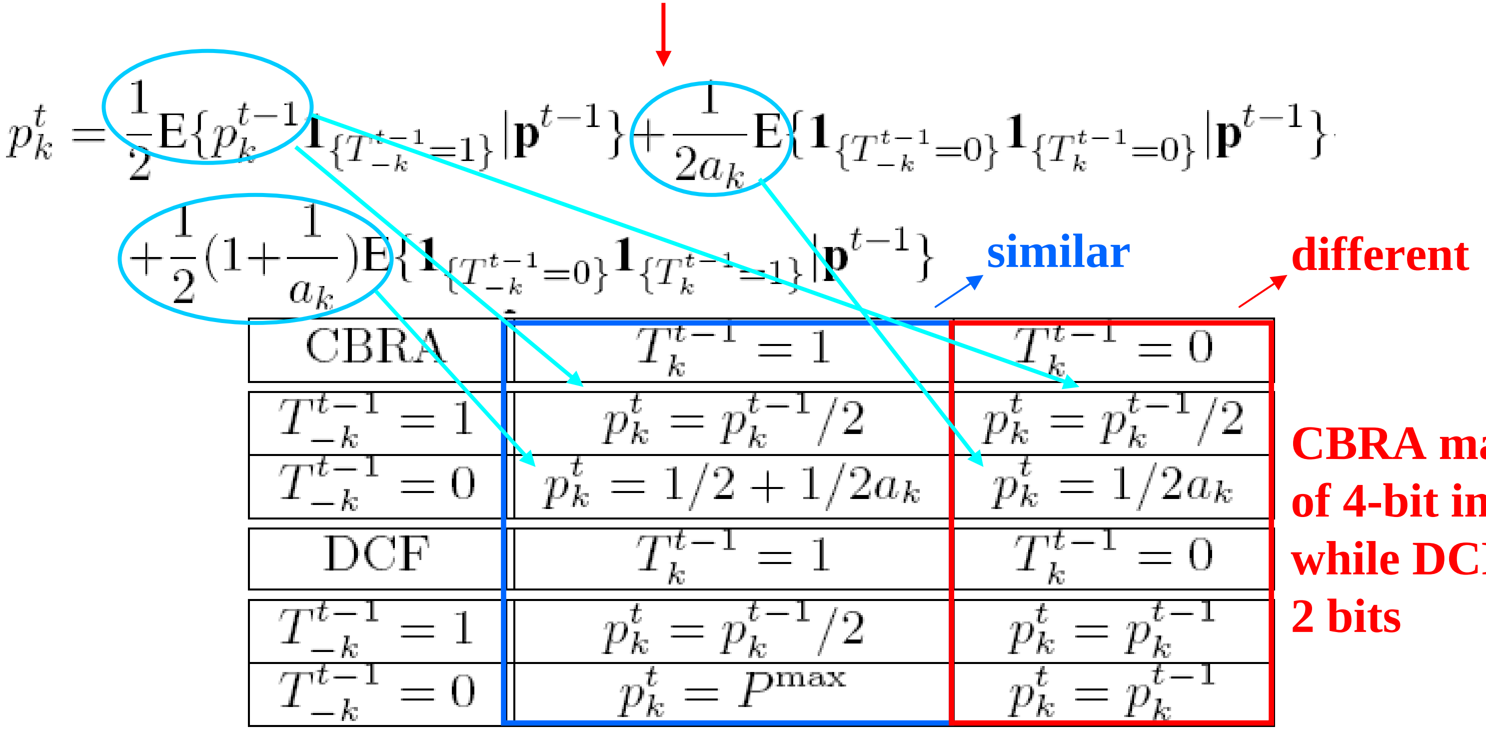
different

CBRA	$T_k^{t-1}=1$	$T_k^{t-1}=0$
$T_{-k}^{t-1}=1$	$p_k^t=p_k^{t-1}/2$	$p_k^t=p_k^{t-1}/2$
$T_{-k}^{t-1}=0$	$p_k^t=1/2+1/2a_k$	$p_k^t=1/2a_k$
DCF	$T_k^{t-1}=1$	$T_k^{t-1}=0$
$T_{-k}^{t-1}=1$	$p_k^t=p_k^{t-1}/2$	$p_k^t=p_k^{t-1}$
$T_{-k}^{t-1}=0$	$p_k^t=P^{\max}$	$p_k^t=p_k^{t-1}$

Engineering interpretation

- DCF vs. the best response update
  - re-design the random access protocol

$$p_k^t = \arg\max_{p_k \in P_k} u_k^t(\tilde{s}_k^t(p_k), p_k) = \min\left\{\frac{p_k^{t-1}}{2} + \frac{\prod_{i\in\mathcal{K}\setminus\{k\}}(1-p_i^{t-1})}{2a_k}, 1\right\}$$



$$p_k^t = \frac{1}{2}\mathrm{E}\{p_k^{t-1}\mathbf{1}_{\{T_{-k}^{t-1}=1\}}|\mathbf{p}^{t-1}\} + \frac{1}{2a_k}\mathrm{E}\{\mathbf{1}_{\{T_{-k}^{t-1}=0\}}\mathbf{1}_{\{T_k^{t-1}=0\}}|\mathbf{p}^{t-1}\}$$
$$+\frac{1}{2}(1+\frac{1}{a_k})\mathrm{E}\{\mathbf{1}_{\{T_{-k}^{t-1}=0\}}\mathbf{1}_{\{T_k^{t-1}=1\}}|\mathbf{p}^{t-1}\}$$

similar

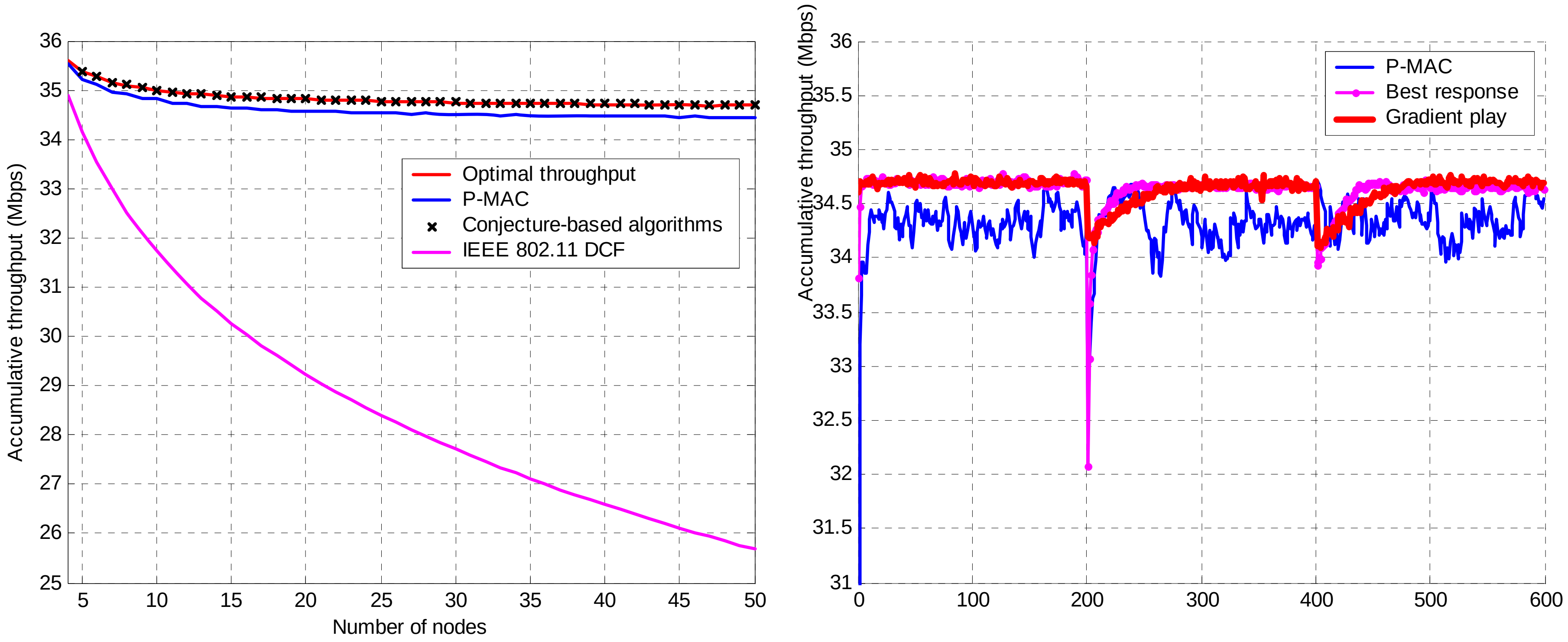
different

CBRA	$T_k^{t-1} = 1$	$T_k^{t-1} = 0$
$T_{-k}^{t-1} = 1$	$p_k^t = p_k^{t-1}/2$	$p_k^t = p_k^{t-1}/2$
$T_{-k}^{t-1} = 0$	$p_k^t = 1/2 + 1/2a_k$	$p_k^t = 1/2a_k$
DCF	$T_k^{t-1} = 1$	$T_k^{t-1} = 0$
$T_{-k}^{t-1} = 1$	$p_k^t = p_k^{t-1}/2$	$p_k^t = p_k^{t-1}$
$T_{-k}^{t-1} = 0$	$p_k^t = P^{\max}$	$p_k^t = p_k^{t-1}$

CBRA makes use of 4-bit information, while DCF only uses 2 bits

Simulation results

- Throughput
- Stability and convergence



DCF: low throughput;
P-MAC: needs to know the number of nodes

P-MAC: instability due to the online estimation

Conventional solutions in Type II games

- Utility function

$$u_n(\mathbf{a}) = a_n^{\beta_n} \cdot (\mu - \sum_{m=1}^{N} \tau_m a_m).$$

- Nash equilibrium

$$a_n^{NE} = \frac{\beta_n \mu}{\tau_n (1 + \sum_{m=1}^{N} \beta_m)}, \forall n \in \mathcal{N}$$

- Pareto boundary

$$\max_{\mathbf{a}} \sum_{n=1}^{N} \omega_n \log[u_n(\mathbf{a})] \longrightarrow \quad a_n^{PB} = \frac{\omega_n \beta_n \mu}{\tau_n (1 + \sum_{m=1}^{N} \omega_m \beta_m)}, \forall n \in \mathcal{N}$$

- Efficiency loss

$$(1 + \sum_{n=1}^{N} \omega_n \beta_n) \cdot \log \frac{(1 + \sum_{n=1}^{N} \omega_n \beta_n)^2}{(1 + \sum_{n=1}^{N} \omega_n^2 \beta_n)(1 + \sum_{n=1}^{N} \beta_n)} < \sum_{n=1}^{N} \omega_n \log \frac{u_n(\mathbf{a}^{NE})}{u_n(\mathbf{a}^{PB})} < 0.$$

Best response dynamics in Type II games

Observed state

Linear belief

- At stage *t*,

$$u_n^t(\tilde{s}_n^t(a_n), a_n) = a_n^{\beta_n} \cdot \left[\mu - \sum_{m=1}^{N} \tau_m a_m^{t-1} - \lambda_n(a_n - a_n^{t-1})\right]$$

- **Theorem 5**: A necessary and sufficient condition for the best response dynamics to converge is

$$\sum_{n=1}^{N} \frac{\tau_n \beta_n}{\lambda_n(1+2\beta_n)} < 1.$$

Determine the eigenvalues of the Jacobian matrix

Stability of the Pareto boundary

- **Theorem 6**: All the operating points on the Pareto boundary are globally convergent CE under the best response dynamics. The belief configurations $\{\lambda_n\}_{n=1}^{N}$ lead to Pareto-optimal operating points if and only if

$$\sum_{n=1}^{N} \frac{\tau_n}{\lambda_n} = 1$$

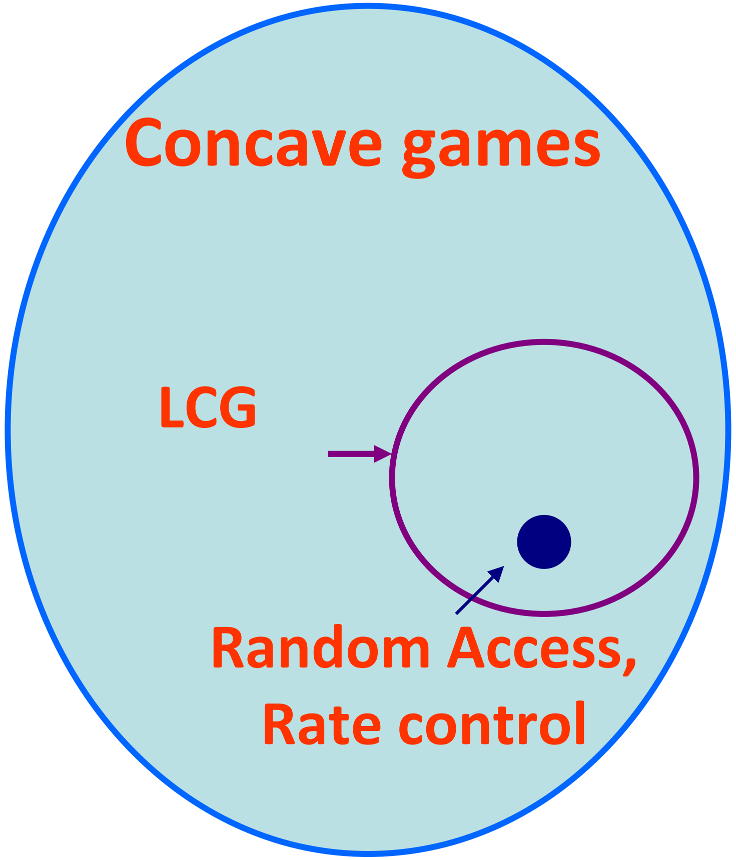
  - $\frac{\tau_n}{\lambda_n}$: the ratio between the immediate performance degradation and the conjectured long-term effect

Theorem 5 and expressions of Pareto boundary and CE

Pricing vs. conjectural equilibrium

- Pricing mechanism in communication networks [Kelly][Chiang]
  - Users repeatedly exchange coordination signals
- Conjectural equilibrium for linearly coupled games
  - Coordination is implicitly implemented when the participating users initialize their belief parameters
  - Pareto-optimality can be achieved solely based on local observations on the states
  - No message passing is needed during the convergence process
  - The key problem is how to design belief functions

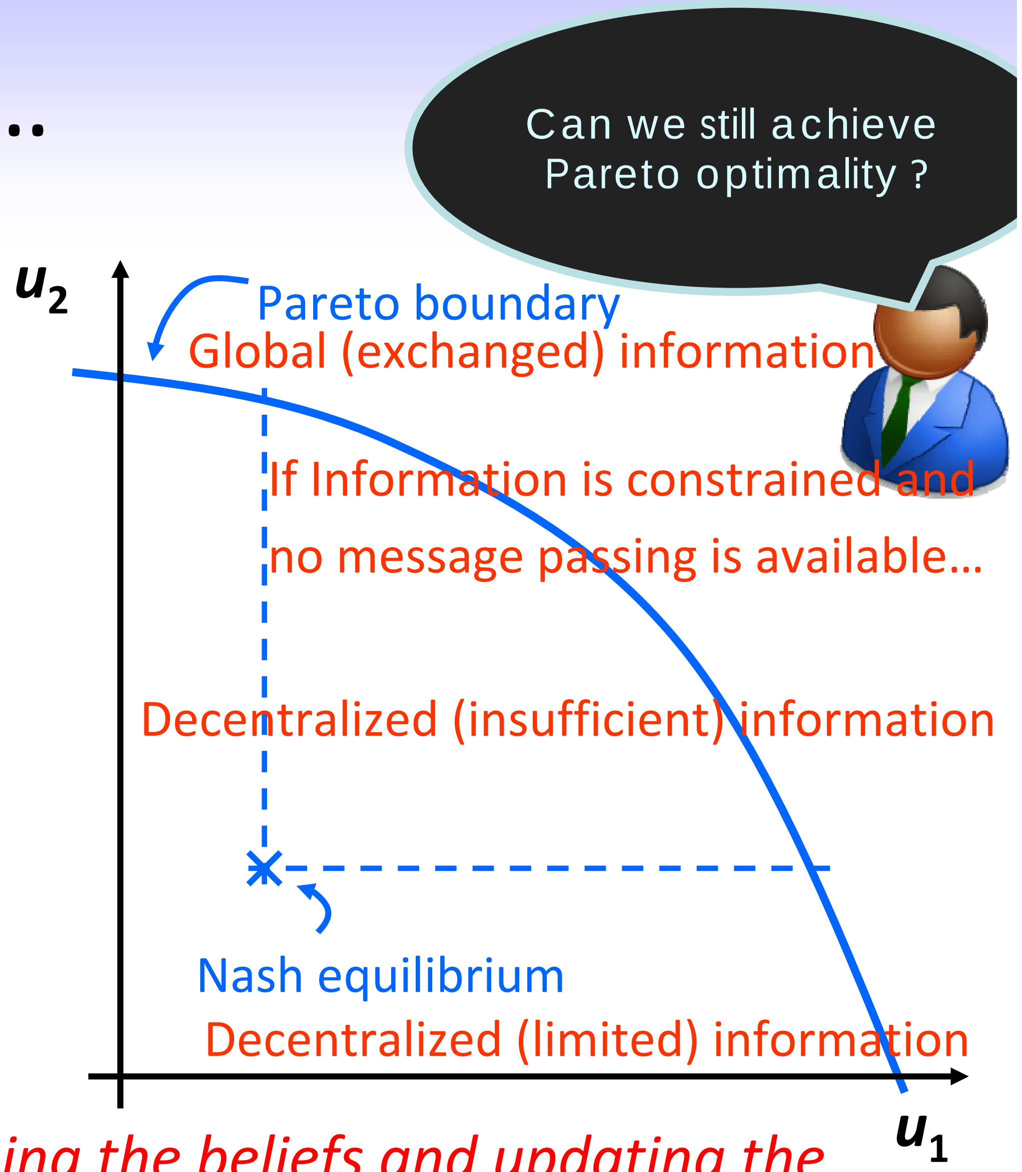
Conclusions so far…



Concave games

LCG

Random Access, Rate control



Can we still achieve Pareto optimality ?

u_2

Pareto boundary

Global (exchanged) information

If Information is constrained and no message passing is available…

Decentralized (insufficient) information

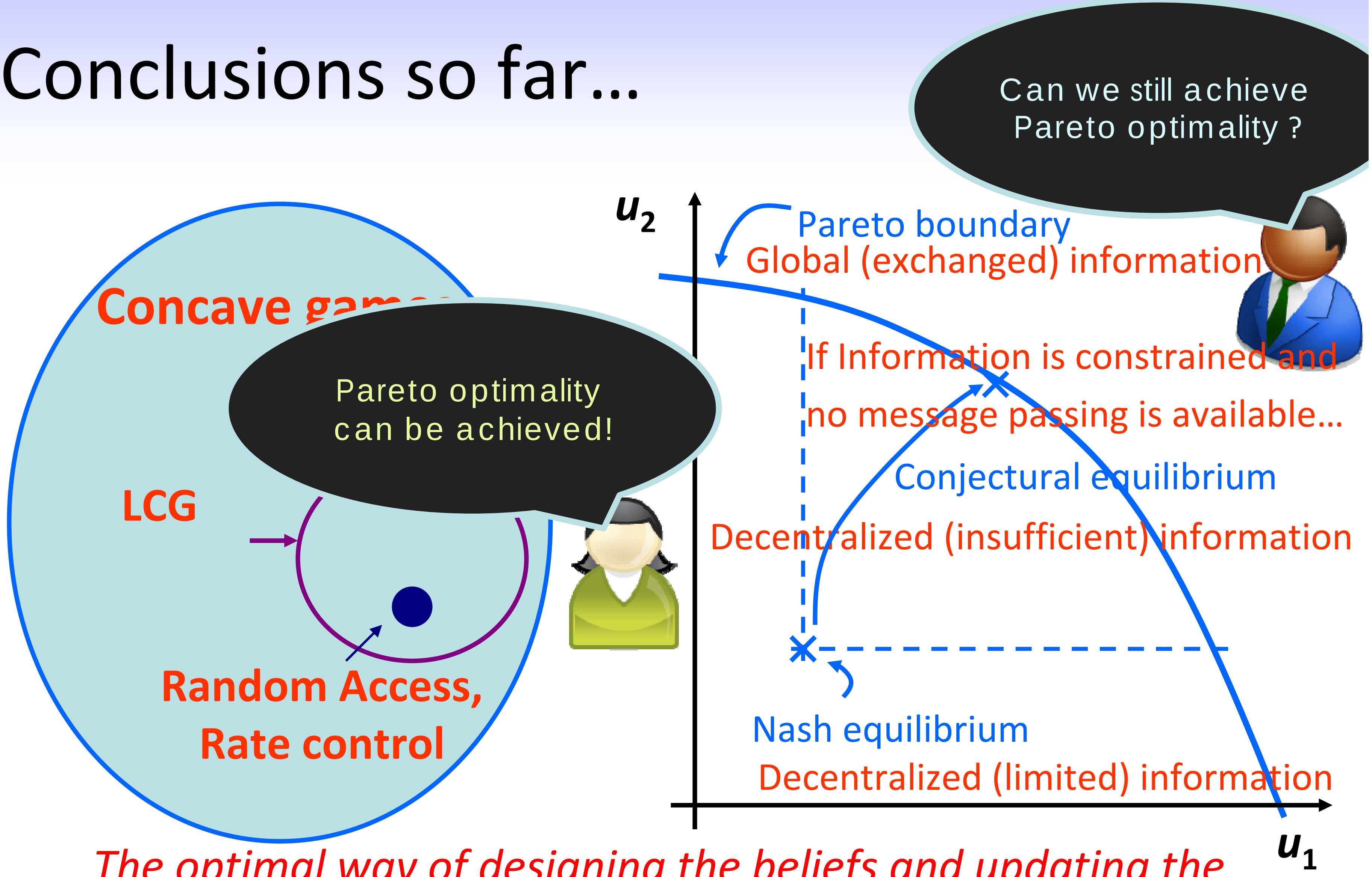
Nash equilibrium

Decentralized (limited) information

u_1

The optimal way of designing the beliefs and updating the actions based on conjectural equilibrium is addressed

Conclusions so far…



Can we still achieve Pareto optimality ?

Pareto boundary
Global (exchanged) information

Concave gam

Pareto optimality can be achieved!

LCG

Random Access, Rate control

If Information is constrained and no message passing is available…

Conjectural equilibrium
Decentralized (insufficient) information

Nash equilibrium
Decentralized (limited) information

The optimal way of designing the beliefs and updating the actions based on conjectural equilibrium is addressed

Conclusions

- We define new classes of games emerging in multi-user communication networks and investigate the information and efficiency trade-off
  - Provide sufficient convergence conditions to NE
  - Suggest a conjectural equilibrium based approach to improve efficiency
  - Quantify the performance improvement

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Linear convergence

- A sequence $x^{(k)}$ with limit x^* is linearly convergent if there exists a constant $c \in (0,1)$ such that

$$|x^{(k)} - x^*| \leq c|x^{(k-1)} - x^*|$$

for *k* sufficiently large.

Solutions with information exchange

- Users aim to solve

$$\max_{\mathbf{a}\in\mathcal{A}} \sum_{n=1}^{N} u_n(\mathbf{a})$$

user n's impact over user m's utility

- They can pass coordination messages

$$\pi_{mn}^{k}(a_m, \mathbf{a}_{-m}) = -\frac{\partial u_m(\mathbf{a})}{\partial a_n^k}$$

and user n behaves according to

$$\max_{\mathbf{a}_n\in\mathcal{A}_n} \quad u_n(\mathbf{a}) - \sum_{k=1}^{K} a_n^k \Big(\sum_{m\neq n} \pi_{mn}^{k} \Big)$$

Solutions with information exchange

- Gradient play $\pi_{mn}^{k,t-1} = -\frac{\partial u_m(\mathbf{a}^{t-1})}{\partial a_n^k}$

$$a_n^{\prime k,t} = a_n^{k,t-1} + \kappa\Big(\frac{\partial u_n(a_n, \mathbf{a}_{-n}^{t-1})}{\partial a_n^k} - \sum_{m \neq n} \pi_{mn}^{k,t-1}\Big).$$

$$\mathbf{a}_n^t = [a_n^{1,t} a_n^{2,t} \cdots a_n^{K,t}] = \Big[a_n^{\prime 1,t} a_n^{\prime 2,t} \cdots a_n^{\prime K,t}\Big]_{\mathcal{A}_n}^{\|\cdot\|_2}$$

Theorem 3: If $\forall n, k, \mathbf{x}, \mathbf{y} \in \mathcal{A}_{-n},$

$$\inf_x \frac{\partial^2 h_n^k(x)}{\partial^2 x} > -\infty, \text{ and } \Big\| \bigtriangledown g_n^k(\mathbf{x}) - \bigtriangledown g_n^k(\mathbf{y}) \Big\| \leq L' \|\mathbf{x} - \mathbf{y}\|,$$

gradient play converges for a small enough stepsize.

Lipschitz continuity and gradient projection algorithm

Solutions with information exchange

- Jacobi update $\pi_{mn}^{k,t-1} = -\frac{\partial u_m(\mathbf{a}^{t-1})}{\partial a_n^k}$

$$B(\mathbf{a}_{-n}^{t-1}) = \arg\max_{\mathbf{a}_n \in \mathcal{A}_n} \; u_n(\mathbf{a}, \mathbf{a}_{-n}^{t-1}) - \sum_{k=1}^{K} a_n^k \Big(\sum_{m \neq n} \pi_{mn}^{k,t-1} \Big)$$

$$a_n^{k,t} = a_n^{k,t-1} + \kappa\{[B(\mathbf{a}_{-n}^{t-1})]_n^k - a_n^{k,t-1}\}$$

Theorem 4: If $\inf_x \frac{\partial^2 h_n^k(x)}{\partial^2 x} > -\infty, \; \sup_x \frac{\partial^2 h_n^k(x)}{\partial^2 x} < 0,$

and $\Big\| \bigtriangledown g_n^k(\mathbf{x}) - \bigtriangledown g_n^k(\mathbf{y}) \Big\| \leq L' \|\mathbf{x} - \mathbf{y}\|,$

Jacobi update converges for a small enough stepsize.

Lipschitz continuity, descent lemma, and mean value theorem

Solutions with information exchange

- Convergence to an operating point that satisfies the KKT conditions is guaranteed

- Total utility is monotonically increasing

- Global optimality is guaranteed if the original problem is convex, otherwise not

- Developed for general non-convex problem in which convex NUM solutions may not apply in general

Stackelberg equilibrium

- Definition
  - Leader (foresighted): only one
  - Follower (myopic): the remaining ones
  - Strategy profile $\left(a_n^*, NE\left(a_n^*\right)\right)$ that satisfies

$$u_n\left(a_n^*, NE\left(a_n^*\right)\right) \geq u_n\left(a_n, NE\left(a_n\right)\right), \forall a_n \in \mathcal{A}_n$$

- Existence and computation of SE in the power control games [SuTWC'09]

A two-user formulation

- Bi-level Programming

upper level problem

$$\left\{\begin{array}{ll} \max\limits_{\boldsymbol{P}_1} \sum\limits_{k=1}^{K} \ln\left(1+\dfrac{P_1^k}{N_1^k+\alpha_2^k P_2^k}\right) & (a) \\ s.t. \ \sum_{k=1}^{K} P_1^k \le \mathrm{P}_{\mathbf{1}}^{\mathbf{max}}, \ P_1^k \ge 0, & (b) \end{array}\right.$$

lower level problem

$$\left\{\begin{array}{ll} \boldsymbol{P}_2 = \arg\max\limits_{\boldsymbol{P}_2'} \sum\limits_{k=1}^{K} \ln\left(1+\dfrac{P_2'^k}{N_2^k+\alpha_1^k P_1^k}\right) & (c) \\ \quad s.t. \ \sum_{k=1}^{K} P_2'^k \le \mathrm{P}_{\mathbf{2}}^{\mathbf{max}}, \ P_2'^k \ge 0. & (d) \end{array}\right.$$

where $N_1^k = \sigma_1^k \big/ \left|H_{11}^k\right|^2, \alpha_1^k = \left|H_{12}^k\right|^2 \big/ \left|H_{22}^k\right|^2, N_2^k = \sigma_2^k \big/ \left|H_{22}^k\right|^2, \alpha_2^k = \left|H_{21}^k\right|^2 \big/ \left|H_{11}^k\right|^2$

Problems with the SE formulation

- Computational complexity
  - intrinsically hard to compute
- Information required for playing SE
  - Global information

$$\left\{\alpha_{ij}^{k}\right\},\left\{\sigma_{i}^{k}\right\},\left\{\mathbf{P_i^{max}}\right\}$$

- Realistic assumption
  - Local information

$$\sum\nolimits_{n=2}^{N}\alpha_{n1}^{k}P_{n}^{k}+\sigma_{1}^{k},\mathbf{P_1^{max}}$$

  - Any appropriate solutions other than SE and NE?

Weighted Fairness

- Priority-based fair medium access control
  - Traffic classes with positive weights

$$SU_i = p_i \cdot (1-p_i)^{|f_i|-1} \cdot \prod_{j \neq i} (1-p_j)^{|f_j|}$$

$$\forall i, j \in \{1, \cdots n\}, \quad \frac{SU_i}{\phi_i} = \frac{SU_j}{\phi_j}$$

$$\Longleftrightarrow \quad \forall i, j \in \{1, \cdots n\}, \quad \boxed{\frac{p_i(1-p_j)}{\phi_i} = \frac{p_j(1-p_i)}{\phi_j}}$$

- Conjecture-based protocol

$$\forall i, j \in \{1, 2, \cdots, N\}, \phi_i p_i (1-p_i) = \phi_j p_j (1-p_j) \Rightarrow \boxed{\frac{\phi_i p_i}{1-p_i} \approx \frac{\phi_j p_j}{1-p_j}}$$

Some distributed iterative algorithms

- Best response

$$\mathbf{a}_n^{BR,t} = \arg\max\nolimits_{\mathbf{a}_n \in \mathcal{A}_n} u_n(\mathbf{a}_n, \mathbf{a}_{-n}^{t-1})$$

- Jacobi update

$$\mathbf{a}_n^{JU,t} = (1 - \kappa)\mathbf{a}_n^{t-1} + \kappa \mathbf{a}_n^{BR,t}$$

- Gradient play

stepsize

$$\mathbf{a}_n^{GP,t} = \mathbf{a}_n^{t-1} + \kappa \nabla_n u_n(\mathbf{a}_n, \mathbf{a}_{-n}^{t-1})$$