Coalitional Games With Intervention: Application to Spectrum Leasing in Cognitive Radio

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***Abstract*—We consider a spectrum leasing system in which secondary networks offer offload services to a primary network (PN) in exchange for temporary access to the PN's spectrum. When the coverage areas of several secondary access nodes (SANs) overlap, they compete for primary users (PUs), which benefits the PN, except when the SANs collude and coordinate their prices, forming a cartel. As a result, the PN obtains lower transmission rates for the serviced PUs. Our coalitional game analysis shows that stable cartels always exist and can form easily. To protect the spectrum owner's interests and enforce market regulation, we propose an intervention framework in which an intervention manager counteracts cartel formation. The specific features that make wireless systems different from conventional markets enable the manager to modify the set of achievable outcomes. The intervention capability is limited; thus, the objective is to design an intervention rule maximizing the PN transmission rate within the given constraints. Importantly, the intervention can solely act as a threat or a warning that does not need to be executed in practice. To reduce the computational effort, we also propose a low-complexity intervention rule that performs similarly to the optimal one in terms of assurable PN rate increment and outperforms other effective approaches.**

***Index Terms*—Spectrum leasing, cooperative secondary spectrum access, coalitional game theory, intervention game.**

I. Introduction

SPECTRUM leasing is considered one of the most promising paradigms to increase the efficiency of wireless spectrum occupation. By this mechanism, a licensed operator or primary network (PN) allows unlicensed or secondary networks (SNs) to temporarily use part of its spectrum in exchange for monetary payments and/or some type of service provided by the SNs to the spectrum owner, assuring the absence of harmful interference at the primary users (PUs). The PN improves its revenue, its performance, or both, while the SNs gain access to spectrum resources, achieving a *win-win* situation [1], [2].

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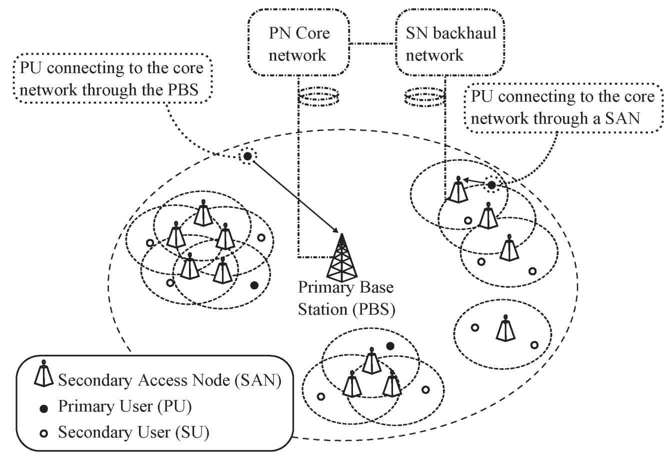

Fig. 1. Schematic representation of the system considered. Each SN comprises one SAN and one or several SUs. The PUs can be connected to the PN core network by means of a SAN, which receives part of the PU's spectral resources in return.

We focus on a scenario in which the SNs provide offloading services to PUs and, in return, each serving SN is granted access to part of the wireless bandwidth of the served PU. The SNs are infrastructure-based, each one comprising a secondary access node (SAN) and some secondary users (SUs). As in [3], [4] and [6], each SAN can provide high-quality wireless links to nearby PUs, and connect them to the core of the PN by means of the SAN's backhaul connection. Fig. 1 shows a schematic representation of the system considered.

As in many other trading scenarios, the outcome of a spectrum leasing system, with or without payments involved, can change significantly if competing agents reach cooperative agreements so that they no longer compete. This situation is known as collusion. For example, in the system considered, a set of SANs with overlapping coverage areas may agree to make coordinated offers to the PUs. When all the competing SANs collude, they form a *cartel*, allowing them to offer their offload services in exchange of higher amounts of bandwidth. Compared to a fully competitive situation, cartel formation may be beneficial for its members, but costly for the entities to whom the service is offered. In our example, the service provided by the cartel of SANs would *cost* more spectrum to the PNs. In economic theory this effect is known as *cartel overcharge*, and has been widely reported both in theory and practice in the economic and legal literature [7]–[9].

In this paper, we use coalitional game theory to characterize collusive behaviors in strategic games, and we extend the game theoretic framework of *intervention* [11], [12] to coalitional games with the goal of minimizing cartel overcharge in a

TABLE I
COMPARISON OF WORKS ISSUING COOPERATIVE AND/OR COLLUSIVE BEHAVIOR IN SPECTRUM TRADING SYSTEMS

	[3]	[13]	[2]	[14]	[17]	[19]	[1]	[18]	[16]	[20]	[15]	our work
Coalitional TU	yes	yes	yes	no	no	no	no	no	no	no	no	yes
Coalitional NTU	yes	no	yes	no	no	no	no	no	no	no	no	yes
Coalitional Partition	yes	no	no	no	no	no	no	no	no	no	no	no
Bargaining	no	no	no	yes	no	no	yes	no	no	no	no	no
Competitive equilibrium	no	no	yes	no	no	no	no	no	yes	no	yes	no
Auction	no	no	no	yes	yes	yes	no	yes	no	no	no	no
Collusion type	no	no	no	(a)	(a)	(a),(b)	no	(a)	no	(c)	(b)	(d)
Anti-collusion	no	no	no	(1)	(1)	no	no	(2)	no	(3)	no	(4)

(a): bidding rings, (b): lose, sublease, kickout collusion, (c): cutting price, (d): **cartel**.
(1): reserve price, (2): sealed bid knapsack auction, (3): alternative coalitions, (4): **intervention**.

TABLE II
COMPARISON OF EXISTING WORKS ON SPECTRUM IN EXCHANGE OF SERVICE

	[24][27][30]	[29]	[4]	[25]	[2]	[26][1]	[3]	[33]	our work
Infrastructure-based SNs	no	no	yes	no	no	no	yes	no	yes
Spectrum owners	1	N	N	1	N	1	N	1	1
Payment transfer	no	no	yes	yes	yes	no	no	yes	no
SN strategic in resource alloc.	no	no	yes	yes	yes	yes	no	no	yes
SN-PN cooperative game	no	no	no	no	yes	no	yes	no	no
SN-SN strategic game	no	no	no	yes	no	no	no	yes	yes
SN-SN cooperative game	no	no	no	no	yes	no	no	no	yes
SN collusion	no	no	no	no	no	no	no	no	yes

spectrum leasing system. In the next subsection we explain in more detail the motivations of our work and how it relates to previous ones.

A. Motivation and Related Works

1) Cooperation and Collusion in Spectrum Leasing: Although coalition formation has been studied in previous works on spectrum leasing (e.g. coalitional games with transferable utility (TU) [2], [3], [13], with non-transferable utility (NTU) [2], [3], in partition form [3], bargaining games [1], [14], and competitive equilibrium [2], [15], [16]), the formation of cartels of SNs has not been considered so far. Collusive behaviors among the secondary users have only been addressed in the framework of spectrum auctions [14], [18], to prevent the formation of coalitions among the bidders (SNs) that could reduce the profit of the auctioneer (PN). Auction theoretic techniques are not applicable to our scenario, where the SANs are *sellers* that exchange their services for spectrum. Table I summarizes the works in spectrum leasing where some type of cooperative/collusive behavior is considered.

Cartel formation in spectrum leasing has been considered as a strategy for competing PNs to avoid the *price war* associated to competition [15], [16], [19]. However, in sharp contrast to our approach, this behavior is not only allowed [16], but sometimes supported by mechanisms assuring the stability of the cartel [15], [19]. Our work is also PN-centered, in the sense that it is aimed to minimize the cartel overcharge for the PN when competing SANs collude. However, our approach is based on the consideration that cartel formation is restricted by anti-trust regulations [21] and therefore, the intervention framework could be applied to counteract PN cartels as well.

Our approach is also connected to previous works on cartel formation and detection from the economic literature. For example, [7] describes the features making a market prone to cartel formation, which are also present in the system considered. Detecting collusive behavior is also an active area of research in economics [9], [21]–[23]. Our approach relies on previous results regarding cartel detection on the basis of the evolution of the prices [9], and detection carried out by the buyers instead of an antitrust agency [22], [23]. However, our strategy for counteracting cartels is different from the proposals for conventional markets, based on fines and leniency programs [10], which are not applicable in automated systems.

2) Spectrum in Exchange for Service: The specific spectrum trading scenario that we consider in this work is similar to [3], [4], where infrastructure-based SNs offer offload services to PUs in exchange of spectrum. These services can also be offered in exchange for monetary payments [5], [6], although this case is not addressed in our paper. In other works [1], [24], [25], [27], [29], [30], the SUs act as wireless relays for PU transmissions, generally using *amplify-and-forward* or cooperative ARQ schemes. In all these works, the spectral resources of the served PU are split between the PU and the serving SN. In [24], [27], [29], [30] it is the PN (either the PU or the primary base station) who determines the amount of resources allocated to the SN's own transmissions, i.e. the SNs are non-strategic with respect to resource allocation, which is an important difference with our work.

When the SNs are strategic and negotiation can be done between the PN and each SN individually, this allocation can be the result of a bargaining process [1], or a contract negotiation [26]. Those approaches, however, are not applicable in our system, in which multiple self-interested SNs compete in several overlapped coverage areas, each area having a different set of competitors (multiple coupled oligopolies). Table II summarizes the characteristics of previous works in this area and are compared to our proposed framework. Noticeably, collusive behaviors have not yet been considered in these systems. Cooperative (not collusive) behavior of the SNs was studied in [2], but requiring monetary transfers among the agents (like in [4], [25], [33]). Our scenario does not involve payments or any type of payoff transfers.

B. Our Contribution

We focus on a system where the SANs are strategic with respect to resource allocation, competing among them for the bandwidth obtained from servicing the PUs. The competition among the SANs should be formulated first as a strategic game but, when collusion is analyzed, a coalitional game analysis is called for. Because no payment is involved, and SANs do not share bandwidth, a non-transferable utility (NTU) formulation is required.

What makes spectrum leasing different from conventional trading scenarios is that it is performed among wireless agents, which allows us to use intervention mechanisms to mitigate cartel effects. This framework, proposed initially for strategic [11] and repeated games [12], requires the existence of a device that can observe the agents actions and modify, to a certain extent, each agent's payoff function. This is, however, the only similarity of [11] and [12] with the present work, because the substantial differences between the solution concepts of strategic and coalitional games require a completely new formulation of the intervention framework. The main challenges to be faced are: to design an intervention rule that is effective as a threat against cartels; to efficiently exploit the limited intervention capabilities imposed by hardware constraints; to manage the problem's computational complexity.

The contributions of this work are:

1) In Section III we characterize, by using an NTU coalitional game approach, the cartel formation problem in a family of games for which the spectrum leasing system is a particular case.
2) We develop, in Section IV, a game intervention framework to reduce cartel overcharge that efficiently exploits the intervention capabilities of the system, and is effective without needing to be exerted.
3) In Section V we propose a low-complexity intervention algorithm suitable for larger sets of competing SANs, which is numerically evaluated in Section VI, where we also study the influence of the number of SANs, the PU traffic distribution, and the network topology on the intervention performance.

Next section describes the system under study and the model characterizing it.

II. System Model and Game Formulation

A. Motivational Scenario

We will refer to the system studied as a Service in Exchange for Spectrum (SES) system. It involves two main types of entities, a *primary base station* (PBS), managed by a wireless operator which has the license to use a certain spectrum band, and a set $\mathcal{N} = \{1, \ldots, N\}$ of secondary access nodes (SANs), or *agents*. The PBS connects the operator's subscribers (PUs) to the operator's core network. The PBS assigns a wireless *channel* to each active PU within its coverage area, consisting of

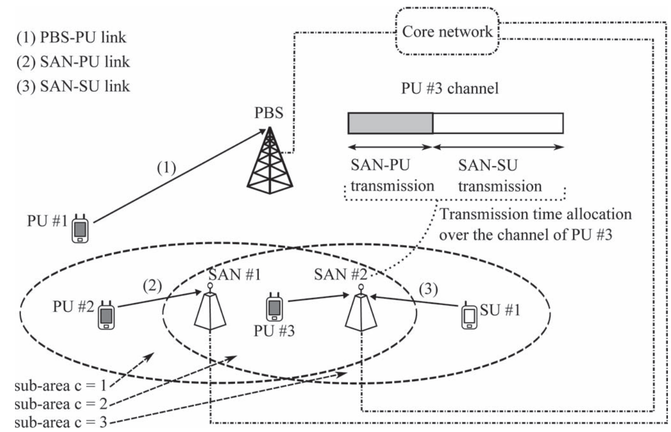

Fig. 2. Example of a SES system comprising a set of two SANs $\mathcal{N} = \{1, 2\}$, covering three sub-areas. The vectors $\mathbf{w}_1 = (1, 1, 0)^\mathsf{T}$, and $\mathbf{w}_2 = (0, 1, 1)^\mathsf{T}$ indicate the sub-areas covered by each SAN.

some transmission time T within the PBS transmission frame, and some bandwidth W from the operator's licensed spectrum band. These channel resources are used to establish the PBS-PU link, comprising uplink and downlink transmissions.

The SANs cover a small part of the area covered by the PBS. They can be seen as micro or even as femtocells. The SANs are neither owned nor controlled by the operator, and their objective is to provide wireless access to a different type of terminals, the SUs. The SANs have a high bandwidth wired connection to a backhaul infrastructure (e.g. DSL, cable or optic fiber) but very limited spectrum resources (e.g. an unlicensed and possibly congested spectrum band). To obtain additional spectral resources, each SAN can provide offload services to the PUs located within their coverage areas and, in return, a serving SAN is granted the right to use part of the served PU channel resources, W and T. The service provided consists of establishing a connection between the PU and the operator's core network over the SAN's wired connection. Part of the PU channel will be used for the SAN-PU wireless link, and the remaining part will be used by the SAN for its own transmissions. Because of the short link distance, the quality of the SAN-PU link can provide higher transmission rate than the PBS-PU link, even if only a fraction of the PU channel is used. Moreover, because these channel resources are confined to a small area, the PBS could reassign them to another and sufficiently distant SAN-PU link, increasing the spectrum's spatial reuse efficiency. Fig. 2 illustrates a simple system with 2 SANs.

The area covered by $\mathcal{N}$ is divided into a set of sub-areas $\mathcal{C} = \{1, \ldots, C\}$. For each $i \in \mathcal{N}$, the column vector $\mathbf{w}_i \in \mathbb{R}^C$ determines the sub-areas where i provides coverage, so that $\mathbf{w}_i(c) = 1$ if the i-th SAN has coverage on c, and $\mathbf{w}_i(c) = 0$ otherwise. Each sub-area $c \in \mathcal{C}$ is characterized by a unique subset $\mathcal{N}_c \subseteq \mathcal{N}$ of SANs providing coverage in c, defined as $\mathcal{N}_c = \{i \in \mathcal{N} | \mathbf{w}_i(c) = 1\}$. For every $i \in \mathcal{N}$ there exist at least one $j \in \mathcal{N} \setminus \{i\}$ and one $c \in \mathcal{C}$ such that $\mathbf{w}_i(c) = \mathbf{w}_j(c) = 1$, i.e. $\mathcal{N}$ does not contain disjoint coverage "islands". In the system of Fig. 2, $\mathcal{N} = \{1, 2\}$, $\mathcal{C} = \{1, 2, 3\}$. The vectors defining the coverage sub-areas of each SAN are $\mathbf{w}_1 = (1, 1, 0)^\mathsf{T}$, and $\mathbf{w}_2 = (0, 1, 1)^\mathsf{T}$, where "$^\mathsf{T}$" denotes the transpose operation.

For the i-th SAN, a_i is the fraction of the PU channel that this SAN is willing to devote to PU's data transmission over the SAN-PU link, so that the remaining fraction $(1 - a_i)$ will be occupied by SAN-SU transmissions as long as the SAN-PU link remains active. The offer[1] a_i made by the i-th SAN belongs to a discrete set of values $\mathcal{A}_S = \{a_{\min}, a_{\min} + \delta_a, a_{\min} + 2\delta_a \ldots, a_{\max}\}$, where δ_a is a fixed increment,[2] the minimum value $a_{\min} > 0$ guarantees that the PU always obtains a positive rate increment with the service, and the maximum value $a_{\max} < 1$ assures that it is worth for the SAN to service the PU.

Let $\gamma(d)$ denote the average SNR of a SAN-PU link of length d. We assume that all the SANs are equal, and therefore, for a given SAN-PU link distance, γ is equal for every $i \in \mathcal{N}$. The expected achievable transmission rate of a SAN-PU link on a given area c is $R_{PU}(c) = E[\kappa W \log_2(1 + \gamma(d))]$ where the expectation is obtained over the PU location in c, and $\kappa < 1$ is a proportionality factor respect to the AWGN Shannon capacity. Similarly, the expected achievable rate in the PBS-PU link in c is defined as $R_0(c) = E[\kappa W \log_2(1 + \gamma_0(d_0))]$, where $\gamma_0(d_0)$ is the SNR of the PBS-PU link for a d_0 PBS-PU distance. Because the coverage area of the SANs is assumed to be very small compared to the PBS coverage area, we can consider that d_0 is approximately constant in $\mathcal{C}$ and therefore $R_0(c) \approx R_0$ for every c. Given a_i, the expected transmission rate of the SAN-PU link provided by $i \in \mathcal{N}_c$ is $a_i R_{PU}(c)$.

Let $D_i(\mathbf{a}, c) = P(\text{select } i|\mathbf{a}, c)$ denote the probability of the PN selecting SAN i in subarea c given $\mathbf{a} = (a_i)_{i\in\mathcal{N}}$. It is assumed that the PN chooses the best offer at each subarea c, denoted by $a_c^* = \max\{a_i, i \in \mathcal{N}_c\}$. Let $\mathcal{N}_c^* = \{i \in \mathcal{N}_c | a_i = a_c^*\}$ be the number of SANs offering a_c^* in subarea c. Therefore, $D_i(\mathbf{a}, c) = 1/|\mathcal{N}_c^*|$ if $i \in \mathcal{N}_c^*$ and $D_i(\mathbf{a}, c) = 0$ otherwise.

We define $\mathbf{p} = (p_1, \ldots, p_C)$ as the vector containing the normalized PU traffic intensities on each sub-area, e.g. $p_c = 0.1$ indicates that 10% of the PU service requests are generated at sub-area c. It is assumed that $p_c > 0$ for all $c \in \mathcal{C}$, i.e. we do not include the sub-areas where the PUs never make any service request. Therefore, given $\mathbf{a}$, the payoff obtained by the PN from the offload services of $\mathcal{N}$ is defined as the expected increment, δ_R, on the transmission rate of each serviced PU terminal:

$$\begin{aligned} E_{\mathbf{a},p}[\delta_R] &= \sum_{c\in\mathcal{C}} \sum_{i\in\mathcal{N}_c} \left(a_i R_{PU}(c) - R_0\right) D_i(\mathbf{a}, c) p_c \\ &= \sum_{c\in\mathcal{C}} \left(a_c^* R_{PU}(c) - R_0\right) p_c \end{aligned} \tag{1}$$

Table III summarizes the most relevant notation used in this and the following sections.

B. Strategic Game Formulation

In this subsection we formulate the strategic game model in normal form played by the SANs in $\mathcal{N}$, referred to as the *agents* of the game. Agent $i \in \mathcal{N}$ selects the offer made to the PU terminals from the agent's action space $\mathcal{A}_i$. We assume that $\mathcal{A}_i = \mathcal{A}_S$ for every $i \in \mathcal{N}$. Therefore, $a_i \in \mathcal{A}_i$ is a pure action for agent i, and the offers made by all the agents in $\mathcal{N}$, $\mathbf{a} = (a_1, \ldots, a_N)$, is a pure action profile. The set of pure action profiles is denoted by $\mathcal{A} = \mathcal{A}_1 \times \cdots \times \mathcal{A}_N$. As in previous systems where spectrum access is granted in exchange for service to the PU terminals (see related works listed in previous section), the SAN can only use the spectral resources of a PU during the time in which the PU is served by the SAN, i.e. while the SAN-PU link is active. The payoff of each agent $i \in \mathcal{N}$ is given by a function $u_i : A \to \mathbb{R}$ defined as follows

$$u_i(\mathbf{a}) = (1 - a_i) R_{SU} \sum_{c\in\mathcal{C}} p_c D_i(\mathbf{a}, c) \tag{2}$$

where R_{SU} is the expected transmission rate achievable by a SAN-SU link using the PU channel resources. The function $u_i(\mathbf{a})$ characterizes the *expected additional transmission rate obtained by agent i from each PU service request arriving to the SES system*. For a given $\mathbf{a}$, the outcome vector for the set of agents $\mathcal{N}$ is defined as $\mathbf{u}(\mathbf{a}) = (u_1(\mathbf{a}), \ldots, u_N(\mathbf{a}))$.

Because the SANs are self-interested, we can define the finite strategic game played by the agents as $\Gamma = \langle \mathcal{N}, \mathcal{A}, (u_i)_{i\in\mathcal{N}} \rangle$.

A randomized or mixed action profile is any probability distribution over the set of available action profiles $\mathcal{A}$. The set of all probability distributions over $\mathcal{A}$ is represented by $\Delta(\mathcal{A})$. We denote a randomized action profile as $\sigma = (\sigma(\mathbf{a}))_{\mathbf{a}\in\mathcal{A}}$, where $\sigma(\mathbf{a})$ is the probability of choosing $\mathbf{a}$. For an action profile $\sigma \in \Delta(\mathcal{A})$, the payoff for the i-th agent in Γ is given by $u_i(\sigma) = \sum_{\mathbf{a}\in\mathcal{A}} u_i(\mathbf{a})\sigma(\mathbf{a})$. Similarly, for a mixed action profile σ, the expected rate increment, δ_R, for the PN is given by $E_{\sigma,p}[\delta_R] = \sum_{a\in\mathcal{A}} E_{\mathbf{a},p}[\delta_R]\sigma(\mathbf{a})$. Considering mixed action profiles, the set of possible outcomes of Γ is given by the convex hull of the outcome vectors $\mathbf{u}(\mathbf{a})$ for all $\mathbf{a} \in \mathcal{A}$: $\mathcal{U} = \text{co}\{\mathbf{u}(\mathbf{a}), \mathbf{a} \in \mathcal{A}\}$.

TABLE III
SUMMARY OF THE MOST RELEVANT NOTATION

Notation	Definition
$\mathcal{C}$	set of coverage sub-areas
$\mathbf{p}$	normalized traffic intensities on each $c \in \mathcal{C}$
$\mathbf{w}_i$	i-th SAN coverage indicator vector over $\mathcal{C}$
$\mathbf{d}_i$	probability of a PU selecting i-th SAN over $\mathcal{C}$
$\mathcal{N}, \mathcal{N}_c$	set of agents, set agents covering $c \in \mathcal{C}$
$\mathcal{S}$	coalition of agents ($\mathcal{S} \subset \mathcal{N}$)
a_i	pure action of agent i
$\mathbf{a}, \mathbf{a}_\mathcal{S}$	pure action profiles of $\mathcal{N}$ and $\mathcal{S}$
σ	mixed action profile
$\mathcal{A}$	set of pure action profiles
$f, \tilde{f}$	intervention rule, null intervention
$\mathcal{A}_0(\mathbf{a})$	feasible set of intervention actions for $\mathbf{a}$
$\mathcal{F}$	set of intervention rules
u_0	manager's payoff
$\overline{u_0}$	pre-cartel manager's payoff
Γ_f, Γ	strategic game played by $\mathcal{N}$ with and without f
$u_i(f, \mathbf{a}), u_i(\mathbf{a})$	agent i payoff for $\mathbf{a}$ with and without f
$\mathbf{u}_f, \mathbf{u}$	outcome vector of Γ_f and Γ
$\mathcal{U}_f, \mathcal{U}$	sets of outcomes of Γ_f and Γ
$\mathcal{U}_f^+, \mathcal{U}^+$	sets of efficient outcomes of Γ_f and Γ
V_f, V	coalition values with and without f
$\mathcal{K}_f, \mathcal{K}$	core of the coalitional games V_f and V
$\mathcal{G}_f^+$	sustainable set of actions
$\mathcal{F}^+$	set of effective intervention rules
$\mathcal{A}_{u_0}$	set $\mathcal{A}$ ordered in manager's preference ($\geq_{u_0}$)
$\mathbf{a}^j$	j-th element of $\mathcal{A}_{u_0}$
A	set of indexes of $\mathcal{A}_{u_0}$

[1] The term *offer* refers to a_i, and is equivalent to considering $(1 - a_i)$ as the *price*, in terms of PU channel resources, that the i-th SAN demands for its service.

[2] The increment δ_a, and more generally the structure of $\mathcal{A}_S$, depend on how the PU channel can be divided. For example, in an OFDMA frame each PU channel consists of a number of subcarriers.

We will use $\mathcal{U}^+$ to denote the set of efficient (not dominated) outcomes of $\mathcal{U}$.

The type of game just described is usually referred to as *price war* [16], [20]. The equilibrium resulting from Γ is not efficient, in general. In fact, it is typically far from $\mathcal{U}^+$. However, if the agents agree to avoid price competition forming a grand coalition or cartel, they can obtain an efficient outcome. Let us consider the action profile where all the agents select the smallest channel fraction for the PU, $\underline{\mathbf{a}} = (a_{\min})_{i\in\mathcal{N}}$. It is straightforward to check that this action would maximize the aggregate utility of the SANs and minimize the payoff for the PN, i.e. $\underline{\mathbf{a}}$ maximizes the cartel overcharge for the PN.

Several questions arise: Are the cartels stable, in the sense that no agent would benefit by deviating from the grand coalition? How can cartels form? How can they be detected? And, how can an intervention device counteract the cartel effects on the PN? Next sections address these questions.

III. Cartels of Secondary Access Nodes

A. Coalitional Game Analysis: Cartel Stability

For cartel formation to be considered an issue, cartels must be sustained long enough to be harmful for the operator. This can only happen if cartels are stable, that is, if no agent or group of agents can obtain a better payoff by deviating from the grand coalition. This subsection presents two results based on a coalitional game analysis of Γ showing that cartels are always stable in the SES system considered.

We formulate Γ as a non-transferable utility (NTU) coalitional game, corresponding to the case where the payoff obtained by an agent i in $\mathcal{N}$, in our case transmission rate or throughput, cannot be transferred to other agents.[3] Let $\mathcal{S} \subseteq \mathcal{N}$ be a subset of SANs. The vector $\mathbf{a}_{\mathcal{S}} = (a_i)_{i\in\mathcal{S}}$ represents an action profile for $\mathcal{S}$. The set of action profiles available for a set $\mathcal{S}$ is denoted by $\mathcal{A}_{\mathcal{S}}$. Similarly, $\sigma_{\mathcal{S}} \in \Delta(\mathcal{A}_{\mathcal{S}})$ denotes a mixed action profile for the SANs in $\mathcal{S}$. To construct an NTU coalitional game representation of a strategic game, we have to characterize the payoff allocations that the members of each $\mathcal{S} \subseteq \mathcal{N}$ can guarantee for themselves independently of the action profile selected by the agents in $\mathcal{N} \setminus \mathcal{S}$. The two more extended ways to do so were suggested by Aumann and Peleg [35] and are known as the *assurable* and *unpreventable* representations. It can be shown that both are equivalent in the system under study. Therefore we will provide the definition for the most general of them (classic game theoretic references e.g. [36] develop this issue in detail).

Definition 1: An allocation vector $\mathbf{x}$ in $\mathbb{R}^{|\mathcal{S}|}$ is *assurable* in Γ for a coalition $\mathcal{S}$ if and only if there exists an action profile $\sigma_{\mathcal{S}}$ such that, for every action profile $\sigma_{\mathcal{N}\setminus\mathcal{S}}$, $u_i(\sigma_{\mathcal{S}}, \sigma_{\mathcal{N}\setminus\mathcal{S}}) \geq x_i, \forall i \in \mathcal{S}$.

That is, $\mathbf{x}$ is assurable for $\mathcal{S}$ if and only if all the players in $\mathcal{S}$ can guarantee that they all can get at least as much as in $\mathbf{x}$ when they choose their action profile $\sigma_{\mathcal{S}}$*before* the players in $\mathcal{N} \setminus \mathcal{S}$.

[3]The intervention framework could be extended to the transferable utility case, in which colluding SANs are equipped with a signaling mechanism allowing them to inform each other of the spectrum resources obtained, and to occupy them in a coordinated way. This is a future research issue.

The *assurable representation* of Γ is the NTU coalitional game V such that $V(\mathcal{S}) = \{\mathbf{x} | \mathbf{x} \text{ is assurable in } \Gamma \text{ for } \mathcal{S}\},\ \forall \mathcal{S} \subset \mathcal{N}$.

It can be checked that the $V(\mathcal{S})$ sets are closed, convex, bounded, superadditive, compactly generated and comprehensive (see [36]).

For the grand coalition, $V(\mathcal{N})$ represents the set of payoff allocations guaranteed for each member $i \in \mathcal{N}$. That is, $V(\mathcal{N})$ contains $\mathbf{x} \in \mathbb{R}^N$ if and only if there exists a $\sigma \in \Delta(\mathcal{A})$ such that $u_i(\sigma) \geq x_i$, for every $i \in \mathcal{N}$. The set $V(\mathcal{N})$ is given by

$$V(\mathcal{N}) = \left\{ \mathbf{x} = \sum_{\mathbf{a}\in\mathcal{A}} \theta_{\mathbf{a}} \mathbf{u}(\mathbf{a}) \,\middle|\, \theta_{\mathbf{a}} \geq 0,\ \sum_{\mathbf{a}\in\mathcal{A}} \theta_{\mathbf{a}} \leq 1 \right\} \tag{3}$$

We will use the more compact expression $V(\mathcal{N}) = \mathrm{co}^e\{\mathbf{u}(\mathbf{a}), \mathbf{a} \in \mathcal{A}\}$, denoting the convex hull of the outcome vectors including the projections of each $\mathbf{u}(\mathbf{a})$ on the basis vectors $\mathbf{e}_i \in \mathbb{R}^N$, for $i \in \mathcal{N}$, where $\mathbf{e}_i(i) = 1$ and $\mathbf{e}_i(j) = 0$ for $j \neq i$. It is straightforward to check that the set of strictly efficient allocations in $V(\mathcal{N})$ is equal to $\mathcal{U}^+$.

The core of V, denoted by $\mathcal{K}$, is defined as the set of all undominated allocations, i.e. $\mathbf{x} \in \mathcal{K}$ if and only if

1) $\mathbf{x} \in V(\mathcal{N})$
2) there is no $\mathcal{S} \subset \mathcal{N}$, $\mathcal{S} \neq \emptyset$, and $\mathbf{y} \in V(\mathcal{S})$ such that $y_i > x_i$ for all $i \in \mathcal{S}$.

In coalitional games, the core plays a similar role to Nash equilibrium in strategic games: it provides the set of outcomes that the agents in $\mathcal{N}$ could obtain when forming a grand coalition (GC) such that no agent or group of agents could obtain a higher payoff by deviating from the grand coalition, i.e. forming an smaller coalition.

However, in many cases, computing the core of a coalitional game, or even finding one element in the core is a hard problem [36]. Moreover, the core is sometimes empty, meaning that the grand coalition is not stable. As we will see, in the system under study this is not the case. We present no two important properties characterizing a general type of payoff functions, $u_i : \mathcal{A} \to \mathbb{R}$, for which the core is always not empty.

Property 1: Uniqueness of the minmax policy. For all $\mathcal{S} \subset \mathcal{N}$, there exists a pure action profile $\overline{\mathbf{a}}_{\mathcal{N}\setminus\mathcal{S}} \in \mathcal{A}_{\mathcal{N}\setminus\mathcal{S}}$ such that $u_i(\mathbf{a}_{\mathcal{S}}, \overline{\mathbf{a}}_{\mathcal{N}\setminus\mathcal{S}}) \leq u_i(\mathbf{a}_{\mathcal{S}}, \mathbf{a}_{\mathcal{N}\setminus\mathcal{S}})$, for all $\mathbf{a}_{\mathcal{S}} \in \mathcal{A}_{\mathcal{S}}$, all $\mathbf{a}_{\mathcal{N}\setminus\mathcal{S}} \in \mathcal{A}_{\mathcal{N}\setminus\mathcal{S}}$, and all $i \in \mathcal{S}$, with strict inequality in some i.

Property 2: Strict upper bound for the sum of minmax outcomes. There exists a pure action profile $\underline{\mathbf{a}} \in \mathcal{A}$, such that, for every $\mathcal{S} \subseteq \mathcal{N}$ the following holds

$$\sum_{i\in\mathcal{S}} u_i(\mathbf{a}_{\mathcal{S}}, \overline{\mathbf{a}}_{\mathcal{N}\setminus\mathcal{S}}) < \sum_{i\in\mathcal{S}} u_i(\underline{\mathbf{a}}),\ \text{for every } \mathbf{a}_{\mathcal{S}} \in \mathcal{A}_{\mathcal{S}}, \tag{4}$$

where $\overline{\mathbf{a}}_{\mathcal{N}\setminus\mathcal{S}}$ is the unique minmax policy of Property 1.

Next proposition shows that these two properties are held by the SES system under study.

Proposition 1: The payoff functions $u_i : \mathcal{A} \to \mathbb{R}$ defined in (2) satisfy Property 1, with $\overline{\mathbf{a}}_{\mathcal{N}\setminus\mathcal{S}} = (a_{\max})_{j\in\mathcal{N}\setminus\mathcal{S}}$ for every $\mathcal{S} \subset \mathcal{N}$, and Property 2, with $\underline{\mathbf{a}} = (a_{\min})_{i\in\mathcal{N}}$.

Proof: See Appendix.

It is also straightforward to check that properties 1 and 2 are also satisfied for mixed action profiles, i.e. $u_i(\sigma_{\mathcal{S}}, \overline{\mathbf{a}}_{\mathcal{N}\setminus\mathcal{S}}) \leq u_i(\sigma_{\mathcal{S}}, \mathbf{a}_{\mathcal{N}\setminus\mathcal{S}})$, and $\sum_{i\in\mathcal{S}} u_i(\sigma_{\mathcal{S}}, \overline{\mathbf{a}}_{\mathcal{N}\setminus\mathcal{S}}) < \sum_{i\in\mathcal{S}} u_i(\underline{\mathbf{a}})$, for all $\sigma_{\mathcal{S}} \in \Delta(\mathcal{A}_{\mathcal{S}})$.

Proposition 1 allows us to define $V(\mathcal{S})$. First, let us note that, if $\mathcal{S}$ selects its mixed action profile $\sigma_{\mathcal{S}} \in \Delta(\mathcal{A}_{\mathcal{S}})$, either before $\mathcal{N} \setminus \mathcal{S}$, or after $\mathcal{N} \setminus \mathcal{S}$, by Property 1, the payoff that can be guaranteed for each agent $i \in \mathcal{S}$ is $u_i(\sigma_{\mathcal{S}}, \overline{\mathbf{a}}_{\mathcal{N}\setminus\mathcal{S}})$ in both cases. Therefore, an allocation vector $\mathbf{x}$ is assurable (and unpreventable) if and only if $x_i \leq \max_{\sigma_{\mathcal{S}} \in \Delta(\mathcal{A}_{\mathcal{S}})} u_i(\sigma_{\mathcal{S}}, \overline{\mathbf{a}}_{\mathcal{N}\setminus\mathcal{S}})$, for every $i \in \mathcal{S}$. Let us define the outcome vectors $\mathbf{u}_{\mathcal{S}}(\mathbf{a}) = (u_i(\mathbf{a}))_{i\in\mathcal{S}}$. The mapping V is then given by $V(\mathcal{S}) = \mathrm{CO}^e\left\{\mathbf{u}_{\mathcal{S}}(\mathbf{a}_{\mathcal{S}}, \overline{\mathbf{a}}_{\mathcal{N}\setminus\mathcal{S}}), \mathbf{a}_{\mathcal{S}} \in \mathcal{A}_{\mathcal{S}}\right\}$.

The existence of $\mathcal{K}$ is assured by the following proposition.

Proposition 2: The NTU coalitional game representation V of a game $\Gamma = \langle \mathcal{N}, \mathcal{A}, (u_i)_{i\in\mathcal{N}}\rangle$, in which functions $(u_i)_{i\in\mathcal{N}}$ satisfy Property 1 and Property 2, has a nonempty core, and $\mathbf{u}(\underline{\mathbf{a}}) \in \mathcal{K}$.

Proof: We will make use of the following result [37]: A sufficient condition for the game V to have a nonempty core is that the inclusion $\bigcap_{j=1}^{m} V(\mathcal{S}_j) \subset V(\mathcal{N})$ always holds for any balanced collection of sets $\{\mathcal{S}_1, \mathcal{S}_2, \ldots, \mathcal{S}_m\}$. Because $u_i(\mathbf{a}_{\mathcal{S}}, \overline{\mathbf{a}}_{\mathcal{N}\setminus\mathcal{S}}) \leq u_i(\mathbf{a}_{\mathcal{S}}, \mathbf{a}_{\mathcal{N}\setminus\mathcal{S}})$ for every $i \in \mathcal{N}$, with strict inequality in some i, there always exists an action profile $\mathbf{a}_{\mathcal{N}\setminus\mathcal{S}}$ such that $\mathbf{u}_{\mathcal{S}}(\mathbf{a}_{\mathcal{S}}, \mathbf{a}_{\mathcal{N}\setminus\mathcal{S}})$ weakly dominates $\mathbf{u}_{\mathcal{S}}(\mathbf{a}_{\mathcal{S}}, \overline{\mathbf{a}}_{\mathcal{N}\setminus\mathcal{S}})$, for each $\mathcal{S}$. Let us define $\mathcal{D} = \mathrm{co}^e\{\mathbf{u}_{\mathcal{S}}(\mathbf{a}_{\mathcal{S}}, \mathbf{a}_{\mathcal{N}\setminus\mathcal{S}}), \text{for all } \mathcal{S} \subset \mathcal{N}\}$, as the (extended) convex combination of outcomes that weakly dominate each outcome $\mathbf{u}_{\mathcal{S}}(\mathbf{a}_{\mathcal{S}}, \overline{\mathbf{a}}_{\mathcal{N}\setminus\mathcal{S}})$. Because $\mathcal{D}$ contains points that are in $V(\mathcal{N})$ but not in any $V(\mathcal{S})$, we have that $\bigcap_{j=1}^{m} V(\mathcal{S}_j) \subset \mathcal{D} \subseteq V(\mathcal{N})$ for any collection of m subsets $\mathcal{S}_j$. Finally, because the functions u_i satisfy Property 2, $\sum_{i\in\mathcal{S}} x_i < \sum_{i\in\mathcal{S}} u_i(\underline{\mathbf{a}})$, with $(x_i)_{i\in\mathcal{S}} \in V(\mathcal{S})$, and $\mathcal{S} \subset \mathcal{N}$. Therefore, there is no allocation $\mathbf{u}_{\mathcal{S}} \in V(\mathcal{S})$ that dominates $(u_i(\underline{\mathbf{a}}))_{i\in\mathcal{S}}$, and because $\mathbf{u}(\underline{\mathbf{a}}) \in V(\mathcal{N})$ we have that $\mathbf{u}(\underline{\mathbf{a}}) \in \mathcal{K}$. ■

By propositions 1 and 2 we know that, for the SES system under study, the grand coalition (cartel) is stable and the collusive offer $\underline{\mathbf{a}}$ provides the agents with an efficient payoff vector in the core. Let us see a simple example illustrating the cartel overcharge in a system with two SANs.

Example 1: Let us see a simple example for the system in Fig. 2. Let us assume that the expected transmission rates are $R_{PU}(c) = 4$ Mb/s for $c = 1, 2, 3$, $R_{SU} = 4$ Mb/s, and $R_0 = 1.5$ Mb/s, the PU service request distribution is given by $\mathbf{p} = (0.25, 0.5, 0.25)$, and the SANs can select up to 10 actions between $a_{\min} = 0.4$, and $a_{\max} = 0.8$. The unique Nash equilibrium (σ_{NE}) of Γ has an output $\mathbf{u}_{\mathrm{NE}} = (0.65, 0.65)$ Mb/s for the SANs, while the SAN outcome obtained with $\underline{\mathbf{a}} = (0.4, 0.4)$ is $\mathbf{u}(\underline{\mathbf{a}}) = (1.2, 1.2)$ Mb/s, which is in the core as shown in Fig. 3. However, while the expected δ_R for the PN in the Nash equilibrium is $E_{\sigma_{\mathrm{NE}},p}[\delta_R] \approx 1.2$ Mb/s, when the action profile is $\underline{\mathbf{a}}$, it is equal to $E_{\underline{\mathbf{a}},p}[\delta_R] = 0.1$. In this case, if the agents make the joint offer $\underline{\mathbf{a}}$, the PN suffers a cartel overcharge, with respect to the NE, equal to $E_{\sigma_{\mathrm{NE}},p}[\delta_R] - E_{\underline{\mathbf{a}},p}[\delta_R] \approx 1.1$ Mb/s. □

B. Cartel Formation and Detection

Because the core is always non-empty and contains the action profile $\underline{\mathbf{a}}$, cartels can be formed very easily with little or even no communication requirements. The competing agents only need to simultaneously select $a_{\min}$ at some point of time. As an example let us consider the following *distributed cartel formation algorithm*: at every decision stage, each agent randomly switches to $a_{\min}$ with probability p_i. If the agent does not improve its payoff, it returns to its price war strategy with probability q_i. Otherwise, it adopts $a_{\min}$ as its new strategy.

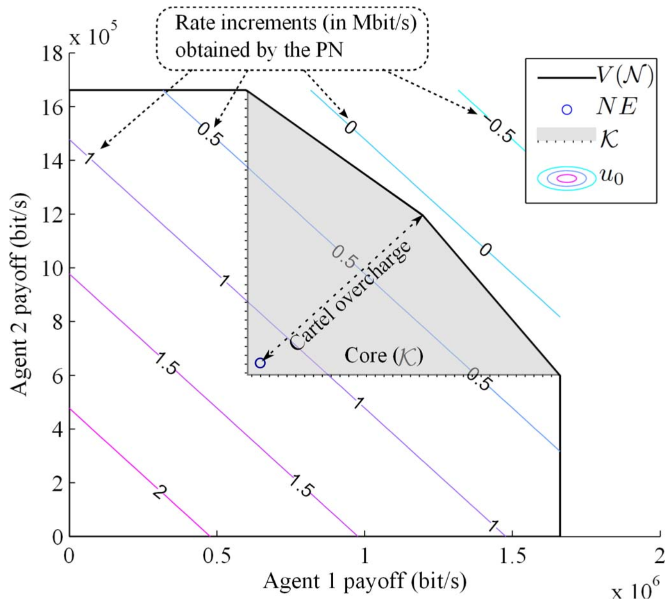

Fig. 3. Outcomes in a two SANs example, Nash equilibrium of the strategic game, $V(\mathcal{N})$ and core of the NTU coalitional representation.

Lemma 1: Let $\Gamma = \langle \mathcal{N}, \mathcal{A}, (u_i)_{i\in\mathcal{N}}\rangle$, be a price war game with $(u_i)_{i\in\mathcal{N}}$ satisfying Properties 1 and 2, and let σ^P be a NE of Γ such that $u_i(\sigma^P) < u_i(\underline{\mathbf{a}})$ for every $i \in \mathcal{N}$. If the *distributed cartel formation algorithm* is executed at every $i \in \mathcal{N}$ with $p_i > 0$ and $q_i < 1$ for all i, then: (i) for every $t > 0$ there is a positive probability of achieving a stable cartel with action profile $\underline{\mathbf{a}}$, and (ii) it is achieved with probability 1 when $t \to \infty$.

Sketch of a Proof: At each stage, $t = 0, 1, 2, \ldots$, the action profile σ^t induced by the algorithm can be characterized by a discrete time Markov chain. This chain has a single recurrent class containing only one state $\underline{\mathbf{a}}$ (single absorbing state), from which (i) and (ii) follow. ■

It should be highlighted that specialized literature on cartels identify several features that make a market prone to cartel formation [7]: homogeneity of the products, market concentration, small number of sellers and inelastic demand. Note that all of them are fulfilled by the SES system.

The issue of cartel detection is currently an area of active research in economics. Theoretical and empirical results support the statement that, when a market moves from competitive to collusive behavior, the price tends to increase and its variability tends to decrease [9]. In consequence, cartel detection mechanisms usually rely on measuring the first two moments of the price. Regarding who performs the detection, one current approach is that it is not necessarily carried out by an antitrust authority but by buyers themselves, especially in the case of industrial buyers [22], [23]. In our SES system, where offers are done automatically among wireless devices and there are no externalities affecting prices (such as raw materials prices, taxes, production costs, and so forth), collusive behavior can be easily detected by the primary network by following the price dynamics, or equivalently, the payoff obtained from the offload services. A sustained decrease of the PN payoff is therefore a sign of collusive behavior by the SANs.

IV. Intervention Strategy for Counteracting Cartels

A. General Intervention Framework

The idea of intervention, introduced in [11] and [12] for strategic and repeated games respectively, relies on the existence of a manager or intervention device capable of observing the action profiles and modifying, to some extent, the agents' payoffs. In the SES system, it is the primary network the entity fulfilling these requirements: it knows the amount of spectrum that each SAN is obtaining per PU terminal, and it can also occupy this spectrum in order to reduce the payoff obtained by each SAN. Therefore, the roles of manager and intervention device are performed by the primary network, in particular the primary base station, PBS acts as the manager, and the set of PU terminals in the area covered by the SES system is the intervention device. As discussed in previous section, a noticeable and sustained decrease of the primary network's payoff triggers intervention. In this subsection, we formulate an intervention framework to counteract collusive behavior in general and, in the next subsection, we particularize for the SES system.

Let $\mathcal{A}_0$ denote the set of all possible intervention actions. The strategy for the manager is defined as a mapping $f : \mathcal{A} \to \mathcal{A}_0$. The set of all possible intervention rules is denoted by $\mathcal{F}$. With intervention, the payoff function is redefined as $u_i : \mathcal{A}_0 \times \mathcal{A} \to \mathbb{R}$. The payoff vector for action $\mathbf{a}$ and intervention rule f, is given by $\mathbf{u}_f = (u_i(f, \mathbf{a}))_{i\in\mathcal{N}}$. Let $\tilde{f}$ denote the absence of intervention. Therefore, $u_i(\tilde{f}, \sigma) = u_i(\sigma)$, and $\mathbf{u}_{\tilde{f}}(\sigma) = \mathbf{u}(\sigma)$. The strategic finite game induced by the manager is $\Gamma_f = \langle \mathcal{N}, \mathcal{A}, (u_i(f, \cdot))_{i\in\mathcal{N}}\rangle$, and the associated NTU coalitional game is V_f, whose core is denoted as $\mathcal{K}_f$.

In the SES system, the payoff function of the manager is given by the PN's payoff defined in (1): $u_0(\mathbf{a}) = E_{\mathbf{a},p}[\delta_R] = \sum_{c\in\mathcal{C}}(a_c^* R_{\mathrm{PU}}(c) - R_0)p_c$. Note that the manager's payoff may also depend on the intervention rule f, and then we use the notation $u_0(f, \mathbf{a})$. The pre-cartel manager's payoff is denoted by $\overline{u_0}$. Therefore, the manager is only allowed to intervene if $u_0(\mathbf{a}) < \overline{u_0}$.

Definition 2: The *sustainable set* of an intervention rule $f \in \mathcal{F}$ is defined as $\mathcal{G}_f^+ = \{\sigma | u_f(\sigma) \in \mathcal{U}_f^+\}$.

Definition 3: An intervention rule $f \in \mathcal{F}$ is said to be *effective* if $\mathbf{u}_f(\sigma) = \mathbf{u}(\sigma)$ for every $\sigma \in \mathcal{G}_f^+$.

The effectiveness property is highly desirable for an intervention rule because it means that the manager can enforce action profiles in the sustainable set without the need to execute any action in $\mathcal{A}_0$. That is $f(\mathbf{a}) = \tilde{f}$ for every $\mathbf{a} \in \mathcal{G}_f^+$. As in [11] and [12] the intervention is understood as a punishment, which every agent, including the manager, do not wish to be executed. The set of effective intervention rules is denoted by $\mathcal{F}^+$.

Definition 4: We say that u_0 is an *attainable bound* if there exists an action profile $\mathbf{a} \in \mathcal{A}$, and an effective intervention rule $f \in \mathcal{F}^+$ such that $u_0 = u_0(\mathbf{a}) \le u_0(\sigma)$ for all $\sigma \in \mathcal{G}_f^+$.

An attainable bound represents the smallest payoff that an intervention rule can guarantee to the manager. A pair $(f, \mathbf{a})$ is attainable if $u_0(\mathbf{a})$ is an attainable bound and $f \in \mathcal{F}^+$. The intervention rule design problem consists on obtaining an attainable pair $(f, \mathbf{a})$ providing the maximum attainable bound, u_0^*. The general formulation of this problem is

$$\begin{aligned}
&\max_{(\mathbf{a},f)} \ u_0(\mathbf{a}) \\
&\text{s.t.} \\
&f \in \mathcal{F}^+ \\
&u_0(\mathbf{a}) \le u_0(\sigma), \ \text{for all } \sigma \in \mathcal{G}_f^+ \\
&u_0(\mathbf{a}) \le \overline{u_0}
\end{aligned} \tag{5}$$

Note that there may be multiple pairs $(f, \mathbf{a})$ solving this problem, or no solutions apart from the trivial one $(\tilde{f}, \underline{\mathbf{a}})$. It depends on the intervention capabilities determined by $\mathcal{A}_0$, and on the definition of the possible intervention rules $\mathcal{F}$. In fact, part of the appeal of the intervention framework is given by the flexibility to specify $\mathcal{F}$, and more specifically $\mathcal{F}^+$, which can be considered the key issue in intervention design. Next, we explain how the general framework described can be adapted to an SES system.

B. Intervention Framework for SES Systems

Several strategies could be used for intervention: i) reducing the throughput of SAN transmissions by interfering them with jamming signals from the served PU terminals, ii) continue data transmission of the served PU terminal during part of the channel fraction assigned to the SAN, iii) randomly rejecting service from SANs with some probability. Options (ii) and (iii) require a different $u_0(f, \mathbf{a})$ formulation, in which the effect on f must be included. Here we focus on option (i), but emphasizing that the framework is also applicable to the others.

One important issue with the jamming-based intervention is energy consumption at the PU terminals. On the one hand, these terminals do not really need to spend energy on intervention provided that the intervention rule f is *effective* (definition 3), i.e. it acts as a dissuasive threat, capable to counteract collusion without being exerted. On the other hand, for the threat to be *credible*, the intervention action set $\mathcal{A}_0$ should be compatible with the power limitations of the terminals. In the definition of $\mathcal{A}_0$ it should also be assured that, even with interference, every SAN obtains a throughput that is above the minimum that justifies serving the PU terminals. Fig. 4 illustrates the intervention operation in an SES system.

Intervention could be executed following two alternatives: the on-off scheme or the noise scheme. In the on-off model, the intervention device corresponding to the i-th SAN, generates a jamming signal only during a fraction of the time $\alpha_i \in (0, 1)$ that the SAN devotes to SU communication. The SAN's achievable rate under the reduced SINR caused by jamming is $R'_{SU} < R_{SU}$. Therefore, for the i-th SAN we have that $\epsilon_i R_{SU} = \alpha_i R'_{SU} + (1 - \alpha_i)R_{SU}$. Note that, depending on the transmission power of the jamming signal, it is possible that $R'_{SU} \approx 0$, therefore $\epsilon_i = (1 - \alpha_i)$. Fig. 4 illustrates the on-off scheme. In the noise scheme, the intervention device adjusts

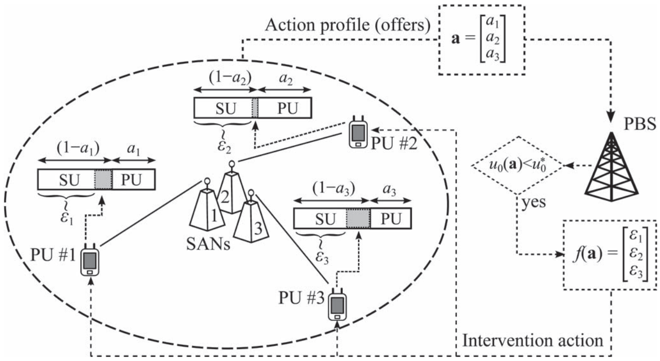

Fig. 4. Example of an intervention execution sequence. When the i-th terminal is instructed by the PBS to exert intervention, it transmits over the part of the channel occupied by the SN transmission, reducing the payoff of the i-th SAN by a factor of ϵ_i.

the power of the jamming signal so that the SAN's achievable rate is reduced to a certain value, i.e. $\epsilon_i R_{SU} = R'_{SU,i}$. Let $\underline{R'}_{SU}$ denote the minimum SAN transmission rate that any intervention device can impose, which depends on the transmission power and the maximum length of a SAN-PU link. We define the minimum reduction factor according to this technological limit as $\epsilon_{\min} = R_{SU}/\underline{R'}_{SU}$. For each $i \in \mathcal{N}$, the intervention device determines the reduction factor $\epsilon_i \in [\epsilon_{\min}, 1]$ by changing the transmission power level (noise scheme) or the fraction of time in which the jamming signal is transmitted (on-off scheme).

Let us recall that the SAN action $a_{\max}$ was defined as the channel share for which the SAN obtains the minimum throughput increment to justify serving a PU terminal. Therefore, for a SAN selecting action $a_i \in \mathcal{A}_i$, the reduction factor $\epsilon_i \in [\epsilon_{\min}, 1]$ should satisfy $\epsilon_i(1 - a_i) \geq 1 - a_{\max}$.

For each action $a_i \in \mathcal{A}_i$, the intervention capability is defined by the set $\mathcal{E}(a_i)$:

$$\mathcal{E}(a_i) = \left\{ \epsilon \,\middle|\, \max\left\{ \epsilon_{\min}, \frac{1 - a_{\max}}{1 - a_i} \right\} \leq \epsilon \leq 1 \right\} \tag{6}$$

We can define the set of feasible intervention actions for each $\mathbf{a} \in \mathcal{A}$ as $\mathcal{A}_0(\mathbf{a}) = \mathcal{E}(a_1) \times \cdots \times \mathcal{E}(a_N)$. The intervention rule is given by $f(\mathbf{a}) = (\epsilon_i)_{i \in \mathcal{N}}$, where $\epsilon_i \in \mathcal{E}(a_i)$, for each $i \in \mathcal{N}$, and the payoff function for each i under intervention is $u_i(f, \mathbf{a}) = \epsilon_i u_i(\mathbf{a})$.

Let us summarize the intervention framework: (i) The PN computes the intervention rule f, and the maximum attainable bound u_0^*. (ii) The PN detects collusive behavior when $u_0 < u_0^*$, implying that the SANs have deviated from the price-war situation (since $u_0^* \leq \overline{u_0}$,). (iii) The PN selects the intervention action $f(\mathbf{a})$ and (iv) *announces* the intervention. If the intervention is effective, the reaction of rational agents will cause the desired effect without needing to *execute* $f(\mathbf{a})$ in practice, and the PN receives at least u_0^*. Next subsection explains how to design such an intervention rule making also an efficient use of the energy required for intervention.

C. Intervention Rule Design Problem for SES Systems

Let us define the ordered set $\mathcal{A}_{u_0} = \{\mathbf{a}^1, \mathbf{a}^2, \ldots, \mathbf{a}^{|\mathcal{A}|}\}$, with $\mathbf{a}^j \in \mathcal{A}$, and $u_0(\mathbf{a}^j) \leq u_0(\mathbf{a}^{j+1})$, for $j = 1, \ldots |\mathcal{A}| - 1$. The notation $g <_{u_0} h$ denotes a pair of indexes $g < h$ in $\mathcal{A}_{u_0}$. We will use $A = \{1, 2, \ldots, |\mathcal{A}|\}$ to refer to the set of indexes in $\mathcal{A}_{u_0}$, and the notation a_i^j to refer to the i-th element of $\mathbf{a}^j$. The following proposition gives a sufficient condition for $(f, \mathbf{a}^j)$ being an attainable pair.

Proposition 3: Given the set $\mathcal{A}_{u_0}$, if the intervention rule $f(\mathbf{a}^k) = (\epsilon_i^k)_{i \in \mathcal{N}}$ satisfies the following conditions

$$\begin{array}{ll} \epsilon_i^k = 1 \text{ (no intervention)}, & \text{for all } i \in \mathcal{N}, \text{ and } k \geq_{u_0} j \\ \epsilon_i^k u_i(\mathbf{a}^k) < u_i(\mathbf{a}^g), & \text{for some } g \geq_{u_0} j \\ & \text{at each } k <_{u_0} j, \text{ and all } i \in \mathcal{N}. \end{array} \tag{7}$$

then $(f, \mathbf{a}^j)$ is an attainable pair.

Proof: We will first prove that f is effective $(f \in \mathcal{F}^+)$, and then that $u_0(\mathbf{a}^j)$ is an attainable bound, i.e. $u_0(\mathbf{a}^j) \leq u_0(\sigma)$ for all $\sigma \in \mathcal{G}_f^+$. Both facts are proved by contradiction.

For $k \geq_{u_0} j$, $\epsilon_i^k = 1$ which by definition corresponds to $f = \tilde{f}$. We have to consider the outcomes for $k <_{u_0} j$. Let assume that f satisfying (7) is not effective. Then, there exists an $\mathbf{a}^k$, with $k <_{u_0} j$, such that $u(f, \mathbf{a}^k) = (\epsilon_i^k u_i(\mathbf{a}^k))_{i \in \mathcal{N}}$ belongs to $\mathcal{U}_f^+$. This implies that $u(f, \mathbf{a}^k)$ is not weakly dominated by any $u \in V(\mathcal{N})$. However, by (7) there exists at least one $\mathbf{a}^g$, with $g \geq_{u_0} j$ such that $\epsilon_i^k u_i(\mathbf{a}^k) < u_i(\mathbf{a}^g)$, for all $i \in \mathcal{N}$, which is a contradiction, and therefore $u(f, \mathbf{a}^k) \notin \mathcal{U}_f^+$.

Let us now assume that $u_0(\mathbf{a}^j)$ is not an attainable bound, therefore there should exist some $\mathbf{a}^k \in \mathcal{G}_f^+$ such that $u_0(\mathbf{a}^j) > u_0(\mathbf{a}^k)$. Because $\mathbf{a}^k \in \mathcal{G}_f^+$ then, by definition, $u(f, \mathbf{a}^k) \in \mathcal{U}_f^+$, which implies that $f = \tilde{f}$, and therefore $\epsilon_i^k = 1$, for every $i \in \mathcal{N}$. Let consider the case for $k \geq_{u_0} j$. Then, $\epsilon_i^k = 1$ (7), but $k \geq_{u_0} j$ implies that $u_0(\mathbf{a}^k) \geq u_0(\mathbf{a}^j)$, which is a contradiction. Let us now consider $k <_{u_0} j$, in this case $u_0(\mathbf{a}^k) < u_0(\mathbf{a}^j)$ but, since $\epsilon_i^k = 1$, the second inequality in (7) is $u_i(\mathbf{a}^k) < u_i(\mathbf{a}^g)$, implying that $\mathbf{u}(\mathbf{a}^k)$ is dominated by $\mathbf{u}(\mathbf{a}^g)$, so $\mathbf{a}^k \notin \mathcal{G}_f^+$, which is a contradiction. ■

This proposition provides an entire set of effective intervention rules for each $\mathbf{a}^j \in \mathcal{A}_{u_0}$. With a slight abuse of notation, we will denote this set by $\mathcal{F}^+(\mathbf{a}^j)$. Note that $\mathcal{F}^+(\mathbf{a}^j)$ does not guarantee that $\mathcal{K}_f \neq \emptyset$. Nevertheless, the intervention goal is not to eliminate the possibility for the agents to establish a grand coalition, but to assure that, if they do, the manager payoff is at least equal to the maximum attainable bound, u_0^*. Note that, for any action profile $\mathbf{a}$ such that $u_0(\mathbf{a}) < u_0^*$ the manager can always announce at least one alternative action $\mathbf{a}^g$ such that $u_i(\mathbf{a}^g) > u_i(f, \mathbf{a})$ and $u_0(\mathbf{a}^g) \geq u_0^*$.

We have seen that the intervention capabilities are constrained by the sets $\mathcal{A}_0(\mathbf{a})_{\mathbf{a} \in \mathcal{A}}$. Moreover, executing intervention is costly in terms of energy consumption at the PU terminals. In order to fully exploit the intervention capabilities and the energy of the terminals, we must find a minimal intervention, which is defined as follows.

Definition 5: Given an action profile $\mathbf{a}^j$, f^*, defined as $f^*(\mathbf{a}^k) = (\epsilon_i^{k*})_{i \in \mathcal{N}}$ for every $k \in A$, is a *minimal intervention rule* of $\mathcal{F}^+(\mathbf{a}^j)$ if and only if $f^* \in \mathcal{F}^+(\mathbf{a}^j)$ and for every $f \in \mathcal{F}^+(\mathbf{a}^j)$, such that $f(\mathbf{a}^k) = (\epsilon_i^k)_{i \in \mathcal{N}}$, the following holds

$$\min_{i \in \mathcal{N}} \left(\epsilon_i^{k*}\right) \geq \min_{i \in \mathcal{N}} \left(\epsilon_i^k\right), \text{ for each } k <_{u_0} j. \tag{8}$$

Based on the above definition we define the more practical concept of δ-*minimal* intervention rule. If f^* is a minimal intervention rule of $\mathcal{F}^+(\mathbf{a}^j)$, and $\delta > 0$, we say that that $f^\delta = (\epsilon_i^{k\delta})_{i\in\mathcal{N}}$ is a δ-*minimal* intervention rule if and only if $f^\delta \in \mathcal{F}^+(\mathbf{a}^j)$ and

$$\min_{i\in\mathcal{N}}\left(\epsilon_i^{k*}\right) < \min_{i\in\mathcal{N}}\left(\epsilon_i^{k\delta}\right) + \delta, \text{ for each } k <_{u_0} j. \tag{9}$$

A δ-minimal intervention guarantees that ϵ_i (i.e. the ratio $u_i(f^\delta, \mathbf{a})/u_i(\mathbf{a})$) is always greater than or equal to the left hand side of (9) minus a small positive value δ. Recall that the smaller ϵ_i is, the more interference power is required.

Next proposition provides a useful necessary and sufficient condition for f^δ being a δ-minimal intervention rule of $\mathcal{F}^+$.

Proposition 4: Given an action profile $\mathbf{a}^j$, the function $f^\delta(\mathbf{a}^k) = (\epsilon_i^{k\delta})_{i\in\mathcal{N}}$, is a δ-*minimal* intervention rule of $\mathcal{F}^+(\mathbf{a}^j)$ if and only if for each $k <_{u_0} j$, $\epsilon_i^{k\delta} = \min\{u_i(\mathbf{a}^g)/u_i(\mathbf{a}^k) - \delta, 1\}$ for $g = \underline{g}(j,k) = \arg\min_{h\geq_{u_0} j} \|\mathbf{u}(\mathbf{a}^k) - \mathbf{u}(\mathbf{a}^h)\|_\infty$.

Proof: By (7), any $f(\mathbf{a}^k) = (\epsilon_i^k)_{i\in\mathcal{N}}$ satisfies, for every $i \in \mathcal{N}$ and $k <_{u_0} j$

$$\epsilon_i^k < \frac{u_i(\mathbf{a}^g)}{u_i(\mathbf{a}^k)}, \text{ for some } g \geq_{u_0} j. \tag{10}$$

Let $\underline{\epsilon}^k = \min_{i\in\mathcal{N}}(\epsilon_i^k)$.

$$\underline{\epsilon}^k < \min_{i\in\mathcal{N}} \frac{u_i(\mathbf{a}^g)}{u_i(\mathbf{a}^k)}, \text{ for some } g \geq_{u_0} j \tag{11}$$

Let $f^*(\mathbf{a}^k) = (\epsilon_i^{k*})_{i\in\mathcal{N}}$ denote the minimal intervention rule, and $\underline{\epsilon}^{k*} = \min_{i\in\mathcal{N}}(\epsilon_i^{k*})$. Because f^* is the intervention rule having the highest minimum intervention factor for every $k <_{u_0} j$, by maximizing both sides of (11) we obtain

$$\underline{\epsilon}^{k*} < \max_{g\geq_{u_0} j}\left\{\min_{i\in\mathcal{N}} \frac{u_i(\mathbf{a}^g)}{u_i(\mathbf{a}^k)}\right\} \tag{12}$$

In case the right hand side of the above inequality is greater than or equal to 1, $\epsilon_i^k = 1$, meaning that no intervention is needed for $\mathbf{a}^k$, so that $\tilde{f}$ is trivially minimal for $\mathbf{a}^k$. In the nontrivial case of $\min_{i\in\mathcal{N}}\{u_i(\mathbf{a}^g)/u_i(\mathbf{a}^k)\} < 1$ for every g, the following equations provide the maximizing value g^*:

$$\begin{aligned} g^* &= \arg\max_{h\geq_{u_0} j}\left\{\min_{i\in\mathcal{N}} \frac{u_i(\mathbf{a}^h)}{u_i(\mathbf{a}^k)}\right\} \\ &= \arg\min_{h\geq_{u_0} j}\left\{\max_{i\in\mathcal{N}}\left(1 - \frac{u_i(\mathbf{a}^h)}{u_i(\mathbf{a}^k)}\right)\right\} \\ &= \arg\min_{h\geq_{u_0} j}\left\{\max_{i\in\mathcal{N}}\left|u_i(\mathbf{a}^k) - u_i(\mathbf{a}^h)\right|\right\} \\ &= \arg\min_{h\geq_{u_0} j}\left\|\mathbf{u}(\mathbf{a}^k) - \mathbf{u}(\mathbf{a}^h)\right\|_\infty \end{aligned} \tag{13}$$

Then $\underline{\epsilon}^{k*} < \frac{u_i(\mathbf{a}^{g^*})}{u_i(\mathbf{a}^k)}$, for every $i \in \mathcal{N}$. Therefore, $\underline{\epsilon}^{k*} < \min_{i\in\mathcal{N}} \frac{u_i(\mathbf{a}^{g^*})}{u_i(\mathbf{a}^k)} = \min_{i\in\mathcal{N}} \frac{u_i(\mathbf{a}^{g^*})}{u_i(\mathbf{a}^k)} - \delta + \delta = \underline{\epsilon}^{k\delta} + \delta$. In consequence $\min_{i\in\mathcal{N}}(\epsilon_i^{k*}) < \min_{i\in\mathcal{N}}(\epsilon_i^{k\delta}) + \delta$. ■

We can obtain an attainable pair $(f, \mathbf{a})$ providing the maximum attainable bound, u_0^* by solving the following optimization problem, the Intervention Rule Design Problem (IRDP):

$$\begin{aligned} &\max_{j\in A}\ u_0(\mathbf{a}^j) \\ &\text{s.t.} \\ &\epsilon_i^k = 1, && \text{for } i \in \mathcal{N}, \text{ and } k \geq_{u_0} j \\ &\epsilon_i^k = \min\left\{\frac{u_i\left(\mathbf{a}^{\underline{g}(j,k)}\right)}{u_i(\mathbf{a}^k)} - \delta, 1\right\}, && \text{for } i \in \mathcal{N}, \text{ and } k <_{u_0} j \\ &\epsilon_i^k \geq \max\left\{\epsilon_{\min}, \frac{1-a_{\max}}{1-a_i}\right\}, && \text{for } i \in \mathcal{N}, \text{ and } k <_{u_0} j \\ &u_0(\mathbf{a}^j) \leq \overline{u_0} \end{aligned} \tag{14}$$

where $\underline{g}(j,k) = \arg\min_{h\geq_{u_0} j} \|\mathbf{u}(\mathbf{a}^k) - \mathbf{u}(\mathbf{a}^h)\|_\infty$. By removing the factors ϵ_i^k from (14) we can formulate the IRDP more compactly as follows:

$$\begin{aligned} &\max_{j\in A} u_0(\mathbf{a}^j) \\ &\text{s.t.} \\ &\frac{u_i\left(\mathbf{a}^{\underline{g}(j,k)}\right)}{u_i(\mathbf{a}^k)} - \delta \geq \max\left\{\epsilon_{\min}, \frac{1-a_{\max}}{1-a_i^k}\right\}, \\ &\qquad \text{for } i \in \mathcal{N}, \text{ and } k <_{u_0} j \\ &u_0(\mathbf{a}^j) \leq \overline{u_0} \end{aligned} \tag{15}$$

If $j \in A$ is the argument solving the above problem, the resulting intervention rule $f(\mathbf{a}^k) = (\epsilon_i^k)_{i\in\mathcal{N}}$ is given by

$$\begin{aligned} &\epsilon_i^k = 1, && \text{for } i \in \mathcal{N}, \text{ and } k \geq_{u_0} j \\ &\epsilon_i^k = \min\left\{\frac{u_i\left(\mathbf{a}^{\underline{g}(j,k)}\right)}{u_i(\mathbf{a}^k)} - \delta, 1\right\}, && \text{for } i \in \mathcal{N}, \text{ and } k <_{u_0} j \end{aligned} \tag{16}$$

which, by proposition 3, assures that no intervention action is executed at any efficient outcome of the coalitional game V_f and, by proposition 4, that f is δ-minimal for $\mathcal{F}^+(\mathbf{a}^j)$.

Example 2: Let us evaluate the intervention framework in the SES system of example 1. Let us assume that the intervention capability allows $\epsilon_{\min} = 0.7$, which means that the punishment signal can reduce the SAN throughput to, at most, 70% of its nominal throughput. Applying the intervention rule solving the IRDP with $\delta = 0.01$, the attainable bound is $u_0 =$ 0.6 Mb/s, while it was equal to 0.1 Mb/s in absence of intervention. Fig. 5 shows the sets $V(\mathcal{N})$ and $V_f(\mathcal{N})$ corresponding to the coalitional representations *without* intervention, and *with* intervention respectively. The figure also depicts the payoff vectors of the intervened action profiles before intervention $(\mathbf{u})$, and after intervention $(\mathbf{u}_f)$.

V. Low Complexity Implementation

Despite its apparent simplicity, the IRDP (15) may be computationally intractable when $\mathcal{N}$ is large, because the dimension of A grows exponentially with N ($|A| = |\mathcal{A}_S|^N$), making it more difficult to obtain and store the sorted set $\mathcal{A}_{u_0}$ and to compute $\underline{g}(j,k)$. Note that, in the worst case, solving the IRDP implies $\sum_{j=1}^{|A|} j(|A| - j)$ iterations, thus having a complexity $O(|A|^3) = O(|\mathcal{A}_S|^{3N})$. In this section we present a low-complexity algorithm that provides an effective intervention rule, close to the optimal one, and not requiring the storage of massive data.

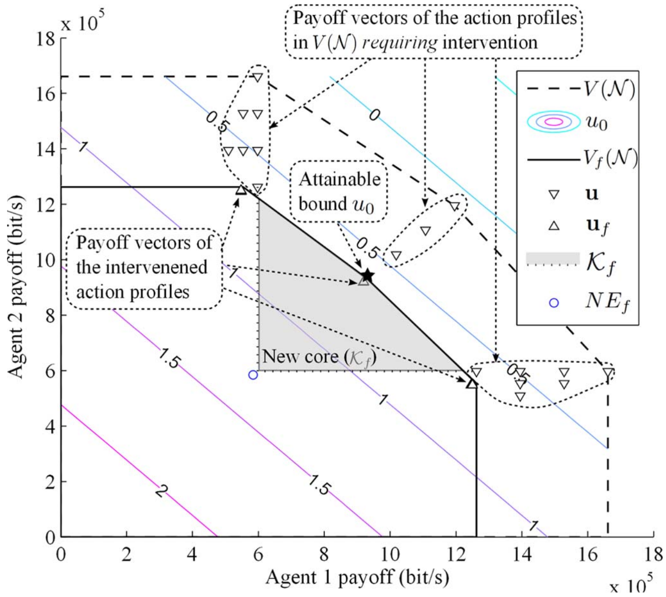

Fig. 5. Outcomes in a two SANs example with $\epsilon_{\min} = 0.7$. The intervention reduces the achievable outcomes $V_f(\mathcal{N})$ such that no intervention is executed at any efficient outcome. The figure also shows the outcomes, $\mathbf{u}$, of the action profiles requiring intervention, and the corresponding outcomes, $\mathbf{u}_f$, of these action profiles under intervention.

The main idea of this algorithm is to solve a Modified IRDP (MIRDP) in which the optimal j is found over a smaller subset of values, and $\mathbf{a}^{g(j,k)}$ is replaced by an easier to compute $\mathbf{a}^g$. Let us define $\mathcal{A}' = \{\mathbf{a} : a_i = a_n, i, n \in \mathcal{N}\}$, as the set of action profiles where all the agents select the same action. We also define the ordered set $\mathcal{A}'_{u_0}$ and the associated set of indexes A' in a similar way to $\mathcal{A}_{u_0}$ and A. Note that $|A'| = |\mathcal{A}_S|$. The rationale behind the use of A' instead of A is that, as the coalitional analysis showed, if the core is non-empty, it contains at least one action profile with all the agents making the same offer.

To obtain $\mathbf{a}^g$, we define a function $\Psi : \mathcal{A} \times \mathcal{A} \to \mathcal{A}$ such that if $\mathbf{a}^g = \Psi(\mathbf{a}^k, \mathbf{a}^j)$, then $u_0(\mathbf{a}^g) \geq u_0(\mathbf{a}^j)$. The objective is to obtain an action profile whose payoff vector $\mathbf{u}(\mathbf{a}^g)$ is as close as possible to $\mathbf{u}(\mathbf{a}^k)$ (small distance $\|\mathbf{u}(\mathbf{a}^k) - \mathbf{u}(\mathbf{a}^g)\|_\infty$), but with small computation and storage requirements compared to finding $\mathbf{a}^{g(j,k)}$. Algorithm 1 obtains $\mathbf{a}^g = \Psi(\mathbf{a}^k, \mathbf{a}^j)$ by iteratively adding increments δ_a to the actions in $\mathbf{a}^g$, while keeping the structure of $\mathbf{a}^k$ (thus preserving the sets $\mathcal{N}_c^*$) whenever possible, until $u_0(\mathbf{a}^g) \leq u_0(\mathbf{a}^i)$. Note that Algorithm 1 performs, at most, $|\mathcal{A}_S|$ iterations ($O(|\mathcal{A}_S|)$).

Algorithm 1 Computation of $\mathbf{a}^g = \Psi(\mathbf{a}^k, \mathbf{a}^j)$

$\mathbf{a}^g \leftarrow \mathbf{a}^k$
while $u_0(\mathbf{a}^g) > u_0(\mathbf{a}^k)$ **do**
  $i' = \arg\min_{i \in \mathcal{N}}\{a_i^g\}$ s.t. $a_i^g + \delta_a < a_n^g$ for all $n \in \mathcal{N} \setminus \{i\}$
  $\mathcal{I} = \{i \in \mathcal{N} : a_i^g \leq a_{i'}^g\}$
  if $\mathcal{I} = \emptyset$ **then**
    $a_i^g \leftarrow a_i^g + \delta_a$ for all $i \in \mathcal{I}$
  else
    $a_i^g \leftarrow \min\{a_i^g + \delta_a, a_{\max}\}$ for all $i \in \mathcal{N}$
  end if
end while

Solving the MIRDP for all possible action profiles still requires to compute $\mathcal{A}_{u_0}$ and perform, in the worst case, $\sum_{j=1}^{|A|} j|\mathcal{A}_S|$ iterations ($O(|\mathcal{A}_S|^{2N})$). The problem can be notably simplified by solving for the maximum achievable payoff that an agent can obtain when only this agent deviates from $\underline{\mathbf{a}}$. It can be shown that, for the u_i functions defined in (2), the maximum achievable payoff for an agent i is either $u_i(\underline{\mathbf{a}})$, or $u_i(\underline{\mathbf{a}}_{\mathcal{N}\setminus\{i\}}, a_{\min}^+)$, where $\underline{\mathbf{a}}_{\mathcal{N}\setminus\{i\}} = (a_{\min})_{n \in \mathcal{N}\setminus\{i\}}$, and $a_{\min}^+ = a_{\min} + \delta_a$. In the first case, the action profile determined by Ψ is $\mathbf{a}^j$, and, in the second case, it is $(\mathbf{a}^j_{\mathcal{N}\setminus\{i\}}, a_i^{j+})$, where $a_i^{j+} = a_i^j + \delta_a$. Algorithm 2 solves the MIRDP for the maximum achievable payoffs, having a complexity $O(|\mathcal{A}_S|)$.

Algorithm 2 Approximate solution of the MIRDP

$j = 2, j^* = j$, continue=true
while $(j+1) \in A'$ **and** continue=true **do**
  if $\min_{i \in \mathcal{N}}\{u_i(\mathbf{a}^j_{\mathcal{N}\setminus\{i\}}, a_i^{j+})/u_i(\underline{\mathbf{a}}_{\mathcal{N}\setminus\{i\}, a_{\min}^+})\} \geq \max\{\epsilon_{\min}, (1-a_{\max})/(1-a_{\min}^+)\} + \delta$ **and** $\min_{i \in \mathcal{N}}\{u_i(\mathbf{a}^j)/u_i(\underline{\mathbf{a}}))\} \geq \max\{\epsilon_{\min}, (1-a_{\max})/(1-a_{\min})\} + \delta$ **and** $u_0(\mathbf{a}^j) \leq \overline{u_0}$ **then**
    $j^* = j$
    $j \leftarrow j + 1$ (with $(j+1) \in A'$)
  else
    continue=false
  end if
end while
$\mathbf{a}^j = \mathbf{a}^{j^*}$

Starting with the $\mathbf{a}^j$ provided by Algorithm 2, Algorithm 3 computes the intervention actions $f(\mathbf{a}^k)$ for each action profile $\mathbf{a}^k$ selected by the agents during system's operation. If $\mathbf{a}^k$ is considered collusive, then the manager computes $\mathbf{a}^g$ and announces it. As explained in previous section, each SAN obtains with $\mathbf{a}^g$ a payoff that is strictly higher than the payoff obtained with $\mathbf{a}^k$ under intervention, and therefore is preferable to all of them. If SANs are rational, the manager does not need to execute intervention. In case the agents select an action profile for which the current f is not effective, the manager upgrades $\mathbf{a}^j$. The only signaling required by this algorithm is the announcement of the action profiles $\mathbf{a}^j$ and $\mathbf{a}^g$, when collusive behavior is detected. Once the intervention has made effect, no additional signaling is required. One advantage of this implementation is that it does not require the precomputation and storage of all the factors ϵ_i^k for $i \in \mathcal{N}$ and $k \in A$. Summarizing, the computational costs associated to the low-complexity intervention are: $O(|\mathcal{A}_S|)$ for initializing[4] $\mathbf{a}^j$, and $O(|\mathcal{A}_S|)$ for computing $\mathbf{a}^g$.

[4]Since initialization is done only once, an heuristic search, more exhaustive than Algorithm 3 could be conducted. This would mitigate the possible inaccuracies of using an approximate solution for the MIRDP.

Algorithm 3 Intervention rule operation and upgrading

Initialize $\mathbf{a}^j$ (2), announce $\mathbf{a}^j$
for each $\mathbf{a}^k$ **do**
  if $u_0(\mathbf{a}^k) \geq u_0(\mathbf{a}^j)$ **then**
    $f = \tilde{f}$
  else
    $\mathbf{a}^g = \Psi(\mathbf{a}^k, \mathbf{a}^j)$
    Announce $\mathbf{a}^g$
    $\epsilon_i^k = \min\{(u_i(\mathbf{a}^g)/u_i(\mathbf{a}^k)) - \delta, 1\}$, for $i \in \mathcal{N}$
    $\mathcal{A}_0(\mathbf{a}^k) = \{(\epsilon_i)_{i\in\mathcal{N}} | \max\{\epsilon_{\min}, (1 - a_{\max}/1 - a_i^k)\} \leq \epsilon_i \leq 1\}$
    if $(\epsilon_i)_{i\in\mathcal{N}} \in \mathcal{A}_0(\mathbf{a}^k)$ **then**
      $f = (\epsilon_i^k)_{i\in\mathcal{N}}$
    else
      $f = \tilde{f}$
      while $(\epsilon_i^k)_{i\in\mathcal{N}} \notin \mathcal{A}_0(\mathbf{a}^j)$ **do**
        $\mathbf{a}^j \leftarrow \mathbf{a}^{j-1}$ (with $(j-1) \in A'$)
        $\mathbf{a}^g = \Psi(\mathbf{a}^k, \mathbf{a}^j)$
        $\epsilon_i^k = \min\{(u_i(\mathbf{a}^g)/u_i(\mathbf{a}^k)) - \delta, 1\}$, for $i \in \mathcal{N}$
      end while
      announce $\mathbf{a}^j$
    end if
  end if
end for

VI. Numerical Evaluation

In this section we evaluate the performance of the low-complexity intervention rule presented in previous section. First, we will evaluate the sub-optimality gap, between the intervention rules provided by IRDP (exact solution) and MIRDP, in terms of the manager's attainable bound u_0, for a two SANs scenario similar to the one in Examples 1 and 2. Recall that u_0 denotes the PN's rate increment attainable with the intervention rule. We will also include, as a reference, the Simplest Effective Intervention Rule (SEIR), consisting on using $\mathbf{a}^g = \mathbf{a}^j$. Therefore, computing SEIR for each $\mathbf{a}^k$ selected by the SANs has complexity $O(1)$.

The average signal to noise ratio for a given SAN-PU link length, d, is computed by means of a two ray model $\gamma(d) = (p_{\rm tx}K/WN_0d^4)$, where $p_{\rm tx}$ is the transmission power (which is set to 2 W), W is the channel bandwidth (set to 1 MHz), N_0 is the noise spectral density (set to 10^{-9} W/Hz), and K is a constant depending on the antenna gains and heights. The distance between the SANs is 50 meters. A PU requesting a service can be located at any of the three sub-areas with equal probability. Fig. 6 compares the intervention rules under different intervention capabilities, determined by $\epsilon_{\min}$. The results of MIRDP are notably close to the optimal intervention rule and clearly outperform the EIRP scheme. We can also observe that, for sufficiently high intervention capabilities (small $\epsilon_{\min}$ values), the MIRDP and EIRP curves *saturate* at the same maximum value. This maximum value is the pre-cartel payoff $\underline{u_0}$, which determines one of the constraints of the intervention design problem (5).

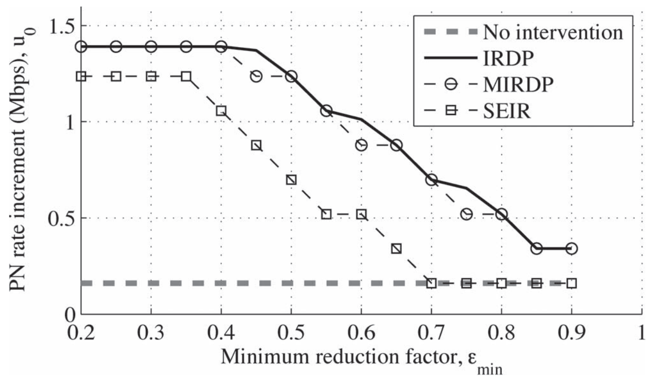

Fig. 6. Minimum attainable PN rate increment, u_0, versus $\epsilon_{\min}$ in a 2 SANs system. The figure shows the results for the IRDP intervention rule (exact solution), the MIRDP intervention rule, and the SEIR.

A. Evaluating the Influence of the System's Parameters

In this subsection we evaluate the influence of 3 relevant system's parameters on the intervention performance: (i) the number of competing SANs, N, (ii) the PU traffic intensity in the covered sub-areas, and (iii) the intervention capability determined by the parameter $\epsilon_{\min}$.

For this purpose, we extend the 2 SAN scenario of Fig. 6 by adding additional SANs. All the N SANs provide coverage in one sub-area, referred to as the overlap region, and numbered as $c = 1$. Additionally each SAN also provides coverage on its own sub-area, where it does not compete with other SANs. It is clear that the effects of competition will be affected by N and by the share of the total PU traffic coming from the overlap region, p_1.

The system is simulated for $N = 2,\ 4,\ 6$ and 8 SANs, with p_1 ranging from 0.1 to 0.9 with a 0.1 step size. The pre-cartel equilibrium providing $\overline{u_0}$ is obtained by averaging the results of 30 fictitious play simulation runs. For computing the intervention rule at each parameter combination, we use Algorithm 2 to obtain the approximate MIRDP solution, and Algorithm 3 to execute and adjust the intervention rule if needed. Up to 10^6 action profiles requiring intervention are randomly generated for each case. Finally, to evaluate the effect of the intervention capability limits, all the simulations are done for $\epsilon_{\min}$ ranging from 0.2 to 0.8 in steps of 0.1. Fig. 7 shows the attainable PN rate increment u_0 for 3 different values of $\epsilon_{\min}$.

Let us first discuss the effects of N and p_1 in the intervention performance. From the curves in Fig. 7 it is clear that u_0 increases with both N and p_1. To understand this effect, note that, in a pre-cartel situation, when more SANs compete they tend to make better offers (a_i) to the PUs, increasing the average rate increment for these PUs. Similarly, when the size of the shared market is larger (i.e. when p_1 increases) the SANs also tend to make better offers. This behavior is consistent with well-known theoretical and empirical results of market economics: increasing the degree of competition benefits the customer. In consequence, larger N and p_1 result in higher pre-cartel PN payoff $\overline{u_0}$, loosening the constraint $u_0(\mathbf{a}) \leq \overline{u_0}$ and therefore giving the intervention rule a higher margin to improve u_0.

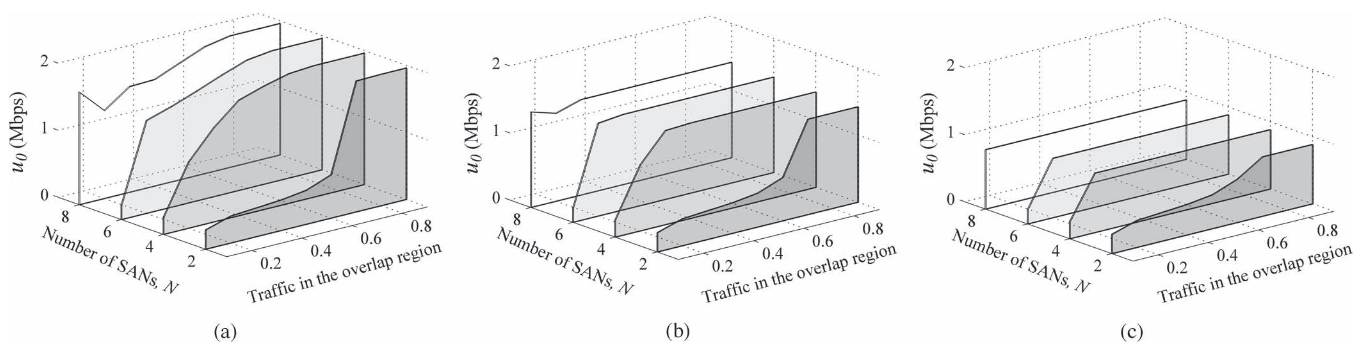

Fig. 7. Minimum attainable rate increment for the PN, u_0, for $N = 2,\ 4,\ 6,\ 8$. The ratio of the total PU traffic coming from the overlapping region ranges from 0.1 to 0.9. Figures (a) (b) and (c) correspond to $\epsilon_{\min} = 0.3,\ 0.5,$ and 0.7 respectively. (a) $\epsilon_{\min} = 0.3$. (b) $\epsilon_{\min} = 0.5$. (c) $\epsilon_{\min} = 0.7$.

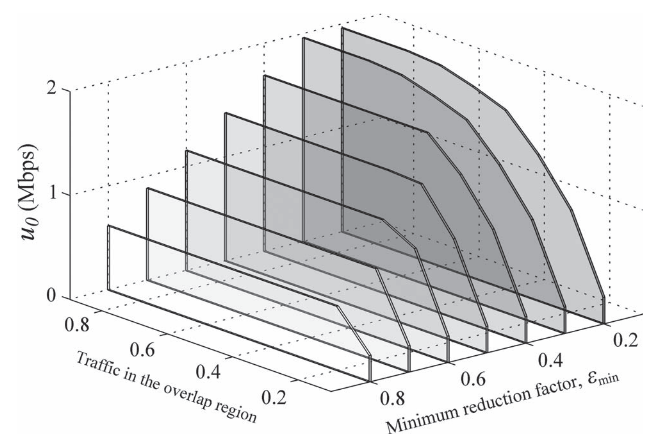

Fig. 8. Effect of the intervention capability and the traffic intensity on the overlap region for $N = 4$.

The other parameter with a relevant impact on the attainable PN payoff is the minimum reduction factor $\epsilon_{\min}$ that can be applied on the SANs' payoffs. Note that a smaller $\epsilon_{\min}$ requires more jamming power in case intervention is exerted. As expected, less available power (and therefore higher $\epsilon_{\min}$) implies a smaller attainable u_0. To illustrate this effect more clearly, Fig. 8 shows how the intervention capability determined by $\epsilon_{\min}$ affects the intervention performance in a $N = 4$ system, for different PU traffic intensities in the overlap region.

B. Comparing Performance Under Different Underlying Topologies

By underlying topology of the SES system we refer to the vector $\mathbf{p}$ containing the normalized PU traffic intensity on each sub-area, and the set of vectors $\mathbf{w}_i$ for $i = 1, \ldots, N$, indicating the sub-areas covered by each SAN. We consider 3 representative cases: (i) N SANs whose coverages overlap partially, but all SANs cover a sub-area where 60% of the overall PU traffic is generated. This topology is equivalent to the one in previous subsection and is referred to as *symmetric* topology. (ii) N SANs whose coverages overlap partially, but the PU traffic is unevenly distributed so that one of the SANs covers 60% of the overall PU traffic. We refer to this topology as *asymmetric*. (iii) N SANs located in line, thus each SAN overlaps only with the adjacent SANs. Traffic is evenly distributed. This is the *linear* topology. N is set to 4 in all cases. Fig. 9 contains a diagram of each topology. This figure also compares the results obtained on each topology by the two solution methods previously discussed (MIRDP and SEIR), under different intervention capabilities $(\epsilon_{\min})$.

The attainable performance u_0 in the symmetric topology is slightly better than u_0 in the asymmetric one. And similarly, u_0 achieves higher values in the asymmetric topology than in the linear one. As in previous subsection, the reason is that when the degree of competition is higher, the achievable pre-cartel payoff for the PN $\overline{u_0}$ is also higher. Clearly, the symmetric topology implies more competition than the asymmetric one, while the topology with the least degree of competition is the linear one. We see that MIRDP clearly outperforms SEIR, as expected. We also see that, contrary to MIRDP, the performance of SEIR decreases for topologies implying higher competition degree. This is because, when competition is harder, the pre-cartel payoffs of the SANs are smaller, and therefore the payoff increment under collusion is higher for each SAN. Since SEIR makes a less efficient use of the energy than MIRDP, it has more trouble counteracting high payoffs in every SAN, and attains lower values of u_0.

VII. Conclusion

This paper presents an intervention framework for coalitional games to counteract cartel formation effects. The idea of intervention requires the existence of a manager capable of observing the actions of the agents and modifying the payoff of these agents. The framework is applied to a spectrum leasing system in which several secondary access nodes offer offload services to a network operator, in exchange for bandwidth from the serviced PUs. In this case, cartel overcharge implies that the PUs obtain lower increments of the transmission rate. In this deployment scenario, coalitional game analysis showed that cartel formation is possible because the core of the game is always nonempty and it is very easy to find at least one payoff vector belonging to the core. In the design of an intervention rule, the objective is to maximize the minimum attainable bound for the manager's payoff with the premise

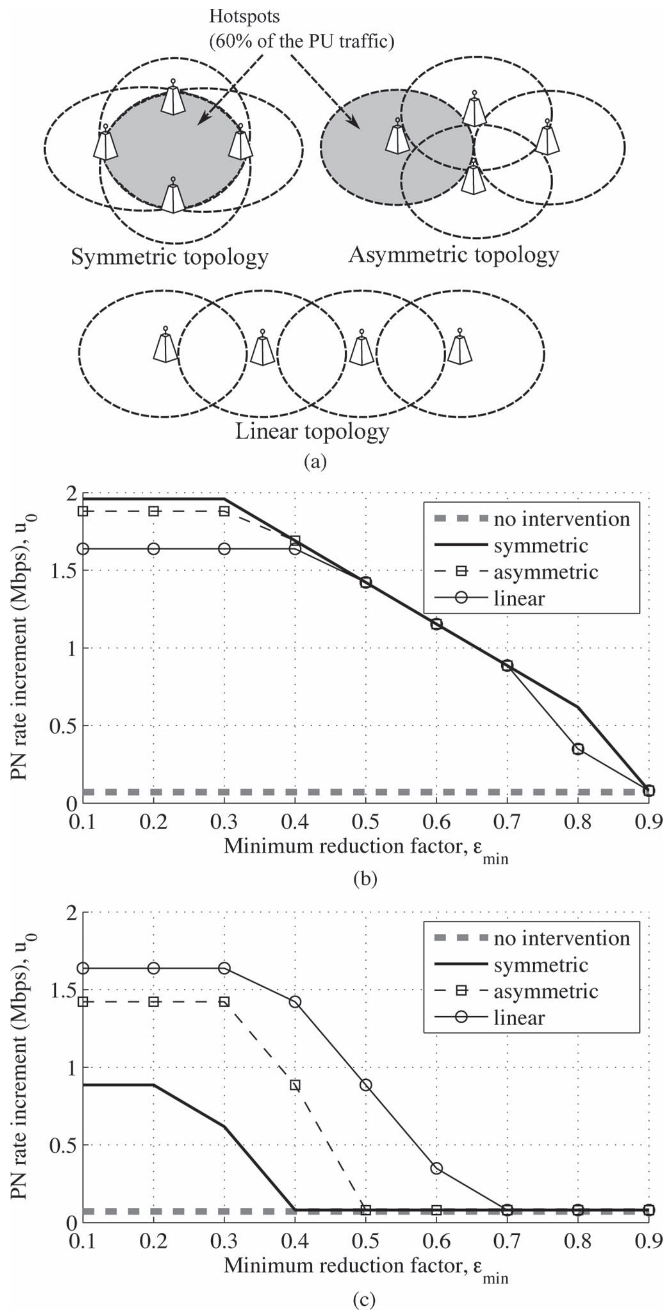

Fig. 9. Minimum attainable rate increment for the PN, u_0, vs. the intervention capability given by $\epsilon_{\min}$. Figure (a) shows the schematic views of the different topologies. Figures (b) and (c) correspond to MIRDP and SEIR solution schemes respectively, for $N = 4$. (a) 3 representative topologies. (b) **MIRDP** performance vs. $\epsilon_{\min}$. (c) **SEIR** performance vs. $\epsilon_{\min}$.

that the intervention should be effective without needing to be exerted. Moreover, the intervention rule needs to make an efficient use of the limited intervention capabilities, which are, in general, subject to technical and regulatory constraints. An exact rule fulfilling this characteristics can be found by solving an optimization problem. However, when the number of competing agents increase, the computational and storage requirement of the problem increase exponentially. Therefore, a low-complexity intervention algorithm is proposed to overcome this limitation.

Appendix
Proof of Proposition 1

Property 1. Let $\mathbf{a}' = (\mathbf{a}_{\mathcal{S}}, \overline{\mathbf{a}}_{\mathcal{N}\setminus\mathcal{S}})$, then $u_i(\mathbf{a}') = (1 - a_i) R \sum_{c\in\mathcal{C}} p_c D_i(\mathbf{a}', c)$. Let us define $\mathcal{C}_{\mathcal{N}\setminus\mathcal{S}} = \{c \in \mathcal{C} | \mathbf{w}_j(c) = 1, j \in \mathcal{N} \setminus \mathcal{S}\}$, and $\mathcal{C}_i = \{c \in \mathcal{C} | \mathbf{w}_i(c) = 1\}$. We consider two cases:

1) If $\mathcal{C}_i \cap \mathcal{C}_{\mathcal{N}\setminus\mathcal{S}} = \emptyset$, then no agent in $\mathcal{N} \setminus \mathcal{S}$ shares a coverage sub-area with i, and we have that $u_i(\mathbf{a}') = u_i(\mathbf{a}_{\mathcal{S}}, \mathbf{a}_{\mathcal{N}\setminus\mathcal{S}})$, for any $\mathbf{a}_{\mathcal{N}\setminus\mathcal{S}}$.
2) If $\mathcal{C}_i \cap \mathcal{C}_{\mathcal{N}\setminus\mathcal{S}} \neq \emptyset$, we have $D_i(\mathbf{a}', c) = 0$, if $a_i < a_{\max}$, $c \in \mathcal{C}_i \cap \mathcal{C}_{\mathcal{N}\setminus\mathcal{S}}$, and $D_i(\mathbf{a}', c) = (1/|\mathcal{N}_c^*|)$, if $a_i = a_{\max}$, $c \in \mathcal{C}_i \cap \mathcal{C}_{\mathcal{N}\setminus\mathcal{S}}$.

If $\mathbf{a}_{\mathcal{N}\setminus\mathcal{S}} \neq \overline{\mathbf{a}}_{\mathcal{N}\setminus\mathcal{S}}$, then $a_j \leq a_{\max}$ for $j \in \mathcal{N} \setminus \mathcal{S}$ with strict inequality in some j. Let $\mathcal{N}_c'^*$ denote the number of agents of agents selecting $a_{\max}$ at each c with this action profile, and $\mathbf{a}'' = (\mathbf{a}_{\mathcal{S}}, \mathbf{a}_{\mathcal{N}\setminus\mathcal{S}})$ such that $\mathbf{a}_{\mathcal{N}\setminus\mathcal{S}} \neq \overline{\mathbf{a}}_{\mathcal{N}\setminus\mathcal{S}}$. We have that $D_i(\mathbf{a}'', c) \geq 0$, if $a_i < a_{\max}$, $c \in \mathcal{C}_i \cap \mathcal{C}_{\mathcal{N}\setminus\mathcal{S}}$, and $D_i(\mathbf{a}'', c) = (1/|\mathcal{N}_c'^*|) \geq (1/|\mathcal{N}_c^*|)$, if $a_i = a_{\max}$, $c \in \mathcal{C}_i \cap \mathcal{C}_{\mathcal{N}\setminus\mathcal{S}}$, with at least one strict inequality. Therefore $D_i(\mathbf{a}', c) \leq D_i(\mathbf{a}'', c)$ for each $i \in \mathcal{S}$, with some strict inequalities in some i, which proofs property 1.

Property 2. Let us recall that $\underline{\mathbf{a}} = (a_{\min})_{i\in\mathcal{N}}$, and $\mathbf{a}' = (\mathbf{a}_{\mathcal{S}}, \overline{\mathbf{a}}_{\mathcal{N}\setminus\mathcal{S}})$, where $\overline{\mathbf{a}}_{\mathcal{N}\setminus\mathcal{S}} = (a_{\min})_{j\in\mathcal{N}\setminus\mathcal{S}}$. Because $a_c^* \leq a_{\max}$, $D_j(\mathbf{a}', c) \geq D_j(\underline{\mathbf{a}}, c)$ for all $j \in \mathcal{N} \setminus \mathcal{S}$. Therefore, the following inequalities hold

$$\begin{aligned} \sum_{j\in\mathcal{N}\setminus\mathcal{S}} D_j(\mathbf{a}', c) &\geq \sum_{j\in\mathcal{N}\setminus\mathcal{S}} D_j(\underline{\mathbf{a}}, c) \\ 1 - \sum_{j\in\mathcal{N}\setminus\mathcal{S}} D_j(\mathbf{a}', c) &\leq 1 - \sum_{j\in\mathcal{N}\setminus\mathcal{S}} D_j(\underline{\mathbf{a}}, c) \\ \sum_{i\in\mathcal{S}} D_i(\mathbf{a}', c) &\leq \sum_{i\in\mathcal{S}} D_i(\underline{\mathbf{a}}, c) \end{aligned} \tag{17}$$

where last inequality comes from the fact that, for any $\mathbf{a}$, $\sum_{i\in\mathcal{N}} D_i(\mathbf{a}, c) = \sum_{i\in\mathcal{S}} D_i(\mathbf{a}, c) + \sum_{j\in\mathcal{N}\setminus\mathcal{S}} D_j(\mathbf{a}, c) = 1$. If $\mathbf{a}_{\mathcal{S}} \neq (a_{\max})_{i\in\mathcal{S}}$ last inequality in (17) is strict. We have that $(1 - a_i) \leq (1 - a_{\min})$ for every $i \in \mathcal{S}$, with strict inequality if $\mathbf{a}_{\mathcal{S}} \neq (a_{\min})_{i\in\mathcal{S}}$. Therefore, we have $\sum_{i\in\mathcal{S}} (1 - a_i) D_i(\mathbf{a}', c) < \sum_{i\in\mathcal{S}} (1 - a_{\min}) D_i(\underline{\mathbf{a}}, c)$, $R \sum_{c\in\mathcal{C}} p_c \sum_{i\in\mathcal{S}} (1 - a_i) D_i(\mathbf{a}', c) < R \sum_{c\in\mathcal{C}} p_c \sum_{i\in\mathcal{S}} (1 - a_{\min}) D_i(\underline{\mathbf{a}}, c)$, $R \sum_{i\in\mathcal{S}} (1 - a_i) \sum_{c\in\mathcal{C}} p_c D_i(\mathbf{a}', c) < R \sum_{i\in\mathcal{S}} (1 - a_{\min}) \sum_{c\in\mathcal{C}} p_c D_i(\underline{\mathbf{a}}, c)$, which is equivalent to $\sum_{i\in\mathcal{S}} u_i(\mathbf{a}_{\mathcal{S}}, \mathbf{a}_{\mathcal{N}\setminus\mathcal{S}}) < \sum_{i\in\mathcal{S}} u_i(\underline{\mathbf{a}})$.

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