Revenue Maximization and Distributed Power Allocation in Cognitive Radio Networks

Invited Paper

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ABSTRACT

Cognitive radio is an enabling technology that allows unlicensed users to opportunistically access the spectrum in order to enhance the spectrum efficiency. In this paper, we consider a cognitive system wherein there exists a primary relay network and a secondary network. In order to efficiently exploit the available spectrum and gain revenues whenever the primary relay infrastructure is not utilized, the primary network leases its unused bandwidth and the idle relay node to the secondary users. As a reimbursement, the secondary users make payments to the primary network based on the service they receive. We first characterize the interactions between the primary and secondary users using a buyer/seller model. Specifically, the price is determined by the primary network such that the revenue is maximized. On the buyer side, given the specified price, the secondary users competitively access the spectrum and employ the primary relay node to forward their packets. Then, we model each secondary user as a selfish player, which aims at maximizing its own benefit through power allocation, and analyze the competition among the secondary users within the framework of non-cooperative game theory. It is shown that, in the game played by the secondary network, there always exists a unique Nash equilibrium point that can be achieved through distributed iterations. Next, we propose a low-complexity algorithm, in which the primary network charges the secondary users at a sub-optimal price and gains close-to-optimal revenues. Extensive simulations are conducted to verify the performance of the proposed methods from both a primary as well as a secondary network perspective.

Categories and Subject Descriptors

C.2.1 [**Computer-Communication Networks**]: Network Architecture and Design–*Wireless Communications*

General Terms

Algorithm, Design, Economics

Keywords

Cognitive radio, game theory, power allocation

1. INTRODUCTION

The increasing demand for wireless applications has driven significant development of wireless technologies over the past few years. However, the continuing growth of wireless networks is largely hindered by the spectrum scarcity. As a promising approach to exploit the under-utilized spectrum, cognitive radio technology defines a new wireless paradigm that allows the unlicensed users to opportunistically access the spectrum [5]. Generally speaking, cognitive radio systems can be further categorized into three classes, i.e., underlay, overlay and interweave [5]. In this paper, an interweave cognitive system is considered wherein the spectrum is initially allocated to the licensed users which are also referred to as primary users. The unlicensed users, or secondary users, are permitted to occupy the spectrum provided that the licensed spectrum is unused, i.e., when the primary users are idle, and hence, the spectrum resource is more efficiently utilized from the system perspective.

Pricing-based resource allocation has been advocated as an effective and flexible mechanism to share the system resources among users in a communication network. In particular, given a network of selfish users whose selfishness nature can be well captured and modeled by the non-cooperative game theory [10, 13], it has been demonstrated in the literature that the introduction of an appropriate pricing technique can create various resource allocation policies including, but not limited to, system-wide optimization [7], user fairness guarantee [8] and revenue maximization [2]. Interested readers are referred to [1] for a good survey on the game-theoretic resource allocation and pricing mechanisms. In a cognitive radio system without a proper compensation framework, the primary network may have no incentives to accommodate the secondary users in its licensed spectrum. Hence, pricing becomes a useful and efficient mechanism that reimburses the primary network in the form of *revenue* and spurs it to lease its unused spectrum to the secondary users [2].

For many wireless networks, the transmission between two distant users may have to be accomplished with the help of an intermediate node, i.e., relay, due to transmit power or other constraints [11]. In this paper, we consider a cognitive system wherein there exists a primary relay network and a secondary network. In the absence of primary transmission activities, the unused bandwidth and the idle relay can be leased to the secondary users, who will then make payments to the primary network as a reimbursement, such that the spectrum is more efficiently utilized. By adopting a pricing mechanism, we first characterize the interactions between the primary and secondary users using a buyer/seller model. In particular, the price is first determined by the primary network, and on the buyer side, the secondary users competitively access the spectrum and employ the primary relay node to forward their packets, and correspondingly, make payments to the primary network based on the receive signal to interference plus noise ratio (SINR) and the the specified price. Then, the price is adjusted such that the revenue of the primary network is further increased. To take into account the selfishness nature, we model each secondary user as a selfish player, which aims at maximizing its own benefit through power allocation, and analyze the competition among the secondary users using the non-cooperative game theory. Specifically, with the knowledge of its local channel state information (CSI), each secondary user myopically maximizes its utility by optimally choosing its power level in response to the power allocation strategies of the other users. This process iterates until convergence. It is shown that, in the non-cooperative game played by the secondary network, there always exists a unique Nash equilibrium point (NEP) that can be achieved through the distributed iterative power allocation process. Next, in order to reduce the communication overheads of the pricing mechanism, we propose a low-complexity algorithm, using which the primary network charges the secondary users at a sub-optimal price and gains close-to-optimal revenues. Finally, extensive simulations are conducted to verify the performance of the proposed methods from both a primary as well as a secondary network perspective.

The rest of this paper is organized as follows. Section II describes the system model and problem formulation. In Section III, a distributed power allocation algorithm along with a low-complexity pricing algorithm are developed for the considered cognitive network. Simulation results are shown in Section IV and literature review is provided in V. Finally, concluding remarks are offered in Section VI.

2. SYSTEM MODEL

Consider a cognitive system consisting of a primary relay network and a secondary network, as illustrated in Fig. 1. The secondary network contains Q source-destination pairs[1], indexed by $\mathcal{S}_i$ and $\mathcal{D}_i$, respectively, for $i = 1, 2 \cdots Q$. The primary relay node is represented by $\mathcal{R}$.

2.1 Network Model

We mainly focus on the distributed transmissions of the secondary users with the help of the primary relay node when the other primary users are idle and the licensed spectrum becomes available. The channel coefficients for the $\mathcal{S}_i - \mathcal{R}$ and the $\mathcal{R} - \mathcal{D}_i$ channels are denoted by g_i and h_i, respectively, for $i = 1, 2 \cdots Q$. The transmit powers of $\mathcal{S}_i$ and $\mathcal{R}$ are p_i and $p_{\mathcal{R}}$, respectively. Local CSI, i.e., g_i and h_i, is accurately obtained by secondary user i, and neither g_j nor h_j is known to secondary user i, if $j \neq i$, due to the distributed nature of the considered communication problem. Furthermore, we assume the zero-mean complex additive white Gaussian noise (AWGN) at each node to have a variance[2] of N_0. Due to the half-duplex constraint, we consider orthogonal relaying transmissions, e.g., the secondary source nodes and the primary relay node transmit in two non-overlapping time slots. The direct link between $\mathcal{S}_i$ and $\mathcal{D}_i$ is neglected due to, for instance, the shadowing effects [11]. To forward the data from the source to the destination, we adopt the classical amplify-and-forward strategy [9] as the relaying operation, which has been shown to be an appealing technique due to its low cost and easy implementation as compared to the decode-and-forward protocol [3].

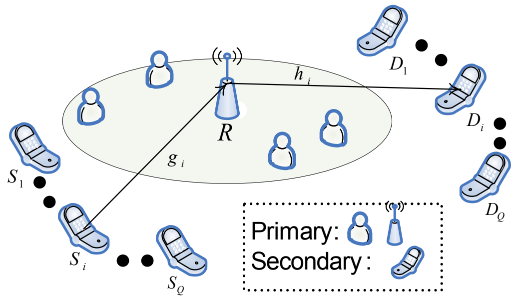

Figure 1: Network model

When the licensed spectrum is unused by the primary network (e.g., the primary users become idle), the secondary source nodes are activated to transmit simultaneously to the primary relay node which then forwards its received signal to the secondary destination nodes. Hence, the signals received at $\mathcal{R}$ and $\mathcal{D}_i$ can be written, respectively, as

$$y_{\mathcal{R}} = \sum_{j=1}^{Q} g_j \sqrt{p_j} x_j + n_{\mathcal{R}} \text{ and } y_i = \alpha h_i y_{\mathcal{R}} + n_i, \quad (1)$$

where x_i is the unit-variance transmit signal from $\mathcal{S}_i$ to $\mathcal{D}_i$, α is the amplification factor of $\mathcal{R}$, $n_{\mathcal{R}}$ and n_i are the statistically-independent AWGN terms at $\mathcal{R}$ and $\mathcal{D}_i$, respectively. The amplification factor α, which is public information available to all the secondary users, is chosen to satisfy the power constraint at the primary relay, i.e., $\alpha = \sqrt{\frac{p_{\mathcal{R}}}{\sum_{j=1}^{Q} |g_j|^2 p_j + N_0}}$. Assuming that $\mathcal{D}_i$ is only interested in the signal x_i and treats the multiuser interference as noise, we can then express the receive SINR at $\mathcal{D}_i$ as

$$\gamma_i = \frac{|g_i|^2 |h_i|^2 p_{\mathcal{R}} p_i}{|g_i|^2 N_0 p_i + (|h_i|^2 p_{\mathcal{R}} + N_0) \cdot \left(\sum_{j=1, j\neq i}^{Q} |g_j|^2 p_j + N_0 \right)}. \quad (2)$$

In general, the utility function is increasing and concave in the receive SINR [6]. Particularly, we adopt in the sequel the

[1]Throughout this paper, we interchangeably use the term "secondary user i" to represent the the i-th secondary source-destination pair.

[2]This assumption is imposed only for the convenience of notation, as in [15], and can be relaxed without affecting the analysis in this paper.

following achievable rate[3] as the utility function of secondary user i

$$R_i(p_i;p_{-i}) = \frac{1}{2}\log(1+\gamma_i) \quad (3)$$

where the scaling factor 1/2 is due to the fact that $\mathcal{S}_i$ transmits x_i only for half of the frame, γ_i is given in (2), and $p_{-i} = (p_1 \cdots p_{i-1}, p_{i+1} \cdots p_Q)$ is the vector of power allocation strategies of all the users except for user i.

2.2 Problem Formulation

It is clear from (2) that the receive SINR is partially determined by the primary relay's power. Furthermore, it is the SINR that measures the quality of the received signal and thus influences the utility of each secondary user. Hence, it is reasonable to assume that the payment made to the primary network is a function of the receive SINR which reflects, in an indirect manner, both the quality of services the secondary users enjoy and the power level of the primary relay. Mathematically, the payment that secondary user i needs to make to the primary system, which sets the price π, is determined by $\pi\gamma_i$. This payment rule charges each secondary user in proportion to its receive SINR, which was similarly referred to as "SINR auction" in [6]. Other similar payment rules can be found in [2, 4]. Given the payment rule, the net utility function of secondary user i can therefore be expressed as the following surplus

$$u_i(p_i;p_{-i}) = \frac{1}{2}\log(1+\gamma_i) - \pi\gamma_i, \quad (4)$$

where the first term is the achievable rate that represents the quality of services provided to user i. Now, at the secondary-user level, we mathematically capture the competition and user selfishness using the following the non-cooperative game

$$\mathcal{G}_{\mathrm{s}} = \{\Omega,\ \{\mathcal{P}_i\}_{i\in\Omega},\ \{u_i(p_i;p_{-i})\}_{i\in\Omega}\} \quad (5)$$

where $\Omega \triangleq \{1, 2 \cdots Q\}$ is the set of active secondary users (i.e., $\mathcal{S}_i - \mathcal{D}_i$ pair), $\mathcal{P}_i$ is the set of admissible power allocation strategies of secondary user i defined as $\{p_i : 0 \le p_i \le p_i^{\max}\}$ and $u_i(p_i;p_{-i})$ is the net utility function of user i given in (4). The optimal power of secondary user i in response to the power levels of all the other secondary users is referred to as the *best response* function denoted by $\mathcal{B}_i(p_{-i})$. In the non-cooperative game played by the secondary users, the NEP is achieved when secondary user i, given p_{-i}, cannot increase its net utility $u_i(p_i;p_{-i})$ by unilaterally changing its own power p_i, for all $i \in \Omega$. Mathematically, the NEP, denoted by $\mathbf{p}^* = (p_1^*, p_2^* \cdots p_Q^*)$, of the secondary user game $\mathcal{G}_{\mathrm{s}}$ in (5) is formally defined as follows [10]

$$u_i(p_i^*;p_{-i}^*) \ge u_i(p_i;p_{-i}^*), \quad \forall\, p_i \in \mathcal{P}_i,\ \forall\, i \in \Omega\,. \quad (6)$$

From the primary network perspective, in order to maximize the revenue collected from the secondary users when the game $\mathcal{G}_{\mathrm{s}}$ reaches the NEP, the primary network needs to set an optimal price π^* such that

$$\pi^* = \arg\max_{\pi\ge 0}\left(\pi\sum_i^Q \gamma_i(p_i^*;p_{-i}^*)\right). \quad (7)$$

[3]Note that the achievable rate is a widely-used utility function (see, e.g., [2]) and the analysis herein can be applied, after minor modifications, to other forms of utility functions as well.

3. JOINT SECONDARY NETWORK AND PRIMARY NETWORK OPTIMIZATION

In this section, we jointly consider the problem of distributed power allocation in the secondary network and revenue maximization in the primary network.

3.1 Distributed Power Allocation

In a non-cooperative game, NEP is a critical operating point at which no user can improve its utility by unilaterally changing its strategy and the outcome of the game becomes stabilized. In particular, we have the following theorem regarding the existence of NEP in the secondary user game.

THEOREM 1. *Given any price $\pi \ge 0$ set by the primary network, there always exists at least one NEP in the non-cooperative game $\mathcal{G}_{\mathrm{user}}$ played by the secondary users.*

PROOF. By showing that, given any π, there always exists a unique $\mathcal{B}_i(p_{-i})$ for all $i \in \Omega$, we prove the quasi-concavity of the net utility function $u_i(p_i;p_{-i})$ with respect to p_i. In particular, the best response function $\mathcal{B}_i(p_{-i})$ can be expressed in a compact form as

$$\mathcal{B}_i(p_{-i}) = \left[\frac{\delta_i(\pi)\left(|h_i|^2 p_{\mathcal{R}} + N_0\right)\left(\sum_{j=1,j\ne i}^{Q}|g_j|^2 p_j + N_0\right)}{|g_i|^2\cdot\left[|h_i|^2 p_{\mathcal{R}} - N_0\cdot\delta_i(\pi)\right]}\right]_0^{p_i^{\max}} \quad (8)$$

where $[\ \cdot\ ]_a^b = \max\{\min\{\ \cdot\ , b\}, a\}$ and $\delta_i(\pi)$ is a non-negative and continuously non-increasing function of π defined as

$$\delta_i(\pi) = \begin{cases} 0, & \text{if } \frac{1}{2} < \pi, \\ \frac{1}{2\pi} - 1, & \text{if } (1+\gamma_i(p_i^{\max};\mathbf{0}))^{-1} < 2\pi \le 1, \\ \gamma_i(p_i^{\max};\mathbf{0}), & 0 \le 2\pi \le (1+\gamma_i(p_i^{\max};\mathbf{0}))^{-1}, \end{cases} \quad (9)$$

in which $\gamma_i(p_i^{\max};\mathbf{0})$ is obtained by plugging $(p_i;p_{-i}) = (p_i^{\max};\mathbf{0})$ into (2). Then, following the equilibrium existence theorem [10], the existence of NEP in $\mathcal{G}_{\mathrm{s}}$ is proved. The detailed proof can be found in a longer version of this paper [12]. □

In addition to the existence of NEP in the game $\mathcal{G}_{\mathrm{s}}$, whether and how the non-cooperative game can eventually arrive at the NEP is another question we have yet to answer. To this end, we present an iterative distributed algorithm that requires only limited information at each secondary user and reaches the unique NEP of $\mathcal{G}_{\mathrm{s}}$, given any price set by the primary network. The distributed algorithm generates, at each iteration, the best response of each secondary user to the power strategies of the others and can be formally described as follows.

Algorithm: Iterative Distributed Power Allocation

Step 1: $n = 0$; **choose any feasible** $\mathbf{p}^0 = (p_1^0, p_2^0 \cdots p_Q^0)$
Step 2: $p_i^{(n+1)} = \mathcal{B}_i(p_{-i}^n)$, **for** $i = 1, 2 \cdots Q$
Step 3: $n = n + 1$; **go to Step 2 until convergence**

To complete the algorithm description, we give Theorem 2 as follows regarding the convergence of the proposed algorithm.

THEOREM 2. *Given any price $\pi \ge 0$ set by the primary network and starting from any initial point $\mathbf{p}^0 \in \mathcal{P} \triangleq \mathcal{P}_1 \times$*

$\mathcal{P}_2 \cdots \times \mathcal{P}_Q$, *the iteration specified by* $p_i^{(n+1)} = \mathcal{B}_i(p_{-i}^n)$, *for* $i \in \Omega$, *always converges to the unique NEP of the secondary user game* $\mathcal{G}_{\mathrm{s}}$ *as* $n \to \infty$.

Proof. The proof is mainly based on the *standard* interference function that was first proposed for distributed power control in cellular networks [14]. The details are omitted due to space limitations and can be found in [12]. □

Before concluding this section, we note that the distributed nature of the algorithm stems from the fact that the information required to compute $\mathcal{B}_i(p_{-i})$ at secondary user i can be locally observed. Specifically, as shown in (8), the information needed by secondary user i includes the local CSI (i.e., g_i and h_i), the relay's transmit power $p_{\mathcal{R}}$, the price π set by the primary network and the multi-user interference plus noise $\sum_{j=1,j\neq i}^{Q} |g_j|^2 p_j + N_0$. The primary relay node can broadcast to the secondary network its amplification factor α such that secondary user i, for $i \in \Omega$, acquires the value of $\sum_{j=1,j\neq i}^{Q} |g_j|^2 p_j + N_0$ by computing $\frac{p_{\mathcal{R}}}{\alpha^2} - |g_i|^2 p_i$. Therefore, the proposed algorithm can be applied in a distributed manner.

3.2 Revenue Maximization

From the primary network perspective, it aims at maximizing the revenue by leasing the unused spectrum and the idle relay node to the secondary users. Nevertheless, due to the lack of private information of the secondary users, e.g., power strategy space, the primary network cannot analytically compute the NEP of the game $\mathcal{G}_{\mathrm{s}}$ and directly set an optimal price such that $\pi^* = \arg\max_{\pi \geq 0} \left(\pi \sum_i^Q \gamma_i(p_i^*; p_{-i}^*)\right)$. Hence, an iterative process that adjusts the price is needed to identify the optimal price. A naive idea is that the primary network exhaustively searches for the optimal price over the (quantized) interval of all the feasible prices and, for each candidate price, computes the revenue when the secondary users iteratively reach the NEP of the game $\mathcal{G}_{\mathrm{s}}$. Unfortunately, the average total number of iterations required by this method is $m\bar{N}$, where m is the number of quantized candidate prices and is typically a large value, and $\bar{N}$ is the average number of iterations needed by the distributed power allocation algorithm to converge. Given that it is computationally prohibitive and mathematically involved to find the optimal price through the exhaustive search, we alternatively propose a low-complexity algorithm that can yield a close-to-optimal price. Before stating the algorithm, we first define $\pi_1 = \frac{1}{2}\min_{i\in\Omega}\{1+\gamma_i(\mathbf{p}^{\max})\}^{-1}$ and $\pi_2 = \frac{1}{2}\max_{i\in\Omega}\{1+\gamma_i(\mathbf{p}^{\max})\}^{-1}$, and summarize some instrumental properties of the revenue function[4], i.e., $\rho(\pi) = \pi\sum_{i=1}^{Q}\gamma_i(\pi)$, in the following theorem.

Theorem 3. *The revenue function has the following properties*[5]*:*

1. $\rho(\pi) \geq 0$;
2. $\rho(\pi) = 0$ *if* $\pi = 0$ *or* $\pi \geq \frac{1}{2}$;
3. $\rho(\pi) < \infty$ *if the number of users, Q, is finite.*
4. $\rho(\pi) = \pi\sum_{i=1}^{Q}\gamma_i(\mathbf{p}^{\max})$ *when* $0 \leq \pi \leq \pi_1$;
5. *There exists a certain value of price* $\hat{\pi}$ *satisfying*

$$\begin{cases} \hat{\pi} < \pi_2, & \exists i,j \in \Omega \ s.t. \ \gamma_i(\mathbf{p}^{\max}) \neq \gamma_j(\mathbf{p}^{\max}) \\ \hat{\pi} = \pi_2, & \forall i,j \in \Omega \ s.t. \ \gamma_i(\mathbf{p}^{\max}) = \gamma_j(\mathbf{p}^{\max}) \end{cases}, \tag{10}$$

such that $\rho(\pi) = Q \cdot \left(\frac{1}{2} - \pi\right)$;

Proof. Property 1–3 directly follows the best response function in (8). The lengthy proof of Property 4 and 5 is available in [12]. □

Theorem 3 can be simply interpreted as follows: <1> The receive SINR is always non-negative and thus, the revenue is also non-negative; <2> The revenue of the primary network becomes vanishing when the service of the primary network, i.e., spectrum lease and packet forwarding, is free or the price is too high; <3> The maximum revenue of the primary network is finite as long as the number of secondary users is finite; <4> and <5> The optimal price of the primary network lies in a certain interval that depends on the channel conditions and transmit power constraints. Based on these desirable properties of the revenue function, we conclude the following statement.

Corollary 1. *There exists an optimal finite price* π^* *such that* $\pi_1 \leq \pi^* \leq \hat{\pi} \leq \pi_2$, *and the corresponding maximum revenue* $\rho(\pi)$ *is finite and positive. The equalities are activated simultaneously if and only if* $\gamma_i(\mathbf{p}^{\max}) = \gamma_j(\mathbf{p}^{\max})$ *for* $i,j \in \Omega$. □

Corollary 1 states that the optimal price is upper and lower bounded by $\hat{\pi}$ and π_1, respectively. As a special case, if $\hat{\pi} = \pi_1$ holds, the optimal price π^* is then clearly π_1. Based on this fact, we propose a low-complexity algorithm that gives the primary network a sub-optimal price. Specifically, if we artificially increase π_1 and decrease $\hat{\pi}$ simultaneously until they meet at $\overline{\pi}$ and assume that

$$\rho(\pi) = \begin{cases} \pi \cdot \sum_{i=1}^{Q}\gamma_i(\mathbf{p}^{\max}), & \text{if } 0 \leq \pi \leq \overline{\pi} \\ Q \cdot \left(\frac{1}{2} - \cdot\pi\right), & \text{if } \overline{\pi} < \pi \leq \frac{1}{2} \end{cases}, \tag{11}$$

we can easily obtain the "optimal" price as

$$\pi^* = \overline{\pi} = \frac{Q}{2\left[\sum_{i=1}^{Q}\gamma_i(\mathbf{p}^{\max}) + Q\right]}. \tag{12}$$

Generally speaking, setting (12) as the price can only result in a sub-optimal revenue for the primary network. Nevertheless, the high computational complexity incurred by the exhaustive search is avoided and only limited information is needed to calculate (12): the number of active users in the network, i.e., Q, and the value of $\sum_{i=1}^{Q}\gamma_i(\mathbf{p}^{\max})$. The primary network can set two different and sufficiently low prices π_a and π_b, given which the NEPs are both $\mathbf{p}^{\max}$, and find $\sum_{i=1}^{Q}\gamma_i(\mathbf{p}^{\max})$ by computing $\frac{\rho(\pi_a)-\rho(\pi_b)}{\pi_a-\pi_b}$. Moreover, we shall show in numerical results that the loss of revenue is not significant when the primary network chooses (12), rather than the optimal one, as its price. It should also be noted that the proposed sub-optimal pricing algorithm is re-executed only when the network condition changes, e.g., channel coefficients vary or additional secondary users enter the system.

When the number of users in the secondary network is large, the sub-optimality of (12) can be further explained as follows. It is natural that the level of interference observed by secondary user i, i.e., $\sum_{j=1,j\neq i}^{Q}|g_j|^2 p_j$, increases when there are more active secondary users. Hence, given a large value of Q, $\max_{i=1,2\cdots Q}\gamma_i(\mathbf{p}^{\max})$ becomes a small non-negative number due to the strong interference caused by the other secondary users. Correspondingly, the difference between the lower bound and the upper bound on the

[4] The SINR is an explicit function of the price π which affects the net utility and the power allocation of secondary users.
[5] $\gamma_i(\mathbf{p}^{\max})$ is obtained by plugging $\mathbf{p}^{\max}$ into (2).

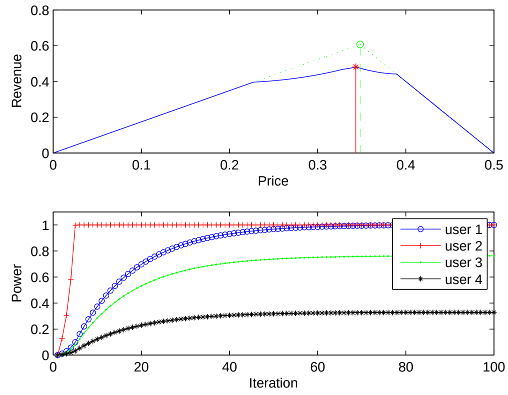

Figure 2: Sub-optimal Pricing and Convergence

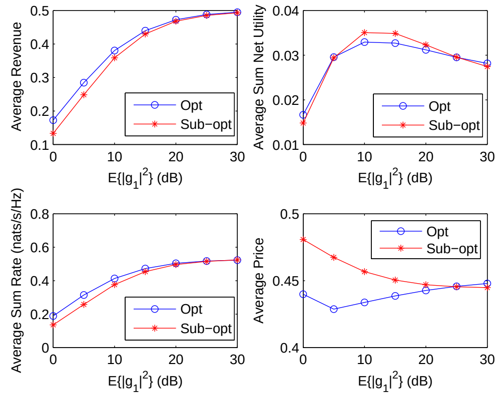

Figure 3: Effects of Channel Gains

optimal price is not significant, i.e., $\hat{\pi} - \pi_1$ is a small number. Thus, the sub-optimal price (12), which lies between π_1 and $\hat{\pi}$, is close to the optimal one. Similar statements can also be made when the secondary network operates in low SINR regions.

4. NUMERICAL RESULTS

For the convenience of illustration, g_i and h_i are modeled as independently and identically Rayleigh distributed random variables, for $i \in \Omega$. The transmit power of the primary relay node and the maximum transmit power of each secondary source node are normalized to one[6].

First, considering a simple four-user secondary network, we randomly generate the channel gains and illustrate in Fig. 2 the convergence of the proposed distributed power allocation algorithm and the sub-optimal pricing algorithm. The upper plot shows that the sub-optimal price (dashed line) is reasonably close to the optimal price (solid line) obtained through exhaustive search, which validates the use of (12) as the price by the primary network.

4.1 Effects of Channel Gains

We consider a ten-user secondary network and examine the effects of channel gains on the performance in Fig. 3. As intuitively expected, the revenue of the primary network and the average sum rate of the secondary network increases as the channel condition becomes better. However, the average sum net utility may not always increase with the channel gain. This is because when the channel gain increases, the SINR also increases, and so does the payment made to the primary network. As a result, the net utility does not necessarily increase even when the channel becomes better. The upper left plot demonstrates that the revenue loss due to the sub-optimality of the price is negligible. The bottom right plot indicates that the gap between the sub-optimal price and the optimal one is sufficiently small, which again verifies the proposed pricing algorithm.

4.2 Effects of Number of Secondary Users

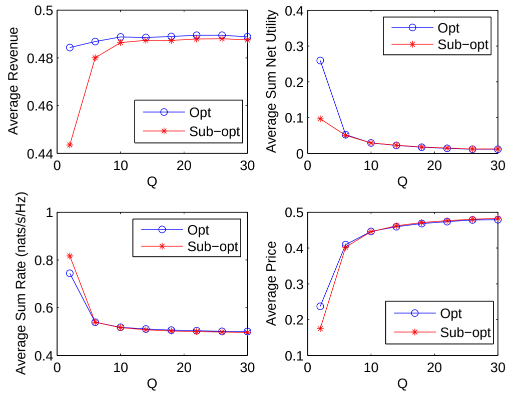

Figure 4: Effects of Number of Secondary Users

In Fig. 4, we fix the average channel gain as 25dB and vary the number of active secondary users. It shows that, when there are more secondary users competing for the spectrum usage, strong interferences limit the receive SINR of each secondary user and thus, the average sum rate decreases. However, both the average optimal and sub-optimal prices are increasing in the number of secondary users, which can also be observed from (12). Therefore, the average revenue of the primary network remain relatively stable even though the receive SINR decreases as the number of secondary users becomes large. Fig. 4 also indicates that the sub-optimal revenue of the primary network gained by setting (12) is sufficiently close to the optimal one.

5. RELATED WORKS

Power allocation, both with and without pricing, has been extensively studied in wireless networks. Before concluding this paper, we list a few related works as follows.

Following a joint user-centric and network-centric optimization approach, the authors in [4] propose a distributed power control and revenue optimization framework in conventional cellular networks. Specifically, the network controller, e.g., base station, charges each user in accordance

[6]Note that the analysis can be applied to any other parameters as well.

with its throughput while the users transmit over an interference channel and maximize the energy efficiency. In [6], an auction-based spectrum sharing protocol is proposed such that the each user submits an optimal bid to the network manager to maximize the utility minus the payment. Two payment rules, i.e., SINR and power, are considered and it is shown that, with logarithmic utilities, the power auction outperforms the SINR auction in terms of the revenue from the network perspective. Focusing on the classic Gaussian interference channel, [15] introduces the notion of "taxation" which summarizes the effect of one user's power allocation on the others', and presents a modified iterative water-filling algorithm to maximize the sum capacity.

In the context of a cognitive wide-band uplink network, [2] proposes a differentiated pricing algorithm that charges different secondary users at different prices to maximize the revenue of the service provider. The authors in [16] adopt the hierarchical Stackelberg game-theoretic framework wherein the primary user, as the leader, selects some secondary users as the cooperative relay nodes and in return, grants the spectrum usage to the participating secondary users for their own data transmissions. As followers, the secondary users decide the payment made to the primary user to gain the channel access time and maximize their own utilities. In contrast with the existing literature, we propose that the secondary users competitively utilize the primary relay infrastructure to accomplish their own transmissions whenever the primary users are idle and, for the sake of fairness, the primary network charges all the active secondary users at a unified price. Furthermore, both the competition among the secondary users and the revenue maximization at the primary network are addressed.

6. CONCLUSION

In this paper, we considered a cognitive system consisting of a primary relay network and a secondary network. First, using a buyer/seller model, the interactions between the primary and secondary users were appropriately captured. We then modeled each secondary user as a selfish player, which aims at maximizing its own benefit by choosing the optimal transmit power, and analyzed the competition among the secondary users using the notion of non-cooperative game theory. It was proved that, in the non-cooperative game played by the secondary network, there always exists a unique desirable operating point, i.e., NEP, which can be achieved in a distributed manner. Next, we proposed a low-complexity algorithm, in which the primary network charges the secondary users at a sub-optimal price and yet gains close-to-optimal revenues. Extensive simulations were finally conducted to verify the analysis from both a primary as well as a secondary network perspective.

7. ACKNOWLEDGMENTS

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