

Pricing and Investment for Online TV Content Platforms

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Abstract—Online television (TV) market has been expanding rapidly over the last few years and provided TV studios with a cost-effective and reliable channel for the delivery of high-quality TV content. To maximize profit by setting up an online TV content platform, two major challenges are faced by the platform owner: what is the optimal investment (e.g., how many hosting servers, bandwidth acquisition) and how to price TV content producers who utilize the platform as a channel to distribute their content. To address these two challenges, we first derive the optimal pricing policy based on the widely-adopted “pay-per-usage” model, and then formalize and solve the optimal investment decision problem. Rationality of self-interested TV content producers and audiences is also taken into account. Specifically, we first use a model with a representative content viewer to determine how many times a TV content with a certain quality is watched. Then, by modeling the content providers as self-interested agents making independent production decisions, we show that for any price charged by the platform, there always exists a unique equilibrium in the content production stage, which makes it possible for the platform owner to maximize its profit without uncertainties because of the unique outcome in the content producers’ decision stage. Finally, we develop an algorithm to derive the optimal price and then formalize the investment decision problem to maximize the platform’s profit.

Index Terms—Online TV content platforms, pricing, two-sided market.

I. INTRODUCTION

WEB-BASED television (TV) has become an important new paradigm for the delivery of high-quality TV content. In particular, TV content distribution networks are being transformed from TV stations in a traditional TV value chain to online TV service websites which have distinctive advantages in offering socialized and personalized TV content. This trend is evidenced by a recent survey showing that over 20% of the TV audiences choose to watch primetime programs online and this percentage is expected to continue to grow [1].

Due to unaffordable marketing fees, costs and/or the lack of required technical skills, many TV content producers (especially small TV studios) now participate in a large content platform (e.g., Amazon Instant Video, NetFlix) and sell their

produced content by leveraging the content platform’s selling power. By doing so, TV content producers can focus on content production, while content storage, distribution as well as promotion and marketing are all taken care of by the content platform. That is, all TV studios (and even individual TV content producers) have an opportunity to reach a mass of online TV audiences by selling their produced content on “virtual shelves” provided by the content platform. This is analogous to customer-to-customer business mode adopted by e-commerce websites (e.g., eBay), where sellers utilize the websites as a marketing channel to increase their volumes of sales [16]. In summary, selling TV content on content platforms has the following features that are particularly attractive to medium and small TV studios [28].¹

Large selling power: Popular content platforms, such as Amazon Instant Video and NetFlix, have millions of customers and the ability of make niche, or “hard to find”, content (often produced by medium and small TV studios) visible to their huge base of customers (e.g., through content recommendation systems [17]).

Low cost: By applying advanced resource management schemes (e.g., workload consolidation) [8], content platform significantly reduces the cost incurred for TV content distribution, which therefore allows the platform to charge TV content producers a lower price (or “commission fee”) for using the platform to sell their content.

Reliability: With large-scale data centers housing tens of thousands of high-performance servers, TV content sold on a large content platform is stored and replicated in multiple servers, minimizing the chance of outages and/or content loss even though some servers may go down due to fatal errors [10].

Providing a simple and efficient way to distribute TV content, TV content platforms make it possible for every TV content producer, whether it is big or small, to make profits by selling its produced content online. While some TV content platforms (e.g., NetFlix) uses “subscription-based” pricing which charges users a flat fee regardless of how many TV episodes they watch, other TV content platforms (e.g., Amazon Instant Video) provide “pay-per-view” pricing in which users are charged only when they purchase and watch certain TV content.² In the “pay-per-view” model, TV content producers can set prices for their own content, whereas in the “subscription-based” model the content platform will have the power of setting subscription

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¹We note that content delivery networks also provide low-cost and reliable TV content distribution, whereas they cannot offer a large base of potential audiences to TV content producers.

²In practice, some content platforms allow a content viewer to watch its purchased content several times while paying for it only once [28]. In our current study, we treat such “multiple content views” as one content view, since the content viewer only purchases the content once.

TABLE I
SUMMARY OF DECISION MAKING

Decision Maker	Decision	Real-life Example
Content platform	Investment and pricing	Amazon decides the number of hosting servers, content encoding format, and the price (per content view) charged from TV content producers
Content producers	TV content production	Universal Television (or other TV studios) decides whether or not to produce a certain TV content and distribute it through Akamai
Content viewers	TV content viewing	TV audiences on the Internet decide which TV content to watch

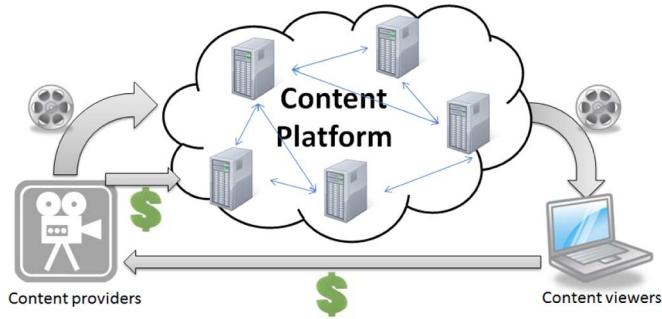


Fig. 1. Illustration of TV content distribution platforms.

fees. In this paper, we mainly consider the “pay-per-view” charging model for online audiences, as it allows TV content producers to have more control of their content [28]. Moreover, although the content platform may charge online TV audiences for watching the content on behalf of the content producers, it will only keep a certain (commission) fee while giving the rest of the payment to the respective TV content producers. Thus, essentially, it is TV content producers who charge the online TV audiences and then pay commission fees to the content platform.

Despite the huge potential in online TV markets, there are two important issues that the owner of a content platform has to address before setting up its content platform: what is the optimal investment (e.g., how many hosting servers, bandwidth acquisition) and how to charge TV content producers who utilize the platform as a channel to sell their content. To answer these two questions, we focus on a TV content platform that charges TV content producers based on the widely-adopted “pay-per-view” model. Specifically, a TV content producer charges its audiences (or the content platform charges the audiences on behalf of the TV content producer) and pays to the content platform whenever its content is viewed by online audiences (i.e., whenever it is “sold”). This practice has been adopted by several leading online TV content platforms such as Amazon Instant Video [28]. For notational convenience, we sometimes use *platform* to refer to as the content platform wherever applicable, and TV content is briefly referred to as content throughout the remainder of this paper unless otherwise stated. We also refer to Internet/Online TV audiences as content viewers. A diagram illustrating our considered model is available in Fig. 1. In our study, we explicitly take into account self-interested decisions made by TV content producers and audiences. The decisions made by content viewers, content producers, and the platform are summarized in Table I. In addition to pricing and investment decisions, what adds to the platform owner’s design challenges is that decisions made by various parties are closely related to and coupled with

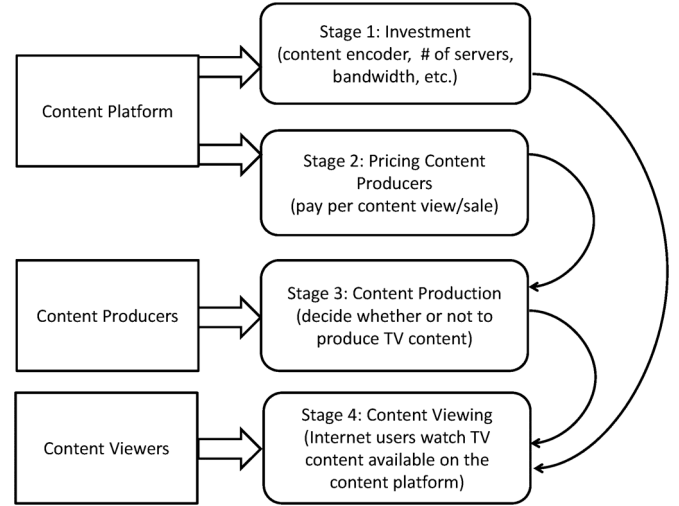


Fig. 2. System diagram of a TV content distribution platform.

each other. We show the order of decisions and their inter-dependencies in Fig. 2, where curves with arrows going from A to B indicate that A directly affects B. For instance, the platform’s investment will directly affect the quality of content delivered to content viewers (e.g., more servers and better encoders will lead to higher-resolution content with a higher reliability [31]), while the platform’s price decision influences the content producers’ production decisions (e.g., a high price may discourage content producers from selling their produced content on the platform).

In general, the platform’s investment (in infrastructure) is difficult to alter once made³ and hence, it is a long-term strategy for the platform. In contrast, the platform can adjust its pricing plans offered to the content producers over the lifespan of its investment, although the pricing plans are typically not updated as frequently as the content producers decide their content production or the content viewers make their viewing decisions.⁴ By using backward induction, we shall consider a flow of analysis illustrated in Fig. 3. In other words, the platform can be regarded as the leader, followed by the content producers and then by the content viewers. First, we use a model with a representative content viewer, which is a collection of all the individual content viewers, to determine how many times each content is viewed (i.e., content views) by solving a utility maximization problem. Then, we study the content production decisions made by self-interested content producers. It is shown that

³Once built, a data center will be used for at least several years.

⁴Unlike the general two-sided market research [30], our model takes into account content substitution, inter-group network externalities and intra-group negative network externalities (i.e., competition among the content producers for the content viewer’s attention), which are key features of TV content platforms.

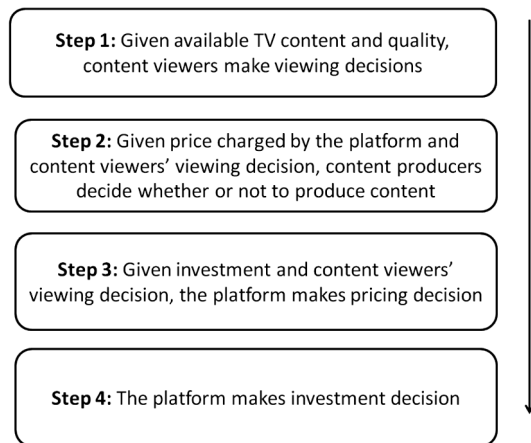


Fig. 3. Flow of 4-step analysis.

there always exists a unique equilibrium point at which no content provider can gain by changing its production decision. This enables the platform to maximize its profit without any uncertainties, since there is a unique outcome in the content production stage given the platform's price. We also propose a best-response dynamics to model the content producers' decision process. Next, by considering the quality-adjusted Dixit-Stiglitz utility function for the representative content viewer [24], [25], we develop an algorithm to derive the optimal price and then formalize the investment decision problem to maximize the platform's profit. Finally, we conduct comprehensive simulations to complete our analysis. Our results highlight that the proposed formalism can enable the optimal design (in pricing and investment) of TV content platforms in terms of profit maximization. Our proposed framework is also incentive compatible for content producers and content viewers, as self-interested decisions made by these two parties are explicitly taken into account. Our study can provide the online TV content platform with a quantitative guidance on its pricing and investment decision, if the parameters in the model can be appropriately set (e.g., by conducting market surveys and using data mining techniques). For instance, to maximize its profit, the platform owner can decide based on our study its video codec and bandwidth acquisition (which is related to the investment decision) as well as its price that should be charged per content view/sale to content producers.

The rest of this paper is organized as follows. Related work is reviewed in Section II. Section III describes the model. In Section IV, we study the decisions made by the content viewers and content producers, and derive the optimal price and formalize the optimal investment decision maximizing the platform's profit. Numerical results are provided in Section V, and finally, concluding remarks are offered in Section VI.

II. RELATED WORKS

The platform's decision consists of two components: price and investment decisions. In this section, we discuss the existing related works from these two aspects.

Pricing has been studied extensively in both engineering and economics. For instance, [2] studied a multi-cell wireless multimedia system and derived the optimal price that the service provider should charge wireless users to maximize its revenue.

Considering a general two-sided market, the authors in [18] studied the tradeoffs between the merchant mode and the platform mode, and showed the conditions under which the merchant or platform mode is preferred. Focusing on the Internet markets, [19] revealed that a neutral network is inferior to a non-neutral one in terms of social welfare when the ratio between advertising rates and end user price sensitivity is either too high or too low. The authors in [20] studied the broadband communications market based on a two-sided model, and proposed pricing the content producers to maximize the service provider's profit. In the context of TV markets, different payment schemes have been proposed. For instance, to combine the advantages of both pay-per-channel (PPC) and pay-per-view in a Pay-TV broadcasting system, the authors in [5] proposed a new payment model, referred to as Flexible-PPC, which allows a subscriber to subscribe freely to his/her favorite channels, and unsubscribe/change his/her subscription at any time. They also propose a conditional access system with a four-level key hierarchy to implement Flexible-PPC. The proposed conditional access system was later improved by [7], and the Pay-TV broadcasting system was extended to cellular networks based on a mobile conditional access system [6]. Advertising is a major source of revenue for some content platforms (especially user-generated content platforms such as YouTube). Reference [14] studied the problem of advertising impact optimization of program grids, and proposed an evolution solution based on semantic similarity measures between the descriptions of the TV contents to maximize the satisfaction of advertisers and content viewers.

To provide cost-effective and reliable services, a large TV content platform (e.g., Amazon Instant Video) typically has its own data centers to host the content and (possibly) content delivery networks, which take up a large portion of the investment. A lot of attention has been devoted to research on data center and content delivery network optimization, which may be applied when the platform invests in its own infrastructure and software. [8] introduced the components of the Prism architecture—content management, content discovery, content-aware redirection—which can support high-quality streaming media services in an IP-based content distribution network. Focusing on the transport layer, [9] designed a new framework to ensure the scalability, quality of delivered content and reliability of content delivery networks (in particular, Akamai). By modeling the server replica placement problem as the well-known minimum p -median problem, [10] proposed a heuristic algorithm to improve the system performance measured in terms of client request latency and total network bandwidth consumption. Reference [12] proposed a general model and architecture for an open content delivery network that aims at reducing response time experienced by users through locating multiple servers in the edge of the network. To improve the quality of delivered content, both the multimedia nature of the content and the diversity of client platforms need to be addressed when designing content delivery protocols. The authors in [15] presented a universal access system that can adapt multimedia content on the web based on the capabilities of client devices requesting the content.

III. MODEL

Focusing on a TV content platform where the content is provided by independent content producers and viewed by content

TABLE II
LIST OF NOTATIONS

Notation	Description
α	Investment level
w	Lump-sum deployment cost
b	Distribution cost per content view
θ	Price charged to content producers per content view
\bar{x}	Total content views
s	Price charged to content viewers per content view
c	Content production cost
q	Content desirability
$x(q)$	Content views for content with desirability q
q_a	Aggregate outside content quality
σ	Elasticity of content substitution
T	Total budget

viewers, we specify in this section the modeling details of the platform, content producers and content viewers. Key notations are listed in Table II.

A. Platform

As we have stated, we consider a charging model specified as that the content producers pay to the platform whenever their content is viewed by content viewers and the total payment is proportional to the total number of content views. Note that the platform does not set prices for the content. On the one hand, if the platform charges a high price, few content producers will use the service to deliver their content. On the other hand, the platform may incur a loss in profits if it charges a low price, even though a large number of content producers may choose to sell their content on the platform. Thus, in order to maximize the platform's profit, an optimal price needs to be identified. Moreover, before making its pricing decision, the platform needs to first make investment decisions, which will eventually affect the qualities of content delivered to content viewers. The order of decision making has been described in Fig. 2.

To formally state our model, we use $\alpha \in (0, \bar{\alpha}]$ and $\theta \geq 0$ to represent the platform's investment decisions and price charged to content producers per content view, respectively, where $\bar{\alpha}$ is the maximum possible value of α indicating the platform's investment budget constraint. The investment decision includes, but is not limited to, the number of servers to purchase, encoders to adopt, and bandwidths to allocate [10]. Naturally, a larger investment will lead to a higher quality of experience for content viewers (e.g., faster streaming, less delay jitter) [13]. In our model, we use α to indicate how the quality of content is scaled, and a larger value of α means that the platform invest more in its infrastructure and can deliver higher-quality content to content viewers (while incurring a larger bandwidth cost) [11], [31]. Thus, throughout the paper, we refer to α as the platform's *investment level*. In our model, we use $w(\alpha)$ as the lump-sum cost (e.g., server purchase) when the platform chooses α , and $b(\alpha)$ as the (average) distribution cost (mainly bandwidth cost) incurred when delivering a copy of content to content viewers [4], [8]. Naturally, higher-quality content delivery (i.e., larger α) incurs a higher cost. Thus, both $w(\alpha)$ and $b(\alpha)$ are increasing in $\alpha \in (0, \bar{\alpha}]$ and, following the order of decisions described in Section I, the price θ is also an implicit function of α .⁵ To keep

⁵We can remove the assumption that $w(\alpha)$ and $b(\alpha)$ are both increasing in $\alpha \in (0, \bar{\alpha}]$ without affecting our analysis.

the notations succinct, we simply use w , b and θ to represent $w(\alpha)$, $b(\alpha)$ and $\theta(\alpha)$, respectively, without causing ambiguity. We denote \bar{x} as the total content views of all the content on the platform. It should be pointed out that since α affects the delivered content quality and hence content views too, \bar{x} is also indirectly subject to α . For the convenience of analysis, we assume that, given α , b is constant regardless of \bar{x} , i.e., the (average) distribution cost is independent of the content views. The platform charges θ per content view from the respective content producers. Neglecting its recurring fixed operational cost, we can express the platform's profit as

$$\Pi = (\theta - b) \cdot \bar{x} - w. \quad (1)$$

We assume throughout the paper that b is exogenously determined and fixed given a fixed value of α (i.e., the shape of $b(\alpha)$ is predetermined),⁶ and shall focus on deriving the optimal θ and α that maximize the platform's profit.

B. Content Producers

There are a large number of (possibly small) content producers in today's TV content market. To capture this fact, we use a continuum model and assume that the mass of content producers is normalized to one. Each content provider can produce content of a certain desirability while incurring a production cost. We further assume that content producers produce differentiated content, or in other words, no two content producers can produce completely identical content. Note that the content desirability can be different across content producers, although we assume that the production cost is the same for all content producers.⁷ In practice, content production cost includes, but is not limited to, TV taping costs and compensation paid to TV crews. The (original) content desirability is represented by a scalar and treated as an internal feature of content (e.g., how fun/informative the content is) [3]. Mathematically, we denote $q_i \geq 0$ and $c > 0$ as the desirability of content produced by content provider i and the production cost, respectively. Without causing ambiguity, we occasionally use content q_i to refer to the content with a desirability q_i . To characterize heterogeneity in the content desirability, we assume that the content desirability follows a cumulative distribution function (CDF), denoted by $F(q)$, across the unit mass of content producers. In other words, $F(q)$ denotes the number or fraction of content producers whose content has a desirability less than or equal to $q \geq 0$. Although a content may have a high original desirability, the quality of experience of content viewers for this content also depends on the platform's investment level [31]. For instance, if the number of servers is too small, congestion may be intolerant for content viewers and the quality of content delivery is significantly degraded [4], [8]. Mathematically, given the investment level of α , we assume that the actual quality of a content with a desirability of q will be $\alpha \cdot q$ for content viewers. Essentially, the overall content quality is determined by two factors: content desirability, which is the content's inherent feature, and the platform's investment, which affects the content's quality of experience such as PSNR, delay jitters, etc. [13], [14], [31].

⁶Note that b (e.g., bandwidth usage) can be reduced by using sophisticated resource allocation algorithms [3].

⁷Extension to heterogeneous production costs can be found in the full version of this paper [29].

Content producers set prices and can directly or indirectly charge the content viewers for viewing their content. Nevertheless, in the presence of a large number of content producers, perfect competition is observed in the market [24], [30]. That is, (almost) all the content producers will set the same price for their content due to the intensive competition. For instance, on Amazon Instant Video, most of the TV shows are priced at US\$1.99 per episode. Therefore, we shall assume a uniform price, denoted by s , for all the content. It is further assumed that each content provider i is self-interested and can strategically make a binary decision: produce or not produce. The decision making is described in Table I and Fig. 2. Denote by $x(q_i) \geq 0$ the number of views for content q_i (i.e., content views, or volumes of sales). If content provider i produces content on the platform, it can derive a utility (with a monetary unit) expressed as

$$\pi_i = (s - \theta) \cdot x(q_i) - c \quad (2)$$

where $s \geq 0$ is the price per content view (i.e., per content sale) received from the content viewers, θ is the charge per content view determined by the platform, and c is the production cost. Content provider i obtains zero utility if it chooses not to produce and sell content. By the assumption of rationality, content provider i chooses to produce content if and only if its utility is non-negative.

In what follows, we assume that the content desirability q follows a uniform distribution in a normalized interval $[0, 1]$ and the probability density distribution (PDF) is given by $f(q) = 1$ for $q \in [0, 1]$.⁸ Scaling the interval $[0, 1]$ to $[0, \bar{q}]$ does not affect the analysis, but will only complicate the notations. It is intuitively expected that a content with a higher desirability will attract more content views (and yield a higher utility for its content provider, too) than the one with a lower desirability. Thus, the production decision of the content producers has a threshold structure. In particular, there exist *marginal* content producers whose content has a desirability denoted by $q_m \in [0, 1]$, and those content producers whose content desirability is greater (less) than q_m will (not) choose to produce content on the platform. We refer to q_m as the marginal content desirability.

C. Content Viewers

In this subsection, we model the decisions made by content viewers (see Table I and Fig. 2 for illustration). Despite that the content viewers are diverse in terms of preferences towards the content, the aggregate content viewing decisions of all the content viewers can be conveniently characterized by the decision of a representative content viewer, which is a virtual decision maker on behalf of all the content viewers [24]. Thus, we adopt the widely-used representative agent model to determine how the total content views are allocated across a variety of content [25]. Specifically, the representative content viewer optimally allocates its total budget, denoted by T , across the available content to maximize its utility. Note that T can be interpreted as the online TV market size (i.e., total expenditure of online TV audiences). In addition to viewing the content provided on the considered platform, content viewers may also have access to content provided on other platforms (e.g., traditional TV chan-

nels and other online TV content platforms), which we refer to as *outside* content. Focusing on the platform's optimal pricing and investment decision, we do not consider the details of how the outside content is produced. Instead, we assume that the mass of outside content is $n_a \geq 0$ and the outside content quality (also determined by the content desirability and delivery quality) follows a certain CDF $\tilde{F}(q)$ with support $q \in [q_l, q_h]$, where $0 \leq q_l < q_h$ are the lowest and highest content quality on the other platforms, respectively. For the convenience of notation, throughout the paper, we alternatively represent the outside content using a unit mass of content with an *aggregate* quality of q_a , without affecting the analysis. Note that q_a is a function of $n_a \geq 0$, $\tilde{F}(q)$ and the utility function of the representative content viewer. In particular, given a uniform distribution of outside content quality and the quality-adjusted Dixit-Stiglitz utility for the representative content viewer (which we shall define later), we can readily obtain

$$q_a = \frac{1}{\alpha^{\frac{1}{\sigma}}} \cdot \left[\frac{n_a (q_h^{\sigma+1} - q_l^{\sigma+1})}{1 + \sigma} \right]^{\frac{1}{\sigma}} \quad (3)$$

where $\sigma > 1$ measures the content substitutability [24]. Recalling that $q_m \in [0, 1]$ is the marginal content desirability above which the content producers choose to produce content on the platform, we write the representative content viewer's utility function as $U(x(q), x_a | q_m, q_a)$, where $x(q)$ denotes the content view for content $q \in [q_m, 1]$ and x_a is the content view allocated to the aggregate outside content q_a . Note that although there is outside content available on the other platforms, we implicitly assume that the considered platform is a monopolist in the online TV content market and the content producers under consideration, if they choose to produce content,⁹ can only provide content on the considered platform [22]. In other words, the amount of outside content is constant and all the content producers under consideration are affiliated with the considered platform. Thus, x_a is essentially interpreted as "outside activity" of the content viewers, i.e., how much time content viewers spend in watching outside content. Note that $x(q)$ can be rewritten as $x(q | q_m, q_a)$, although we use the succinct notation $x(q)$ throughout the paper whenever applicable. If q_m increases (decreases), there will be less (more) content on the platform. Because of the continuum model, we allow $x(q)$ and x_a to take non-integer values, and $x(q)$ actually represents the content view *density* allocated to a continuum of content with desirability $q \in [q_m, 1]$, i.e., $x(q)$ is the content view that an *individual* content producer with a content desirability of q receives. Next, by using a *quality-adjusted* version of the well-known Dixit-Stiglitz function [24], [25] as the utility function, we formulate the utility maximization problem for the representative content viewer as follows:

$$\begin{aligned} & U(x(q), x_a | q_m, q_a) \\ &= \left[\int_{q_m}^1 \alpha \cdot q \cdot x(q)^{\frac{\sigma-1}{\sigma}} dq + q_a \cdot x_a^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}} \quad (4) \\ & \text{s.t.,} \quad \int_{q_m}^1 s \cdot x(q) dq + s \cdot x_a \leq T \end{aligned}$$

⁸The uniform distribution has been widely applied to model the diversity of various factors, such as opportunity cost [18] and valuation of quality-of-service [21].

⁹Throughout the paper, we use "produce content" and "produce and sell content" interchangeably.

where $\sigma > 1$ measures the elasticity of substitution between different content and α , determined by the platform's investment decision, is the factor scaling the content desirability to the actual content quality of experiences of the content viewers. In the extreme case, the content is perfectly substitutable when $\sigma = \infty$, i.e., watching content A and content B makes no difference except for the quality difference [24]. Note that although in practice content viewers may not need to pay for "outside" viewing activities, content viewers still incur other costs (e.g., time) for these activities and, to limit the number of parameters, we assume that the cost can be converted to an equivalent cost which is the same as s , i.e., payment for viewing content on the considered platform. We can also choose other values of outside viewing cost, and it does not affect our analysis except for that the aggregate outside content quality may be changed. It is also worth mentioning that an implicit assumption underlying the problem (4) is that the aggregate quality of the outside content is independent of the platform's pricing or investment decision and other variables in the model such as $q_m, x(q), x_a$. This can be justified by noting that there are many "outside" content platforms (including traditional TV stations) besides the considered platform and changes on one content platform have a negligible impact on the others. Before performing further analysis, we discuss the following properties satisfied by the utility function $U(x(q), x_a | q_m, q_a)$ in (4).

Property 1 (Diminishing Marginal Utility): $U(x(q), x_a | q_m, q_a)$ is increasing and strictly (jointly) concave in $x(q)$ and x_a , for $q \in [0, 1]$.

Property 2 (Preference Towards Diversified Bundle of Content): $\max_{x(q) \geq 0, x_a \geq 0} U(x(q), x_a | q_m, q_a)$ is decreasing in $q_m \in [0, 1]$.

Property 3 (Negative Externalities): Denote by $x^*(q | q_m, q_a)$, for $q \in [0, 1]$, the optimal solution to (4). If content q is produced, then $x^*(q | q_m, q_a)$ is positive. Moreover, it is continuous and strictly increasing in $q_m \in [0, 1]$, increasing in $q \in [0, 1]$, and decreasing in q_a for $q_a \in [0, \infty)$. In particular, $x^*(0 | q_m, q_a) = 0$ for all $q_m \in [0, 1]$ and $q_a \geq 0$.

Property 4 (More Content Leading to More Content Views): $\bar{x} = \int_{q_m}^1 x^*(q | q_m, q_a) dq$ is decreasing in $q_m \in [0, 1]$.

We briefly discuss the above properties. Property 1 captures the effects of diminishing marginal utility when the representative content viewer views more content [24]. Property 2 models the phenomenon that content viewers will typically benefit from the participation of content producers on the platform. This is particularly true for TV content platforms, where the content viewers prefer to view a diversified bundle of content. Thus, when $q_m \in [0, 1]$ increases, i.e., fewer content producers provide content on the platform, the representative content viewer's (maximum) utility decreases [14]. Property 3 reflects the "crowding effects", i.e., lower q_m or more content production increases competition among the content producers. Specifically, an individual content provider will attract fewer content views if more content producers choose to produce content on the platform or the aggregate outside content quality is higher [30]. The last property ensures that more content views are devoted to the platform if there is more content available on the platform. Note that although we focus on the utility function defined in (4), our analysis of content viewing and content production applies to any other utility functions that satisfy Properties 1–4.

D. Discussions

Before proceeding with the analysis, we provide the following remarks regarding our model.

Remark 1: While content producers do not pay to the platform for content storage, the considered pricing scheme complies with the common practice of "pay per use" on the Internet [19]. Specifically, content producers pay to the platform for using it as a sales channel, i.e., "pay-per-view" or "pay-per-sale",¹⁰ which has been adopted by leading content platforms such as Amazon [28]. As an initial study, we assume uniform θ and constant cost b across all the content, which is valid if different content has (almost) the same length and size. This is particularly true for most TV shows [28], [31]. The cost term b mainly captures the bandwidth cost (and possibly energy cost) incurred in delivering the content [8], while storage cost is not implicitly considered and may be absorbed into the lump-sum cost w (since the storage cost does not increase linearly with content views). Note that considering more general prices and costs (e.g., different prices and costs for different content) is left for future work.

Remark 2: Although we have assumed uniformly distributed content desirability q in $[0, 1]$ throughout the paper, the assumption is only needed to derive explicitly the optimal price θ^* . A finite (but possibly large) number of content producers with content qualities denoted by q_1, q_2, \dots, q_N , respectively, can also be considered. In such cases, the unit mass of content producers corresponds to N content producers,¹¹ and we can formulate the utility maximization problem for the representative content viewer as

$$\begin{aligned} & U(x_1, x_2, \dots, x_N x_a | q_m, q_a) \\ &= \left[\sum_{q_i \geq q_m, i=1,2,\dots,N} \alpha \cdot q_i \cdot x_i^{\frac{\sigma-1}{\sigma}} + q_a \cdot x_a^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}} \quad (5) \\ & \text{s.t.}, \quad \sum_{q_i \geq q_m} s \cdot x_i + s \cdot x_a \leq T \end{aligned}$$

where x_i denotes the content views for content q_i , for $i = 1, 2, \dots, N$. Although the content production analysis becomes involved, the optimal content viewing decision can be easily derived by solving (5). If there are multiple representative content viewers (each of which represents a group of content viewers), then they can be further consolidated into one representative content viewer.

Remark 3: In our model, the scalar α reflects the investment decision and, when multiplied with the original content desirability q , indicates the quality of experiences for the content. While the model is simple, it captures the fact that the actual quality of experience of content viewers depend on the original content desirability (e.g., how fun/informative the content is) as well as the quality of content delivery, measured by, for instance, PSNR and delay jitter, which are closely related to the platform's investment decisions [13], [14], [31]. This is similar

¹⁰As we have noted earlier, one "content view" and one "content sale" are treated as the same in our current study, although on some content platforms a user may watch its purchased content multiple times while paying for it only once [28].

¹¹The aggregate outside content quality q_a shall be appropriately scaled depending on N .

in spirit to economic models in which the users' utilities are derived by multiplying the quality of service with a scalar indicating the users' valuations of QoS [18], [21]. In our future work, we shall consider more sophisticated models to characterize the actual quality of experience of content viewers.

Remark 4: We briefly discuss how to consider non-uniform s and non-uniform θ . To explain how the assumption of a uniform price s can be relaxed, we consider a scenario that the TV content price is expressed as a function $s(q)$ in terms of the content desirability.¹² To limit the number of free parameters, we assume that the price for outside TV content is normalized to 1. Hence, for the representative content viewer, the budget constraint becomes $\int_{q_m}^1 x(q) \cdot s(q) dq + x_a \leq T$, while the objective function remains unchanged. Then, the representative content viewer will pay more attention to TV content that has higher values of "desirability/price" (i.e., $q/s(q)$) instead of higher values of q . Moreover, according to the distribution of TV content desirability, we can easily derive the distribution of $q/s(q)$. As a result, we can view $q/s(q)$ as if it were the original TV content desirability " q ". Note that a content producer's profit may not always increase with its content views. To tackle this problem, we can normalize the content producers' profits with respect to their own net profits per TV content view without affecting the binary production decisions. For instance, if the profits of content producer A and content producer B are $(s_A - \theta) \cdot x_A - c$ and $(s_B - \theta) \cdot x_B - c$, then the corresponding normalized profits are $x_A - c/(s_A - \theta)$ and $x_B - c/(s_B - \theta)$, respectively, where s_A and x_A are content producer A's TV content price (per view) and content views, respectively, and similar definitions for content producer B. By comparing $x_A - c/(s - \theta)$ (i.e., normalized profit with homogeneous content production cost) with $x_A - c/(s_A - \theta)$, we see that the analysis of content production decisions can be performed following the study of "heterogeneous production costs" (see [29] for details). Given a non-uniform s , there may not exist a closed-form expression for the optimal price θ^* to maximize the platform's profit, and we may need to resort to numerical methods to find it. Next, we discuss non-uniform θ . The analysis of content production and content viewing can still be performed, despite it is much more complicated. Specifically, by comparing $x(q) - c/(s - \theta)$ (i.e., normalized profit with homogeneous production cost and uniform price θ) with $x(q) - c/(s - \theta(q))$ (i.e., normalized profit with non-uniform price θ), we see that the analysis of content production decisions can be performed following the study of "heterogeneous content production costs". Since $\theta(q)$ is not directly related to the decision of the representative content viewer, the content viewing decision is not explicitly affected by $\theta(q)$.

Remark 5: In this paper, we implicitly assume that the outside TV content platforms are operated by agents that do not strategically change their decisions. This assumption has been widely considered in the existing literature (e.g., [18]) and makes the analysis tractable without violating "inter-group positive network externalities", i.e., more content available on the considered TV content platform can attract more content views and increase the utility derived by the content viewers, which are important in two-sided markets such as TV content platforms

¹²We can also consider that TV content of the same desirability may have different prices, but this significantly complicates the notations and explanation.

[30]. Moreover, we assume that the considered TV content platform has a fixed mass of "loyal" content producers who do not provide content to other platforms (i.e., no multihoming) [18]. In other words, if the content producers can publish content on multiple platforms, we separate the content production decisions made by a content producers for different content platforms. Explicitly considering the strategic competition among multiple TV content platforms and multihoming is left as part of our future work.

IV. PLATFORM'S PROFIT MAXIMIZATION

In this section, based on the model described above, we study the problem of optimizing pricing and investment level, i.e., θ and α , to maximize the platform's profit in the presence of self-interested content producers and content viewers. Given the timing of decision making described in Fig. 2, we proceed with our analysis using backward induction as illustrated in Fig. 3.

A. Optimal Content Viewing

As in Fig. 3, the first step in our analysis is the optimal content viewing decision made by the content viewers. This step determines how many content views (i.e., volumes of sales) a certain content can receive if it is sold on the platform. If a content cannot be sold to/viewed by a sufficiently large number of content viewers, then its content producer may lose profit and choose not to produce the content. Hence, the analysis of content viewing decision will serve as a basis for the content producers to make content production decisions. By considering the quality-adjusted Dixit-Stiglitz utility defined in (4) and uniform distribution of the content desirability, we can obtain explicitly the closed-form solution as follows:

$$x^*(q) = \frac{T(\sigma + 1)q^\sigma}{s \left[(\sigma + 1) \cdot \left(\frac{q_a}{\alpha}\right)^\sigma + (1 - q_m^{\sigma+1}) \right]} \quad (6)$$

for $q \in [q_m, 1]$, $x^*(q) = 0$ for $q \in [0, q_m)$, and $x_a^* = T(\sigma + 1)(q_a/\alpha)^\sigma / s \left[(\sigma + 1) \cdot (q_a/\alpha)^\sigma + (1 - q_m^{\sigma+1}) \right]$. The details of deriving (6) are omitted for brevity. After plugging $x^*(q)$ and x_a^* into (4), the maximum utility derived by the representative content viewer is given by

$$U^*(x^*(q), x_a^*) = \alpha^{\frac{\sigma}{\sigma-1}} \cdot \frac{T}{s} \cdot \left[\left(\frac{q_a}{\alpha}\right)^\sigma + \frac{1 - q_m^{\sigma+1}}{\sigma + 1} \right]^{\frac{1}{\sigma-1}} \quad (7)$$

which is decreasing in $q_m \in [0, 1]$.

B. Equilibrium Content Production

Based on the content viewing decision, we now analyze the content production decisions made by self-interested content producers (i.e., step 2 in Fig. 3). Due to rationality, content producers will not choose to produce and provide content if they cannot obtain non-negative utilities. Essentially, interaction among the content producers can be formalized as a non-cooperative game with an infinite number of players, the solution to which is (Nash) equilibrium (i.e., steady-state outcome in the content production stage given the platform's price) [33]. From a system point of view, the platform's profit will become stabilized if the content production stage reaches an equilibrium.

Thus, the existence of an equilibrium point is important and relevant for the platform to maximize its long-term profit. At an equilibrium, if any, no content producers can gain higher benefits by deviating from their decisions. In other words, the fraction of content producers choosing to produce content on the platform does not change at the equilibrium, or equivalently, the marginal content desirability $q_m \in [0, 1]$ becomes invariant. Next, we study the equilibrium content production by specifying the equilibrium marginal content desirability denoted by q_m^* .

If $q_m^* = 1$, then no (or a zero mass of) content producers can receive a non-negative utility by producing and selling content on the platform. This implies that, with $q_m^* = 1$, we have $x^*(1|1, q_a) \cdot (s - \theta) - c \leq 0$. If there are some content producers choosing to produce content at the equilibrium (i.e., $q_m^* \in [0, 1)$), we have $x^*(q_m^*|q_m^*, q_a) \cdot (s - \theta) - c = 0$. Hence, we can show that $q_m^* \in [0, 1]$ satisfies

$$q_m^* \triangleq Q(q_m^*) = \left[\left\{ \frac{c \cdot s \cdot \left[(\sigma + 1) \cdot \left(\frac{q_a}{\alpha} \right)^\sigma + 1 - (q_m^*)^{\sigma+1} \right]}{T(\sigma + 1)(s - \theta)} \right\}^{\frac{1}{\sigma}} \right]_0^1 \quad (8)$$

where $[\nu]_0^1 = \max\{1, \min\{0, \nu\}\}$. Thus, an equilibrium point of content production exists if and only if the mapping $Q(q_m^*)$, defined in (8), has a fixed point. Next, we formally define the equilibrium marginal content desirability in terms of q_m^* as below.

Definition 1: q_m^* is an *equilibrium* marginal content desirability if it satisfies $q_m^* = Q(q_m^*)$.

We establish the existence and uniqueness of an equilibrium marginal content desirability in Theorem 1, whose proof can be found in [29].

Theorem 1: For any $\theta \in [b, s]$, there exists a unique equilibrium $q_m^* \in (0, 1]$ in the production decision stage. Moreover, q_m^* satisfies

$$\begin{cases} q_m^* = 1, & \text{if } x^*(1|1, q_a) \cdot (s - \theta) \leq c \\ q_m^* \in (0, 1), & \text{otherwise} \end{cases} \quad (9)$$

where $x^*(1|1, q_a)$ is obtained by solving (4) with $q_m \rightarrow 1$.¹³ \square

Theorem 1 guarantees the existence of a unique equilibrium point. This result is important for the platform to design its optimal price, since once its optimal price is set, the platform's profit is guaranteed to be maximized without any uncertainties due to the unique outcome in the production stage. Moreover, Theorem 1 shows that if the content provider with the highest quality cannot obtain a positive utility (due to high production cost and/or price charged by the platform), then no content producers choose to produce and provide content on the platform at equilibrium. For notational convenience, we denote the value of θ that satisfies $x^*(1|1, q_a) \cdot (s - \theta) = c$ by

$$\bar{\theta} \triangleq s - \frac{c}{x^*(1|1, q_a)} = s - \frac{c \cdot s}{T} \cdot \left(\frac{q_a}{\alpha} \right)^\sigma. \quad (10)$$

Then, it follows from Theorem 1 that the platform can gain a positive profit if and only if $\theta \in [b, \bar{\theta}]$. Nevertheless, if $\bar{\theta} \leq b$, then the platform's profit is always zero. Hence, we assume $\bar{\theta} >$

b throughout the paper. Based on the uniqueness of q_m^* for any $\theta \in [-s, b]$, we can express $q_m^* = q_m^*(\theta)$ as a function of $\theta \in [-s, b]$. While there exists no simple closed-form expression of $q_m^*(\theta)$, it can be easily shown that $q_m^*(\theta) \in (0, 1)$ is strictly increasing in $\theta \in [b, \bar{\theta}]$ (i.e., fewer content producers choose to produce and sell content on the platform when the price θ increases) and $q_m^*(\theta) = 1$ for $\theta \in [\bar{\theta}, s]$.

In practice, the content producers do not have complete information regarding each other (e.g., they do not know whether a certain content producer will produce content or not) and hence, they may not make directly the decisions that arrive at an equilibrium. In such a scenario, the content producers may use an adjustment process to update their decisions based on limited information [33]. A natural and well-studied approach to modeling an adjustment process is the best-response dynamics, in which each decision maker chooses the best action in response to the decisions made by the others [33]. In this paper, we consider the best-response dynamics based on naive (or static) expectation [23]. Specifically, at the end of (discrete) time $t = 1, 2, 3 \dots$, content producer i assumes that the decisions made by the other content producers at time $t + 1$ remain the same as those at time t , and expects $x_{t+1}(q_i) = x^*(q|q_{m,t}, q_a)$, where $x^*(q|q_{m,t}, q_a)$ is the solution to (4) and $q_{m,t} \in [0, 1]$ is the marginal content desirability at time t . Note that a content provider with a content desirability less than $q_{m,t}$ may also choose to produce and sell content at time $t + 1$, if it believes that there is not much high-desirability content on the platform (i.e., $q_{m,t} \in [0, 1]$ is large) and it can receive a non-negative utility. Similar decision processes have been adopted in the existing literature (e.g., [21] and references therein). The best-response decision model implies that the sequence $q_{m,t}$, for $t = 0, 1, 2 \dots$, evolves as follows:¹⁴

$$q_{m,t+1} = Q(q_{m,t}) \quad (11)$$

where $Q(\cdot)$ is defined in (8). Essentially, the dynamics in (11) is a fixed point iteration for $Q(\cdot)$ and it converges regardless of the initial point if $|Q'(q)| < 1$ for $q \in [0, 1]$ [27]. For brevity, we omit the derivation of convergence conditions. It should be noted that the dynamics specified by (11) requires that all the content producers update production decisions at the end of each time period. In practice, if only a fraction $\epsilon \in (0, 1]$ of the content producers make decisions each time, then the sequence becomes $q_{m,t+1} = (1 - \epsilon)q_{m,t} + \epsilon Q(q_{m,t})$ without affecting the equilibrium analysis while the convergence rate is slowed down.

To summarize, we have studied how content producers make self-interested and independent decisions (in particular equilibrium decisions) in response to the platform's price θ . This will serve as a basis for the platform to optimize its price, as when making the price decision in practice, the platform has to take into account the corresponding responses of content producers.

C. Optimal Price

Based on decisions made by the content viewers and content producers, we study the optimal price θ that maximizes the platform's steady-state profit (i.e., profit obtained when the content production decision stage reaches equilibrium). This analysis

¹³When $q_m \rightarrow 1$, only a negligible fraction of content producers choose to produce and sell content on the platform.

¹⁴If there exists no $q \in [0, 1]$ such that $x(q|q_{m,t}, q_a) \cdot (\theta + s) - c \geq 0$, then we set $q_{m,t+1} = 1$.

corresponds to the third step in Fig. 3. Mathematically, we formalize the profit maximization problem as

$$\theta^* = \arg \max_{\theta \in [b, \bar{\theta}]} (\theta - b) \cdot \bar{x} - w \quad (12)$$

where $\bar{x} = \int_{q_m^*}^1 x^*(q|q_m^*, q_a) dq$ denotes the total content views (or volumes of sales) of the content on the platform. The decision interval is shrunk to $[b, \bar{\theta}]$, since $\theta \in (\bar{\theta}, s]$ always results in a zero profit for the platform, where $\bar{\theta}$ is defined in (10). By Property 4 stated in Section III-C, $\bar{x} = \int_{q_m^*}^1 x^*(q) dq$ is decreasing in $q_m^* \in [0, 1]$. Then, recalling that $q_m^*(\theta)$ is strictly increasing in $\theta \in [b, \bar{\theta}]$, we can see \bar{x} is decreasing in $\theta \in [b, \bar{\theta}]$. In other words, decreasing $\theta \in [b, \bar{\theta}]$ will encourage more content producers to produce content on the platform and hence lead to more content sales in total.

In the following analysis, a closed-form optimal price $\theta^* \in [b, \bar{\theta}]$ is explicitly obtained and shown in Theorem 2.

Theorem 2: The unique optimal price $\theta^* \in [b, \bar{\theta}]$ that maximizes the platform's profit is given by

$$\theta^* = \frac{c \cdot s \cdot [(\sigma + 1) \cdot (\frac{q_a}{\alpha})^\sigma + 1 - z^{\sigma+1}]}{T(\sigma + 1) \cdot z^\sigma} - s \quad (13)$$

where $z \in [q_m^*(b), 1]$ is the unique solution to the equation¹⁵

$$\frac{T \cdot (\frac{q_a}{\alpha})^\sigma \cdot (b + s)}{s \cdot [(\sigma + 1) \cdot (\frac{q_a}{\alpha})^\sigma + 1 - z^{\sigma+1}]^2} = \frac{c}{(\sigma + 1)^3} \cdot \frac{\sigma + z^{\sigma+1}}{z^{2\sigma+1}}. \quad (14)$$

Proof: The proof is provided in [29]. \square

Finally, we conclude this section by discussing two extreme cases, $q_a \rightarrow 0$ and $\sigma \rightarrow \infty$. When $q_a \rightarrow 0$, the aggregate outside content quality is negligible (e.g., very low quality or little outside content available). In other words, almost all the content views are devoted to content on the platform. Therefore, the platform can charge the content producers by setting $\theta^* \rightarrow s$ and its profit can be arbitrarily close to $(s - b) \cdot (T/s)$. When $\sigma \rightarrow \infty$, the content becomes perfectly substitutable. Naturally, all the content views will be attracted by the content with the highest quality. This can also be verified by taking the limit $\sigma \rightarrow \infty$ in (6). Therefore, if $q_a > \alpha$ and $\sigma \rightarrow \infty$ (which is equivalent to $q_h > \alpha$ when $\sigma \rightarrow \infty$),¹⁶ then the content produced on the platform will receive no content views and the platform cannot possibly obtain a positive profit by varying θ . On the other hand, if $q_a < \alpha$ (which is equivalent to $q_h < \alpha$ when $\sigma \rightarrow \infty$), then the content with a desirability of 1 can receive almost all the content views and the platform can set $\theta^* \rightarrow s$ to make its profit arbitrarily close to $(s - b) \cdot (T/s)$. When $q_a = \alpha$ and $\sigma \rightarrow \infty$, the outside content with a quality of $q_h = \alpha$ will attract some content views while the remaining content views will be devoted to the content with a desirability of 1 on the considered platform. Moreover, after some simple mathematical manipulations, we can show that in this case the platform's profit is $(s - b) \cdot (T/(s(n_a + 1)))$, where n_a is the total amount of outside content. To sum up, when $q_a \rightarrow 0$ or

¹⁵ $q_m^*(b)$ is the equilibrium point in the content production stage when $\theta = b$.

¹⁶When $\sigma \rightarrow \infty$, we see from (3) that $q_a = \lim_{\sigma \rightarrow \infty} n_a^{1/\sigma} \cdot \lim_{\sigma \rightarrow \infty} q_h^{(\sigma+1)/\sigma} \cdot \lim_{\sigma \rightarrow \infty} [1 - (q_l/q_h)^{\sigma+1}]^{1/\sigma} \cdot \lim_{\sigma \rightarrow \infty} (1/(1 + \sigma))^{1/\sigma} = q_h$. Thus, when $\sigma \rightarrow \infty$, $q_a > \alpha$ if and only if the highest content quality $q_h > \alpha$.

$\sigma \rightarrow \infty$ with $q_a < \alpha$, the platform can almost fully extract the revenue source, i.e., payment from content producers.

D. Optimal Investment Level

Finally, based on the decision making of content viewers, content producers and the platform's price decision, the platform needs to choose the optimal investment level to maximize its profit (i.e., step 4 in Fig. 3). In practice, the investment decision corresponds to the purchase of servers, selection of encoders, bandwidth acquisition and allocation, etc., which all affect the quality of experiences of content viewers [10], [31]. For instance, it is clear that faster servers will provide better quality of experience while costing more money. Mathematically, we need to solve the following problem:

$$\alpha^* = \arg \max_{\alpha \in (0, \bar{\alpha}]} [\theta - b(\alpha)] \cdot \bar{x} - w(\alpha). \quad (15)$$

In general, it is difficult to express $b(\alpha)$ and $w(\alpha)$ explicitly, as it involves many factors such as server prices, bandwidth costs, etc. Thus, in practice, the platform can conduct simulations to fit $b(\alpha)$ and $w(\alpha)$ as certain functions in terms of $\alpha \in (0, \bar{\alpha}]$. That is, given that the actual quality of experience is the product of α_0 and content desirability q , then the platform estimates the values of $b(\alpha_0)$ and $w(\alpha_0)$. Hence, by changing the values of α_0 , a relation between $b(\alpha)$, $w(\alpha)$, and α can be obtained. We note that even with simple expressions of $b(\alpha)$ and $w(\alpha)$, e.g., $b(\alpha) = w(\alpha) = \alpha$, the optimization in (15) loses analytical tractability and it is rather challenging to analytically derive the optimal value of α , because, as shown in (13), the optimal price θ^* is very complicated. Similar challenges also exist in other problems involving sequential decision-making steps (e.g., [26]). Therefore, we resort to numerical methods to find the optimal α^* to solve (15), which provides the platform with an approach to making the optimal investment decision.

V. NUMERICAL RESULTS

In this section, we provide numerical results to illustrate the impacts of the price θ and investment level α on the platform's profit. Because of the nature of our research, it is prohibitive to conduct simulations or build prototypes. Nevertheless, as in other related works [18], our study provides a quantitative guidance for the platform to make pricing and investment decisions to maximize its profit, and once market data is available, the platform owner may set up its content platform based on our analysis.

The default settings are as follows. The total budget of the representative content viewer (i.e., market size) is set to $T = 50$, the elasticity of substitution between different content is $\alpha = 2$ (i.e., the content is mostly non-substitutable), the production cost per content is $c = 1.0$, and the aggregate outside content quality is $q_a = (1/\alpha^{1/\sigma}) \cdot [(n_a(q_h^{\sigma+1} - q_l^{\sigma+1})/(1 + \sigma))]^{1/\sigma} = (4/\alpha^{1/\sigma})$. Note that these values are only for the purpose of illustration and our analysis applies to any other settings. For instance, with all the other parameters being the same, a larger value of T indicates that the online TV market size is bigger (i.e., online TV audiences spend more money in watching TV). In practice, the platform needs to obtain market settings by conducting market surveys and data analysis [18]. As there are no explicit expressions for $b(\alpha)$ or $w(\alpha)$ in terms of $\alpha \in (0, \bar{\alpha}]$,

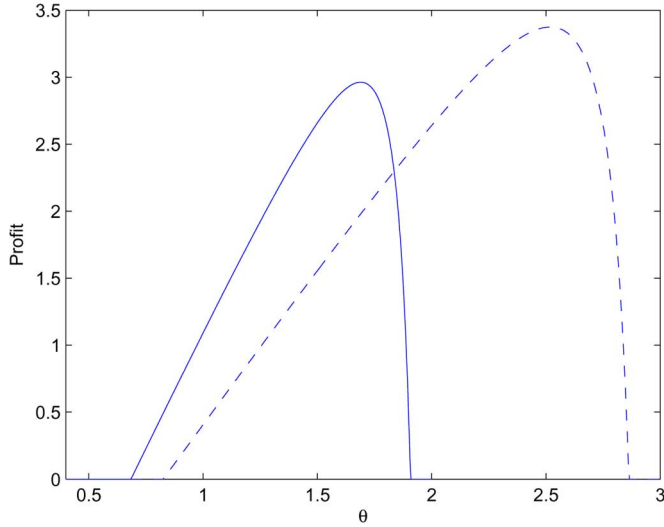


Fig. 4. Profit versus price θ with $\alpha = 2.0$. $s = 2$ and $s = 3$ are represented by solid lines and dashed lines, respectively.

we assume for illustration purposes that the bandwidth cost per content distribution and lump-sum investment are $b(\alpha) = 0.2\alpha$ and $w(\alpha) = 0.5\alpha$, respectively, where α indicates the platform's investment level. This assumption means that the investment cost grows linearly with α , which then affects the users' quality of experience by scaling the content desirability. Although the linear expressions of $b(\alpha) = 0.2\alpha$ and $w(\alpha) = 0.5\alpha$ are simple, they capture the fact that a larger investment in the infrastructure will result in a better quality of experience for content viewers.¹⁷ Next, we provide a simple example to explain this point. It is known that PSNR is an objective parameter indicating the video quality, and the resulting PSNR is an increasing function of the allocated bandwidth/rate [13], [31]. If we assume that the actual quality of experience is the product of (normalized) PSNR and the content desirability (i.e., α is proportional to PSNR), then the average bandwidth cost, which is a major component in the cost term $b(\alpha)$ per content view for the platform, will grow with α (and the growth is approximately linear in high PSNR regions [31]). The lump-sum cost $w(\alpha)$, e.g., server expenditure, also increases with PSNR (although the increase is not necessarily linear), since in general better servers are required to support a higher PSNR [11], [12], [31]. Finally, note that in our figures, "zero" values (e.g., zero profit, zero price) are assumed if the platform cannot obtain a positive profit.

A. Fixed Investment Level $\alpha = 2$

In this subsection, we examine the impacts of prices on the equilibrium outcome under a fixed investment level $\alpha = 2$. We first plot in Fig. 4 the profits under various prices θ . It shows that, when the price per content s is larger, the platform can derive a larger profit, as the platform can charge a higher price θ per content view from the content producers, although the total content views decrease. In order to obtain a positive profit, we observe from Fig. 4 that the platform's minimum price θ per content view is higher when s is larger. This can be explained

¹⁷Our analysis is not affected if we consider other expressions of $b(\alpha)$ and $w(\alpha)$.

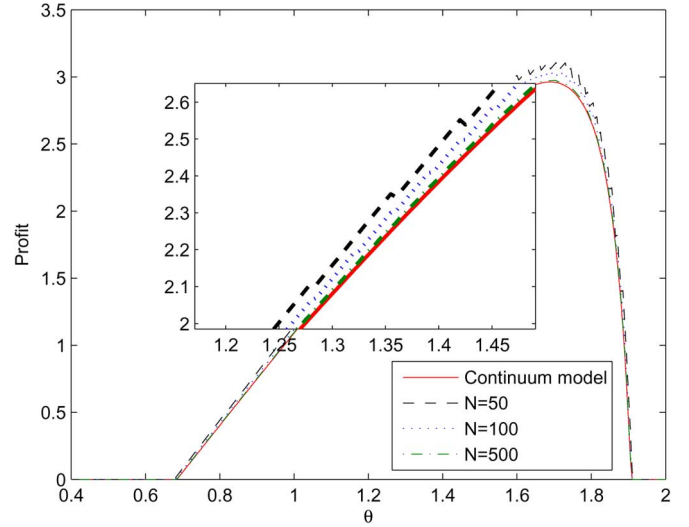


Fig. 5. Profit versus price θ with $\alpha = 2.0$ and $s = 2$ under different models.

by the fact that the increase of s leads to a decrease of the total content views in the entire TV market and hence, the total content views attracted by the content on the platform may also decrease. Therefore, in order to recover the lump-sum investment, the platform should increase θ (even though increasing θ will further decrease the total content views of the content on the platform). Another observation made from Fig. 4 is that the platform cannot set θ higher than s , since otherwise no content producers can make profits out of producing and providing content on the platform. To validate the continuum model, we show in Fig. 5 that when the number of content producers (denoted by N) is greater than or equal to 50, the long-term profit obtained under the discrete model is very close to that obtained under the continuum model.

Although we focus on only one TV content platform without explicitly considering strategic competition among multiple content platforms, we note that the outside content quality q_a can be viewed as an aggregate parameter that conveniently captures all aspects of the outside content platforms (e.g., outside content price, how much outside content platforms charge the content producers, etc.). For example, if the outside content platforms invest more in their infrastructure, the outside content quality q_a will naturally increase. Thus, we can use the outside content quality q_a as a proxy to investigate the impacts of the actions taken by outside content platforms on the pricing and investment decisions of the considered TV content platform. For instance, with a fixed investment decision, we show in Fig. 6(a) the profit of the considered TV content platform given different values of q_a . We see that as q_a increases (e.g., the outside content platforms make larger investment and/or charge lower prices from the outside content producers), the considered TV content platform's profit decreases and its optimal price θ^* also decreases.

In Fig. 6(b), we show the total amount of content available on the platform under different values of θ . Given the same value of θ , more content producers choose to produce content on the platform if s is larger, since they can receive a larger benefit per content view although the total content views in the entire market are smaller. Fig. 6(b) also confirms that fewer content producers

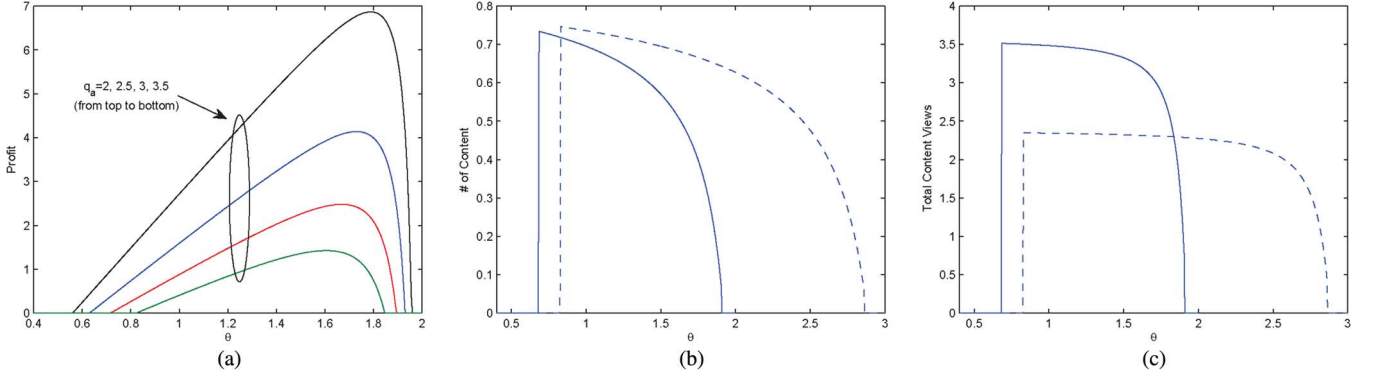


Fig. 6. Fixed investment level. (a) Profit versus price θ with different aggregate outside content quality q_a . (b) $(1 - q_m^*)$ versus price θ . $s = 2$ and $s = 3$ are represented by solid lines and dashed lines, respectively. (c) Total content views versus price θ . $s = 2$ and $s = 3$ are represented by solid lines and dashed lines, respectively.

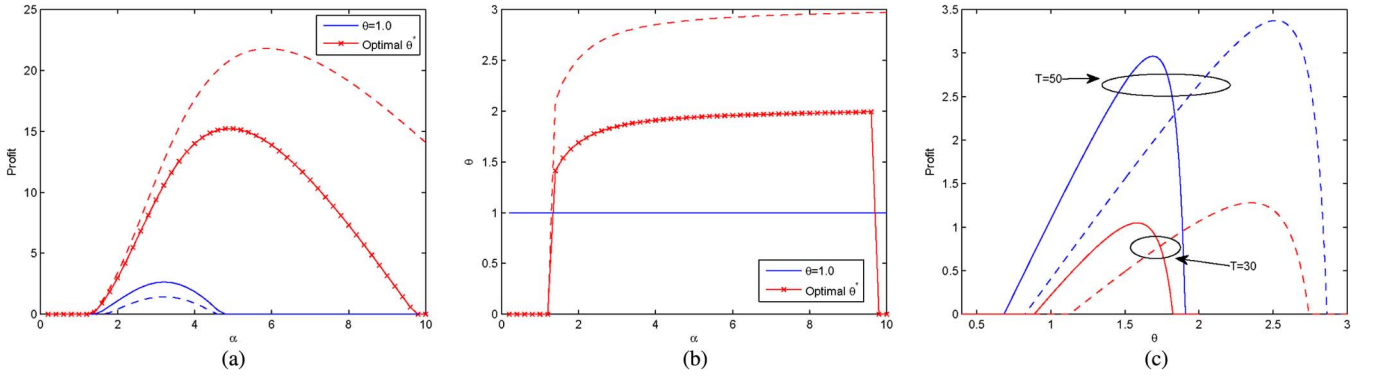


Fig. 7. Various investment levels and sensitivity study. (a) Profit versus investment level α . $s = 2$ and $s = 3$ are represented by solid lines and dashed lines, respectively. (b) Price versus investment level α . $s = 2$ and $s = 3$ are represented by solid lines and dashed lines, respectively. (c) Sensitivity analysis: profit versus price θ with $\alpha = 2.0$. $s = 2$ and $s = 3$ are represented by solid lines and dashed lines, respectively.

choose to produce and provide content on the platform if they are charged a higher price by the platform. In particular, if θ is too large such that no content producers can make profits or θ is too small such that the platform cannot make profits, then no content producers will provide content or the platform will not be established. As shown in Fig. 6(b), in either case, the amount of content on the platform is zero. Next, we show in Fig. 6(c) the total content views devoted to the content on the platform under various values of θ . When s increases, the total content views in the entire market shrink given a fixed total budget of T . Thus, the platform can only attract fewer content views if s increases when θ is not large. Nevertheless, when θ increases, very few content producers will choose to produce content on the platform if they cannot receive enough benefits from content views or sales (i.e., s is small) and hence, due to the lack of content on the platform, the representative content viewer will only allocate a small portion of its budget to the content on the platform.

B. Various Investment Levels

In this subsection, we examine the impacts of investment levels on the platform's decision under both the optimal price θ^* and a fixed price $\theta = 1.0$. First, in Fig. 7(a), we plot the profits under different investment levels, showing that the investment level significantly affects the platform's equilibrium profit. Specifically, if the platform adopts a fixed price of $\theta =$

1.0 which is far from being optimal, then the platform cannot invest a lot in its infrastructure, as it can only receive limited payment from the content producers. In contrast, if the platform can choose the optimal θ^* , then it is willing to invest much more in its infrastructure and provide higher quality content delivery to attract more content views from the content viewers. Nevertheless, the investment should not be too much if the platform aims at maximizing its profit. This is because, when the platform make more investment than needed, then the total content views that are devoted to the content on the platform will only increase marginally while the platform incurs a higher cost in bandwidth and lump-sum investment. In particular, when the investment level α is around 6 (i.e., original content desirability q is scaled by 6 after being processed and delivered to content viewers by the platform), the aggregate outside content quality is already very low and most content viewers prefer to viewing content on the platform, and hence the platform does not need to invest more, which merely increases its cost without attracting more content views. Fig. 7(a) also demonstrates that the platform's profit increases when the price s per content view becomes higher. The reason is that the platform's investment level is sufficiently high such that it can attract most content views. That is, the platform has a very large marketing power and hence, can charge the content producers a higher price θ per content view without

compromising its market share too much. Fig. 7(b) shows the optimal prices under different investment levels. It verifies that when the platform's investment level is sufficiently high (e.g., $\alpha \geq 6$), most content viewers will choose to view the content on the platform. Thus, the platform can announce a very large θ to the content viewers as long as θ is still below the price s per content view (i.e., content producers can still make profits from content views).

Finally, we show in Fig. 7(c) that our previous qualitative discussions for Fig. 4 are also applicable for a different value of T . Note that, under different settings, our discussions are still valid, while the corresponding numerical results are not shown because of the space limitation.

VI. CONCLUSION

In this paper, we focused on a TV content platform and studied two issues for the platform owner: pricing and investment decisions. To formalize the profit maximization problem, we proposed a sequential decision-making model, in which the platform first makes its investment decision and then pricing decision, followed by the content producers' production decisions and then by the content viewers' viewing decisions. Using backward induction, we first used the *representative* content viewer model to determine how the content views are allocated across a variety of content. Then, we showed that there always exists a unique equilibrium point at which no content producer can gain by changing its production decision, and proposed a best-response dynamics to model the distributed decision-making process. Next, we formalized the platform's profit maximization problem and, by using the quality-adjusted Dixit-Stiglitz utility function, derived the closed-form optimal price explicitly. Finally, we formalized the platform's investment decision problem and solved it numerically. Our results provide the platform with a quantitative guidance with respect to its pricing policy for the content producers and its long-term investment decision, in the presence of self-interested content producers and content viewers. Future research directions include, but are not limited to: 1) differentiated pricing in which different content producers may be charged differently; 2) a scenario where content producers can vary their own content quality and choose to produce on more than one platforms; and 3) optimal price and investment level maximizing social welfare.

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