To Tax or To Subsidize: The Economics of User-Generated Content Platforms

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Abstract

As the Internet has been penetrating every aspect of our lives, we have witnessed a significant expansion of online platforms for sharing user-generated content during the past decade. Well-known examples are YouTube, Flickr, and Yahoo! Answers. On these platforms, users can view content for free while they post content voluntarily. Recently, turning the user-generated content into profit has become increasingly important for platform owners, as the operational cost (e.g., bandwidth, energy, marketing) for managing user-generated content platforms is skyrocketing. In this chapter, we study the economics of user-generated content platforms from the perspective of maximizing profit of platform owners, particularly focusing on the question of whether the platform owner should subsidize content producers for their contribution or tax them for using the platform's service. Based on formal analysis of a three-stage game modeling the interactions among the platform owner, content producers and content viewers, we derive the optimal profit-maximizing payment scheme and give our recommendation as to when the platform owner should subsidize or tax content producers for profit maximization.

1 Introduction

Enabled by ubiquitous broadband connectivity and seamlessly accessible wireless connections, user-generated content platforms have witnessed in the past few years an explosive growth, allowing everyone to conveniently publish information online and share their knowledge, opinions, experiences, etc., with the rest of the world. Every day, millions of people "tweet" on Twitter, update their status on Facebook, ask and answer questions on Yahoo! Answers, and share their videos on YouTube. As user-generated content platforms are becoming an integral part of our lives, both Internet users and platform owners are eager to see the continuing growth of such platforms, which are nonetheless largely hindered by a number of obstacles, notably low-quality content and lack of revenue sources. Typically, users can view the content for free while they post content on user-generated content platforms voluntarily. Thus, users often have little incentive to devote their efforts into improving content quality, resulting in low-quality content published on the platforms. To tackle this, various mechanisms have been proposed to promote high-quality content produced by users. For example, a common approach is to eliminate or hide low-quality content as adopted by popular websites such as Yahoo! Answers [1]. While incentivizing high-quality content contributions on user-generated content platforms still remains an active research area (see [1, 2] and references therein), turning the user-generated content into profit has become increasingly important for platform owners, as advertising accounts for a major (and possibly the only) revenue source while the operational cost (e.g., bandwidth, energy, marketing) for managing user-generated content platforms is skyrocketing. Even though sophisticated algorithms may be employed to increase the advertising revenue (e.g., displaying more relevant advertisement or use auction mechanisms for selling advertisement space), the continuously increasing operational cost still catapults the exploration of alternative revenues as one of the key challenges for user-generated content platform owners, thereby motivating our study of the economics of user-generated content platforms from the perspective of profit maximization.

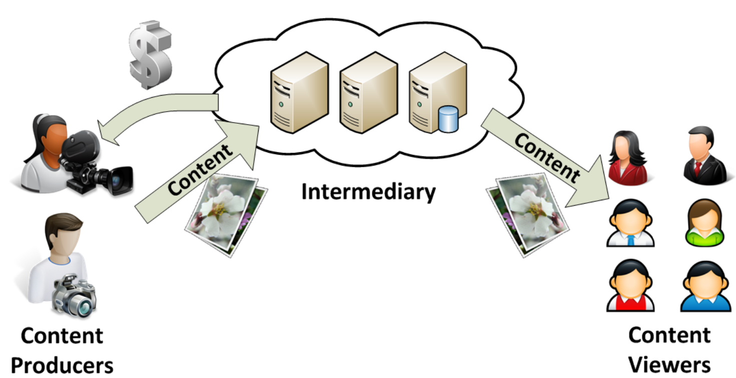


Figure 1: Illustration of user-generated content platform

On a user-generated content platform as illustrated in Fig. 1, the platform owner is referred to as an *intermediary*, while the users can be classified as content producers and content viewers, although a user can be both a producer and a viewer. While content producers post content on the platform voluntarily without any obligation, content producers still receive *benefits* by publishing their content: millions of users engaging daily in Internet activities such as blogs, for which they receive no monetary rewards, suggest that content producers may simply derive satisfaction from attracting content viewers' attention (see, for example, [1, 3, 17]). In addition to social satisfaction, the intermediary may also provide economic incentives by subsidizing (i.e., paying) content producers for their contribution. The logic behind "subsidizing" is that the intermediary gives away some of its advertising revenue to content producers in the hope that the total advertising revenue may increase due to the increased amount of content available on the platform. Opposite to subsidizing, the intermediary may tax (i.e., charge) content producers for using the platform's service (e.g., storage space, bandwidth): if content producers have intrinsic social incentives to produce content, taxing content producers will give the intermediary an additional revenue source while still attracting some content producers, although the intermediary may suffer from reduced advertising revenue due to decreased user traffic resulting from less content production. In general, subsidizing content producers can be considered as a reward for providing content, whereas taxing content producers can be considered as a usage fee for utilizing the intermediary's resources. While, in practice, "subsidizing content producers" can be observed more often (e.g., YouTube Partner, Squidoo) than "taxing content producers" (e.g., Google Picasa, which taxes its users for storage exceeding the free quota), there is no simple answer to the question of which strategy maximizes the intermediary's profit, which is crucial for the long-term growth of user-generated content platforms.

Concluding whether subsidizing or taxing content producers is maximizes the intermediary's profit requires an appropriate model as the foundation. Naturally, with both content producers and content viewers as participants, user-generated content platform can be modeled as a two-sided market, where two user groups (i.e., content producers and content viewers) interact and provide each other with network benefits [4]. Besides user-generated content platforms, two-sided market can also model a variety of other markets such as a broadband communications market with content producers and end-users [8] and payment card industry with merchants and customers [9]. Unlike other two-sided markets, however, user-generated content platforms have the following characteristics that have not been adequately addressed by the existing research. First, content producers compete for the content viewers' attention, i.e., intra-group negative externalities, which have been neglected by most existing research on two-sided markets (see [4, 5] for a survey). Second, content viewers tend to have *love for variety*, i.e., inter-group positive externalities, which have been incorporated by the existing literature [6] but not well suited to user-generated content platforms. Last but not least, user-generated content exhibits a diverse content quality and substitutability, i.e., different content often has different qualities but may substitute each other to a certain extent from the content viewers' perspective. In what follows, in order to provide a formal analysis on when subsidizing or taxing content producers is profit-maximizing for the intermediary, we provide a new model for user-generated content platform, which captures all the above three characteristics. Since subsidizing content producers per content view is a common practice in the Internet industry (e.g., YouTube Partner) while not taxing users for viewing content is also a common practice (e.g., YouTube, Yahoo! Answers). we focus on a class of payment schemes in which the intermediary subsidizes or taxes content producers on a basis of per content view while it provides the service for free to content viewers. Our analysis provides a formal guidance for the intermediary to decide its optimal payment to the content producers for profit maximization. In particular, we recommend that the intermediary should subsidize the content producers under the following circumstances: (1) there are few content viewers; (2) content production cost is high; (3) content producers do not receive strong social satisfaction; or (4) there are more popular user-generated content platforms where the content viewers can go and view content.

The rest of this chapter is organized as follows. The model is described in Chapter 2. In Chapter 3, we analyze the problems of content viewers, content producers, and the intermediary. In Chapter 4, we outline how our analysis can be extended to the case of heterogeneous production costs. Finally, concluding remarks are offered in Chapter 5.

2 Model

We consider an online user-generated content platform owned by a profit-maximizing intermediary. Content on the platform is produced and viewed by individual users. Our model is a three-stage game that is played by the intermediary, content producers, and content viewers in the following order.

- *Stage 1:* The intermediary sets a payment rate per content view, paid to content producers.
- *Stage 2:* Given the payment rate chosen by the intermediary, each content producer chooses whether or not to produce content on the platform.
- *Stage 3:* Given available content, content viewers, consolidated as a representative content viewer, allocate their total content views over available content.

While a single user can be a content producer and a content viewer at the same time, it is assumed in our analysis that the decision making of a user as a content producer is separable from that as a content viewer. Below, we describe the problems of the intermediary, content producers, and content viewers in detail.

2.1 Intermediary

It is well-known that advertising is one of the most prevailing revenue sources in the Internet industry, especially for online content platforms such as YouTube and Yahoo! Answers. Hence, we consider a scenario in which the intermediary monetizes its content platform by displaying contextual advertisement to content viewers. Although a platform may use different bases (e.g., pay per click, pay per sale, and pay per impression) to charging advertisers, the advertising revenue is in general increases with the number of times that content with advertisement is viewed (or content views). For simplicity, we assume that the intermediary's advertising revenue is proportional to the total number of content views in the platform, as in [7].

The intermediary chooses the amount of money, denoted by θ, which it subsidizes a content producer for each time the content producer's content is viewed. We refer to θ as the *payment rate* (to content producers per content view). We allow θ to be any real number. The intermediary subsidizes content producers if $\theta > 0$, while it taxes them if $\theta < 0$. Let b be the intermediary's profit per content view without accounting for payment to content producers, and b can be interpreted as advertising revenue minus operation cost per content view. Note that b can be negative if the operation cost is larger than the advertising revenue. We assume that b is an exogenously given as a constant independent of the total number of content views, while the intermediary may increase its advertising revenue per content view by developing algorithms for displaying relevant advertisement or auction mechanisms for selling advertisement space (which is outside of the scope of this chapter). Let $\bar{x}(\theta)$ be the total number of content views, which is determined by the decisions of users, that the intermediary obtains when it chooses θ. Neglecting fixed operation cost, we can express the intermediary's profit as

$$\Pi(\theta) = (b - \theta)\bar{x}(\theta). \tag{1}$$

The intermediary's problem is to maximize its profit by choosing a payment rate.

2.2 Content Producers

As evidenced by the exploding number of YouTube users, a popular user-generated content platform can attract a huge number of content producers. To capture this fact, we consider a continuum of content producers of mass one as in prior work [12, 22, 20]. We index content producers by i (and sometimes j without ambiguity), where i is uniformly distributed over the interval $[0, 1]$. Content producer i can produce content of quality $q_i \geq 0$ while incurring production cost $c > 0$. The quality of content is represented by a nonnegative scalar, and we treat it as an internal feature of content (e.g., how fun or informative content is). Also, the production cost is the same for all content producers, and this assumption will be relaxed in Chapter 4. Content producers produce differentiated content, or in other words, no two content producers can produce identical content. We sometimes refer to content produced by content producer i as content i.

Given the payment rate chosen by the intermediary, content producers make decisions regarding whether to produce content or not. Once a content producer produces content, it posts the content on the intermediary's platform. In other words, the intermediary's platform is the only platform on which the content producers can provide their content. We assume that a user cannot provide content produced by others, for example, due to copyright restrictions. That is, content i can be posted on the platform only by content producer i. We represent the decision of content producer i by a binary variable y_i, which takes value 0 if content producer i chooses not to produce and 1 otherwise. The decisions of all the content producers are summarized in $y = (y_i)_{i\in[0,1]} \in \{0,1\}^{[0,1]}$. Note that y determines content available on the platform.

There are two major types of benefits that a content producer obtains by posting its content on the platform. One is payment from the intermediary, which can also be interpreted as a cost when the payment rate is negative, while the other is social satisfaction. Millions of users engaging daily in Internet activities such as blogs, for which they receive no monetary rewards, suggest that content producers may simply derive satisfaction from attracting content viewers' attention (see, for example, [1, 3, 17]). We use content views to quantify the amount of received attention and assume that a content producer's satisfaction is proportional to the number of content views of its content. Let $s > 0$ be the social benefit, measured in a monetary unit, per content view that a content producer derives from content viewers' attention. Let $x_i(y)$ be the number of content views that content i attracts given available content on the platform determined by y. Note that $(x_i(y))_{i\in[0,1]}$ is determined by content viewers. The payoff function of content producer i is given by

$$\pi_i(\theta, y) = \begin{cases} (\theta + s)x_i(y) - c & \text{if } y_i = 1, \\ 0 & \text{if } y_i = 0. \end{cases} \tag{2}$$

Content producers make production decisions simultaneously to maximize their own payoffs given the payment rate chosen by the intermediary.

Remark: In our model, each content producer can produce only one piece of content. However, if there is a content producer who can produce up to $m \geq 1$ pieces of content, it can be treated as m separate content producers in our model as long as the cost of producing any $\tilde{m} \leq m$ pieces is $\tilde{m}c$ (i.e., constant returns to scale, as assumed in [6]).

2.3 Content Viewers

We can expect that content viewers have diverse preferences towards content. In order to study their aggregate content viewing behavior conveniently, we adopt the widely-used representative agent model [14].

That is, we analyze the optimal decision of a representative content viewer to determine the content views that each piece of available content receives. The representative content viewer has a fixed number of content views to be allocated to available content, which is denoted by $T > 0$. Note that T can be interpreted as the size of the representative content viewer or the market size on the viewer side. Besides the intermediary's platform, there are other platforms that offer (differentiated) content. For convenience, we assume that "outside" content has a fixed aggregate quality $q_a > 0$. In Chapter 3.2, we illustrate how the aggregate quality can be derived from a distribution of individual qualities. The assumption that q_a is independent of y can be justified by noting that there are many content platforms on the Internet and thus changes on a single content platform have negligible impacts on the other platforms.

Let x_i be the number of content views of content i for $i \in [0,1]$ and x_a be the total number of content views of outside content (i.e., content on the other platforms). We use $x = ((x_i)_{i\in[0,1]}, x_a) \in \mathbb{R}_+^{[0,1]} \times \mathbb{R}_+$ to denote a content view allocation of the representative content viewer. Let $U(x)$ be the utility of the representative content viewer when its content view allocation is x. Then the representative content viewer's problem given the decisions of the content producers, y, can be written as

$$\max_x U(x) \tag{3}$$

$$\text{s.t.,} \int_0^1 x_i di + x_a = T, \tag{4}$$

$$x_i \geq 0 \text{ for all } i \in [0,1], x_a \geq 0, \tag{5}$$

$$x_i = 0 \text{ for all } i \text{ such that } y_i = 0. \tag{6}$$

The constraint in (4) requires that the total number of content views be equal to T. The constraints in (5) are nonnegativity constraints for numbers of content views. The constraints in (6) impose that the content viewer cannot allocate a positive number of content views to unavailable content.

3 Profit Maximization on User-Generated Content Platforms

3.1 Definition of Equilibrium

In this chapter, we analyze the optimal decision making of the intermediary, the content producers, and the representative content viewer. In Chapter 2, we have modeled the interaction among them as a three-stage game (with perfect information). Hence, we adopt the solution concept of subgame perfect equilibrium. The strategy of the intermediary is the payment rate θ, that of content producer i is the production decision $y_i(\theta)$ as a function of θ, and that of the content viewer is the content view allocation $x(\theta, y)$ as a function of (θ, y). Since the payment rate θ does not affect the represent content viewer's problem (3)–(6) directly, we restrict attention to the strategies of the content viewer that depend only on y, writing its strategy as $x(y)$. Below, we define the equilibrium of the game played by the intermediary, the content producers, and the representative content viewer.

Definition 1. $(\theta^*, y^*(\theta), x^*(y))$ *is an* equilibrium *if*

(i) $x^*(y)$ *is an optimal solution to the content viewer's problem* (3)–(6) *given* y,

(ii) For each content producer $i \in [0,1]$, $y_i^*(\theta)$ *is an optimal production decision given* θ, $x_i^*(y)$, *and* $(y_j^*(\theta))_{j\neq i}$, *that is,*

$$(\theta + s)x_i^*(y^*(\theta)) - c \geq 0 \quad \textit{if } y_i^*(\theta) = 1 \tag{7}$$

and

$$(\theta + s)x_i^*(1, y_{-i}^*(\theta)) - c \leq 0 \quad \textit{if } y_i^*(\theta) = 0, \tag{8}$$

where $(1, y_{-i}^*(\theta))$ *is the production decision profile equal to* $y^*(\theta)$ *except that* $y_i = 1$, *and*

(iii) θ^* *is an optimal payment rate for the intermediary given* $y^*(\theta)$ *and* $x^*(y)$, *that is,*

$$(b - \theta^*)\bar{x}(\theta^*) \geq (b - \theta)\bar{x}(\theta) \quad \textit{for all } \theta \in \mathbb{R}, \tag{9}$$

where $\bar{x}(\theta) = \int_0^1 x_i^*(y^*(\theta))di$.

In the remainder of this chapter, we characterize the equilibrium by analyzing each agent's problem in detail.

3.2 Optimal Content Viewing

For analytical tractability, we impose the following assumptions on the utility function U of the representative content viewer.

Assumption 1. $U(x) = U(x')$ *for all* x *and* x' *such that* $x_i = x'_i$ *for almost all* $i \in [0,1]$ *(with respect to the Lebesgue measure) and* $x_a = x'_a$.

Assumption 2. *U is continuous on its domain and twice continuously differentiable on the interior of its domain. It is additively separable in its arguments (i.e., all of its cross partial derivatives are zero).*

Assumption 3. *(i) For all* $i \in [0,1]$ *such that* $q_i = 0$*,* $\partial U/\partial x_i = 0$ *for all* $x_i > 0$*.*

(ii) For all $i \in [0,1]$ *such that* $q_i > 0$*,* $\partial U/\partial x_i > 0$ *and* $\partial^2 U/\partial x_i^2 < 0$ *for all* $x_i > 0$*, while* $\partial U/\partial x_i \to +\infty$ *as* $x_i \to 0$ *and* $\partial U/\partial x_i \to 0$ *as* $x_i \to +\infty$*.*

(iii) $\partial U/\partial x_a > 0$ *and* $\partial^2 U/\partial x_a^2 < 0$ *for all* $x_a > 0$*, while* $\partial U/\partial x_a \to +\infty$ *as* $x_a \to 0$ *and* $\partial U/\partial x_a \to 0$ *as* $x_a \to +\infty$*.*

Assumption 4. $\partial U/\partial x_i$ *is continuous and increasing in* q_i *for all* $i \in [0,1]$*, and* $\partial U/\partial x_a$ *is continuous and increasing in* q_a*.*

We briefly discuss the above assumptions. Assumption 1 means that the content views of a single piece of content have an infinitesimal effect on the utility. Additive separability in Assumption 2 excludes complementarity between two pieces of content. Assumption 3(i) says that the marginal utility from content of quality 0 is 0. Assumption 3(ii) and (iii) have that the utility from content of a positive quality is increasing in its content views at a diminishing rate. Assumption 4 states that the higher quality of content leads to the larger marginal utility from content.

Assumptions 2 and 3 guarantee that for each $y \in \{0,1\}^{[0,1]}$ there exists an optimal content view allocation $x^*(y)$ satisfying the following optimality conditions:

$$x_i^*(y) > 0 \text{ for all } i \text{ such that } q_i > 0 \text{ and } y_i = 1, x_a^*(y) > 0, \tag{10}$$

$$x_i^*(y) = 0 \text{ for all } i \text{ such that } q_i = 0 \text{ or } y_i = 0, \tag{11}$$

$$\int_0^1 x_i^*(y)di + x_a^*(y) = T, \tag{12}$$

$$\frac{\partial U}{\partial x_i}(x^*(y)) = \frac{\partial U}{\partial x_a}(x^*(y)) \text{ for all } i \text{ such that } q_i > 0 \text{ and } y_i = 1. \tag{13}$$

By Assumption 4, we have $x_i^*(y) > x_j^*(y)$ for all i and j such that $q_i > q_j$ and $y_i = y_j = 1$, i.e., content of a higher quality attracts more content views. Consider a content view allocation $x'(y)$ satisfying $x_i'(y) = x_i^*(y)$ for *almost all* $i \in [0,1]$ such that $y_i = 1$, $x_i'(y) = 0$ for all i such that $y_i = 0$, and $x_a'(y) = x_a^*(y)$. Then $x'(y)$ satisfies all the constraints of the content viewer's problem while achieving the same utility as $x_a^*(y)$ by Assumption 1. Thus, $x'(y)$ is also an optimal content view allocation. In this chapter, we focus on optimal solutions of the kind $x^*(y)$ in which the optimality conditions are satisfied for *all* $i \in [0,1]$ such that $y_i = 1$. Note that such an optimal solution $x^*(y)$ is unique for each y under our assumptions.

A particular form of a utility function satisfying Assumptions 1–4, which we shall use extensively as an illustration, is the quality-adjusted version of the well-known Dixit-Stiglitz utility function [15], defined as

$$U(x) = \left[\int_0^1 q_i x_i^{\frac{\sigma-1}{\sigma}} di + q_a x_a^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}}, \tag{14}$$

where $\sigma > 1$ measures the elasticity of substitution between different pieces of content. In the extreme case where $\sigma \to +\infty$, content becomes perfectly substitutable [15], whereas when $\sigma \to 1$, content becomes unsubstitutable. Dixit-Stiglitz utility function provides an effective and tractable means of capturing constant elasticity of substitution in a market. More precisely, it provides a rigorous characterization of product diversity through the parameter $\sigma > 1$ and combines multiple types of product consumption into an aggregate utility, which is particularly suitable for user-generated content platforms where content producers produce diversified content. Next, by using the optimality conditions (10)–(13), we obtain the optimal solution

$$x_i^*(y) = \frac{Tq_i^\sigma}{q_a^\sigma + \int_0^1 y_j q_j^\sigma dj} y_i \quad \text{for all } i \in [0,1], \text{ and} \tag{15}$$

$$x_a^*(y) = \frac{Tq_a^\sigma}{q_a^\sigma + \int_0^1 y_j q_j^\sigma dj} = \frac{T}{1 + \frac{\int_0^1 y_j q_j^\sigma dj}{q_a^\sigma}}, \tag{16}$$

assuming that the integral $\int_0^1 y_j q_j^\sigma dj$ exists. Note that $[\int_0^1 y_j q_j^\sigma dj]^{1/\sigma}$ can be interpreted as the aggregate quality of content on the intermediary's platform, which reflects not only the overall quality but also the quantity of available content.[1] Also, $[q_a^\sigma + \int_0^1 y_j q_j^\sigma dj]^{1/\sigma}$ can be interpreted as the aggregate quality of all available content. The total content views in the intermediary's platform at the optimal solution are given by

$$\bar{x}(y) = \frac{T\int_0^1 y_j q_j^\sigma dj}{q_a^\sigma + \int_0^1 y_j q_j^\sigma dj} = \frac{T}{1 + \frac{q_a^\sigma}{\int_0^1 y_j q_j^\sigma dj}}, \tag{17}$$

and the indirect utility of the content viewer is

$$U^*(y) = U(x^*(y)) = T\left(q_a^\sigma + \int_0^1 y_j q_j^\sigma dj\right)^{\frac{1}{\sigma-1}}. \tag{18}$$

We can see that the optimal number of the content views of content i, if produced, is increasing in its quality, q_i, and the size of the content viewer, T, while it is decreasing in the aggregate quality of all available content. Let us treat the platforms other than the intermediary's platform as a single platform. Then the optimal total number of the content views in a platform (i.e., $x_a^*(y)$ and $\bar{x}(y)$) is increasing in the size of the content viewer and the aggregate quality of content available on the platform, while it is decreasing in the aggregate quality

[1] Similarly, the aggregate quality of content on other platforms, q_a, can be derived from a distribution of individual qualities. We index content producers providing content on the other platforms by k, and suppose that the content producers on other platforms are uniformly distributed over $[0,1]$ with total mass n_a. Then the Dixit-Stiglitz utility function U can be modified as $U(x) = \left[\int_0^1 q_i x_i^{\frac{\sigma-1}{\sigma}} di + n_a \int_0^1 q_k x_k^{\frac{\sigma-1}{\sigma}} dk\right]^{\frac{\sigma}{\sigma-1}}$. If we set $q_a = (n_a \int_0^1 q_k^\sigma dk)^{1/\sigma}$, assuming that the integral exists, we obtain the same optimal number of content views in the intermediary's platform in both approaches, while x_a represents the total number of content views in the other platforms, i.e., $x_a = \int_0^1 x_k dk$.

of content available on the other platform. Finally, the per-capita indirect utility, $U^*(y)/T$ is increasing in the aggregate quality of all available content.

3.3 Equilibrium Content Production

At equilibrium, no content producer can gain by unilaterally changing its production decision. In other words, equilibrium content production decisions $y^*(\theta)$ can be considered as a Nash equilibrium of the game played by the content producers given the payment rate θ and anticipating the optimal content view allocation $x^*(y)$ of the content viewer. In the following lemma whose proof is omitted for brevity, we provide some basic properties of equilibrium content production decisions.

Lemma 1. *Let $y^*(\theta)$ be an equilibrium strategy profile of the content producers.*
(i) If $\theta \le -s$, then $y_i^*(\theta) = 0$ for all $i \in [0,1]$.
(ii) If $\theta > -s$, $y_i^*(\theta) = 1$ and $q_j > q_i$, then $y_j^*(\theta) = 1$.

Lemma 1(i) shows that if the intermediary chooses $\theta \le -s$, no content producer will produce content and thus the intermediary will obtain no content views, i.e., $\bar{x}(\theta) = 0$. Hence, $\Pi(\theta) = 0$ for any $\theta \le -s$. Now consider the case where $\theta > -s$. Let us define $\tilde{I}_q = \{i \in [0,1] : q_i \le q\}$ for all $q \in \mathbb{R}$. Assuming that $\tilde{I}_q$ is (Lebesgue) measurable for every q, we define $F(q)$ to be the measure of $\tilde{I}_q$. Then $F(\cdot)$ can be considered as the cumulative distribution function of the qualities of content that can be produced by the content producers. We assume that $F(\cdot)$ has a continuous probability distribution function $f(\cdot)$ with support $[0,1]$.[2] Lemma 1(ii) implies that for any equilibrium strategy profile $y^*(\theta)$ of the content producers there exists a threshold, or sometimes called the marginal content quality, $q_m^*(\theta) \in [0,1]$, such that $y_i^*(\theta) = 1$ if $q_i > q_m^*(\theta)$ and $y_i^*(\theta) = 0$ if $q_i < q_m^*(\theta)$. If $q_m^*(\theta) \in (0,1)$, a content producer who can produce content of quality $q_m^*(\theta)$ is indifferent between producing and not producing given payment rate θ. For convenience, we restrict attention to threshold strategies of the content producers in which all indifferent producers choose to produce, although given Assumption 1 and the assumption on $F(\cdot)$ the behavior of indifferent producers does not affect the utility of the content viewer as well as the profit of the intermediary.

At the optimal content view allocation, the content viewer allocates the same number of content views to two pieces of available content of equal quality. That is, if $q_i = q_j$ and $y_i = y_j = 1$, then $x_i^*(y) = x_j^*(y)$. For each $q \in [0,1]$ and $q_m \in [0,1]$, we define $z^*(q|q_m) = x_i^*(y)$ where $q = q_i$, $y_i = 1$, and $y_j = 1$ if and only if $q_j \ge q_m$ for all $j \ne i$. In other words, $z^*(q|q_m)$ is the number of content views that a content producer receives if it produces content of quality q while all the other content producers use threshold q_m. We can obtain the following properties of $z^*(q|q_m)$.

[2] More generally, we can have a finite support $[\underline{q}, \overline{q}]$ where $0 \le \underline{q} < \overline{q} < +\infty$. We can think of using 1 instead of $\overline{q}$ as a normalization, while we use 0 instead of sufficiently small $\underline{q}$ to simplify our analysis.

Lemma 2. *For all $q_m \in [0,1]$, $z^*(0|q_m) = 0$ and $z^*(q|q_m) > 0$ for all $q > 0$. $z^*(q|q_m)$ is continuous and increasing in each of q and q_m.*

With the Dixit-Stiglitz utility function in (14), we have

$$z^*(q|q_m) = \frac{T(\sigma+1)q^\sigma}{(\sigma+1)q_a^\sigma + (1-q_m^{\sigma+1})}, \tag{19}$$

which obviously satisfies the properties in Lemma 2.

By Lemma 2, we have $(\theta+s)z^*(0|q_m) - c < 0$ for all $q_m \in [0,1]$. If $(\theta+s)z^*(1|q_m) - c \geq 0$, then there exists a unique minimum element of the set $\{q \in [0,1] : (\theta+s)z^*(q|q_m) - c \geq 0\}$ because $z^*(q|q_m)$ is continuous and increasing in q. We define a mapping $Q_\theta : [0,1] \to [0,1]$ by

$$Q_\theta(q_m) = \begin{cases} \arg\min\{q \in [0,1] : (\theta+s)z^*(q|q_m) - c \geq 0\} & \text{if } (\theta+s)z^*(1|q_m) - c \geq 0, \\ 1 & \text{otherwise.} \end{cases} \tag{20}$$

$Q_\theta(q_m)$ is the threshold of the optimal production decisions given payment rate θ when the content producers expect that only producers who can produce content of quality higher than q_m choose to produce. Thus, if q_m^* is a fixed point of the mapping Q_θ, i.e., $q_m^* = Q_\theta(q_m^*)$, then no content producer can gain from unilateral deviation when the content producers use threshold q_m^*.

Definition 2. *q_m^* is an* equilibrium marginal content quality (or equilibrium production threshold) given payment rate θ *if it satisfies $q_m^* = Q_\theta(q_m^*)$.*

By Lemma 2, $z^*(1|1)$ is the maximum number of content views that a content producer can attract in the platform. Hence, for $\theta > -s$, if $(\theta+s)z^*(1|1) - c \leq 0$, a content producer can never obtain a positive payoff from producing content. The condition $(\theta+s)z^*(1|1) - c \leq 0$ can be rewritten as $\theta \leq \underline{\theta}$, where

$$\underline{\theta} = \frac{c}{z^*(1|1)} - s. \tag{21}$$

We establish the existence and uniqueness of an equilibrium marginal content quality in Proposition 1, whose proof can be found in [23] and references therein.

Proposition 1. *For any $\theta > -s$, there exists a unique equilibrium marginal content quality given payment rate θ, $q_m^*(\theta)$. $q_m^*(\theta) = 1$ for $\theta \leq \underline{\theta}$ and $q_m^*(\theta) \in (0,1)$ for $\theta > \underline{\theta}$. Moreover, $q_m^*(\theta)$ is continuous and decreasing in θ on $[\underline{\theta}, +\infty)$ and approaches zero as $\theta \to +\infty$.*

Proposition 1 guarantees the existence of a unique equilibrium threshold and shows that if the the content producer who can produce content of the highest quality can never obtain a positive payoff (due to high production cost, low payment rate from the intermediary, or low satisfaction from attracting content views), then (almost) no content producers choose to produce content at equilibrium.

In practice, the content producers may not have complete information regarding each other (e.g., about the distribution of qualities), and hence they may not be able to make decisions that strike the equilibrium at one shot. In such a scenario, the content producers may use an adjustment process to update their decisions based on limited information. A natural and well-studied approach to modeling an adjustment process is the best-response dynamics, in which each decision maker chooses the best action in response to the decisions made by the others. In this chapter, we consider the best-response dynamics based on naive (or static) expectation. Specifically, time is discrete and labeled as $t = 1, 2, \ldots$. Let $q_{m,t}$ be the threshold used by the content producers in period $t = 1, 2, \ldots$. At the end of each period $t = 1, 2, \ldots$, the content producers observe $q_{m,t}$ (or the qualities of all content available on the platform in period t) and expect that $q_{m,t}$ is used when they make production decisions in period $t+1$. That is, each content producer i expects to attract $z^*(q_i|q_{m,t})$ content views if it chooses to produce in period $t+1$. Then content producer i chooses to produce in period $t+1$ if $(\theta + s)z^*(q_i|q_{m,t}) - c \geq 0$, and given an initial belief $q_{m,0} \in [0, 1]$, the best-response dynamics induces a sequence of thresholds $\{q_{m,t}\}_{t=0}^{\infty}$ that evolve following the relationship

$$q_{m,t+1} = Q_\theta(q_{m,t}) \tag{22}$$

for $t = 0, 1, \ldots$. Similar decision processes have been adopted in the existing literature (e.g., [12, 22, 20] and references therein). Essentially, the dynamics defined by (22) is a fixed point iteration for $Q_\theta(\cdot)$, and it converges regardless of the initial point if $Q_\theta(\cdot)$ is a contraction mapping [16]. Following the contraction mapping theorem, we can easily specify further a sufficient condition for convergence, and we omit the details here for brevity. It should be noted that, by considering the dynamics specified by (22), we implicitly assume that content produced in the previous periods has little value and will not significantly affect the content views in the current period (e.g., news content). Moreover, the dynamics specified by (22) requires that all the content producers update production decisions in every period. If only a randomly chosen fraction $\epsilon \in (0, 1]$ of the content producers can update their decisions in each period, the sequence of thresholds is generated by $q_{m,t+1} = (1 - \epsilon)q_{m,t} + \epsilon Q_\theta(q_{m,t})$. This modification does not affect the equilibrium analysis although it slows down the convergence.

3.4 Optimal Payment Rate

From Proposition 1, we can see that $\bar{x}(\theta) > 0$ if and only if $\theta > \underline{\theta}$. Also, from (1), we can see that $\Pi(\theta) > 0$ only if $\theta < b$. Suppose that $b \leq \underline{\theta}$. Then $\Pi(\theta) = 0$ for $\theta \leq \underline{\theta}$ and $\Pi(\theta) < 0$ for $\theta > \underline{\theta}$. Hence, the maximum profit of the intermediary is zero, and any $\theta \leq \underline{\theta}$ is an optimal payment rate. In the remainder of this chapter, we shall assume $\underline{\theta} < b$. In this case, $\Pi(\theta) = 0$ for $\theta \leq \underline{\theta}$, $\Pi(\theta) > 0$ for $\theta \in (\underline{\theta}, b)$, and $\Pi(\theta) \leq 0$ for $\theta \geq b$. Hence, when searching for an optimal payment rate, we can restrict attention to the interval $(\underline{\theta}, b)$. Then the problem of the intermediary to find an optimal payment rate θ^* can be written as

$$\max_{\theta \in (\underline{\theta}, b)} (b - \theta)\bar{x}(\theta), \tag{23}$$

where $\bar{x}(\theta) = \int_{q_m^*(\theta)}^1 z^*(q|q_m^*(\theta))dF(q)$.

Using Lemma 2, we can show that $\int_{q_m}^1 z^*(q|q_m)dF(q)$ is continuous and decreasing in q_m on $[0, 1]$, and thus by Proposition 1, $\bar{x}(\theta)$ continuous and increasing in θ on $[\underline{\theta}, +\infty)$. That is, an increase in θ on $[\underline{\theta}, +\infty)$ will encourage more content producers to produce content and make the intermediary to attract more content views from the content viewer. Then it follows that the intermediary's problem (23) has an optimal solution by extending the region of θ into a compact set $[\underline{\theta}, b]$. Moreover, if we assume that the objective function $\Pi(\theta)$ is strictly concave on $(\underline{\theta}, b)$, the intermediary's problem has a unique solution. In this case, the unique optimal payment rate θ^* satisfies the first-order optimality condition

$$\Pi'(\theta) = -\bar{x}(\theta) + (b - \theta)\bar{x}'(\theta) = 0, \tag{24}$$

assuming that $\bar{x}$ is differentiable. Moreover, if $0 \in (\underline{\theta}, b)$, we can find out whether the intermediary should subsidize or tax the content producers by examining the sign of $\Pi'(0)$. Specifically, θ^* has the same sign as that of $\Pi'(0)$. If $\Pi'(0) > 0$, or $(b - \theta)\bar{x}'(0) > \bar{x}(0)$, the first-order gain from increased content views dominates the first-order loss from payment to the content producers as the intermediary increases θ from 0. Thus, in this case, it is optimal for the intermediary to subsidize the content producers, i.e., $\theta^* > 0$. On the other hand, if $\Pi'(0) < 0$, or $\bar{x}(0) > (b - \theta)\bar{x}'(0)$, it is optimal for the intermediary to tax the content producers, i.e., $\theta^* < 0$.

In the remainder of this chapter, to gain insights on the optimal payment rate, we focus on the quality-adjusted Dixit-Stiglitz utility function and uniform distribution of content qualities. In this case, $z^*(1|1) = T/q_a^\sigma$ and thus $\underline{\theta} = cq_a^\sigma/T - s$. In Proposition 2, whose proof is given in [23] and references therein, we study the optimal payment rate and its sign.

Proposition 2. *Suppose that the utility function $U(x)$ of the representative content viewer is given by the Dixit-Stiglitz utility function in* (14) *and that the qualities of content that can be produced by the content producers are uniformly distributed on* $[0, 1]$. *Then there exists a unique optimal payment rate $\theta^* \in (cq_a^\sigma/T - s, b)$ that maximizes the intermediary's profit. The equilibrium marginal content quality given θ^*, denoted by*

q_m^{**}, *is the unique root of the following equation in the variable* q_m *on* $(q_m^*(b), 1)$*:*

$$-\frac{T(b+s)q_a^\sigma}{[(\sigma+1)q_a^\sigma+1-q_m^{\sigma+1}]^2}+\frac{c(\sigma+q_m^{\sigma+1})}{(\sigma+1)^3 q_m^{2\sigma+1}}=0, \tag{25}$$

and θ^* *is given by*

$$\theta^*=\frac{c\left[(\sigma+1)q_a^\sigma+1-(q_m^{**})^{\sigma+1}\right]}{T(\sigma+1)(q_m^{**})^\sigma}-s. \tag{26}$$

Moreover, if $0\in(cq_a^\sigma/T-s,b)$*, then*

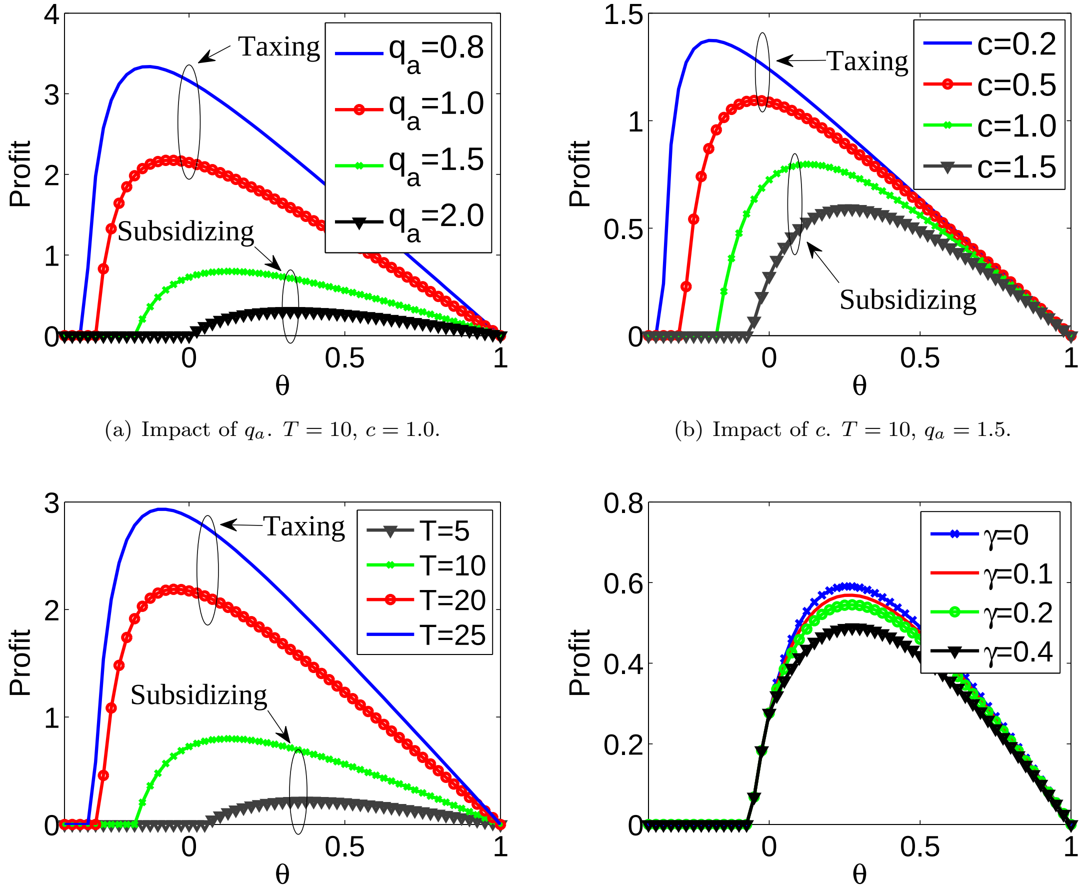
$$\begin{cases}\theta^*\in(0,b) & \text{if } \frac{\sigma}{q_m^*(0)}+[q_m^*(0)]^\sigma<\frac{c(b+s)(\sigma+1)q_a^\sigma}{Ts^2},\\ \theta^*=0 & \text{if } \frac{\sigma}{q_m^*(0)}+[q_m^*(0)]^\sigma=\frac{c(b+s)(\sigma+1)q_a^\sigma}{Ts^2},\\ \theta^*\in\left(\frac{cq_a^\sigma}{T}-s,0\right) & \text{if } \frac{\sigma}{q_m^*(0)}+[q_m^*(0)]^\sigma>\frac{c(b+s)(\sigma+1)q_a^\sigma}{Ts^2}.\end{cases} \tag{27}$$

Proposition 2 provides conditions under which the profit-maximizing intermediary subsidizes or taxes the content producers. We can see that the intermediary should subsidize the content producers in the following cases:

1. The total number of content views T is sufficiently small;
2. The production cost c is sufficiently large;
3. Social satisfaction per content view s is sufficiently small;
4. The aggregate quality of content on the other platforms q_a is sufficiently large;
5. Outside profit per content view b is sufficiently large.

The first four cases lead to a situation in which it is difficult for a content producer to make a profit from providing content on the intermediary's platform. Thus, in these cases, the intermediary tends to subsidize the content producers in order to encourage content production. The statements can also be mathematically verified. Let us take the first case as an example. When T is sufficiently small, $q_m^*(0)$ will approach 1, making the left-hand side of the first inequality in (27) reach $\sigma+1$. Thus, the first inequality in (27) will hold when T is sufficiently small. In the last case, the intermediary derives high outside profit per content view, for example, from advertisement. Hence, it subsidizes off for the intermediary to subsidize the content producers in order to increase content production on its platform, while it has more revenue to share with them.

Numerical results illustrating the impacts of q_a, c, and T are plotted in Fig. 2. It can be seen that the intermediary can improve its profit significantly by using the proposed payment scheme compared to the case in which it does not use a payment scheme at all (i.e., the case where $\theta=0$). For example, we observe from Fig. 2(b) that by optimally choosing the payment rate, the intermediary's profit increases from



(a) Impact of q_a. $T = 10$, $c = 1.0$.

(b) Impact of c. $T = 10$, $q_a = 1.5$.

(c) Impact of T. $c = 1.0$, $q_a = 1.5$.

(d) Impact of γ. $T = 10$, $c = 1.5$, $q_a = 1.5$.

Figure 2: Profit versus price θ. $\sigma = 2$, $b = 1$, $s = 0.4$.

approximately 0.21 to 0.5 (a nearly 150% increase) when $c = 1.5$.

Finally, we conclude this chapter by discussing two extreme cases, $q_a \to 0$ and $\sigma \to +\infty$. First, consider the case where $q_a \to 0$. In this case, the aggregate quality of content on the other platforms is negligible (e.g., very low quality or little content available), and the intermediary becomes virtually a monopolist in the market. Since the content producers who can produce content of the highest quality 1 have the strongest incentive to produce, almost all the T content views will be devoted to content on the intermediary's platform as long as there is some content on the platform. Note that $\bar{x}(\theta) > 0$ if and only if $\theta > \underline{\theta}$ and that $\underline{\theta} \to -s$ as $q_a \to 0$. Thus, as $q_a \to 0$, the optimal payment θ^* converges to $-s$ while the maximum profit converges to $(b+s)T$. Next, consider the case where $\sigma \to +\infty$. In this case, content becomes perfectly substitutable, and almost all the content views will be devoted to content of the highest quality among content on all the platforms. This can be verified by taking the limits of $x_i^*(y)$ and $x_a^*(y)$ given in (15) and (16), respectively, as $\sigma \to +\infty$ with $\int_0^1 y_j q_j^\sigma dj = [1-(q_m^*)^{\sigma+1}]/(\sigma+1)$. When $q_a > 1$, we have $x_i^* \to 0$ for all i and $x_a^* \to T$. Thus, content produced on the intermediary's platform will attract no content views and the intermediary cannot obtain a positive profit regardless of the payment rate θ. On the contrary, when $q_a < 1$, we have $x_a^* \to 0$ and $\bar{x} \to T$ with $x_i^*/x_j^* \to +\infty$ for all i, j such that $q_i > q_j > q_a$. When $q_a < 1$, we again have $\underline{\theta} \to -s$ as $\sigma \to +\infty$. Hence, the optimal payment rate θ^* converges to $-s$ while the maximum profit converges to $(b+s)T$. To sum up, when $q_a \to 0$ or $\sigma \to +\infty$ with $q_a < 1$, the intermediary can attract almost all the T content views while extracting almost all the social satisfaction of the content producers.

Remark: Suppose that the aggregate quality of content on the other platforms q_a is derived from a distribution of qualities. Suppose further that the qualities are uniformly distributed on the interval $[q_l, q_h]$ with total mass n_a, where $0 \le q_l < q_h$. Then by the relationship $q_a = (n_a \int_0^1 q_k^\sigma dk)^{1/\sigma}$, we obtain

$$q_a = \left[\frac{n_a\left(q_h^{\sigma+1} - q_l^{\sigma+1}\right)}{1+\sigma}\right]^{\frac{1}{\sigma}} = n_a^{\frac{1}{\sigma}} q_h^{\frac{\sigma+1}{\sigma}} \left[1 - \left(\frac{q_l}{q_h}\right)^{\sigma+1}\right]^{\frac{1}{\sigma}} \left(\frac{1}{1+\sigma}\right)^{\frac{1}{\sigma}}. \tag{28}$$

Since $\lim_{\sigma\to+\infty} n_a^{1/\sigma} = \lim_{\sigma\to+\infty} \left[1 - (q_l/q_h)^{\sigma+1}\right]^{1/\sigma} = \lim_{\sigma\to+\infty} (1/(1+\sigma))^{1/\sigma} = 1$ and $\lim_{\sigma\to+\infty} q_h^{(\sigma+1)/\sigma} = q_h$, we have $q_a \to q_h$ as $\sigma \to +\infty$. Thus, when $\sigma \to +\infty$, the highest quality q_h determines the aggregate quality q_a, and we have $q_a > 1$ if $q_h > 1$ and $q_a < 1$ if $q_h < 1$.

3.5 Overjustification Effects

Now, we briefly discuss overjustification effects on the intermediary's equilibrium profit.

It has been argued that an external incentive such as money or prizes decreases a person's intrinsic motivation to perform a task, and such phenomenon is referred to as overjustification effects [13]. In the context of user-generated content platforms, content producers' internal incentive to produce content may decrease in the long term if the intermediary chooses to subsidize content producers. We model overjustification effects using the following formula

$$s' = \max\left[0, s - \gamma \cdot \max(0, \theta)\right], \tag{29}$$

where s' is the new social satisfaction per content view in the presence of overjustification effects, and $\gamma \in [0, \infty)$ indicates the severity of overjustification effects. Note that if the intermediary taxes content producers, overjustification effects do not exist, i.e., $s' = s$. We show the profits under various values of γ in Fig. 2(d), and observe that even though overjustification effects exist, the intermediary can still increase its profit by applying the proposed payment scheme, as long as overjustification effects are not too strong (e.g., γ is not too large). In the presence of strong overjustification effects (e.g., $\gamma \ge 1$), a content producer's incentive to produce content will even decrease if the intermediary subsidizes content producers for content production (unless the intermediary subsidizes content producers sufficiently high).

4 Extension to Heterogeneous Production Costs

In the analysis so far, it has been assumed that all the content producers incur the same production cost c when they choose to produce content. In this chapter, we extend our model and generalize the preceding analysis by relaxing this assumption and considering a scenario in which the content producers have different production costs. We assume that there are K groups of content producers and that the production cost of a content producer in group k is given by $c_k > 0$ for $k = 1, \dots, K$. We assume that $c_1 < c_2 < \cdots < c_K$. We use I_k to denote the set of the indexes of the content producers in group k, for $k = 1, \dots, K$. Then $\{I_1, \dots, I_k\}$ forms a partition of $[0, 1]$. The mass of the content producers in group k is denoted by n_k so that $\sum_{k=1}^{K} n_k = 1$. We assume that for each k the qualities of content that can be produced by the content producers in group k are distributed according to a cumulative distribution function $F_k(\cdot)$ which has a continuous probability distribution function $f_k(\cdot)$ with support $[0, 1]$.

With heterogeneous production costs, the definition of an equilibrium is modified so that (ii) in Definition 1 becomes as follows: For each content producer $i \in I_k$, $k = 1, \dots, K$, $y_i^*(\theta)$ is an optimal production decision given θ, $x_i^*(y)$, and $(y_j^*(\theta))_{j \neq i}$, that is,

$$(\theta + s)x_i^*(y^*(\theta)) - c_k \geq 0 \quad \text{if } y_i^*(\theta) = 1 \tag{30}$$

and

$$(\theta + s)x_i^*(1, y_{-i}^*(\theta)) - c_k \leq 0 \quad \text{if } y_i^*(\theta) = 0, \tag{31}$$

while (i) and (iii) remain the same. When $\theta > -s$, equilibrium production decisions are characterized by thresholds, one for each group. We use $q_{m,k} \in [0, 1]$ to denote the threshold used by the content producers in group k, for $k = 1, \dots, K$. We also use a vector notation $\mathbf{q}_m = (q_{m,1}, q_{m,2}, \dots, q_{m,K}) \in [0, 1]^K$. As before, we use $z^*(q|\mathbf{q}_m)$ to denote the number of content views that a content producer receives if it produces content of quality q while all the other content producers use thresholds $\mathbf{q}_m$. Lemma 2 can be modified so that the following holds: (i) For all $\mathbf{q}_m \in [0, 1]^K$, $z^*(0|\mathbf{q}_m) = 0$ and $z^*(q|\mathbf{q}_m) > 0$ for all $q > 0$, and (ii) $z^*(q|\mathbf{q}_m)$ is continuous and increasing in q and each element of $\mathbf{q}_m$. We define a mapping $\mathbf{Q}_\theta : [0, 1]^K \to [0, 1]^K$ so that the kth element of $\mathbf{Q}_\theta(\mathbf{q}_m)$ is given by

$$Q_{\theta,k}(\mathbf{q}_m) = \begin{cases} \arg\min\{q \in [0,1] : (\theta+s)z^*(q|\mathbf{q}_m) - c_k \geq 0\} & \text{if } (\theta+s)z^*(1|\mathbf{q}_m) - c_k \geq 0, \\ 1 & \text{otherwise,} \end{cases} \tag{32}$$

for $k = 1, \ldots, K$. Equilibrium content production is characterized by a fixed point of $\mathbf{Q}_\theta$, as defined below.

Definition 3. $\mathbf{q}_m^*$ *is an equilibrium marginal content quality vector (or equilibrium production threshold vector) given payment rate* θ *if it satisfies* $\mathbf{q}_m^* = \mathbf{Q}_\theta(\mathbf{q}_m^*)$.

The following properties of equilibrium thresholds can be readily established.

Lemma 3. *Let* $\mathbf{q}_m^*$ *be an equilibrium marginal content quality vector given payment* $\theta > -s$. *Then* $0 < q_{m,1}^* \leq q_{m,2}^* \leq \cdots \leq q_{m,K}^* \leq 1$, *and*

$$\frac{z^*(q_{m,k}^*|\mathbf{q}_m^*)}{z^*(q_{m,l}^*|\mathbf{q}_m^*)} = \frac{c_k}{c_l}, \tag{33}$$

for all $k, l \leq k^*$, *where* $k^* = \max\{k \in \{1, \ldots, K\} : q_{m,k}^* < 1\}$.

In the previous analysis, we have defined $\underline{\theta}$ as the payment rate that induces the content producers to start producing content. That is, $\underline{\theta}$ was the payment under which a content producer who can produce content of the highest quality 1 is indifferent between producing and not producing. Since there are K heterogeneous groups in the current analysis, we define K payment rates, $\underline{\theta}_1, \ldots, \underline{\theta}_K$, where $\underline{\theta}_k$ is the payment rate that induces the content producers in group k to start producing content. Following $c_1 < c_2 < \cdots < c_K$, it is obvious that $\underline{\theta}_1 < \underline{\theta}_2 < \cdots < \underline{\theta}_K$. First, the indifference condition satisfied at $\theta = \underline{\theta}_1$ is $(\theta + s)z^*(1|(1, \ldots, 1)) = c_1$. Hence, we obtain

$$\underline{\theta}_1 = \frac{c_1}{z^*(1|(1, \ldots, 1))} - s. \tag{34}$$

Now, consider $k = 2, \ldots, K$. At $\theta = \underline{\theta}_k$, there are some content producers who choose to produce in each group $l < k$, and the following indifference conditions should be satisfied.

$$(\underline{\theta}_k + s)z^*(\bar{q}_{m,l}^k|(\bar{q}_{m,1}^k, \ldots, \bar{q}_{m,k-1}^k, 1, \ldots, 1)) = c_l, \text{ for } l = 1, \ldots, k-1, \text{ and} \tag{35}$$

$$(\underline{\theta}_k + s)z^*(1|(\bar{q}_{m,1}^k, \ldots, \bar{q}_{m,k-1}^k, 1, \ldots, 1)) = c_k. \tag{36}$$

Note that (35) and (36) together define a system of k equations in k unknowns, $\bar{q}_{m,1}^k, \ldots, \bar{q}_{m,k-1}^k$, and $\underline{\theta}_k$. To simplify our analysis, we impose the following assumption.

Assumption 5. *For any* $q \in [0,1]$, $q' \in (0,1]$, *and* $\mathbf{q}_m \in [0,1]^K$, *the ratio* $z^*(q|\mathbf{q}_m)/z^*(q'|\mathbf{q}_m^*)$ *is independent of* $\mathbf{q}_m$.

We can verify that $z^*(q|\mathbf{q}_m)$ induced by the Dixit-Stiglitz utility function satisfies Assumption 5. By Assumption 5, we can write $z^*(q|\mathbf{q}_m)/z^*(q'|\mathbf{q}_m^*)$ as $\tilde{z}(q,q')$. By modified Lemma 2, given $q' \in (0,1]$, $\tilde{z}(q,q')$ is continuous and increasing in q with $\tilde{z}(0,q') = 0$ and $\tilde{z}(q',q') = 1$. Hence, there exists unique $\bar{q}_{m,k-1}^k \in (0,1)$ that satisfies $\tilde{z}(\bar{q}_{m,k-1}^k, 1) = c_{k-1}/c_k$. The remaining thresholds, $\bar{q}_{m,1}^k, \ldots, \bar{q}_{m,k-2}^k$, can be obtained by solving $\tilde{z}(\bar{q}_{m,l}^k, \bar{q}_{m,l+1}^k) = c_l/c_{l+1}$ from $l = k-2$ to $l = 1$ recursively. This yields the $k-1$ thresholds $\bar{q}_{m,1}^k, \ldots, \bar{q}_{m,k-1}^k$ such that $0 < \bar{q}_{m,1}^k < \cdots < \bar{q}_{m,k-1}^k < 1$. Then using (36), we obtain

$$\underline{\theta}_k = \frac{c_k}{z^*(1|(\bar{q}_{m,1}^k, \ldots, \bar{q}_{m,k-1}^k, 1, \ldots, 1))} - s \tag{37}$$

for $k = 2, \ldots, K$. Based on the discussion so far, we can obtain the following proposition, which is stated without a proof due to space limitation.

Proposition 3. *For any* $\theta > -s$, *there exists a unique equilibrium marginal content quality vector given payment rate* θ, $\mathbf{q}_m^*(\theta)$. *For each* $k = 1, \ldots, K$, *the following properties hold.* $q_{m,k}^*(\theta) = 1$ *for* $\theta \leq \underline{\theta}_k$ *and* $q_{m,k}^*(\theta) \in (0,1)$ *for* $\theta > \underline{\theta}_k$. $q_{m,k}^*(\theta)$ *is continuous and decreasing in* θ *on* $[\underline{\theta}_k, +\infty)$ *and approaches zero as* $\theta \to +\infty$. *For* $\theta \in [\underline{\theta}_k, \underline{\theta}_{k+1})$ *(*$[\underline{\theta}_K, +\infty)$ *when* $k = K$*), we have* $0 < q_{m,1}^*(\theta) < \cdots < q_{m,k}^*(\theta) \leq 1 = q_{m,k+1}^*(\theta) = \cdots = q_{m,K}^*(\theta)$.

As in Chapter 3.3, we can consider a best-response dynamics for the content producers to adjust their production decisions based on the thresholds used in the previous period. Let $\mathbf{q}_{m,t} = (q_{m,1,t}, q_{m,2,t}, \ldots, q_{m,K,t})$ be the vector of the thresholds used in period $t = 1, 2, \ldots$. Then starting from an initial belief $\mathbf{q}_{m,0}$, the dynamics yields a sequence $\{\mathbf{q}_{m,t}\}_{t=0}^{\infty}$ generated by $\mathbf{q}_{m,t+1} = \mathbf{Q}_\theta(\mathbf{q}_{m,t})$ for $t = 0, 1, \ldots$. More specific results regarding the equilibrium marginal content quality vector and the convergence of the best-response dynamics can be obtained using the Dixit-Stiglitz utility function. The details are omitted for brevity.

Now we turn to the intermediary's problem of finding an optimal payment rate. Note that we have $\bar{x}(\theta) = \sum_{k=1}^{K} n_k \int_{q_{m,k}^*(\theta)}^{1} z^*(q|\mathbf{q}_m^*(\theta)) dF_k(q)$. We assume that $\underline{\theta}_1 < b$. Then $\Pi(\theta) = 0$ for $\theta \leq \underline{\theta}_1$, $\Pi(\theta) > 0$ for $\theta \in (\underline{\theta}_1, b)$, and $\Pi(\theta) \leq 0$ for $\theta \geq b$. Hence, we can restrict attention to the interval $(\underline{\theta}_1, b)$ when searching for an optimal payment rate. Also, if $\underline{\theta}_k \geq b$ for some k, then it is not profitable for the intermediary to induce content production by producers in groups $k, k+1, \ldots, K$, and we can ignore these groups in the analysis. Hence, without loss of generality, we assume that $\underline{\theta}_K < b$. For concreteness, we will focus on the

Dixit-Stiglitz utility function and uniform distribution of qualities for each group. In this case, we have

$$z^*(q|\mathbf{q}_m) = \frac{T(\sigma+1)q^\sigma}{(\sigma+1)q_a^\sigma + \sum_{k=1}^K n_k(1-q_{m,k}^{\sigma+1})}. \tag{38}$$

Thus, we have $\tilde{z}(q,q') = (q/q')^\sigma$, from which we get $\bar{q}_{m,l}^k = (c_l/c_k)^{1/\sigma}$ for $l = 1,\dots,k-1$, for $k = 2,\dots,K$. Hence, we have $\underline{\theta}_1 = c_1 q_a^\sigma/T - s$ and

$$\underline{\theta}_k = \frac{c_k\left\{q_a^\sigma + \frac{1}{\sigma+1}\sum_{l=1}^{k-1} n_l\left[1-\left(\frac{c_l}{c_k}\right)^{\frac{\sigma+1}{\sigma}}\right]\right\}}{T} - s \tag{39}$$

for $k = 2,\dots,K$.

Let $\underline{\theta}_{K+1} = b$. We consider the problems $\max_{\theta\in[\underline{\theta}_k,\underline{\theta}_{k+1}]}\Pi(\theta)$ for $k = 1,\dots,K$. Once we obtain $\Pi_k^* = \max_{\theta\in[\underline{\theta}_k,\underline{\theta}_{k+1}]}\Pi(\theta)$, we have $\max_{\theta\in(\underline{\theta}_1,b)}\Pi(\theta) = \max\{\Pi_1^*,\dots,\Pi_K^*\}$ and an optimal payment rate is the one that achieves the maximum profit. As in the previous analysis, when solving $\max_{\theta\in[\underline{\theta}_k,\underline{\theta}_{k+1}]}\Pi(\theta)$, we replace the choice variable θ with $q_{m,k}^*$, where the corresponding region of $q_{m,k}^*$ is $[q_{m,k}^*(\underline{\theta}_{k+1}), q_{m,k}^*(\underline{\theta}_k)]$. Note that $q_{m,k}^*(\underline{\theta}_k) = 1$ for $k = 1,\dots,K$ and $q_{m,k}^*(\underline{\theta}_{k+1}) = \bar{q}_{m,k}^{k+1}$ for $k = 1,\dots,K-1$. When the intermediary chooses $q_{m,k}^* \in [q_{m,k}^*(\underline{\theta}_{k+1}), q_{m,k}^*(\underline{\theta}_k)]$, we have $q_{m,l}^* = (c_l/c_k)^{1/\sigma} q_{m,k}^*$ for $l < k$ and $q_{m,l}^* = 1$ for $l > k$. The corresponding value of θ is given by

$$\theta = \frac{c_k\left\{q_a^\sigma + \frac{1}{\sigma+1}\sum_{l=1}^{k} n_l[1-(\frac{c_l}{c_k})^{\frac{\sigma+1}{\sigma}}(q_{m,k}^*)^{\sigma+1}]\right\}}{T(q_{m,k}^*)^\sigma} - s, \tag{40}$$

while the total number of content views in the platform is

$$\bar{x} = \frac{T\sum_{l=1}^k n_l[1-(\frac{c_l}{c_k})^{\frac{\sigma+1}{\sigma}}(q_{m,k}^*)^{\sigma+1}]}{(\sigma+1)q_a^\sigma + \sum_{l=1}^k n_l[1-(\frac{c_l}{c_k})^{\frac{\sigma+1}{\sigma}}(q_{m,k}^*)^{\sigma+1}]}. \tag{41}$$

Hence, the problem $\max_{\theta\in[\underline{\theta}_k,\underline{\theta}_{k+1}]}\Pi(\theta)$ can be transformed into $\max_{q_{m,k}^*\in[q_{m,k}^*(\underline{\theta}_{k+1}),q_{m,k}^*(\underline{\theta}_k)]}\tilde{\Pi}_k(q_{m,k}^*)$, where

$$\tilde{\Pi}_k(q_{m,k}^*) = \frac{T(b+s)\sum_{l=1}^k n_l[1-(\frac{c_l}{c_k})^{\frac{\sigma+1}{\sigma}}(q_{m,k}^*)^{\sigma+1}]}{(\sigma+1)q_a^\sigma + \sum_{l=1}^k n_l[1-(\frac{c_l}{c_k})^{\frac{\sigma+1}{\sigma}}(q_{m,k}^*)^{\sigma+1}]} - \frac{c_k\sum_{l=1}^k n_l[1-(\frac{c_l}{c_k})^{\frac{\sigma+1}{\sigma}}(q_{m,k}^*)^{\sigma+1}]}{(\sigma+1)(q_{m,k}^*)^\sigma}. \tag{42}$$

By taking the second-order derivative of $\tilde{\Pi}_k$, we can show that $\tilde{\Pi}_k$ is strictly concave on $(q_{m,k}^*(\underline{\theta}_{k+1}), q_{m,k}^*(\underline{\theta}_k))$. Hence, there exists a unique optimal solution to $\max_{q_{m,k}^*\in[q_{m,k}^*(\underline{\theta}_{k+1}),q_{m,k}^*(\underline{\theta}_k)]}\tilde{\Pi}_k(q_{m,k}^*)$. As in the case of a homogeneous production cost, we can also analyze whether the intermediary should subsidize or tax the content producers. Nevertheless, we omit the result because of its similarity to Proposition 2.

5 Conclusion

In this chapter, we studied the economics of user-generated content platforms by focusing on the profit maximization problem of an intermediary who owns a user-generated content platform. We considered a class of payment schemes in which the intermediary subsidizes or taxes content producers per content view while it provides the service for free to content viewers. We analyzed a three-stage game using backward induction. First, we used the representative content viewer to determine how content viewers' attention is allocated across a variety of available content. Next, after establishing the threshold property of equilibrium production decisions, we showed the existence and uniqueness of an equilibrium production threshold used by the content producers and derived its properties. Lastly, we formalized the intermediary's profit maximization problem and studied the optimal payment rate by using the quality-adjusted Dixit-Stiglitz utility function and the uniform distribution of content qualities as a concrete example. We provided conditions under which the intermediary should subsidize or tax the content producers, and discussed qualitatively the impacts of the aggregate quality of outside content and content substitutability on the intermediary's profit. We also discussed the effectiveness of the proposed payment scheme in the presence of overjustification effects. As an extension of our model, we considered heterogeneity in the content producers' production costs. There are various directions to extend our work, among which we mention only a few as follows. First, we can consider a more general class of payment schemes. In this chapter, we considered anonymous and linear payment schemes, but the intermediary may use personalized (e.g., quality-dependent) and non-linear payment schemes as well. Second, we can consider a scenario where a content producer can choose a production intensity, which determines the production cost and the content quality, and can choose a content platform to post its content among several competing platforms. Third, we can incorporate the intermediary's payment decision into its back-end resource management (e.g., data centers). As shown in our preliminary work [18], jointly optimizing the payment decision and resource management by exploring the interactions among them can significantly enhance the intermediary's profitability. Last but not least, the tree-stage game studied in this chapter can be extended to other application scenarios (e.g., customer-to-customer market [19]) or incorporate additional decisions to model the intermediary's investment strategy (e.g., infrastructure and technology investment on television content platforms [21]).

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