A Unique Beamforming-Based Equilibrium in Multi-User Random Access SIMO Networks

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***Abstract*—In this letter, we tackle the problem of competitive single antenna transmitters accessing a multiple antenna access point using a random access protocol. We show that the access point can help the system to reach a unique Stackelberg equilibrium and prevent network collapse due to the selfish nature of the transmitters.**

***Index Terms*—Stackelberg equilibrium, SIMO networks, beamforming.**

I. INTRODUCTION

WHEN the MAC layer of the WLAN standard was first designed, both the access point (AP) and the transmitters were assumed to follow the standard's rules. However, if a selfish transmitter violates these rules, it will increase its utility (e.g. network bandwidth) with respect to the other users. A game theoretic analysis for the problem has been introduced in several papers. An equilibrium can be reached if the AP can force a pricing technique upon the transmitters [1] or if different selfish transmitters can bargain to reach an agreement [2]. The drawbacks of these techniques are described in [3], where both Nash equilibrium and Stackelberg equilibrium have been studied. In Nash equilibrium, no user can gain by changing his own strategy unilaterally. In Stackelberg equilibrium, unlike the Nash equilibrium, one player leads the game. The leader makes a move first and then the rest of the players (which are referred to as the followers) try to reach a Nash equilibrium based on the leader's move. In [3], a network manager acts as a game leader, which enables the selfish users to reach a Stackelberg equilibrium. There are three major drawbacks to this technique: (1) the game leader will punish the whole network if any transmitter floods the network with his packets, (2) the Stackelberg equilibrium is not unique, and (3) the manager cannot force the users to reach the required equilibrium if they have large deviations. In all previous techniques, the AP and the transmitters are assumed to be single antenna devices. Although several papers have considered the multiple access problem when the AP has multiple antennas (e.g [4]), none of them has addressed the practical case of contention based protocols, where the AP can receive only one packet at a time. That is important since the new generation of WLANs will be based on the 802.11n multiple antenna standard. The new standard's MAC protocol will be a contention based protocol and its multiple antennas capability allows directional signal transmission and reception through beamforming.

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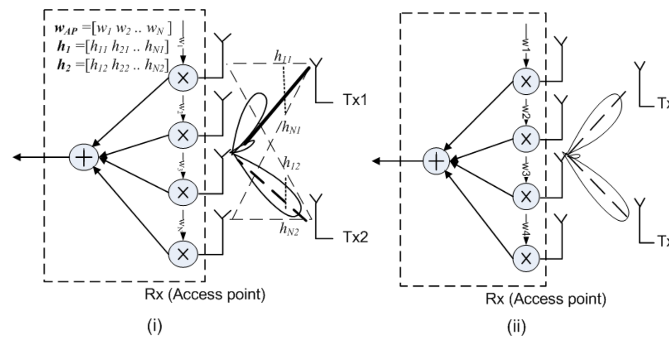

Fig. 1. Basic system setup. (i) AP punishes one transmitter, (ii) AP enables reception from both transmitters.

In this letter, we use game theory to show the existence and uniqueness of a Stackelberg equilibrium when different selfish single antenna transmitters access the same multiple antenna AP. A simple contention based multiple access protocol similar to the one used in [3] is used in this letter. Each transmitter tries to maximize his own utility, which is a weighted difference between his achievable throughput and the cost of a successful transmission. Meanwhile, the AP tries to force the transmitters to access the channel with certain probabilities that maximize the social welfare of the network. The AP is the game leader, but unlike [3], it only punishes the deviating transmitters by using its multiple antennas to block reception from these transmitters for some time. This setup is the major contribution in this letter, since it enables the game to reach a unique Stackelberg equilibrium, in contrast with the multiple equilibrium points reached in [3]. The simplicity of this algorithm makes it easy to implement in practice. Without loss of generality, we will consider two transmitters that try to access one AP. However, the methodology followed in this letter can be followed for more than two transmitters.

II. PROBLEM FORMULATION

The system considered in our work is shown in Fig. 1. Two transmitters access the same AP in a time slotted channel. Similar to [3], transmitters are sending their packets during time slots of the same duration. Each transmitter will retransmit his packet until it is correctly received at the receiver. We will consider two scenarios:

A. Passive AP

In this scenario, the receiver does not use its multiple antenna capability and enables reception of all packets from all users all the time. A packet is received successfully if and only if only one user transmits his packet while the other user does not transmit. The transmitters are competitive; they try to maximize their own utilities in a non-cooperative

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fashion. The system is modeled as a non-cooperative game $\langle K, P_i, u_i \rangle$, where $K = 1, 2$ is the set of players. The strategy of transmitter i is p_i, is the probability that transmitter i transmits his packet during a time slot. $P_i = [0, 1]$ is the strategy space for transmitter i . The utility of transmitter i is defined as

$$u_i(\boldsymbol{p}) = \alpha_i r_i p_i (1 - p_{-i}) + (1 - \alpha_i) \left(- \left(1/(1 - p_{-i})\right)\right) r_i, \quad (1)$$

where $\boldsymbol{p} = [p_1 \; p_2]$, p_{-i} is the probability of transmission of the other transmitter. The first part of the utility function is proportional to the achievable throughput when the transmitters play the game given certain strategy profile. Transmitter i can achieve, on average, $p_i(1 - p_{-i})$ packets per time slot. r_i is the number of bits per packet for transmitter i, which is proportional to its physical layer throughput and the time slot duration. The second part of the utility function is the cost of a successfully received packet for transmitter i. The cost of a packet transmission is proportional to the number of bits per packet r_i. We will assume a unit cost per bit. Consequently, the cost of a packet transmission is r_i. The probability of a successful packet transmission for transmitter i when he transmits his packets is $(1 - p_{-i})$. Therefore, the average number of transmissions necessary for a packet to be successfully received at the receiver is $1/(1 - p_{-i})$ [5]. The cost of a successful transmission will be between r_i, when the other user does not send at all; and ∞ when the other user is sending all the time. To minimize the cost, we need to minimize $1/(1 - p_{-i})$, which corresponds to the maximum of $-1/(1 - p_{-i})$. The two parts of the utility function are linearly combined using α_i. The parameter α_i is used to scale the relative importance of throughput maximization and the cost of a successfully received packet, $0 < \alpha_i < 1$. When $\alpha_i \to 1$, the utilities will be similar to [3]; while if $\alpha_i \to 0$, the transmitters will try to minimize the cost of successful packet transmissions.

B. Active AP

In this scenario, the AP changes its radiation pattern by changing its antenna weights with time to enable reception from any transmitter or block him. We assume perfect knowledge of the channel vectors between the transmitters and the AP ($\boldsymbol{h_1}$,$\boldsymbol{h_2}$). Fig. 1 shows two possible radiation patterns during certain time slot. The radiation pattern in Fig. 1(i) has a null in the direction of the first transmitter to block him, and a peak at the direction of the second one to receive his packets. This happens when the AP adjusts its antenna weights vector $\boldsymbol{w_{AP}}$ to have $\boldsymbol{w_{AP}} \boldsymbol{h_1^T} = 0$ and $\boldsymbol{w_{AP}} \boldsymbol{h_2^T} = 1$, where $\boldsymbol{h_i^T}$ is the transposition of $\boldsymbol{h_i}$ [7].This will be a punishment for the first transmitter if he tries to flood the network with his packets. This setup has been used in multiple antenna MAC protocols [7]. Given this, we will prove in section IV that the AP can play a game with the transmitters and force them to reach target transmission probabilities that maximizes some social welfare criteria for $0 < \tilde{p_i} < 1$. At each time slot, the AP can block reception from transmitter i with probability g_i. The AP will enable reception from both transmitters if they transmit with $\boldsymbol{p} = \tilde{\boldsymbol{p}}$, as in Fig. 1 (ii). In this case, the AP adjusts $\boldsymbol{w_{AP}}$ to have $\boldsymbol{w_{AP}} \boldsymbol{h_1^T} = 1$ and $\boldsymbol{w_{AP}} \boldsymbol{h_2^T} = 1$. The receiver successfully receives a packet from transmitter i if all the following three conditions are satisfied: (1) the transmitter transmits his packet, (2) the AP's radiation pattern is adjusted to receive that packet, and (3) the AP disables reception from the other transmitter or the other transmitter does not transmit any packet. Transmitter i's utility is

$$u_i(g_i, g_{-i}, \boldsymbol{p}) = \alpha_i r_i \left((1 - g_i) p_i (1 - (1 - g_{-i}) p_{-i})\right) - \left(\frac{(1 - \alpha_i) r_i}{(1 - g_i)(1 - (1 - g_{-i}) p_{-i})}\right) \quad (2)$$

The AP's utility function has a maximum at $\boldsymbol{p} = \tilde{\boldsymbol{p}}$ and any deviation from the target probabilities decreases it. The AP's utility function is:

$$u_{AP}(G, \boldsymbol{p}) = -\left((p_1 - \tilde{p_1})^2 + (p_2 - \tilde{p_2})^2\right). \quad (3)$$

Note that the transmission probabilities $\boldsymbol{p}$ depend on the blocking probability vector $G = [g_1 \; g_2]$, as will be shown in section IV.

III. Nash Equilibrium

When the AP takes a passive role, the system reaches Nash equilibrium. We will prove that the players will face the tragedy of commons.

Proposition 1: A strategy profile $\boldsymbol{p}$ is a Nash equilibrium when at least one $p_i = 1$.

Proof: The utility of user i when the AP is passive is shown in (1). First, we consider the case where the probability of transmission of the other user is $p_{-i} < 1 \to (1 - p_{-i}) > 0$. The first part of (1) will be proportional to user i's transmission probability. The second part of (1) will not depend on p_i. As a result, the utility is maximized when $p_i = 1$. Second, when $p_{-i} = 1 \to (1 - p_{-i}) = 0$, the utility function of user i will be $-\infty$ for any value of p_i. So the transmission probability for user i is indifferent between [0,1]. Therefore, at least one of $\boldsymbol{p}$ the elements of has to be equal to 1 to have a Nash equilibrium.

The most probable Nash equilibrium to be played is $p_i = 1$ for both transmitters because it maximizes the utility for user i for any value of p_{-i} . This is expected because the users are selfish and there is neither coordination nor an external manager to the network.

IV. Stackelberg Equilibrium

A Stackelberg equilibrium can be reached when the AP uses its multi-antenna capability. We will first define the relative deviation of user i from the target probability as:

$$h_i = \frac{p_i}{\tilde{p_i}} - 1. \quad (4)$$

The values of the blocking probabilities will be determined by the relative deviations of the individual users from $\boldsymbol{p}$

$$g_i = \min\{\max(h_i, 0), 1\}. \quad (5)$$

When user i sends his packets with $\tilde{p_i}$, the AP will not block any of his packets because $h_i = 0$. Then, it becomes in the self-interest of the user to transmit with the target probabilities of the AP, as will be shown in Proposition 2.

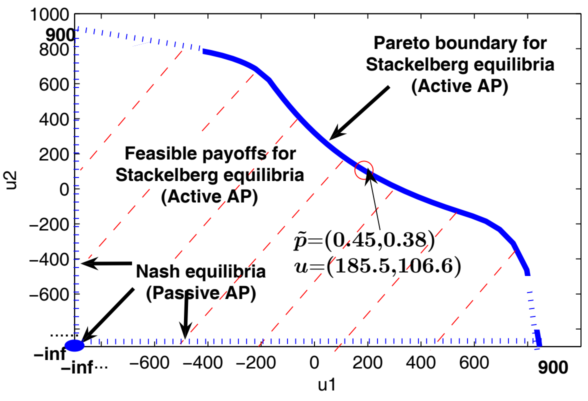

Fig. 2. Users' feasible utilities for the passive and active AP cases.

Proposition 2: The new multiple access game with the AP as a game leader has a unique Stackelberg equilibrium at $\boldsymbol{p}$.

Proof: First, we will prove that $\boldsymbol{p} = \tilde{\boldsymbol{p}}$ is a Stackelberg equilibrium strategy. At the Stackelberg equilibrium, the users should not have any incentive for individual deviation from the equilibrium point. At equilibrium, $p_{-i} = \tilde{p_{-i}}$, and consequently, from (4) and (5), $g_{-i} = 0$. For ease of notation, we set $a_i = \alpha_i r_i$ and $b_i = (1-\alpha_i)r_i$. Equation (2) is simplified to

$$u_i(g_i,p_i) = a_i p_i(1-g_i)(1-\tilde{p_{-i}}) - b_i/\left((1-g_i)(1-\tilde{p_{-i}})\right). \quad (6)$$

Three ranges are possible for user i's transmission probability:

1) $p_i \geq 2\tilde{p}_i$: when transmitter i sends with very high transmission probability, his relative deviation will be greater than 1. The AP will block him by setting $g_i = 1$, and from (6), his utility will be $-\infty$.
2) $p_i \leq \tilde{p}_i$: when transmitter i sends with probability less than the target probability, his relative deviation will be less than 0. The AP will enable reception from him all the time by setting $g_i = 0$. The utility of user i will be

$$u_i(p_i) = a_i p_i(1-\tilde{p_{-i}}) - b_i/(1-\tilde{p_{-i}}). \quad (7)$$

Thus, independent of the value of the target transmission probability of the other user, user i will always try to increase his transmission probability in that range. The maximum value of p_i in this range is $\tilde{p}_i$.
3) $\tilde{p}_i \leq p_i \leq 2\tilde{p}_i$: when the transmitter slightly increases his transmission probability, the AP blocks some of his packets to force him to decrease his transmission probability. In this range, his relative deviation will be $0 \leq h_i \leq 1$, which will lead to $g_i = h_i = (p_i/\tilde{p}_i) - 1$. Equation (6) can be written as

$$u_i(p_i) = a_i\left(2-\frac{p_i}{\tilde{p}_i}\right)p_i(1-\tilde{p_{-i}}) + \left(\frac{-b_i}{(2-p_i/\tilde{p}_i)(1-\tilde{p_{-i}})}\right). \quad (8)$$

The first term is a concave function in p_i with a peak at $\tilde{p}_i$, and the second term is a decreasing function of p_i in the range between $\tilde{p}_i$ and $2\tilde{p}_i$. Its maximum value is also at $\tilde{p}_i$.

From the previous discussion, user i has no incentive to deviate from $\tilde{p}_i$ which proves the existence of an equilibrium at $\tilde{\boldsymbol{p}}$. To prove the uniqueness of the equilibrium, we can write (2) as

$$u_i(g_i,p_i) = a_i p_i(1-g_i)\beta_{-i} + \left(\frac{-b_i}{((1-g_i)\beta_{-i})}\right), \quad (9)$$

where $\beta_{-i} = 1-(1-g_{-i})p_{-i}$ is a factor that does not depend on the response of user i or the AP blocking probability for user i. Equation (9) has the same form as (6), if we change β_{-i} to $(2-\tilde{p_{-i}})$. Note that $0 < \beta_{-i} \leq 1$ for $0 < \tilde{p_{-i}} < 1$ and any value of $0 \leq p_{-i} \leq 1$. Following the same procedure as in the first part of the proof, we find that the maximum of (6) is only at $p_i = \tilde{p}_i$ for all values of $0 < \beta_{-i} \leq 1$. This completes the proof.

Fig. 2 shows the utilities of two 802.11a single antenna transmitters when they access an 802.11n multiple antenna AP in the legacy mode [6]. MCS0, which has 6 Mbps throughput, is transmitted from both transmitters and the time slot duration is 1 ms. Therefore, the number of bits per time slot $r_i = 6000$. A higher preference is assumed for user throughput over the cost per successful transmission ($\alpha_i = 0.95$). Without the AP as a game leader, at least the utility of one of the transmitters will be $-\infty$ and the most probable Nash equilibrium will have $-\infty$ utility for both transmitters. At this point, no packet is received correctly by the AP. When the AP acts as a game leader, the possible values of utilities will span a larger space, as shown in Fig. 2. As an example of the Stackelberg equilibrium, when the AP sets the target probabilities to be $\tilde{\boldsymbol{p}}$=(0.45,0.38), the transmitters reach a Stackelberg equilibrium by sending with $\boldsymbol{p} = \tilde{\boldsymbol{p}}$. In this case, the transmitters' payoffs will be $\boldsymbol{u}$=(185.5,106.6), which is on the Pareto boundary as shown in Fig. 2.

V. Conclusion

We have introduced a novel game theoretic framework to analyze the interaction between wireless transmitters and a multiple antenna access point. We have introduced a blocking probability vector to punish deviating users. This allows the system to reach a unique Stackelberg equilibrium at a certain target transmission probabilities set by the access point.

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