

# Conjecture-Based Channel Selection Game for Delay-Sensitive Users in Multi-Channel Wireless Networks

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*Abstract* – In this paper, we study the problem of multi-user channel selection in multi-channel wireless networks. Specifically, we study the case in which the autonomous users deploy delay-sensitive applications. Existing centralized approaches result in efficient allocations, but require intensive message exchanges among the users (i.e. they are not informationally efficient). Current distributed approaches do not require any message exchange for collaboration, but they often result in inefficient allocations, because users only respond to their experienced contention in the network. Alternatively, in this paper we study a distributed channel selection approach, which does not require any message exchanges, and which leads to a system-wise Pareto optimal solution by enabling a foresighted user to predict the implications (based on their beliefs) of their channel selection on their expected future delays and thereby, foresightedly influence the resulting multi-user interaction. We model the multi-user interaction as a channel selection game and show how users can play an  $\varepsilon$ -consistent conjectural equilibrium by building near-accurate beliefs and competing for the remaining capacities of the channels. We analytically show that when the system has the foresighted user, this self-interested leader can deploy a linear belief function in each channel and manipulates the equilibrium to approach the Stackelberg equilibrium. Alternatively, when the leader is altruistic, the system will converge to the system-wise Pareto optimal solution. We propose a low-complexity learning method based on linear regression for the foresighted user to learn its belief functions.

*Keywords:* autonomous channel selection; foresighted decision making; conjectural equilibrium; distributed resource management; informationally efficient resource management.

## I. INTRODUCTION

In this paper, we provide an autonomous channel selection method for multi-channel MAC protocols that aims to minimize the delays of delay-sensitive users transmitting their packets through a multi-channel wireless network. Since the delay of a user is impacted by the channel selection strategies of the other network users, it is important that users consider the impact of these other users while determining their own channel selection strategy. We model the multi-user interaction as a channel selection game played by users who are capable of making conjectures about how their transmission actions (i.e. their channel selection) will impact other users and eventually impact their future performance. Specifically, we endow the users with the ability to build beliefs about the aggregate response of the other users to their actions (the aggregate response in this paper is the remaining capacity in each channel that can be measured based on the throughput estimation method

[27]) and efficiently minimize their expected future delays in a foresighted manner. We investigate the performance of the resulting  $\varepsilon$ -consistent conjectural equilibrium obtained when these users interact based on their conjectures about the future remaining capacities when selecting channels. The proposed  $\varepsilon$ -consistent conjectural equilibrium is a relaxed version of the conventional conjectural equilibrium [18], which allows us to characterize the equilibrium obtained when network users are able to build near-accurate conjectures.

### A. Related work

The channel selection problem was first studied in cellular networks. Various channel assignment schemes have been proposed (see e.g. [5] for an excellent survey). However, most of these channel assignment schemes are based on centralized solutions, which do not scale to the network size and/or are not suitable for wireless networks without a fixed infrastructure, such as ad hoc wireless networks. Moreover, centralized approaches are especially not desirable for delay-sensitive applications as considered in this paper. The reason is that these centralized solutions require propagating control messages back and forth to a network coordinator, thereby incurring delays that are often unacceptable for delay-sensitive applications [9].

To cope with these challenges, distributed channel selection schemes without a network manager have also been proposed in various types of wireless networks, such as wireless ad hoc networks [1]-[3], wireless mesh networks [4], and cognitive radio networks [6]-[9], etc. For instance, in wireless ad hoc networks, Nasipuri et al. [1] proposed a multi-channel carrier sense multiple access (CSMA) protocol that identifies the set of idle channels and selects the best channel for transmission based on the channel condition observed at the transmitter side. Jain et al. [2] assumed a separate control channel and proposed an alternate multi-channel CSMA protocol that selects the best channel based on the channel condition observed at the receiver side. So and Vaidya [3] proposed a solution that allows users to perform request-to-send (RTS)/clear-to-send (CTS) negotiation without a separate control channel. However, these solutions are myopic, because the users only adapt to their latest network measurement (e.g. idle channel set, channel condition). These solutions can be inefficient, since the users only react to the latest contention measurements experienced in the different wireless channels.

In emerging cognitive radio networks, a key challenge is

how the secondary users can select their transmission channels in order to optimize their performance. Zheng and Cao [7] provided five rule-based spectrum management schemes where users measure local interference patterns and act independently according to the prescribed rules. J. Huang et al. [8] proposed a spectrum sharing scheme where users can select multiple channels to transmit packets and exchange interference prices for each channel. These distributed schemes assume that users cooperate in order to efficiently coordinate their channel selection strategies. However, as discussed in e.g. [13], users can decide to deviate from the rules prescribed by the MAC protocols as long as they derive a higher utility when deviating. That is, users in the network may not have incentives to cooperate and maximize a network/system performance, because this would not maximize their own utilities. Non-cooperative games were proposed to characterize and analyze the performance of self-interested users interacting in different communication systems. For example, Lee et al. [22] showed that the current back-off based MAC protocols can be modeled as a non-cooperative channel access game. The distributed channel selection problem was studied by Felegyhazi et al. [12], who showed that users autonomously selecting channels in non-cooperative multi-channel wireless networks converge to the Nash Equilibrium (NE). However, it is well-known that the NE can often be Pareto-inefficient. For instance, it is possible that some of the selfish users will improve their performance at the cost of degrading the system-wide performance. To optimize the multi-user system utility, a Network Utility Maximization (NUM) framework has been introduced in [21]. It has been shown that by allowing users to exchange messages, they can determine a wireless channel access strategy that reaches a Pareto-efficient solution in a distributed manner. Similar concepts have been proposed in [11] for distributed channel selection, where pricing has been deployed in order to enable users to maximize the system throughput in a distributed manner. To determine the resource price, message exchanges among users are necessary. However, such message exchanges among users can be undesirable due to their increased computational and communication overhead, or simply due to security issues, protocol limitations, etc. Moreover, the incentives for the users to add a penalty term in their utility functions in order to collaborate with each other are not addressed. Alternatively, a distributed channel access scheme using simple random access algorithms without message exchanges was discussed in [23]. However, this solution can only achieve a near optimal system-wide throughput if there are no message exchanges among the participating users.

In this paper, we develop an autonomous channel selection scheme for multi-channel wireless networks using conjecture-based channel selection game. We show that it is possible for users to achieve a system-wise optimal solution without the need for message exchanges when users are able to make foresighted decisions based on their future expected utilities. Their foresighted interaction also provides them the necessary incentives to collaborate, because they can now determine their own performance

benefits resulting from their voluntary collaboration with the other users. We investigate in this paper the multi-user communication scenarios under which a system-wise optimal solution can be reached by the autonomous users.

### B. Contributions and organization of the paper

This paper considers how autonomous users can transmit delay-sensitive traffic over the same multi-channel wireless network. The autonomous users will dynamically select the channels in which they should send their traffic in a distributed and strategic manner, by estimating their expected utilities from taking various transmission actions based on their available conjectures about the communication system.

We are able to analytically show that when the system has a foresighted user, this user can deploy a linear belief function to model the aggregate response of the other users. In [17], a foresighted user is assumed to model the market price also as a linear function of its desired demand. However, we note that using the linear model is purely heuristic in [17]. In this paper, we will show that such a linear belief function is able to capture the specific structure of the considered multi-user interaction. We show that when the foresighted user is altruistic (e.g. whenever it acts as a network leader), it can drive the system to the system-wise Pareto optimal solution by modeling the reactions of the other myopic users. Alternatively, if the foresighted user is self-interested, we show that this user will benefit itself at the expense of (some of) the myopic users increased delays.

The paper is organized as follows. Section II discusses the considered wireless network model and formulates the foresighted channel selection problem as a game. In Section III, we define the conjecture-based channel selection game for the foresighted users and the  $\varepsilon$ -consistent conjectural equilibrium of the game. In Section IV, we investigate the case when there is only one foresighted user in the network. We provide a learning algorithm for the foresighted user to update its belief. The numerical results are shown in Section V and Section VI concludes the paper.

## II. PROBLEM FORMULATION FOR FORESIGHTED CHANNEL SELECTION

### A. Channel selection game model

We assume that there are  $M$  autonomous users sharing the same multi-channel wireless network. Let  $\mathbf{V} = \{v_i, i = 1, \dots, M\}$  represent the set of these users. User  $v_i$  is composed by a source-destination pair, i.e.  $v_i = (v_i^s, v_i^d)$ . We assume that there are  $N$  non-overlapping channels for these users to transmit their delay-sensitive applications. Let  $\mathbf{r} = \{r_j, j = 1, \dots, N\}$  represents the set of all these non-overlapping frequency channels.

We assume that each user  $v_i$  wants to serve an application with traffic rate  $x_i$  (bps). Each frequency

channel  $r_j$  has a capacity  $W_j$ <sup>1</sup> (bps). In this paper, we assume an unsaturated network condition, in which the total capacity is more than the total traffic rate of the users, i.e.  $\sum_{j=1}^N W_j > \sum_{i=1}^M x_i$ . Each wireless channel access can then be modeled as a queue [16]. Such unsaturated condition can ensure that a user can always find an unsaturated channel to transmit its traffic, and hence, the queuing delays can be bounded. The network queuing model is illustrated in Figure 1. For each wireless channel, the maximum channel service rate is  $C_j = W_j/L$  (packets/second), where  $L$  is the average packet length. When more users access the same channel, the channel service rate reduces due to the contention. The resulting service rate is measured by user  $v_i$  when accessing channel  $r_j$  and it is referred to the remaining capacity  $C_{ij}$  in this paper. This is regarded as the local information of user  $v_i$ , e.g. the throughput estimation method proposed in [27], based on which it makes its channel selection decision.

We denote the probability of user  $v_i$  to select the channel  $r_j$  as the action  $a_{ij} \in [0,1]$ . Let  $\mathbf{a}_i = [a_{i1}, \dots, a_{iN}] \in [0,1]^N$  be the channel selection probability distribution of user  $v_i$ , where  $\sum_{j=1}^N a_{ij} = 1$ . The traffic rate from user  $v_i$  through the channel  $r_j$  is denoted as  $\lambda_{ij}$  (packet/second), where  $\lambda_{ij} = x_i a_{ij} / L$  and  $\sum_{j=1}^N \lambda_{ij} = x_i / L$ , and we denote  $\boldsymbol{\sigma}_i = [\lambda_{ij}, \forall r_j] \in \Lambda_i$  as the traffic distribution of user  $v_i$ , and  $\boldsymbol{\sigma}_{-i}$  as the traffic distribution for the other users except  $v_i$  ( $\boldsymbol{\sigma} = [\boldsymbol{\sigma}_i, \boldsymbol{\sigma}_{-i}]$ ). The total traffic rate on the channel  $r_j$  is denoted as  $\lambda_j$  and  $\lambda_j = \sum_{i=1}^M \lambda_{ij}$ .

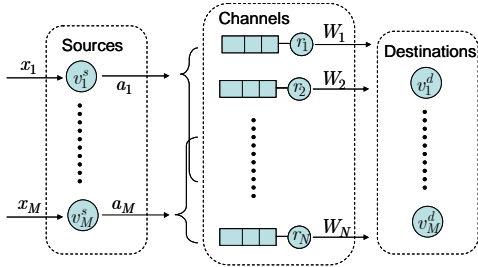


Fig. 1 Considered queuing model for multi-user channel access.

**Definition 1: Channel selection game.** We consider the channel selection game by the following tuple  $\langle \mathbf{V}, \Lambda, \mathbf{U} \rangle$ .

- $\Lambda$  is the action space of the system, where  $\Lambda = \Lambda_1 \times \dots \times \Lambda_M$ . The action of user  $v_i$  is defined as the traffic distribution  $\boldsymbol{\sigma}_i = [\lambda_{ij}, \forall r_j] \in \Lambda_i$ .

<sup>1</sup> For simplicity, we assume that each virtual queue has the same capacity for every user. However, the analysis provided in this paper can be generalized to the case when each virtual queue has different capacities for different users by adopting a more sophisticated queuing model.

- $\mathbf{U}$  is the utility set of the system, where  $\mathbf{U} = \{U_1, \dots, U_M\}$ . We denote the utility of a user  $v_i$  as  $U_i(\boldsymbol{\sigma})$ , which is a function of all the users' actions.

As in [16], we assume that each user deploys an application generating a Poisson packet arrival. We assume that the delay through each frequency channel can be modeled using an M/M/1 queuing model. The expected delay through the channel  $r_j$  can then be expressed as:

$$E[D_j] = \begin{cases} \frac{1}{C_j - \lambda_j}, & \text{if } C_j > \lambda_j \\ \infty, & \text{otherwise} \end{cases}. \quad (1)$$

The delay of user  $v_i$  is defined as:

$$U_i(\boldsymbol{\sigma}_i, \boldsymbol{\sigma}_{-i}) = \sum_{j=1}^N a_{ij} E[D_{ij}] = \frac{L}{x_i} \sum_{j=1}^N \frac{\lambda_{ij}}{C_{ij}(\boldsymbol{\sigma}_{-i}) - \lambda_{ij}}, \quad (2)$$

where  $C_{ij}(\boldsymbol{\sigma}_{-i})$  is the measured remaining capacity (an aggregate response of the other users' channel selection) for a specific user  $v_i$  using channel  $r_j$ . Since in a wireless channel  $E[D_{ij}] = E[D_j]$ , following equation (1) and (2), we have  $C_{ij}(\boldsymbol{\sigma}_{-i}) = C_j - \sum_{i' \neq i} \lambda_{i'j}$ <sup>2</sup>. Note that in the considered network, there is no information exchange among the users. We assume that if user  $v_i$  changes its traffic  $\Delta \lambda_{ij}$  in channel  $r_j$ , another user  $v_{i'}$  can measure the resulting changes in the remaining capacity of channel  $r_j$  as  $C_{i'j}(\Delta \lambda_{ij}) = C_{i'j}^0 - \Delta \lambda_{ij}$ , where  $C_{i'j}^0$  is the remaining capacity when  $\Delta \lambda_{ij} = 0$ .

## B. Achieving Pareto-optimal in a collaborative setting

To achieve Pareto efficient solutions, all the users collaboratively optimize the same "system welfare", e.g. they minimize the weighted summation of users' utilities, i.e.  $U(\boldsymbol{\sigma}) = \sum_{i=1}^M w_i U_i(\boldsymbol{\sigma})$ , where  $w_i$  represents the weighting parameters.

**Definition 2: Pareto boundary.** Given different users' weights  $\mathbf{w} = [w_i, i = 1, \dots, M \mid w_i > 0, \sum_{i=1}^M w_i = 1]$ , the Pareto boundary is formed by the solutions of the following multi-user multi-channel selection problem:

$$\begin{aligned} \boldsymbol{\sigma}^P(\mathbf{w}) &= \arg \min_{\boldsymbol{\sigma}_{\geq 0}} \sum_{i=1}^M w_i U_i(\boldsymbol{\sigma}) \\ \text{s.t. } &\sum_{j=1}^N \lambda_{ij} = x_i / L, \text{ for } \forall v_i \end{aligned}. \quad (3)$$

In order to perform the above optimization, the users need to collect the global network information  $\mathcal{I}_g = [\{C_j, \forall r_j\}, \{x_i, \forall v_i\}, \{w_i, \forall v_i\}]$ . Specifically, in this paper, we define the system-wise utility as

<sup>2</sup> We assume that this remaining capacity can be measured by user  $v_i$  based on the throughput estimation method as in [27]. This value is analytically true when the M/M/1 queuing model in each channel is valid.

$$U^{fair}(\sigma) = \sum_{i=1}^M x_i U_i(\sigma) = \sum_{j=1}^N \frac{L \sum_{i=1}^M \lambda_{ij}}{C_j - \sum_{i=1}^M \lambda_{ij}}. \quad \text{Based on}$$

Little's formula [26], this utility represents the total queue size of these  $N$  M/M/1 queues for the  $N$  channels.

**Definition 3: System-wise Pareto optimal solution.** The system-wise Pareto optimal solution is then defined as:

$$\begin{aligned} \sigma^P &= \arg \min_{\sigma \geq 0} U^{fair}(\sigma) \\ \text{s.t.} \quad &\sum_{j=1}^N \lambda_{ij} = x_i / L, \text{ for } \forall v_i. \end{aligned} \quad (4)$$

The system-wise Pareto optimal solution is on the Pareto boundary where the users' weights are proportional to the traffic rates of the users. However, such collaborative approach may be undesirable in many settings due to the high message overhead required for exchanging the control information. Hence, we consider a non-collaborative setting, where there is no message exchange among the users.

### C. Nash equilibrium in a non-collaborative setting

In a non-collaborative setting, the goal of user  $v_i$  is to minimize its own delay over all possible wireless channels that it chooses. Note that without message exchange in the non-collaborative setting, the traffic distribution of the other users  $\sigma_{-i}$  is not be observable for user  $v_i$ , but  $v_i$  can measure the aggregate response  $C_{ij}(\sigma_{-i})$ . To perform equation (5), user  $v_i$  needs to observe the local information  $\mathcal{I}_i = [\{C_{ij}, \forall r_j\}, \{C_j, \forall r_j\}, x_i]$ . Based on it, the following best response is adopted by every user in the network:

$$\begin{aligned} \pi_i(\mathcal{I}_i) &= \arg \min_{\sigma_i \geq 0} U_i(\sigma_i, \mathcal{I}_i) \\ \text{s.t.} \quad &\sum_{j=1}^N \lambda_{ij} = x_i / L, \end{aligned} \quad (5)$$

where  $\pi_i$  represents the policy for channel selection. The solution to the problem in equation (5) will lead to a unique NE, as proven in [14] for a network routing scenario, similar to the considered channel selection setting. Based on [14], the optimal channel selection probability for user  $v_i$  to transmit in channel  $r_j$  can be expressed as

$$a_{ij}^* = \lambda_{ij}^* L / x_i, \text{ and } \lambda_{ij}^* = \max\{0, C_{ij} - \alpha_{ij} R_i\}, \quad (6)$$

where  $R_i = \sum_{r_j \in \Omega_i} C_{ij} - x_i / L$  represents the overall remaining capacity after user  $v_i$  sends its traffic  $x_i$ ,  $\Omega_i$  represents the set of channels for which  $\lambda_{ij} > 0$ , and

$$\alpha_{ij} = \frac{\sqrt{C_{ij}}}{\sum_{r_j \in \Omega_i} \sqrt{C_{ij}}}$$

represents the optimal fraction (in terms of minimizing  $U_i$ ), based on which  $R_i$  is allocated to channel  $r_j$ . The difference between the measured remaining capacity  $C_{ij}$  and  $\alpha_{ij} R_i$  is the optimal  $\lambda_{ij}^*$  for user  $v_i$  to put on  $r_j$ .

Note that  $\{C_j, \forall r_j\}$  and  $x_i$  is time-invariant, and  $\{C_{ij}, \forall r_j\}$  is time-variant. To reach the NE, users repeatedly measure the remaining capacities  $\{C_{ij}, \forall r_j\}$  and interact with each other using the best response in equation (5). Specifically, user  $v_i$  will update its traffic rate on  $r_j$  as:

$$\lambda_{ij}^t = \min\{0, C_{ij}^{t-1} - \alpha_{ij}(C_{ij}^{t-1})R_i\}, \alpha_{ij} = \frac{\sqrt{C_{ij}^{t-1}}}{\sum_{r_j \in \Omega_i} \sqrt{C_{ij}^{t-1}}}. \quad (7)$$

However, the resulting NE is Pareto inefficient [15]. Hence, in this paper, we investigate how to improve the efficiency of the multi-user interaction to achieve the system-wise Pareto optimal solution in a non-collaborative manner. We endow users with the ability to build belief functions  $\mathbf{B}_i(\sigma_i)$  on the remaining capacities  $C_{ij}$  (instead of using the latest measurement) for user  $v_i$  to take into account the impact of  $\sigma_i$  on  $C_{ij}$ . We refer to this approach as foresighted decision making because it enables users to predict how their channel selection will impact the decision of the other users and thereby, impact the future remaining capacities. Next, we discuss this distributed foresighted resource management approach.

### D. Foresighted decision making for delay-sensitive users

By adopting a belief function  $\mathbf{B}_i(\sigma_i)$ , the distributed optimization in equation (5) is formulated as

$$\begin{aligned} \pi_i^f(\mathbf{B}_i, \mathcal{I}_i) &= \arg \min_{\sigma_i \geq 0} U_i(\sigma_i, \mathbf{B}_i(\sigma_i), \mathcal{I}_i) \\ \text{s.t.} \quad &\sum_{j=1}^N \lambda_{ij} = x_i / L \end{aligned}, \quad (8)$$

where  $\mathbf{B}_i(\sigma_i)$  represents the conjecture (belief) of user  $v_i$  on the expected remaining capacity over each frequency channel when the traffic distribution  $\sigma_i$  is taken. This belief is built based on the measurement history  $o_i^t = \{(\lambda_{ij}^{t-k}, C_{ij}^{t-k}), k = 1, \dots, S, j = 1, \dots, N\}$ , where  $S$  is the observation window size. In Figure 2, we provide a block diagram to highlight the main differences between the myopic channel selection approaches and the proposed foresighted channel selection.

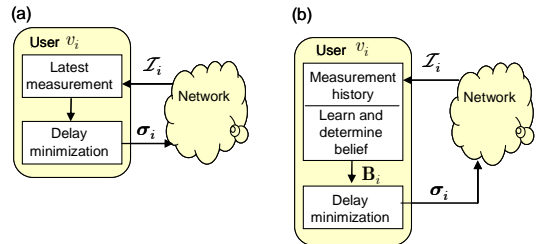


Fig. 2 Block diagram of the (a) myopic channel selection and (b) foresighted channel selection.

## III. CONJECTURE-BASED CHANNEL SELECTION GAME AND THE CONJECTURAL EQUILIBRIUM

In a network, there are users who adopt the myopic

channel selection or adopt the foresighted channel selection. We formalize the multi-user interaction in a multi-channel network using the following repeated game.

*Definition 4: Conjecture-based channel selection game.* We consider the conjecture-based channel selection game as a stage game represented by the following tuple  $\langle \mathbf{V}, \Lambda, \mathcal{S}, \mathbf{U} \rangle$ .

- $\mathbf{V}$  is the set of players (users), and we assume that there are two types of users in the network: a set of foresighted users in  $\mathbf{V}^F$  and a set of myopic users in  $\mathbf{V}^M$ , i.e.  $\mathbf{V} = \{\mathbf{V}^F, \mathbf{V}^M\}$ .
- $\Lambda$  is the action space of the system, where  $\Lambda = \Lambda_1 \times \dots \times \Lambda_M$ . The action of user  $v_i$  is defined as the traffic distribution  $\sigma_i = [\lambda_{ij}, \forall r_j] \in \Lambda_i$ .
- $\mathcal{S}$  is the conjecture space of all the users, i.e.  $\mathcal{S} = \mathcal{S}_1 \times \mathcal{S}_2 \times \dots \times \mathcal{S}_M$ . The *conjecture* of user  $v_i$  is defined as its belief about the expected remaining capacities  $\mathbf{B}_i = [\tilde{C}_{ij}(\lambda_{ij}), \forall r_j] \in \mathcal{S}_i$ . We will discuss how to construct the function  $\tilde{C}_{ij}(\lambda_{ij})$  in Section IV. B. This function models the remaining capacities for user  $v_i$ . Such models implicitly provide the user  $v_i$  with an aggregate belief regarding the coupling of its actions to that of the other users.
- $\mathbf{U}$  is a delay vector of the users, i.e.  $\mathbf{U} = [U_i(\sigma_i, \mathbf{B}_i), \forall v_i]$ .

The stage game is played repeatedly by the users with the following two types of belief updating methods:

a) *Myopic users:* A myopic user  $v_i$  will update its belief function using  $\mathbf{B}_i^t = [C_{ij}^{t-1}, \forall r_j]$  in the repeated game. As a result, user  $v_i$  will select its new action  $\sigma_i^t$  based on the latest measurements obtained about the remaining capacities, using the myopic best response in equation (7).

b) *Foresighted users:* A foresighted user  $v_i$  will update its belief function using  $\mathbf{B}_i^t(\sigma_i^t) = [\tilde{C}_{ij}^t(\lambda_{ij}^t), \forall r_j]$  in the repeated game and select its new action  $\sigma_i^t$  using equation (8). We will discuss how to learn the belief function  $\tilde{C}_{ij}^t(\lambda_{ij}^t)$  in Section IV.

It is easy to verify that the game has a unique NE if all the users are myopic, i.e.  $|\mathbf{V}^F| = 0$ . Note that the actual (real) remaining capacities  $[C_{ij}(\sigma), j = 1, \dots, N]$  depend on  $\sigma$ . However, user  $v_i$ 's conjecture is the expected remaining capacities on the various channels  $\tilde{C}_{ij}(\sigma_i)$  given only  $\sigma_i$ . Based on these conjectures, we can define the concept of a Conjectural Equilibrium (CE) for the considered channel selection game. The CE was first discussed by Hahn in the context of a market model [18]. A general multi-agent framework is proposed in [17] to study the existence of and the convergence to CE in market interactions.

*Definition 5: Conjectural equilibrium of the channel*

*selection game.* Following the definition in [17], the conjectural equilibrium (CE) is defined as  $\sigma^* \in \Lambda$ , if for each user  $v_i \in \mathbf{V}$ , the following two conditions are satisfied:

- The expected remaining capacities at the equilibrium are the actual remaining capacities, i.e.  $\tilde{C}_{ij}^*(\sigma_i^*) = C_{ij}^*(\sigma^*), \forall r_j$ .
- The action at the equilibrium  $\sigma_i^*$  minimizes  $U_i(\sigma_i, \{\tilde{C}_{ij}^*(\sigma_i^*), j = 1, \dots, N\})$ .

The belief function  $\mathbf{B}_i^t(\sigma_i^t)$  may not be perfectly estimated at the equilibrium in practice. However, a user can still keep selecting the same action with imperfect belief estimation, as long as that action consistently minimizes the expected utility. For this, we define an extension to the well-known CE, where users' actions converge to the equilibrium based on their "imperfect" beliefs.

*Definition 6:  $\varepsilon$ -consistent conjectural equilibrium of the channel selection game.* The  $\varepsilon$ -consistent conjectural equilibrium ( $\varepsilon$ -CE) is defined as  $\sigma^* \in \Lambda$ , if for each user  $v_i \in \mathbf{V}$ , the following two conditions are satisfied:

- The expected remaining capacities at the equilibrium approximate the actual remaining capacities, i.e.

$$\max_{v_i \in \mathbf{V}} \max_{r_j \in A_i} (\tilde{C}_{ij}^*(\sigma_i^*) - C_{ij}^*(\sigma^*))^2 \leq \varepsilon. \quad (9)$$

- The action at the equilibrium  $\sigma_i^*$  minimizes its expected delay  $U_i(\sigma_i, \mathbf{B}_i(\sigma_i^*))$ .

Note that as the CE,  $\varepsilon$ -CE may not exist and, even if it exists, it may not be a unique equilibrium [17]. Next, we will discuss how a user should build its conjecture (belief) that leads to the  $\varepsilon$ -CE and compare the resulting performance with the system-wise Pareto optimal solution in various scenarios. In Section IV, we investigate the case when the system has only one foresighted user  $|\mathbf{V}^F| = 1$ .

#### IV. AUTONOMOUS CHANNEL SELECTION WHEN THERE IS A FORESIGHTED USER

##### A. Belief function

In this subsection, we assume that user  $v_1$  is foresighted and the other users are myopic in the conjecture-based channel selection game. We then discuss how to construct the belief function  $\mathbf{B}_1(\sigma_1) = [\tilde{C}_{1j}(\sigma_1), j = 1, \dots, N]$  in equation (8). Given the traffic distribution of the user  $v_1$ , the channel selection game of the other myopic users will reach NE. Note that when user  $v_1$  puts more traffic  $\Delta\lambda_{1j}$  into channel  $r_j$ , the lower remaining capacity  $C_{i,r_j}$  will be measured by the other users, which leads to another NE.

*Proposition 1: Linearity of the belief function in the case of one foresighted user.* The belief function  $\mathbf{B}_1(\sigma_1) = [\tilde{C}_{1j}(\lambda_{1j}), \forall r_j]$  can be approximately modeled as a linear belief function when there is only one foresighted user in the wireless network.

*Proof:* From equation (6), the remaining capacity

$C_{1j}(\Delta\lambda_{1j}) = C_j - \sum_{v_i \neq v_1} \lambda_{i'j}(\Delta\lambda_{1j})$  can be expressed as:

$$\begin{aligned} C_{1j}(\Delta\lambda_{1j}) &= C_j - \sum_{v_i \in \mathbf{V}^M} (C_{i'j}(\Delta\lambda_{1j}) - \alpha_{i'j}(\Delta\lambda_{1j})R_{i'}) \\ &= C_j - \underbrace{\sum_{v_i \in \mathbf{V}^M} C_{i'j}^0}_{\text{constant}} + \sum_{v_i \in \mathbf{V}^M} \Delta\lambda_{1j} + \sum_{v_i \in \mathbf{V}^M} \alpha_{i'j}(\Delta\lambda_{1j})R_{i'} \end{aligned}$$

Note that the last term can be written as follows using the Taylor expansion:

$$\begin{aligned} \alpha_{i'j}(\Delta\lambda_{1j})R_{i'} &= \frac{\sqrt{C_{i'j}(\Delta\lambda_{1j})}}{\sum_{j' \neq j} \sqrt{C_{i'j'}} + \sqrt{C_{i'j}(\Delta\lambda_{1j})}} R_{i'} \\ &\cong \frac{b}{a+b} R_{i'} + \frac{d\alpha_{i'j}(0)}{d\Delta\lambda_{1j}} \Delta\lambda_{1j} R_{i'} + \frac{d^2\alpha_{i'j}(0)}{d^2\Delta\lambda_{1j}} (\Delta\lambda_{1j})^2 R_{i'} + \dots \end{aligned}$$

where  $a = \sum_{j' \neq j} \sqrt{C_{i'j'}}$ ,  $b = \sqrt{C_{i'j}^0}$ . The magnitude of the second order term is bounded as follows:

$$\begin{aligned} \left| \frac{d^2\alpha_{i'j}(0)}{d^2\Delta\lambda_{1j}} (\Delta\lambda_{1j})^2 R_{i'} \right| &= \frac{a}{2b^2} \left( \frac{1}{2b(a+b)^2} + \frac{1}{(a+b)^3} \right) (\Delta\lambda_{1j})^2 R_{i'} \\ &\cong \frac{a}{4(a+b)^2 b^3} (\Delta\lambda_{1j})^2 R_{i'} \leq \frac{R_{i'} (\Delta\lambda_{1j})^2}{4ab^3} \end{aligned}$$

In our network settings, since the value of  $ab^3$  in the denominator is much larger than the value of  $R_{i'}$  in the nominator, it can be shown that all the higher order terms of  $\alpha_{i'j}(\Delta\lambda_{1j})R_{i'}$  can be negligible and only the linear terms are significant. ■

Based on this, we define the linear belief function for the foresighted users.

*Definition 7: Linear belief function for the foresighted user.* The linear belief function on the remaining capacities of a foresighted user  $v_i$  can be expressed by a two-parameter linear function:

$$\tilde{C}_{ij}(\lambda_{ij}) = \beta_{ij}^{(0)} + \beta_{ij}^{(1)}\lambda_{ij}, \quad (10)$$

where  $\beta_{ij} = [\beta_{ij}^{(0)}, \beta_{ij}^{(1)}] \in \mathcal{B}_i$  and  $\mathcal{B}_i$  represents a finite set of positive parameters with  $0 \leq \beta_{ij}^{(1)} < 1$ ,  $0 \leq \beta_{ij}^{(0)} \leq C_j$ . The condition  $0 \leq \beta_{ij}^{(1)} < 1$  implies that when the foresighted user increases the traffic that it transmits through a certain channel  $\lambda_{ij}$ , the other myopic users will avoid using the same channel and move their traffic to other channels. This increases the expected remaining capacity  $\tilde{C}_{ij}$  for the foresighted user  $v_i$ . In the next subsection, we provide a reinforcement learning

method for the foresighted user  $v_i$  to learn these parameters  $\beta_{ij} = [\beta_{ij}^{(0)}, \beta_{ij}^{(1)}]$  based on the measurement history  $o_i^t$ .

### B. Linear regression learning to model the belief function

The foresighted user  $v_i$  repeatedly updates its belief function  $\tilde{C}_{ij}(\lambda_{ij}) = \beta_{ij}^{(0)} + \beta_{ij}^{(1)}\lambda_{ij}$  at every time slot<sup>3</sup>. In this paper, we make the foresighted user update the parameters  $\beta_{ij}^t = [\beta_{ij}^{(0)t}, \beta_{ij}^{(1)t}] \in \mathcal{B}_i$  using the following update rule:

$$\beta_{ij}^t = \arg \min_{\beta_{ij} \in \mathcal{B}_i} \|\beta_{ij} - \hat{\beta}_{ij}\|, \quad (11)$$

$$\text{where } \hat{\beta}_{ij} = (1 - \rho_i) \beta_{ij}^{t-1} + \rho_i \tilde{\beta}_{ij}(o_{ij}^t)$$

$\rho_i$  is the learning rate, which determines how rapidly a user is willing to change its belief on the remaining capacities.  $\tilde{\beta}_{ij}(o_{ij}^t) = [\tilde{\beta}_{ij}^{(0)}, \tilde{\beta}_{ij}^{(1)}]$  will be estimated based on the linear regression from the samples  $o_{ij}^t$ , where  $o_{ij}^t$  represents the latest  $S$  measured remaining capacities and input traffic pairs for a certain channel in  $o_i^t$  (i.e.  $\{(C_{ij}^{t-k}, \lambda_{ij}^{t-k}), k = 1, \dots, S\}$ ). For this, we can adopt standard least square error linear regression [19]. To estimate the error due to deploying a linear model, denote  $e^{t-k}(\tilde{\beta}_{ij}, o_{ij}^t)$  as the residual error of the linear regression at time slot  $t - k$ . The mean residual error is then defined as

$$\bar{e}(\tilde{\beta}_{ij}, o_{ij}^t) = \frac{1}{S} \sum_{k=1}^S e^{t-k}(\tilde{\beta}_{ij}, o_{ij}^t).$$

*Proposition 2: Reaching the  $\varepsilon$ -CE using the linear regression learning.* When there is only one foresighted user, the linear regression learning results in  $\varepsilon$ -CE of the conjecture-based channel selection game with  $\varepsilon \leq \max_{r_j} \{\bar{e}(\tilde{\beta}_{ij}, o_{ij}^t)\}$ .

*Proof:* The foresighted user can determine an optimal action based on the linear belief function using the linear regression learning method. Given the optimal action of the foresighted user, the other myopic users will reach their NE equilibrium. If  $\varepsilon$  is selected as the worst case mean residual error, i.e.  $\varepsilon \leq \max_{r_j} \{\bar{e}(\tilde{\beta}_{ij}, o_{ij}^t)\}$ , the two conditions in

Definition 5 are satisfied. Hence, such equilibrium will be the  $\varepsilon$ -CE. ■

In the simulation section, we also verify that the mean residual error for the belief function linearization is indeed

very small, i.e.  $\bar{e}(\tilde{\beta}_{ij}, o_{ij}^t) \leq \sum_{v_i \in \mathbf{V}^M} \frac{R_{i'} (\Delta\lambda_{1j})^2}{4ab^3} \cong 0$ , when

there is only one foresighted user in the network.

<sup>3</sup> Different time scale can be applied for the foresighted users to make sure that the measured remaining capacities are the stable results of the other myopic users played in the game.

Next, we discuss in more detail the  $\varepsilon$ -CE in two different cases: when the foresighted user is altruistic and when the foresighted user is self-interested.

### C. Altruistic foresighted user

An altruistic foresighted user is usually the leader in a clustered network [6], e.g. the access point in IEEE 802.11 network, or the routing leader in a hierarchical ad hoc network [10]. An altruistic foresighted user will have an objective function that is aligned with the system goal,  $U^{fair}(\sigma)$ . As the foresighted user  $v_i$ 's belief  $\tilde{C}_{ij}(\lambda_{ij})$  reflects the aggregate traffic distribution of all the other users  $\sum_{v_i \in \mathbf{V}^M} \lambda_{ij}$ ,  $U^{fair}(\sigma)$  can be rewritten as:

$$\begin{aligned} \sum_{j=1}^N \frac{\sum_{i=1}^M \lambda_{ij}}{C_j - \sum_{i=1}^M \lambda_{ij}} &= \sum_{j=1}^N \frac{C_j - \tilde{C}_{ij}(\lambda_{ij}) + \lambda_{ij}}{\tilde{C}_{ij}(\lambda_{ij}) - \lambda_{ij}} \\ &= \sum_{j=1}^N \frac{C_j - \beta_{ij}^{(0)} - \beta_{ij}^{(1)} \lambda_{ij} + \lambda_{ij}}{\beta_{ij}^{(0)} + \beta_{ij}^{(1)} \lambda_{ij} - \lambda_{ij}} \equiv U^{fair}(\sigma_i, \mathbf{B}_i(\sigma_i)) \end{aligned} \quad (12)$$

Then, the altruistic foresighted user  $v_i$  performs the following optimization:

$$\begin{aligned} &\underset{\sigma_i \geq 0}{\text{minimize}} U^{fair}(\sigma_i, \mathbf{B}_i(\sigma_i)) \\ &\text{s.t.} \quad \sum_{j=1}^N \lambda_{ij} = x_i / L \end{aligned} \quad (13)$$

while the rest of the myopic users adopt equation (7). Note that only the system-wise Pareto optimal solution on the Pareto boundary can be approached by the altruistic foresighted user<sup>4</sup>. For the other solutions on the Pareto boundary, the foresighted user needs to know the traffic rate  $x_i$  as well as the weights  $w_i$  of the other users. However, the foresighted user adopts a linear belief function in Section IV.B, which provides an imperfect belief by approximating the remaining capacities. There will be a performance penalty (gap) experienced by the foresighted user between the resulting  $\varepsilon$ -CE  $\sigma_{alt}^*$  and the system-wise Pareto optimal solution  $\sigma^P$  based on the user's perfect beliefs, which is defined as:

$$GAP(\sigma_{alt}^*, \sigma^P) = U^{fair}(\sigma_{alt}^*) - U^{fair}(\sigma^P). \quad (14)$$

**Proposition 3: Reaching system-wise Pareto optimal solution when only one user is foresighted.** When there is only one altruistic foresighted user  $v_i$  in the conjecture-based channel selection game, the gap between the resulting  $\varepsilon$ -CE  $\sigma_{alt}^*$  and the  $\sigma^P$  will be bounded by:

$$GAP(\sigma_{alt}^*, \sigma^P) \leq \sqrt{\varepsilon} \sum_{r_j \in \Omega_i} \frac{C_j}{(C_{ij,alt}^* - \lambda_{ij,alt}^*)^2}, \quad (15)$$

<sup>4</sup> For the solution of equation (13) to approach the system-wise Pareto optimal solution, the ratio of the traffic rate of the foresighted user and the total traffic rate is required to be above a certain threshold, which is discussed in [15]. In the following discussion, we assume that the ratio is above such threshold.

where  $\Omega_i$  represents a set of channels whose  $\lambda_{ij} > 0$ .

*Proof:* Since the foresighted user can access all the channels, the foresighted user's action can directly influence all the other myopic users in the network. Since the foresighted user will approximate  $\tilde{C}_{ij}(\lambda_{ij})$  to the actual remaining capacities to satisfy equation (9) at the  $\varepsilon$ -CE  $\sigma_{alt}^*$ , the worst case  $\tilde{C}_{ij}^*(\sigma_i^*) \geq C_{ij}^*(\sigma_{alt}^*) - \varepsilon'$  ( $\varepsilon' = \sqrt{\varepsilon}$ ) can be considered to bound the  $GAP(\sigma_{alt}^*, \sigma^P)$ . The worst case gap is bounded by

$$GAP(\sigma_{alt}^*, \sigma^P) \leq \sum_{r_j \in \Omega_i} \frac{C_j + \lambda_{ij}^* - C_{ij}^* + \varepsilon'}{C_{ij}^* - \lambda_{ij}^* - \varepsilon'} - \sum_{r_j \in \Omega_i} \frac{C_j + \lambda_{ij}^* - C_{ij}^*}{C_{ij}^* - \lambda_{ij}^*}$$

. Let  $K_{ij} = C_j + \lambda_{ij}^* - C_{ij}^*$  and  $J_{ij} = C_{ij}^* - \lambda_{ij}^*$ . For a small  $\varepsilon$ , the first term of the right hand side can be simplified as

$$\sum_{r_j \in \Omega_i} \frac{K_{ij} + \varepsilon'}{J_{ij} - \varepsilon'} \cong \sum_{r_j \in \Omega_i} \frac{K_{ij}}{J_{ij}} + \sum_{r_j \in \Omega_i} \frac{K_{ij} + J_{ij}}{(J_{ij})^2} \varepsilon'$$

and the gap will be bounded by

$$GAP(\sigma_{alt}^*, \sigma^P) \leq \varepsilon' \sum_{r_j \in \Omega_i} \frac{K_{ij} + J_{ij}}{(J_{ij})^2} = \varepsilon' \sum_{r_j \in \Omega_i} \frac{C_j}{(C_{ij}^* - \lambda_{ij}^*)^2}.$$

■

In other words, the foresighted user is able to drive  $\sigma_{alt}^*$  to the system-wise Pareto optimal solution for an arbitrary small  $\varepsilon$ . Proposition 3 also implies that given the same total capacities, i.e.  $\sum_{j=1}^N C_j$  is fixed, the uniform capacities among the frequency channels will result in a minimum gap from the  $\varepsilon$ -CE to the system-wise Pareto optimal solution.

### D. Self-interested foresighted user

Note that reaching the system-wise Pareto optimal solution will not minimize the delay of the foresighted user itself (as will be shown in the Section VI). Thus, a self-interested foresighted user has no incentive to optimize the system-wise delay. Importantly, the foresighted users will have to sacrifice its own delay in order to minimize the system-wise delay. Hence, we now consider the case when the foresighted user is self-interested and only intends to minimize its own delay. If the foresighted user is self-interested, the objective function of the foresighted user is

$$\text{then minimizing } U_i(\sigma_i, \mathbf{B}_i(\sigma_i)) = \frac{L}{x_i} \sum_{j=1}^N \frac{\lambda_{ij}}{\tilde{C}_{ij}(\lambda_{ij}) - \lambda_{ij}}.$$

Specifically, with the linear belief functions, the self-interested foresighted user  $v_i$  performs:

$$\begin{aligned} &\underset{\sigma_i \geq 0}{\text{minimize}} \sum_{j=1}^N \frac{\lambda_{ij}}{\beta_{ij}^{(0)} + \beta_{ij}^{(1)} \lambda_{ij} - \lambda_{ij}} \\ &\text{s.t.} \quad \sum_{j=1}^N \lambda_{ij} = x_i / L \end{aligned} \quad (16)$$

The following proposition provides the optimal action for the self-interested foresighted user.

Proposition 4: Solution of the self-interested foresighted user

Given the belief of the remaining capacity  $\tilde{C}_{ij}(\lambda_{ij}) = \beta_{ij}^{(0)} + \beta_{ij}^{(1)}\lambda_{ij}$ , with  $0 \leq \beta_{ij}^{(1)} < 1$ ,  $0 \leq \beta_{ij}^{(0)} \leq C_j$ , the optimal action that minimizes  $U_i$  for the foresighted user to transmit on channel  $r_j$  is  $a_{ij}^* = \lambda_{ij}^* L / x_i$ ,

$$\lambda_{ij}^* = \max\left\{0, \tilde{D}_{ij} - \alpha_{ij}^{(f)} \left( \sum_{r_j \in \Omega_i} \tilde{D}_{ij} - x_i / L \right)\right\}, \quad (17)$$

where  $\tilde{D}_{ij} = \beta_{ij}^{(0)} / (1 - \beta_{ij}^{(1)})$ . The portion  $\alpha_{ij}^{(f)}$  now becomes  $\kappa_{ij} / \sum_{r_j \in \Omega} \kappa_{ij}$ , where  $\kappa_{ij} = \sqrt{\beta_{ij}^{(0)}} / (1 - \beta_{ij}^{(1)})$  and  $\Omega_i$  represents the channels whose  $\lambda_{ij} > 0$ .

Proof: See Appendix A.

While the other users are myopic, the best performance from the self-interested foresighted user's perspective is to achieve the Stackelberg Equilibrium (SE)  $\sigma^S$  [24]. Note that if the foresighted user is able to build a perfect belief on the remaining capacities (i.e.  $\varepsilon = 0$ ), the resulting conjectural equilibrium is the same as the SE of the game, since the foresighted user knows the exact reactions of the myopic users. Hence, we use the SE  $\sigma^S$  instead of the system-wise Pareto optimal solution  $\sigma^P$  to benchmark the self-interested foresighted user. Denoting the solution in Proposition 3 as  $\sigma_{self}^*$ , the corresponding performance gap is defined as  $GAP(\sigma_{self}^*, \sigma^S) = U_i(\sigma_{self}^*) - U_i(\sigma^S)$ .

Proposition 5: Reaching SE when only one user is foresighted. When there is only one self-interested foresighted user  $v_i$  in the conjecture-based channel selection game, the gap between the resulting  $\varepsilon$ -CE and the SE will be bounded by:

$$GAP(\sigma_{self}^*, \sigma^S) \leq \sqrt{\varepsilon} \sum_{r_j \in \Omega_i} \frac{1}{(C_{ij, self}^* - \lambda_{ij, self}^*)^2}, \quad (18)$$

where  $\Omega_i$  represents a set of channels whose  $\lambda_{ij} > 0$ .

Proof: The gap can be shown to be bounded using a similar proof as Proposition 3. Note that the foresighted user is now minimizing its own delay instead of  $U^{fair}$  in Proposition 3. Hence, the  $GAP(\sigma_{self}^*, \sigma^S)$  is calculated with respect to the foresighted user  $v_i$ 's delay  $U_i$ , and the resulting upper bound changes accordingly.

In other words, the foresighted user is able to drive the  $\varepsilon$ -CE  $\sigma_{self}^*$  to the SE  $\sigma^S$  for an arbitrary small  $\varepsilon$ . Proposition 4 provides the optimal channel selection of the self-interested foresighted user  $v_i$  when applying a linear belief function as described in equation (10) and Proposition 5

implies that the performance of the foresighted user at  $\varepsilon$ -CE can be as good as the SE when the self-interested foresighted user can approximate the future remaining capacities. Algorithm 1 provides the channel selection algorithm that will be followed by the self-interested foresighted user. An illustrative example is given in Figure 3 for the solutions introduced in Section IV.C and IV.D in 2-user case ( $v_i$  is the foresighted user and  $v_{-i}$  is the myopic user). Note that the SE  $\sigma^S$  provides a smaller delay compared to  $\sigma^P$  for the foresighted user  $v_i$  at the cost of increasing the delay of the myopic user. This is because it selfishly minimizes its own delay given that it knows the reaction of the other user, which is the best that a self-interested foresighted user can achieve.

## V. NUMERICAL RESULTS

In this section, we simulate the conjecture-based channel selection game in two network settings, which are shown in Table I. We assume an asymmetric network where the capacities of the channels are  $W_1 = 8$  Mbps and  $W_i = 2$  Mbps,  $i = 2, \dots, N$ . The users have traffic with Poisson arrival rates  $x_1 = 3.8$  Mbps,  $x_i = 0.6$  Mbps,  $i = 2, \dots, M$ . The average packet length is  $L = 1000$  bits.

### A. Single foresighted user scenario

We first simulate the case when there is only one foresighted user. User  $v_1$  is assumed to be the foresighted user, and the rest of the users are myopic users. Figure 5(a) shows the evolution of user  $v_1$ 's action  $\mathbf{a}_1$  (i.e. its channel selection probabilities) until the system reaching the NE in network setting 1 (the large network). Since channel  $r_1$  has a larger capacity, more traffic will be distributed to channel  $r_1$  than to the other channels. Using the learning method proposed in Section IV.B, the foresighted user  $v_1$  can determine its belief functions on the remaining capacities. The circles in Figure 5(b) represent the measured remaining capacities  $C_{11}$  at different channel selection probability  $a_{11}$  (the samples  $a_1^t$ ). The solid line represents the resulting linear regression. The resulting parameters of the linear belief function are  $\beta_{11} = [0.375, 4962]$ . The residual mean square error is 0.051 and the computed bound is

$$\sum_{v_i \in \mathbf{V}^M} \frac{R_i (\Delta \lambda_{ij})^2}{4ab^3} \cong 10.85, \text{ which are in agreement with}$$

Proposition 1. Figure 5(c) shows similar results in channel  $r_2$ . Similarly in network setting 2 (the small network), Figure 5(d) shows again the evolution of  $\mathbf{a}_1$  in a network. The channel selection converges faster in this setting, since the number of users is smaller. The resulting parameters of the linear belief function are  $\beta_{11} = [0.52, 4718]$ . The residual mean square error is 0.012 and the computed bound

$$\text{is } \sum_{v_i \in \mathbf{V}^M} \frac{R_i (\Delta \lambda_{ij})^2}{4ab^3} \cong 4.34.$$



TABLE I. CONSIDERED NETWORK SETTINGS

Network setting	Number of channels $N$	Number of users $M$	Total channel capacities (Mbps)	Total traffic rates (Mbps)
1 (Large network)	10	30	26	21.2
2 (Small network)	2	8	10	8

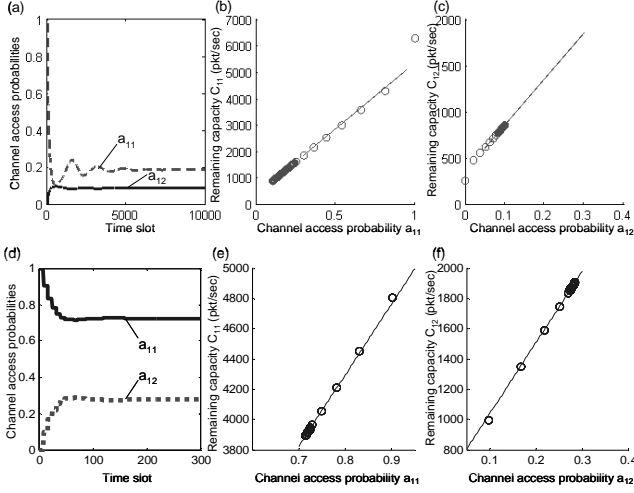


Fig. 5(a)(d) The action of the foresighted user  $v_1$  over time, while participating in the channel selection game. (b)(c)(e)(f) The actual remaining capacity  $C_{1j}$  and the estimated linear belief function  $\tilde{C}_{1j}$ ,  $j = 1, 2$  [(a)(b)(c) in network setting 1, (d)(e)(f) in network setting 2].

In order to show clearly the intuition behind the foresighted channel selection, we now focus on the small network setting. Figure 6 shows the utility domain in terms of delay. The x-axis is the delay of the foresighted user and the Y-axis is the average delay of the myopic users. By using the belief function, the simulation results show that the altruistic foresighted user is able to drive the system from the (system) inefficient NE to the system-wise Pareto optimal solution (in which the system queue size  $U^{fair}$  is minimized) by using the belief function. If the foresighted user is selfish, it will drive the system from NE to SE. Table II shows the results at different equilibriums. When the foresighted user is selfish, it puts more traffic into the efficient channel  $r_1$  and forces the other myopic users to select the other channel, thereby benefiting its own utility. On the contrary, if the foresighted user is altruistic, it puts less traffic into channel  $r_1$  and allows the other users myopically select the efficient channel  $r_1$ , which will result in an optimal system performance.

TABLE II. RESULTS AT DIFFERENT EQUILIBRIUMS

	Action of the foresighted user $a_{11}$	Action of the myopic user $a_{i1}$	Delay of the foresighted user	Average delay of the myopic users	System Performance
NE	0.72	0.97	0.955 ms	0.848 ms	7.19
SE	0.95	0.78	0.914 ms	0.947 ms	7.45
System-wise optimal	0.66	1	1.011 ms	0.752 ms	7.00

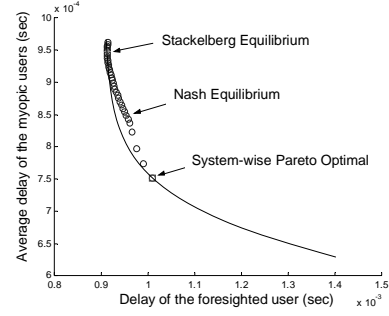


Fig 6 Reaching the system-wise Pareto optimal solution and the Stackelberg Equilibrium.

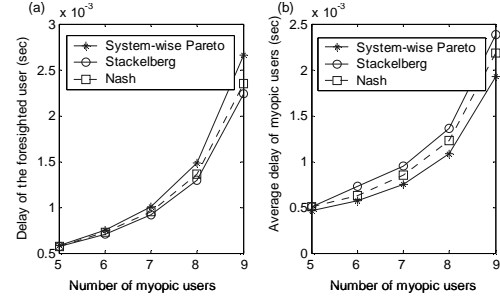


Fig. 7 Delay of the foresighted user at different equilibrium for various numbers of myopic users in the network.

Next, we highlight the impact in terms of delay for the foresighted user and the myopic users, when different numbers of myopic users are active in the network. Figure 7 shows the delay of the foresighted user at different equilibriums when there are various numbers of myopic users in the network. The results show that, as the number of myopic users in the network increases, the altruistic foresighted user will have a higher delay impact to reach the system-wise Pareto optimal solution. Beyond 10 myopic users, the system-wise Pareto optimal solution is not reachable. This situation is also observed in network setting 1 (large network setting). This is because the traffic ratio of the foresighted user to the total traffic in the network is not sufficient enough to drive the equilibrium to the system-wise Pareto optimal solution (as discussed in [15]). On the contrary, the foresighted user can benefit more in terms of delay when the number of the myopic users in the network increases.

## VI. CONCLUSIONS

In this paper, we study the distributed channel selection problem in multi-channel wireless networks. Although we use a multi-channel wireless network setting, it is important to note that the proposed method can be applied to other load balancing resource sharing system. We model the multi-user interaction using a conjecture-based channel selection game where myopic users and foresighted users coexist in the network. In this paper, we show that when there is one foresighted user in the network, the user is able to make an accurate conjecture about the remaining capacity and drive the CE to a system-wise Pareto optimal solution or a SE. We propose an on-line learning procedure for the

foresighted user to build such an accurate conjecture. For the case when there are multiple foresighted users building their own conjectures simultaneously, the same learning procedure may not provide accurate conjecture anymore [17]. This forms our future research to see how system performance degrades if the foresighted users do not know the existence of each other and thereby, using the proposed on-line learning procedure. Also, it is interesting to see, if the foresighted users are able to conjecture the existence of each other, whether they can reach the system-wise Pareto optimal solution in a non-collaborative manner.

#### APPENDIX A

Proof of Proposition 4. First, we see that the objective function is a convex function, given that  $0 \leq \beta_{ij}^{(1)} \leq 1$ ,  $\beta_{ij}^{(0)} \geq 0$ . Assume  $\mu$  as the Lagrange multiplier. For  $\forall r_j \in F_i$ , the optimality conditions:

$$\frac{\beta_{ij}^{(0)}}{(\beta_{ij}^{(0)} + \beta_{ij}^{(1)}\lambda_{ij} - \lambda_{ij})^2} = \mu \Rightarrow \lambda_{ij} = \tilde{D}_{ij} - \sqrt{\frac{1}{\mu}\kappa_{ij}}. \quad (19)$$

From the constraint  $\sum_{j=1}^N \lambda_{ij} = x_i$ , we have

$$\sqrt{1/\mu} = \left( \sum_{r_j \in \Omega} \tilde{D}_{ij} - x_i / L \right) / \sum_{r_j \in \Omega} \kappa_{ij}. \quad (20)$$

By substituting equation (20) into equation (19), we have

$$\lambda_{ij} = \tilde{D}_{ij} - \alpha_{ij}^{(f)} \left( \sum_{r_j \in \Omega} \tilde{D}_{ij} - x_i / L \right) \text{ for } \lambda_{ij} > 0 \text{ case. } \blacksquare$$

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