Intervention in Power Control Games With Selfish Users

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Abstract

We study the power control problem in wireless ad hoc networks with selfish users. Without incentive schemes, selfish users tend to transmit at their maximum power levels, causing significant interference to each other. In this paper, we study a class of incentive schemes based on intervention to induce selfish users to transmit at desired power levels. An intervention scheme can be implemented by introducing an intervention device that can monitor the power levels of users and then transmit power to cause interference to users. We mainly consider first-order intervention rules based on individual transmit powers. We derive conditions on design parameters and the intervention capability to achieve a desired outcome as a (unique) Nash equilibrium and propose a dynamic adjustment process that the designer can use to guide users and the intervention device to the desired outcome. The effect of using intervention rules based on aggregate receive power is also analyzed. Our results show that with perfect monitoring intervention schemes can be designed to achieve any positive power profile while using interference from the intervention device only as a threat. We also analyze the case of imperfect monitoring and show that a performance loss can occur. Lastly, simulation results are presented to illustrate the performance improvement from using intervention rules and compare the performances of different intervention rules.

I. Introduction

A. Motivation and Related Works

Power control is an essential resource allocation scheme to control signal-to-interference-and-noise ratios (SINRs) for efficient transmission in wireless networks. Extensive studies have been done on power control (see [1] and references therein for an overview of the literature in this topic). Earlier works on power control assign a fixed SINR requirement to each user, where each user minimizes its transmit power subject to the fixed minimum SINR requirement [1, Ch. 2] [2] [3]. This formulation is suitable for fixed-rate communications with voice applications. However, with the growth of data and multimedia applications, users are no longer satisfied with a fixed SINR requirement, but seeking to maximize their utility reflecting the quality of service (QoS). To this end, most recent works formulate the problem in a network utility maximization framework. In this framework, a central controller can compute the optimal transmit power levels when the utility functions are such that the network utility maximization problem is convex, and then assigns the optimal power levels to users. Assuming that users are *obedient* to the central controller, the problem can also be solved in a distributed manner [1, Ch. 4] [4] [5].

Besides the network utility maximization framework, many works use noncooperative games to model the distributed power control problem, in which each user maximizes its own utility, instead of maximizing the network utility. In a noncooperative game model, each user tends to transmit at its maximum power level to obtain high throughput, causing significant interference to other users. This outcome may be far from the global optimality of social welfare [1] [4] [6], especially when the interferences are strong [7]. To improve the noncooperative outcome, various incentive schemes based on pricing [8]– [12], auctions [13], mechanism design [14] [15], and repeated games [16] [17] have been proposed. However, most existing game-theoretic works use noncooperative games to model the distributed power control problem while assuming that users are obedient.

Pricing [8]– [12], auctions [13], and other mechanisms [14] [15] impose cost in the objective functions of users by charging for the transmit power or SINR. In all the above works, prices are used as control signals rather than real monetary exchanges. In this regard, the utility functions of users are designed and assigned by the system, and users behave according to the way that the system designer desires. In short, most existing works in distributed power control assume obedient users and aim to find a rule for users to reach a desirable outcome. However, these methods are not sufficient to guarantee a desirable outcome when users are *selfish*. Selfish users are motivated by their own innate objectives and may ignore control signals in their assigned utility functions if they are better off by doing so. Furthermore, selfish users may not disclose their private information, such as the utility function and the channel gains, to the system designer. For this reason, even if the mechanisms in [8]- [13] involve real money exchange to make the cost term real in the utility functions of selfish users, it is difficult, if not impossible, for the designer to design a proper price without the knowledge of the users' utility functions. The mechanisms in [14] [15] require users to follow a designed protocol and broadcast a large amount of signals periodically, which selfish users have no incentive to do.[1]

[1]Another drawback of [14] and [15] is the assumption that each user's utility function is jointly concave in all the users' power levels, which seems to be unrealistic in power control.

In summary, the methods in most existing works are not suitable for power control with selfish users. Selfish behavior of users can arise in many practical scenarios, such as wireless ad hoc networks, where each user transmits information from its own transmitter to its own receiver, and multi-cell cellular networks, where the base station cannot control the interfering mobile stations in other cells. Hence, it is important to design an incentive scheme to induce selfish users to achieve a desirable outcome in power control games. One method to provide incentives for selfish users is to use repeated games [16] [17]. However, incentive schemes based on repeated games requires users to have long-run frequent interactions and to be sufficiently patient [25].

In this paper, we propose a class of incentive schemes, namely intervention schemes [18]- [21], to induce selfish users to transmit at desired power levels. In order to design an intervention scheme, the system designer does not need any information about how users value their SINRs as long as they have monotonically increasing utility functions. We also provide a method based on intervention for the designer to estimate the cross channel gains and noise powers of users without any cooperation from users, such as sending pilot signals for channel estimation and reporting to the designer. With the estimated information, the designer can set a target operating point and configure the intervention rule to achieve the target point as Nash equilibrium (NE).

Different from [18], where intervention schemes have been applied to contention games in medium access control (MAC) layer,[2] and from [19]- [21], where the focus is the impact of monitoring technology and intervention capability in an abstract model, we apply intervention schemes to the specific scenario of power control with selfish users in this paper. To implement an intervention scheme in power control, we need an intervention device operating according to an intervention rule designed by the system. The intervention device estimates the individual transmit power of each user or the aggregate receive power at its receiver, and then transmits at a certain power level, determined as a function of its estimation. If users are transmitting at the desired power levels, the intervention device will transmit minimum, possibly zero, power. Once users deviate, the intervention device will transmit at a positive power level, which causes interference to users and directly reduces the data rates of users. In this way, an intervention scheme can regulate the power transmission of selfish users.

We propose different intervention rules depending on the monitoring technology and intervention capability of the intervention device. The monitoring technology determines what information the intervention device can obtain regarding the power levels of users, while the intervention capability represents

[2] A packet-dropping mechanism using an idea similar to intervention has been proposed for queueing games in [22].

the maximum power that the intervention device can transmit. We first consider a scenario where the intervention device can estimate the individual transmit power of each user without errors. Given a target power profile, we study design principles and intervention capability requirements to obtain an intervention rule that achieves the target power profile as a (unique) NE of the game. We propose a dynamic adjustment process that the designer can use to guide users to the target power profile through intermediate targets. We also consider a scenario where the intervention device can estimate only the aggregate receive power and show that intervention rules can be designed to achieve a given target as a NE but rarely as a unique NE. Finally, we analyze a scenario where estimation of the intervention device has errors. Our analysis provide a systematic design principle based on which a system designer can choose an intervention rule depending on the monitoring technology and intervention capability of its intervention device.

The rest of the paper is organized as follows. In section II, we describe the system model and formulate the design problem of determining an intervention rule. In section III, we propose performance metrics for the intervention mechanism and categorize the intervention rules. In section IV, we study the first-order intervention with perfect monitoring in details. First-order intervention with imperfect monitoring is discussed in Section V. Simulation results are presented in section VI. Finally, section VII concludes the paper.

II. System Model and Design Criteria

We consider a wireless ad hoc network with N users. The set of the users is denoted by $\mathcal{N} \triangleq \{1, 2, \ldots, N\}$. Each user has a transmitter and a receiver. Each user i chooses its transmit power p_i in the set $\mathcal{P}_i \triangleq [0, P_i]$, where $P_i > 0$ for all $i \in \mathcal{N}$. The power profile of the users is denoted by $\mathbf{p} = (p_1, \ldots, p_N) \in \mathcal{P} \triangleq \prod_{i=1}^{N} \mathcal{P}_i$, and the power profile of all the users other than user i is denoted by $\mathbf{p}_{-i}$.

In the network, there is an *intervention device* that consists of a transmitter and a receiver. The receiver of the intervention device can monitor the power profile of the users, while the transmitter can create interference to the users by transmitting power. Once the users choose their power profile, the intervention device obtains a signal $y \in Y$, where Y is the set of all possible signals. The power profile of the users affects the probability distribution of signals, and we use $\rho(\mathbf{p}) \in \Delta(Y)$ to denote the probability distribution of signals given the power profile $\mathbf{p}$, where $\Delta(Y)$ is the set of all probability distributions on Y. After observing a signal, the intervention device chooses its own transmit power p_0 in the set $\mathcal{P}_0 \triangleq [0, P_0]$, where $P_0 > 0$. We call (Y, ρ) the *monitoring technology* of the intervention device, and call P_0 its *intervention capability*. The ability of an intervention device is characterized by its monitoring technology and intervention capability.

We sometimes call the intervention device user 0. Let $\mathcal{N}_0 = \mathcal{N} \cup \{0\}$. For $i, j \in \mathcal{N}_0$, let $h_{ij} > 0$ be the channel gain from user j's transmitter to user i's receiver, and let $n_i > 0$ be the noise power at user i's receiver. When the intervention device chooses a power p_0 and the users choose a power profile $\mathbf{p}$, the signal-to-interference-and-noise ratio (SINR) of user $i \in \mathcal{N}_0$ is given by

$$\gamma_i(p_0, \mathbf{p}) = \frac{h_{ii} p_i}{h_{i0} p_0 + \sum_{j \neq i} h_{ij} p_j + n_i}.^3 \tag{1}$$

We assume that each user $i \in \mathcal{N}$ has monotonic preferences on its own SINR in the sense that it weakly prefers γ_i to γ_i' if and only if $\gamma_i \geq \gamma_i'$. Our analysis does not require any other properties of preferences (for example, preferences do not need to be represented by a concave utility function).

In our setting, the intervention device has a receiver to measure the aggregate receive power from all the users. Furthermore, if the receiver moves and takes measurement at different locations, it can estimate the individual transmit power of each user as well. Thus, in this paper we will focus on two types of monitoring technology with which the intervention device can estimate individual transmit powers $\mathbf{p}$ or an aggregate receive power $\sum_{i=1}^{N} h_{0i} p_i$. We say that estimation is perfect if the intervention device obtains the exact value of $\mathbf{p}$ or $\sum_{i=1}^{N} h_{0i} p_i$, while estimation has errors if the intervention device obtains a noisy signal about the value of $\mathbf{p}$ or $\sum_{i=1}^{N} h_{0i} p_i$. Following the language of the theory of repeated games [25], we say that monitoring is perfect if the intervention device can perfectly estimate $\mathbf{p}$. In the case of perfect monitoring, we have $Y = \mathcal{P}$ and $\rho(\mathbf{p})$ puts probability one at $\mathbf{p}$ and zero elsewhere.

Since the intervention device transmits its power after it obtains a signal, its strategy can be represented by a mapping $f : Y \rightarrow \mathcal{P}_0$, which is called an intervention rule. Henceforth, we assume perfect monitoring until we deal with estimation errors in Section V. The SINR of user i when the intervention device uses an intervention rule f and the users choose a power profile $\mathbf{p}$ is given by $\gamma_i(f(\mathbf{p}), \mathbf{p})$. With an abuse of notation, we will use $\gamma_i(f, \mathbf{p})$ to mean $\gamma_i(f(\mathbf{p}), \mathbf{p})$. Given an intervention rule f, the interaction among the users that choose their own power levels selfishly can be modeled as a non-cooperative game, whose strategic form is given by

$$\Gamma_f = \langle \mathcal{N}, (\mathcal{P}_i)_{i \in \mathcal{N}}, (\gamma_i(f, \cdot))_{i \in \mathcal{N}} \rangle. \tag{2}$$

We can predict the power profile chosen by the users given an intervention rule using the concept of Nash equilibrium.

[3]Throughout the paper, we use $j \neq i$ with the summation operator to mean $j \in \mathcal{N} \setminus \{i\}$, not $j \in \mathcal{N}_0 \setminus \{i\}$.

Definition 1: A power profile $\mathbf{p}^* \in \mathcal{P}$ is a *Nash equilibrium (NE)* of the game Γ_f if

$$\gamma_i(f, \mathbf{p}^*) \geq \gamma_i(f, p_i, \mathbf{p}^*_{-i}) \tag{3}$$

for all $p_i \in \mathcal{P}_i$, for all $i \in \mathcal{N}$.

When a power profile $\mathbf{p}^*$ is a NE of Γ_f, no user has an incentive to deviate from $\mathbf{p}^*$ unilaterally provided that the intervention device uses intervention rule f. Moreover, if $\mathbf{p}^*$ is a unique NE of Γ_f, intervention has added robustness in that we do not need to worry about coordination failure (i.e., the possibility that the users get stuck in a "wrong" equilibrium).

Definition 2: An intervention rule f *(strongly) sustains* a power profile $\mathbf{p}^*$ if $\mathbf{p}^*$ is a (unique) NE of the game Γ_f.

We use $\mathcal{E}(f)$ to denote the set of all power profiles sustained by f.

As a benchmark, suppose that there is no intervention device in the network, i.e., p_0 is held fixed at 0. In this case, γ_i is strictly increasing in p_i, and thus $\mathbf{P} \triangleq (P_1, \dots, P_N)$ is the unique NE of the game $\langle \mathcal{N}, (\mathcal{P}_i)_{i \in \mathcal{N}}, (\gamma_i(0, \cdot))_{i \in \mathcal{N}} \rangle$.[4] We call the transmit powers of the intervention device and the users $(p_0, \mathbf{p}) \in \mathcal{P}_0 \times \mathcal{P}$ an outcome. A desirable property for an outcome is Pareto efficiency, as defined below.

Definition 3: An outcome $(p_0^e, \mathbf{p}^e) \in \mathcal{P}_0 \times \mathcal{P}$ is *Pareto efficient* if there is no $(p_0, \mathbf{p}) \in \mathcal{P}_0 \times \mathcal{P}$ such that $\gamma_i(p_0, \mathbf{p}) \geq \gamma_i(p_0^e, \mathbf{p}^e)$ for all $i \in \mathcal{N}$ and $\gamma_i(p_0, \mathbf{p}) > \gamma_i(p_0^e, \mathbf{p}^e)$ for at least one $i \in \mathcal{N}$.

Since γ_i is strictly decreasing in p_0 for all $i \in \mathcal{N}$, $p_0 = 0$ is necessary for (Pareto) efficiency. Thus, hereafter we deal with efficient power profiles while suppressing p_0, implicitly imposing $p_0 = 0$. We use $\mathcal{B}$ to denote the set of all efficient power profiles. By [5, Theorem 1], $\mathbf{p} \in \mathcal{B}$ if and only if $p_i = P_i$ for at least one $i \in \mathcal{N}$. Thus, the unique NE without intervention, $\mathbf{P}$, is efficient. However, if the designer use other criteria such as sum rates and fairness, $\mathbf{P}$ may be far from the global optimality of the social welfare [1] [4] [6]. If we allow time sharing, which can be achieved by intervention, $\mathbf{P}$ will not even be Pareto efficient when the interferences are strong [7].

In the discussion of this paper, we assume that the designer desires to achieve a target power profile, denoted by $\mathbf{p}^\star$, as in [12], with minimum possible intervention. Thus, the design problem is to find an intervention rule f such that $\mathbf{p}^\star \in \mathcal{E}(f)$ (or $\{\mathbf{p}^\star\} = \mathcal{E}(f)$) and $f(\mathbf{p}^\star) = 0$. If $p_i = 0$, then $\gamma_i(p_0, p_i, \mathbf{p}_{-i}) = 0$ for all $(p_0, \mathbf{p}_{-i})$. Since user i can guarantee a positive SINR by choosing a positive power, it is impossible to provide an incentive for user i to choose $p_i = 0$ using any intervention rule. Thus, we assume that $\mathbf{p}^\star \in \prod_i (0, P_i]$. Also, note that the case where $\mathbf{p}^\star = \mathbf{P}$ is trivial because there is no need for

[4]This is true for any constant intervention rule, where p_0 is chosen independently of the observation of the intervention device. This shows the inability of traditional Stackelberg strategies to provide incentives in our setting.

intervention in order to achieve the outcome $\mathbf{P}$. Hence, our main interest lies in the case where $\mathbf{p}^\star \neq \mathbf{P}$, although our analysis does not exclude the case where $\mathbf{p}^\star = \mathbf{P}$.

We say that $(f, \mathbf{p}^\star)$ is an *equilibrium* if $\mathbf{p}^\star \in \mathcal{E}(f)$ and $f(\mathbf{p}^\star) = 0$. At an equilibrium, no user has an incentive to deviate unilaterally while the designer fulfills his design criteria. Thus, an equilibrium can be considered as a stable outcome. An equilibrium can be achieved following the procedure described below.

1) The designer chooses a target power profile $\mathbf{p}^\star$ and an intervention rule f.
2) The users choose their power profile $\mathbf{p}$, knowing the intervention rule chosen by the designer.[5]
3) The intervention device estimates $\mathbf{p}$ and chooses its power $p_0 = f(\mathbf{p})$.

When the designer chooses a target power profile $\mathbf{p}^\star$ and an intervention rule f that sustains $\mathbf{p}^\star$, it is natural that he takes into account all the parameters in the model, h_{ij}, n_i, and P_i for all $i \in \mathcal{N}_0$ and $j \neq i \in \mathcal{N}_0$. Here we only assume that the designer knows the power budgets of the users P_i for all i as in [14] [15]. This assumption is reasonable, because the maximum power that the users can transmit is the physical limit of the devices, which is usually subject to some standardization. If the designer knows the power budgets of the users, it can measure the cross channel gains h_{ij} and noise powers n_i thanks to some properties of intervention mechanism (see Appendix A for how the designer can measure these parameters). We discuss briefly how the designer can choose a target power profile $\mathbf{p}^\star$, and relate this discussion to the informational advantage of intervention over pricing. One criterion that can be used by the designer is to find a power profile that optimizes the system objective while being sustained by some intervention rule. Suppose that the system objective is defined on the users' SINRs and is represented by a welfare function $U_0(\gamma_1, \ldots, \gamma_N)$. Then the designer can determine $\mathbf{p}^\star$ by solving

$$\max_{\mathbf{p}} \max_{f} \{U_0(\gamma_1(f, \mathbf{p}), \ldots, \gamma_N(f, \mathbf{p})) : \mathbf{p} \in \mathcal{E}(f)\}, \tag{4}$$

assuming that a solution exists. If U_0 is increasing in each of its arguments, our model satisfies Assumption 1 of [20], which implies that an intervention rule solving $\max_f \{U_0(\gamma_1(f, \mathbf{p}), \ldots, \gamma_N(f, \mathbf{p})) : \mathbf{p} \in \mathcal{E}(f)\}$ satisfies $f(\mathbf{p}) = 0$. Note that this approach does not require the designer to know the preferences of the users on their SINRs. Now consider a pricing scheme based on perfect monitoring, in which each user i is charged $\pi_i(\mathbf{p})$ when the users choose a power profile $\mathbf{p}$. Suppose that each user i has quasilinear preferences on its own SINR and payment which are represented by a utility function of the form $u_i(\gamma_i) - \pi_i$. Suppose also that the objective of the designer is to achieve $\mathbf{p}^\star$. Then in order

[5]The intervention rule can be broadcasted to the users, or learned by the users from experimentation.

to find a pricing scheme that sustains $\mathbf{p}^\star$, the designer needs to know u_i for all $i \in \mathcal{N}$ as well as the network parameters. Since intervention affects the users through their SINRs, the designer does not need to know how the users value their SINRs.[6] In contrast, pricing uses an outside instrument to influence the decisions of the users, and thus the designer needs to know how the users value SINRs relative to payments, which is subjective and thus hard to measure.

To execute the above procedure, we may consider an adjustment process (e.g., one based on best-response updates) for the users and the designer to reach an equilibrium, as well as an estimation process for the intervention device to obtain a signal. It is an implicit underlying assumption of our analysis that the time it takes to reach a final outcome (i.e., the duration of the procedure) is short relative to the time for which the final outcome lasts. This justifies that in our model the users fully take into account interference from the intervention device that is realized at the final outcome when they make decisions about their powers. If the duration of the procedure is not negligible relative to that of the final outcome, the users will care about intervention during the procedure as well as that at the final outcome.[7] When a network parameter changes (e.g., some users leave or join the network, or move to different locations), the procedure is repeated to achieve a new equilibrium. Thus, our analysis holds as long as network parameters do not change frequently, whereas providing incentives using a repeated game strategy usually requires an infinite horizon and sufficiently patient players.

III. Performance Characteristics and Classes of Intervention Rules

Given a target power profile $\mathbf{p}^\star$, there are potentially many intervention rules f that satisfy the design criteria $\mathbf{p}^\star \in \mathcal{E}(f)$ and $f(\mathbf{p}^\star) = 0$. Thus, below we propose several performance characteristics with which we can evaluate different intervention rules satisfying the design criteria.

1) Monitoring requirement: The minimum amount of information about the power profile that is required for the intervention device to execute a given intervention rule (assuming perfect estimation).

2) Intervention capability requirement: The minimum intervention capability needed for the intervention device to execute a given intervention rule, i.e., $\sup_{\mathbf{p} \in \mathcal{P}} f(\mathbf{p})$. (Even though there is no intervention at an equilibrium, the intervention device should have an intervention capability $P_0 \geq \sup_{\mathbf{p} \in \mathcal{P}} f(\mathbf{p})$ in order to make the intervention rule f credible to the users.)

[6] Of course, if the system objective depends on the valuation of the users (e.g., to maximize the sum of utilities from SINRs), the designer needs to know the users' utility functions on SINRs.

[7] See [19] for an example of this situation due to the time it takes for the intervention device to collect information about the actions of the users.

3) Complexity: The complexity of a given intervention rule in terms of design, broadcast/learning, and computation.

4) Strong sustainment: Whether a given intervention rule strongly sustains the target power profile $\mathbf{p}^\star$.

Without loss of generality, we can express an intervention rule f satisfying $f(\mathbf{p}^\star) = 0$ as $f(\mathbf{p}) = [g(\mathbf{p})]_0^{P_0}$, where $[x]_a^b = \min\{\max\{x, a\}, b\}$, for some function $g : \mathcal{P} \to \mathbb{R}$ such that $g(\mathbf{p}^\star) = 0$. Also, since the designer desires to achieve $\mathbf{p}^\star$, it is natural to consider functions g that increase as the users deviate from $\mathbf{p}^\star$. Hence, we consider the following simple classes of intervention rules,

$$\mathcal{F}_K(\mathbf{p}^\star) = \left\{ f : f(\mathbf{p}) = \left[\sum_{i=1}^{N} \sum_{k=1}^{K} \alpha_{i,k} |p_i - p_i^\star|^k \right]_0^{P_0} \text{ for some } \alpha_{i,k} \geq 0 \text{ and } P_0 > 0 \right\}, \tag{5}$$

for $K = 1, 2, \ldots$. We call an intervention rule $f \in \mathcal{F}_K(\mathbf{p}^\star)$ a *Kth-order intervention rule with target power profile* $\mathbf{p}^\star$. As K becomes larger, the set $\mathcal{F}_K(\mathbf{p}^\star)$ contains more intervention rules, but at the same time complexity increases. Simple intervention rules are desirable for the designer, the users, and the intervention device. As K is smaller, the designer has less design parameters and less burden of broadcasting the intervention rule; the users can more easily learn the intervention rule and find their best responses during an adjustment process; and the intervention device can more quickly compute the value of the intervention rule at the chosen power profile. Thus, our analysis mainly focuses on first-order intervention rules, the simplest among the above classes.

Let $\tilde{\mathcal{F}}_K(\mathbf{p}^\star)$ ($\tilde{\mathcal{F}}_K^s(\mathbf{p}^\star)$) be the set of all Kth-order intervention rules that (strongly) sustains $\mathbf{p}^\star$, i.e., $\tilde{\mathcal{F}}_K(\mathbf{p}^\star) = \{f \in \mathcal{F}_K(\mathbf{p}^\star) : \mathbf{p}^\star \in \mathcal{E}(f)\}$ and $\tilde{\mathcal{F}}_K^s(\mathbf{p}^\star) = \{f \in \mathcal{F}_K(\mathbf{p}^\star) : \{\mathbf{p}^\star\} = \mathcal{E}(f)\}$. We define the *minimum power budget* for a Kth-order intervention rule to (strongly) sustain $\mathbf{p}^\star$[8] by

$$PB_K(\mathbf{p}^\star) = \inf_{f \in \tilde{\mathcal{F}}_K(\mathbf{p}^\star)} \sup_{\mathbf{p} \in \mathcal{P}} f(\mathbf{p}) \tag{6}$$

and

$$PB_K^s(\mathbf{p}^\star) = \inf_{f \in \tilde{\mathcal{F}}_K^s(\mathbf{p}^\star)} \sup_{\mathbf{p} \in \mathcal{P}} f(\mathbf{p}). \tag{7}$$

Thus, with an intervention capability $P_0 > PB_K(\mathbf{p}^\star)$ ($P_0 > PB_K^s(\mathbf{p}^\star)$), there exists a Kth-order intervention rule that (strongly) sustains $\mathbf{p}^\star$. We set $PB_K(\mathbf{p}^\star) = +\infty$ ($PB_K^s(\mathbf{p}^\star) = +\infty$) if there is no Kth-order intervention rule that (strongly) sustains $\mathbf{p}^\star$ (i.e., $\tilde{\mathcal{F}}_K(\mathbf{p}^\star)$ ($\tilde{\mathcal{F}}_K^s(\mathbf{p}^\star)$) is empty). Since $\mathcal{F}_K(\mathbf{p}^\star) \subset \mathcal{F}_{K'}(\mathbf{p}^\star)$ for all K, K' such that $K \leq K'$, both $PB_K(\mathbf{p}^\star)$ and $PB_K^s(\mathbf{p}^\star)$ are weakly decreasing in K for all $\mathbf{p}^\star$. This suggests a trade-off between complexity and the minimum power

[8]As to be shown later, for strong sustainment, we need P_0 to exceed a certain value. Thus, we actually mean "infimum" power budget by minimum power budget.

budget. Also, since $\tilde{\mathcal{F}}_K^s(\mathbf{p}^\star) \subset \tilde{\mathcal{F}}_K(\mathbf{p}^\star)$, we have $PB_K(\mathbf{p}^\star) \leq PB_K^s(\mathbf{p}^\star)$ for all K and $\mathbf{p}^\star$. The difference $PB_K^s(\mathbf{p}^\star) - PB_K(\mathbf{p}^\star)$ can be interpreted as the price of strong sustainment in terms of the minimum power budget.

Suppose that the target power profile $\mathbf{p}^\star$ is chosen randomly among efficient power profiles following a uniform distribution. In this scenario, without knowing the target power profile in advance, the designer cares about the proportion of efficient power profiles that can be (strongly) sustained by intervention rules in a particular class. Let $\mathcal{F}_K$ be the set of all Kth-order intervention rules, i.e., $\mathcal{F}_K = \cup_{\mathbf{p}^\star \in \mathcal{P}} \mathcal{F}_K(\mathbf{p}^\star)$. Let $\mathcal{E}_K$ ($\mathcal{E}_K^s$) be the set of all power profiles that can be (strongly) sustained by a Kth-order intervention rule, i.e., $\mathcal{E}_K = \cup_{f \in \mathcal{F}_K} \mathcal{E}(f)$ and $\mathcal{E}_K^s = \{\mathbf{p} \in \mathcal{P} : \{\mathbf{p}\} = \mathcal{E}(f) \text{ for some } f \in \mathcal{F}_K\}$. We define the *efficiency range* of Kth-order intervention rules by

$$ER_K = \frac{\Pr(\mathcal{B} \cap \mathcal{E}_K)}{\Pr(\mathcal{B})} \tag{8}$$

and

$$ER_K^s = \frac{\Pr(\mathcal{B} \cap \mathcal{E}_K^s)}{\Pr(\mathcal{B})}, \tag{9}$$

where $\mathbf{p}$ is chosen uniformly on $\mathcal{P}$ when defining $\Pr(\mathcal{P}')$ for $\mathcal{P}' \subset \mathcal{P}$. That is, ER_K (ER_K^s) is the proportion of efficient power profiles that can be (strongly) sustained by a Kth-order intervention rule. It is straightforward to see that both ER_K and ER_K^s are weakly increasing in K, which can be considered as the gain from complexity in terms of the efficiency range. Also, we have $ER_K^s \leq ER_K$ for all K, and the difference $ER_K - ER_K^s$ can be regarded as the price of strong sustainment in terms of the efficiency range.

IV. First-Order Intervention With Perfect Monitoring

A. Design of Intervention Rules

We consider first-order intervention rules of the form

$$f_1^I(\mathbf{p}) = \left[\sum_{i=1}^{N} \alpha_i |p_i - p_i^\star| \right]_0^{P_0}. \tag{10}$$

Under a first-order intervention rule, the intervention device increases its transmit power linearly with the deviation of each user from the target power, $|p_i - p_i^\star|$, in the range of its intervention capability. We call α_i the *intervention rate* for user i, which measures how sensitive intervention reacts to a deviation of user i. Let $\tilde{\mathcal{N}}(\mathbf{p}^\star) = \{i \in \mathcal{N} : p_i^\star < P_i\}$. Without loss of generality, we label the users in such a way that $i \in \tilde{\mathcal{N}}(\mathbf{p}^\star)$ if and only if $i \leq N'$, where $N' = |\tilde{\mathcal{N}}(\mathbf{p}^\star)|$. Since the users have natural incentives to choose their maximum powers in the absence of intervention, we need to provide incentives only for the users in $\tilde{\mathcal{N}}(\mathbf{p}^\star)$. The following theorem shows that when the intervention capability is sufficiently large, the designer can always find intervention rates to have a given target power profile $\mathbf{p}^\star$ sustained by a first-order intervention rule.

Theorem 1: For any $\mathbf{p}^\star \in \prod_i (0, P_i]$, $\mathbf{p}^\star \in \mathcal{E}(f_1^I)$ if and only if

$$\alpha_i \geq \frac{\sum_{j \neq i} h_{ij} p_j^\star + n_i}{p_i^\star h_{i0}} \tag{11}$$

and

$$P_0 \geq \frac{(P_i - p_i^\star)(\sum_{j \neq i} h_{ij} p_j^\star + n_i)}{p_i^\star h_{i0}} \tag{12}$$

for all $i \in \tilde{\mathcal{N}}(\mathbf{p}^\star)$.

Proof: See Appendix B. ■

We can explain the minimum intervention rate for user i, expressed in the right-hand side of (11), as follows. As h_{i0} is larger, intervention causes more interference to user i with the same transmit power, and thus the intervention rate for user i can be chosen smaller to yield the same interference. When $\sum_{j \neq i} h_{ij} p_j^\star + n_i$ is large, interference to user i from other users and its noise power are already strong, and thus the intervention rate for user i should be large in order for intervention to be effective. Hence, $h_{i0}/(\sum_{j \neq i} h_{ij} p_j^\star + n_i)$ can be interpreted as the effectiveness of intervention to user i. Without intervention, the users have natural incentives to increase their transmit powers. Thus, as the target power for user i, $p_i^\star$, is smaller, the incentive for user i to deviate is stronger, and thus a larger intervention rate is needed to prevent deviation. In summary, α_i should be chosen larger as intervention is less effective to user i and user i has a stronger incentive to deviate. Note that $(P_i - p_i^\star)$ is the maximum possible deviation by user i (in the direction where it has a natural incentive to deviate). The minimum intervention capability, expressed in the right-hand side of (12), is increasing with the maximum possible deviation and the strength of the incentive to deviate while decreasing with the effectiveness of intervention. Note that the minimum intervention capability is independent of the choice of intervention rates.

A first-order intervention rule f_1^I satisfying the conditions in Theorem 1 may have a NE other than the target power profile $\mathbf{p}^\star$. For example, if $P_0 \leq \sum_{j \neq i} \alpha_j (P_j - p_j^\star)$ for all $i \in \tilde{\mathcal{N}}(\mathbf{p}^\star)$, $\mathbf{P}$ is also sustained by f_1^I. The presence of this extra NE is undesirable since it brings a possibility that the users still choose $\mathbf{P}$ while the intervention device causes interference to the users by transmitting its maximum power P_0. Obviously, this outcome $(P_0, \mathbf{P})$ is worse for every user than the outcome at the unique NE without intervention $(0, \mathbf{P})$. In order to eliminate this possibility, the designer may want to choose an intervention rule that strongly sustains the target power profile. The following theorem provides a sufficient condition for a first-order intervention rule to strongly sustain a given target power profile.

Theorem 2: For any $\mathbf{p}^\star \in \prod_i (0, P_i]$, $\{\mathbf{p}^\star\} = \mathcal{E}(f_1^I)$ if

$$\alpha_i > \frac{1}{p_i^\star} \sum_{j>i} \alpha_j (P_j - p_j^\star) + \frac{\sum_{j<i} h_{ij} p_j^\star + \sum_{j>i} h_{ij} P_j + n_i}{p_i^\star h_{i0}} \tag{13}$$

and

$$P_0 > \frac{P_i}{p_i^\star} \sum_{j>i} \alpha_j (P_j - p_j^\star) + \frac{(P_i - p_i^\star)(\sum_{j<i} h_{ij} p_j^\star + \sum_{j>i} h_{ij} P_j + n_i)}{p_i^\star h_{i0}} \tag{14}$$

for all $i \in \tilde{\mathcal{N}}(\mathbf{p}^\star)$.[9]

Proof: See Appendix C. ■

By comparing Theorems 1 and 2, we can see that the requirements for the intervention rates and the intervention capability is higher when we impose strongly sustainment. For any given power profile, the intervention rates can be chosen sequentially to satisfy the condition (13) starting from user N' down to user 1. We can set $\alpha_i = 0$ for all $i \notin \tilde{\mathcal{N}}(\mathbf{p}^\star)$. Unlike Theorem 1, the choice of the intervention rates affects the minimum required intervention capability. For strong sustainment, the intervention capability is required to be larger as the designer chooses larger intervention rates. A main reason for the existence of an extra NE is that the region of power profiles on which the maximum intervention power is applied is so wide that the users cannot escape the region by unilateral deviation. Thus, a larger intervention capability is needed to reduce the region and guarantee the uniqueness of NE.

From Theorem 1, we obtain

$$PB_1(\mathbf{p}^\star) = \max_i \frac{(P_i - p_i^\star)(\sum_{j \neq i} h_{ij} p_j^\star + n_i)}{p_i^\star h_{i0}}. \tag{15}$$

Since Theorem 2 gives a sufficient condition for strong sustainment, we obtain an upper bound on $PB_1^s(\mathbf{p}^\star)$,

$$\overline{PB}_1^s(\mathbf{p}^\star) = \sum_{i=1}^{N} \left[\left(\prod_{j=1}^{i-1} \frac{P_j}{p_j^\star} \right) \frac{(P_i - p_i^\star)(\sum_{j<i} h_{ij} p_j^\star + \sum_{j>i} h_{ij} P_j + n_i)}{p_i^\star h_{i0}} \right]. \tag{16}$$

Note that $PB_1(\mathbf{p}^\star) \leq \overline{PB}_1^s(\mathbf{p}^\star)$ with equality if and only if $N' \leq 1$. Combining these results, we can bound $PB_1^s(\mathbf{p}^\star)$ by

$$PB_1(\mathbf{p}^\star) \leq PB_1^s(\mathbf{p}^\star) \leq \overline{PB}_1^s(\mathbf{p}^\star). \tag{17}$$

[9]We define $\sum_{j \in J} x_j = 0$ if J is empty. Similarly, we define $\prod_{j \in J} x_j = 1$ if J is empty.

By Theorems 1 and 2, we have $\mathcal{E}_1 = \mathcal{E}_1^s = \prod_i (0, P_i]$. As argued in Sec. II, it is impossible to provide an incentive for user i to choose $p_i = 0$ by any intervention rule. Thus, we have $\mathcal{E}_K = \mathcal{E}_K^s = \prod_i (0, P_i]$ for all $K = 1, 2, \ldots$. Also, among efficient power profiles, those with $p_i = 0$ for some i have probability measure zero. Hence, we obtain the maximum possible efficiency range 1 by using first-order intervention rules, i.e., $ER_1 = ER_1^s = 1$. This implies that $ER_K = ER_K^s = 1$ for all $K = 1, 2, \ldots$. The discussion suggests that the potential gain from using higher-order intervention rules is not from what they can sustain but how they sustain a target power profile.

Remark 1: An extreme intervention rule f_e^I [20], defined by

$$f_e^I(\mathbf{p}) = \begin{cases} 0, & \text{if } \mathbf{p} = \mathbf{p}^\star, \\ P_0, & \text{if } \mathbf{p} \neq \mathbf{p}^\star, \end{cases} \tag{18}$$

can be considered as the limiting case of first-order intervention rules as each α_i goes to infinity in that the area of the region $\{\mathbf{p} : f_e^I(\mathbf{p}) \neq f_1^I(\mathbf{p})\}$ approaches zero in the limit. With this class of intervention rules, we have $\mathcal{E}(f_e^I) = \{\mathbf{p}^\star, \mathbf{P}\}$ if $P_0 \geq PB_1(\mathbf{p}^\star)$ and $\mathcal{E}(f_e^I) = \{\mathbf{P}\}$ otherwise. Therefore, it is impossible to construct an extreme intervention rule that strongly sustains a target power profile, except in the uninteresting case $\mathbf{p}^\star = \mathbf{P}$. This motivates us to study intervention rules other than extreme intervention rules.

B. *Dynamic Adjustment Processes*

We study a dynamic adjustment process for the designer and the users to reach an equilibrium. The adjustment occurs at discrete steps, labeled as $t = 1, 2, \ldots$. The adjustment process considered in this subsection is based on the myopic best-response updates of the users and is described in Algorithm 1.

We allow that during the adjustment process the designer uses intermediate targets different from the final target $\mathbf{p}^\star$. That is, we have $f^t \in \mathcal{F}_1(\tilde{\mathbf{p}}^t)$, where $\tilde{\mathbf{p}}^t$ is the intermediate target power profile at step t. In the adjustment process, the designer chooses a sequence of intervention rules. Suppose that the designer uses an update rule $\psi : \mathcal{P} \to \mathcal{F}_1$ to determine an intervention rule based on the power profile in the previous step. Then given an initial power profile $\mathbf{p}^0$, an update rule yields a sequence of intervention rules and power profiles $\{(f^t, \mathbf{p}^t)\}_{t=1}^\infty$.[10] We can evaluate an update rule by the following two criteria.

1) *Convergence:* Does the induced sequence reach an equilibrium $(f, \mathbf{p}^\star)$? If so, how many steps are needed?

[10] Since the best response correspondence is non-singleton only in knife-edge cases, we focus on update rules that yield a deterministic sequence.

Algorithm 1 A dynamic adjustment process.

Initialization: $t = 0$

The users choose an initial power profile $\mathbf{p}^0$.

The designer announces the use of first-order intervention rules $f^t \in \mathcal{F}_1$ with power budget P_0.

while $\mathbf{p}^t \neq \mathbf{p}^\star$ or $f^t(\mathbf{p}^t) \neq 0$ **do**

$t \leftarrow t+1$

Given $\mathbf{p}^{t-1}$, the designer chooses and broadcasts the intervention rates α_i^t and the target power profile $\tilde{\mathbf{p}}^t$ for the current time slot t.

Given $f^t(\mathbf{p}) = \left[\sum_{i=1}^N \alpha_i^t \left|p_i - \tilde{p}_i^t\right|\right]_0^{P_0}$, each user i chooses a best response to $\mathbf{p}_{-i}^{t-1}$:

$p_i^t \in BR_i(f^t, \mathbf{p}_{-i}^{t-1}) \triangleq \arg\max_{p_i \in \mathcal{P}_i} \gamma_i(f^t, p_i, \mathbf{p}_{-i}^{t-1})$.

end while

2) *Minimum power budget:* How much power budget is needed to execute $\{f^t\}_{t=1}^\infty$, i.e., $\sup_t \sup_{\mathbf{p}\in\mathcal{P}} f^t(\mathbf{p})$?

The following theorem shows that when the target power profile $\mathbf{p}^\star$ is close to the maximum power profile $\mathbf{P}$ and the intervention capability P_0 is large, we can obtain fast convergence as well as strong sustainment.

Theorem 3: For any $\mathbf{p}^\star \in \prod_i (0, P_i]$ such that $\sum_{i=1}^N (P_i - p_i^\star)/P_i < 1$, there exists $(\alpha_1, \ldots, \alpha_N) \in \mathbb{R}_+^N$ such that

$$\alpha_i > \frac{1}{p_i^\star} \sum_{j\neq i} \alpha_j (P_j - p_j^\star) + \frac{\sum_{j\neq i} h_{ij} P_j + n_i}{p_i^\star h_{i0}} \tag{19}$$

for all $i \in \tilde{\mathcal{N}}(\mathbf{p}^\star)$. Suppose that $f_1^s \in \mathcal{F}_1(\mathbf{p}^\star)$ satisfies (19) and

$$P_0 > \frac{P_i}{p_i^\star} \sum_{j\neq i} \alpha_j (P_j - p_j^\star) + \frac{(P_i - p_i^\star)(\sum_{j\neq i} h_{ij} P_j + n_i)}{p_i^\star h_{i0}} \tag{20}$$

for all $i \in \tilde{\mathcal{N}}(\mathbf{p}^\star)$. Then $\{\mathbf{p}^\star\} = \mathcal{E}(f_1^s)$. Moreover, starting from an arbitrary initial power profile $\mathbf{p}^0 \neq \mathbf{p}^\star$, the adjustment process with $f^t = f_1^s$ for all $t = 1, 2, \ldots$ reaches $(f_1^s, \mathbf{p}^\star)$ in at most two steps (and in one step if $\mathbf{p}^0 \in \prod_i [p_i^\star, P_i]$).

Proof: See Appendix D. ■

The minimum power budget required to execute an intervention rule described in Theorem 3 is given by

$$\widetilde{PB}_1^s(\mathbf{p}^\star) = \frac{1}{1 - \sum_{i=1}^N \frac{P_i - p_i^\star}{P_i}} \sum_{i=1}^N \frac{(P_i - p_i^\star)(\sum_{j\neq i} h_{ij} P_j + n_i)}{P_i h_{i0}}. \tag{21}$$

Since the requirement for α_i in (19) is more stringent than that in (13), we have $\widetilde{PB}_1^s(\mathbf{p}^\star) \geq \overline{PB}_1^s(\mathbf{p}^\star)$ with equality if and only if $N' \leq 1$. The difference $\widetilde{PB}_1^s(\mathbf{p}^\star) - \overline{PB}_1^s(\mathbf{p}^\star)$ can be considered as the price of fast convergence to $\mathbf{p}^\star$ in terms of the minimum power budget. In addition to requiring a larger power budget, Theorem 3 imposes a restriction on the range of target profiles. That is, the target should be close enough to $\mathbf{P}$ for the result of Theorem 3 to hold. However, the target may not satisfy the restriction $\sum_{i=1}^N (P_i - p_i^\star)/P_i < 1$. In this case, the designer need to use intermediate target power profiles that are successively close to one another in order to guide the users to the final target. The use of intermediate target power profiles is also necessary when the intervention device does not have a large enough power budget to strongly sustain a target power profile. In this case, the designer can use a sequence of intervention functions to drive the users to reach the target power profile as the equilibrium. This process requires smaller power budget than that required by strong sustainment, but takes longer time for the system to reach the target power profile.

Define the relative distance from $\mathbf{p}$ to $\mathbf{p}'$ by

$$d(\mathbf{p}, \mathbf{p}') = \sum_{i=1}^N \frac{p_i - p_i'}{p_i}. \tag{22}$$

Using the proofs of Theorems 2 and 3, we can show that, given $\mathbf{p}^{t-1}$, the designer can achieve the intermediate target at step t, i.e., $\mathbf{p}^t = \tilde{\mathbf{p}}^t$ only if $\tilde{\mathbf{p}}^t$ satisfies $d(\mathbf{p}^{t-1}, \tilde{\mathbf{p}}^t) < 1$. This imposes a bound on the relative distance between two successive intermediate targets. Below we provide two different methods for the designer to generate intermediate targets. The first method, which is summarized in Algorithm 2 and analyzed in Theorem 4, produces a sequence of intermediate targets reaching the final target whose successive elements have a relative distance of $\delta \in (0, 1)$ while requiring the minimum power budget in each step. This method can be used in a scenario where the power constraint of the intervention device does not bind; the designer can fix δ sufficiently close to 1, and the method will allow the system to reach the final target in the minimum number of steps. The second method, which is summarized in Algorithm 3 and analyzed in Theorem 5, yields a sequence of intermediate targets with the largest relative distance in each step while satisfying the power constraint. Thus, this method will allow the manager with a limited power budget to reach the final target as fast as possible. The foundation of the first method is presented in the following lemma.

Lemma 1: Given $\delta \in (0, 1)$ and $\mathbf{p}^{t-1} = \tilde{\mathbf{p}}^{t-1} \in \prod_i [p_i^\star, P_i]$ such that $d(\tilde{\mathbf{p}}^{t-1}, \mathbf{p}^\star) \geq 1$, let $\tilde{\mathbf{p}}^t$ be a power profile that minimizes the minimum power budget required by an intervention rule $f^t \in \mathcal{F}_1(\tilde{\mathbf{p}}^t)$ that achieves $\mathbf{p}^t = \tilde{\mathbf{p}}^t$ in the adjustment process subject to $d(\tilde{\mathbf{p}}^{t-1}, \tilde{\mathbf{p}}^t) = \delta$ and $\mathbf{p}^\star \leq \tilde{\mathbf{p}}^t \leq \tilde{\mathbf{p}}^{t-1}$. Then $\tilde{p}_i^t = \mu_i^* \tilde{p}_i^{t-1}$ for all i, where $\mu^* = (\mu_1^*, \ldots, \mu_N^*)$ is an optimal solution of the linear programming (LP) problem:

$$\max_{\mu} \quad \sum_{i=1}^{N} \frac{\sum_{j\neq i} h_{ij}\tilde{p}_j^{t-1} + n_i}{h_{i0}} \mu_i \tag{23}$$

$$s.t. \quad \sum_{i=1}^{N} \mu_i = N - \delta, \tag{24}$$

$$\frac{p_i^\star}{\tilde{p}_i^{t-1}} \leq \mu_i \leq 1, \forall\ i = 1, \ldots, N. \tag{25}$$

Proof: See Appendix E. ■

The condition $d(\tilde{\mathbf{p}}^{t-1}, \mathbf{p}^\star) \geq 1$ guarantees that there exists a feasible solution to the LP problem (23)–(25). An optimal solution of the LP problem can be obtained as follows. First, we set $\mu_i = p_i^\star / \tilde{p}_i^{t-1}$ for all i. At this stage, we have $\sum_{i=1}^{N} \mu_i \leq N - 1$, and thus we need to increase μ_i to satisfy the constraint (24). It is straightforward to see that increasing μ_i with a larger value of $b_i^{t-1} \triangleq (\sum_{j\neq i} h_{ij}\tilde{p}_j^{t-1} + n_i)/h_{i0}$ is more effective in improving the objective value. Hence, we increase μ_i up to 1 starting from i with the largest value of b_i^{t-1} until the constraint (24) is satisfied. Based on the result of Lemma 1, the following algorithm provides a method for the designer to generate intermediate targets in the most power-budget efficient way with a fixed relative distance between two successive targets.

Algorithm 2 An algorithm that generates a sequence of intermediate target power profiles with a fixed relative distance.

Require: Fix $\delta \in (0, 1)$ close to 1

Initialization ($t = 1$): Set $\tilde{\mathbf{p}}^1 = \mathbf{P}$ {This step can be skipped if $\mathbf{p}^0 = \mathbf{P}$} and $\mathcal{M} = \mathcal{N}$
while $\sum_{i=1}^{N} (\tilde{p}_i^t - p_i^\star)/\tilde{p}_i^t \geq 1$ **do**
    $t \leftarrow t + 1$
    Set $\mu_i = p_i^\star / \tilde{p}_i^{t-1}$ for all i
    while $\sum_{i=1}^{N} \mu_i < N - \delta$ **do**
        Choose $i^* \in \arg\max_{i\in\mathcal{M}} b_i^{t-1}$
        Set $\mu_{i^*} = \min\{1, N - \delta - \sum_{j\neq i^*} \mu_j\}$ and $\mathcal{M} \leftarrow \mathcal{M} \setminus \{i^*\}$
    end while
end while
Set $\tilde{\mathbf{p}}^t = \mathbf{p}^\star$

The following theorem shows that the designer can lead the users to the final target by using intermediate targets generated by Algorithm 2 provided that the intervention capability is sufficiently large.

Theorem 4: For any $\mathbf{p}^\star \in \prod_i (0, P_i]$, if $\delta \geq 1 - \min_i(p_i^\star/P_i)$, then Algorithm 2 terminates at a finite step T with $T \leq N' + 1$. Let $(\tilde{\mathbf{p}}^t)_{t=1}^T$ be the sequence of power profiles generated by Algorithm 2. Then there exists a sequence of intervention rules $(f^t)_{t=1}^T$ with $f^t \in \mathcal{F}_1(\tilde{\mathbf{p}}^t)$ such that the adjustment process with $(f^t)_{t=1}^T$ yields $\mathbf{p}^t = \tilde{\mathbf{p}}^t$ for all $t = 1, \ldots, T$ starting from any $\mathbf{p}^0 \in \mathcal{P}$.

Proof: See Appendix F. ∎

Now we consider a scenario where the intervention capability P_0 should be taken into consideration while generating intermediate targets[11]. In this scenario, in order to induce the users to follow intermediate targets during the adjustment process, the intermediate targets $(\tilde{\mathbf{p}}^t)_{t=2}^T$ should satisfy not only

$$\sum_{i=1}^N \frac{\tilde{p}_i^{t-1} - \tilde{p}_i^t}{\tilde{p}_i^{t-1}} < 1 \tag{26}$$

but also

$$P_0^t \triangleq \max_{i \in \tilde{\mathcal{N}}(\tilde{\mathbf{p}}^t)} \left\{ \frac{P_i}{\tilde{p}_i^{t-1}} \frac{\sum_{j=1}^N \frac{(\tilde{p}_j^{t-1} - \tilde{p}_j^t)(\sum_{k \neq j} h_{jk} \tilde{p}_k^{t-1} + n_j)}{\tilde{p}_j^{t-1} h_{j0}}}{1 - \sum_{j=1}^N \frac{\tilde{p}_j^{t-1} - \tilde{p}_j^t}{\tilde{p}_j^{t-1}}} + \frac{(P_i - \tilde{p}_i^{t-1})(\sum_{j \neq i} h_{ij} \tilde{p}_j^{t-1} + n_i)}{\tilde{p}_i^{t-1} h_{i0}} \right\} < P_0 \tag{27}$$

for all $t = 2, \ldots, T$. In order to reach the final target in the minimum number of steps, we need to maximize the relative distance between successive targets while satisfying the constraints (26) and (27). Thus, the problem to obtain $\tilde{\mathbf{p}}^t$ given $\tilde{\mathbf{p}}^{t-1}$ can be written as

$$\max_{\tilde{\mathbf{p}}^t} \quad \sum_{i=1}^N \frac{\tilde{p}_i^{t-1} - \tilde{p}_i^t}{\tilde{p}_i^{t-1}} \tag{28}$$

$$s.t. \quad \max_{i \in \mathcal{N}(\tilde{\mathbf{p}}^t)} \left\{ \frac{P_i}{\tilde{p}_i^{t-1}} \frac{\sum_{j=1}^N \frac{\tilde{p}_j^{t-1} - \tilde{p}_j^t}{\tilde{p}_j^{t-1}} b_j^{t-1}}{1 - \sum_{j=1}^N \frac{\tilde{p}_j^{t-1} - \tilde{p}_j^t}{\tilde{p}_j^{t-1}}} + \frac{P_i - \tilde{p}_i^{t-1}}{\tilde{p}_i^{t-1}} b_i^{t-1} \right\} \leq P_0 - \varepsilon_1 \tag{29}$$

$$\sum_{i=1}^N \frac{\tilde{p}_i^{t-1} - \tilde{p}_i^t}{\tilde{p}_i^{t-1}} \leq 1 - \varepsilon_2 \tag{30}$$

$$\mathbf{p}^\star \leq \tilde{\mathbf{p}}^t \leq \tilde{\mathbf{p}}^{t-1} \tag{31}$$

for small $\varepsilon_1, \varepsilon_2 > 0$. Algorithm 3 formalizes the method to generate a sequence of intermediate target power profiles, which has maximal relative distances (MRD) between successive target power profiles given an intervention capability P_0. We call the sequence generated by Algorithm 3 the MRD sequence. Note that the major complexity in solving the above problem is line 10 in Algorithm 3. This search on $\tilde{p}_{i^*}^t$ can be done efficiently by bisection method, because P_0^t is decreasing with $\tilde{p}_{i^*}^t$. Consider a choice of $\tilde{\mathbf{p}}^t$ as $\tilde{\mathbf{p}}^t = \kappa \mathbf{p}^\star + (1-\kappa)\tilde{\mathbf{p}}^{t-1}$ for some $\kappa \in (0, 1)$. It is straightforward to see that, as we choose κ

[11]Note that to sustain $\mathbf{p}^\star$ as the NE, we require P_0 to satisfy the conditions in Theorem 1.

sufficiently close to 0, the constraints (30) and (31) are satisfied for any given $\varepsilon_2 < 1$. Also, as $\kappa \to 0$, the left-hand side of (29) approaches $\max_{i \in \mathcal{N}(\tilde{\mathbf{p}}^t)} (P_i - \tilde{p}_i^{t-1}) b_i^{t-1} / \tilde{p}_i^{t-1}$, which is bounded by

$$\max_{i \in \mathcal{N}(\tilde{\mathbf{p}}^t)} \frac{P_i - \tilde{p}_i^{t-1}}{\tilde{p}_i^{t-1}} b_i^{t-1} \leq \max_i \left\{ \frac{P_i - p_i^\star}{p_i^\star} \cdot \frac{\sum_{j \neq i} h_{ij} P_j + n_i}{h_{i0}} \right\} \tag{32}$$

for all $\tilde{\mathbf{p}}^{t-1} \in \prod_i [p_i^\star, P_i]$. Hence, the requirement on the power budget in Algorithm 3 guarantees that the optimization problem (28)–(31) has a feasible solution.

Algorithm 3 An algorithm that generates a sequence of intermediate target power profiles with maximal relative distances given an intervention capability.

Require: Small $\varepsilon_1 \in (0,1)$ and $\varepsilon_2 \in (0,1)$; $P_0 - \varepsilon_1 > \max_i \left\{ \frac{P_i - p_i^\star}{p_i^\star} \cdot \frac{\sum_{j \neq i} h_{ij} P_j + n_i}{h_{i0}} \right\}$

Initialization ($t = 1$)**:** Set $\tilde{\mathbf{p}}^1 = \mathbf{P}$ {This step can be skipped if $\mathbf{p}^0 = \mathbf{P}$}
while $\tilde{\mathbf{p}}^t \neq \mathbf{p}^\star$ **do**
  $t \leftarrow t + 1$, $\mathcal{M} = \{i : \tilde{p}_i^{t-1} > p_i^\star\}$, $\tilde{\mathbf{p}}^t = \tilde{\mathbf{p}}^{t-1}$
  repeat
    $i^* = \min_{i \in \mathcal{M}} b_i^{t-1}$, $\tilde{p}_{i^*}^t = \max \left\{ p_{i^*}^\star, (N - 1 + \varepsilon_2 - \sum_{i \neq i^*} \tilde{p}_i^t / \tilde{p}_i^{t-1}) \cdot \tilde{p}_{i^*}^{t-1} \right\}$
    calculate P_0^t by (27)
    if $P_0^t < P_0 - \varepsilon_1$ **then**
      $\mathcal{M} \leftarrow \mathcal{M} \backslash \{i^*\}$
    else if $P_0^t > P_0 - \varepsilon_1$ **then**
      find $\tilde{p}_{i^*}^t \in \left[\max \left\{ p_{i^*}^\star, (N - 1 + \varepsilon_2 - \sum_{i \neq i^*} \tilde{p}_i^t / \tilde{p}_i^{t-1}) \cdot \tilde{p}_{i^*}^{t-1} \right\}, \tilde{p}_{i^*}^{t-1} \right]$ such that $P_0^t = P_0 - \varepsilon_1$
    end if
  until $P_0^t = P_0 - \varepsilon_1$ **or** $\mathcal{M} = \emptyset$
end while
Set $\tilde{\mathbf{p}}^t = \mathbf{p}^\star$

Given the power budget P_0, we are interested in the minimum convergence time for the dynamic adjustment process to reach the target power profile $\mathbf{p}^\star$, defined by $T^\star(\mathbf{p}^\star) = \inf \left\{ T : \tilde{\mathbf{p}}^T = \mathbf{p}^\star \right\}$, where the infimum is taken over the set of sequences satisfying (26) and (27) starting from $\mathbf{p}^1 = \mathbf{P}$. In order to obtain an upper bound for $T^\star(\mathbf{p}^\star)$, we use an upper bound for the convergence time of a geometric sequence of T intermediate target power profiles in the following form:

$$\tilde{p}_i^t = (\eta_i)^{t-1} P_i, \ \forall \ i, \ t = 1, \dots, T, \tag{33}$$

where $\eta_i = (p_i^\star / P_i)^{\frac{1}{T-1}}$, $i = 1, \ldots, N$.

Theorem 5: If $\mathbf{p}^0 \neq \mathbf{p}^\star$, $\sum_{i=1}^N (P_i - p_i^\star)/P_i \geq 1$, and

$$P_0 > \left(\max_i \frac{P_i}{p_i^\star} - 1 \right) \max_i \frac{\sum_{j \neq i} h_{ij} p_j^\star + n_i}{h_{i0}}, \tag{34}$$

then $T^\star(\mathbf{p}^\star) > 2$ and $T^\star(\mathbf{p}^\star)$ satisfies

$$\sum_{i=1}^N \left(\frac{p_i^\star}{P_i} \right)^{\frac{1}{T^\star(\mathbf{p}^\star)-2}} < N - 1 + \frac{1}{C}, \tag{35}$$

where

$$C = \frac{P_0}{\max_i \frac{\sum_{j \neq i} h_{ij} p_j^\star + n_i}{h_{i0}} \cdot \max_i \frac{P_i}{p_i^\star}} + \frac{1}{\max_i \frac{P_i}{p_i^\star}}. \tag{36}$$

Proof: See Appendix G. ∎

The inequality (35) provides an upper bound for $T^\star(\mathbf{p}^\star)$, since the left-hand side of (35) increases in $T^\star(\mathbf{p}^\star)$ and approaches N as $T^\star(\mathbf{p}^\star) \to \infty$ while the right-hand side is smaller than N given (34). From Theorem 5, we can see that the convergence time is small if the power budget P_0 is large, the target power profile is close to the maximum power (i.e. $\max_i \frac{P_i}{p_i^\star}$ is small), or SINR is relatively small compared to the channel gain from the intervention device (i.e. $\max_i \frac{\sum_{j \neq i} h_{ij} p_j^\star + n_i}{h_{i0}}$ is small).

C. Relaxation of Monitoring Requirement

The results in this section so far relies on the ability of the intervention device to estimate individual transmit powers. However, as discussed in Appendix A, estimating individual transmit powers requires larger monitoring overhead for the intervention device than estimating aggregate receive power. In order to study intervention rules that can be executed with the monitoring of aggregate receive power, we consider a class of intervention rules that can be expressed as

$$f_1^A(\mathbf{p}) = \left[\alpha_0 \left| \left(\sum_{i=1}^N h_{0i} p_i \right) - p_A^\star \right| \right]_0^{P_0} \tag{37}$$

for some $\alpha_0 \geq 0$, $P_0 > 0$, and $p_A^\star$. We call an intervention rule in this class a first-order intervention rule based on aggregate receive power or, in short, an intervention rule based on aggregate power. We call α_0 the aggregate intervention rate, and call $p_A^\star$ the target aggregate power, which is set as the aggregate receive power at the target power profile, i.e., $p_A^\star = \sum_{i=1}^N h_{0i} p_i^\star$. We first give a necessary and sufficient condition for an intervention rule based on aggregate power to sustain a target power profile.

Theorem 6: For any $\mathbf{p}^\star \in \prod_i (0, P_i]$, $\mathbf{p}^\star \in \mathcal{E}(f_1^A)$ if and only if

$$\alpha_0 \geq \max_{i \in \tilde{\mathcal{N}}(\mathbf{p}^\star)} \frac{\sum_{j \neq i} h_{ij} p_j^\star + n_i}{h_{0i} p_i^\star h_{i0}} \tag{38}$$

and

$$P_0 \geq \max_{i \in \tilde{\mathcal{N}}(\mathbf{p}^\star)} \frac{(P_i - p_i^\star)(\sum_{j \neq i} h_{ij} p_j^\star + n_i)}{p_i^\star h_{i0}}. \tag{39}$$

Proof: See Appendix H. ■

The minimum intervention capability required to sustain a target profile is not affected by using aggregate receive power instead of individual transmit powers. However, the aggregate intervention rate should be chosen high enough to prevent a deviation of any user, whereas with the monitoring of individual transmit powers the intervention rates can be chosen individually for each user. This suggests that strong sustainment is more difficult with intervention rules based on aggregate power. For example, $\mathbf{P}$ is also sustained by f_1^A if $P_0 \leq \alpha_0 \sum_{j \neq i}(h_{0j} P_j - p_j^\star)$ for all $i \in \tilde{\mathcal{N}}(\mathbf{p}^\star)$, which is weaker than the corresponding condition in the case of intervention rules based on individual powers, $P_0 \leq \sum_{j \neq i} \alpha_j (P_j - p_j^\star)$ for all $i \in \tilde{\mathcal{N}}(\mathbf{p}^\star)$. With the monitoring of individual powers, a deviation of each user can be detected and punished. This leads to the property that the best response of user i is almost always either $p_i^\star$ or P_i under first-order intervention rules based on individual powers. This implies that a power profile sustained by a first-order intervention rule based on individual powers almost always belongs to the set $\prod_i \{p_i^\star, P_i\}$. In contrast, with the monitoring of aggregate power, only an aggregate deviation can be detected. This yields a possibility that an intervention rule based on aggregate power sustains a power profile that is different from the target but yields the same aggregate power. This possibility makes the problem of coordination failure more worrisome because if the users are given only the target aggregate power $p_A^\star$ they may not know which power profile to select among those that yield the aggregate power $p_A^\star$.[12] The problem arising from the increased degree of non-uniqueness can be considered as the cost of reduced monitoring overhead. To state the result formally, let $\alpha_0^i = (\sum_{j \neq i} h_{ij} p_j^\star + n_i)/(h_{0i} p_i^\star h_{i0})$ and $P_0^i = (P_i - p_i^\star)(\sum_{j \neq i} h_{ij} p_j^\star + n_i)/(p_i^\star h_{i0})$ for all $i \in \tilde{\mathcal{N}}(\mathbf{p}^\star)$. Also, let $\bar{\alpha}_0 = \max_{i \in \tilde{\mathcal{N}}(\mathbf{p}^\star)} \alpha_0^i$ and $\bar{P}_0 = \max_{i \in \tilde{\mathcal{N}}(\mathbf{p}^\star)} P_0^i$.

Theorem 7: Suppose that, for $\mathbf{p}^\star \in \prod_i (0, P_i]$, there exist $i, j \in \tilde{\mathcal{N}}(\mathbf{p}^\star)$ such that (i) $\bar{\alpha}_0 = \alpha_0^i > \alpha_0^j$ or $\alpha_0^i, \alpha_0^j < \bar{\alpha}_0$, and (ii) $\bar{P}_0 = P_0^i > P_0^j$ or $P_0^i, P_0^j < \bar{P}_0$. Then for any f_1^A such that $\mathbf{p}^\star \in \mathcal{E}(f_1^A)$ and for any $\epsilon > 0$, there exists $\tilde{\mathbf{p}} \neq \mathbf{p}^\star$ such that $\tilde{\mathbf{p}} \in \mathcal{E}(f_1^A)$, $\sum_{i=1}^N h_{0i} p_i^\star = \sum_{i=1}^N h_{0i} \tilde{p}_i$, and $|\tilde{\mathbf{p}} - \mathbf{p}^\star| < \epsilon$.

Proof: See Appendix I. ■

Theorem 7 provides a sufficient condition under which the strong sustainment of a given target power profile is impossible with intervention rules based on aggregate power. We argue that the sufficient

[12] A way to overcome this problem is to broadcast the target power profile $\mathbf{p}^\star$ to the users in order to make $\mathbf{p}^\star$ as a focal point [26].

condition is mild. First, note that, for almost all $\mathbf{p}^\star \in \prod_i (0, P_i]$, α_0^i's and P_0^i's can be ordered strictly. With strict ordering of α_0^i's and P_0^i's, we can always find a pair of users $i, j \in \tilde{\mathcal{N}}(\mathbf{p}^\star)$ satisfying the condition in Theorem 7 if there are at least three users in $\tilde{\mathcal{N}}(\mathbf{p}^\star)$. That is, strong sustainment is generically impossible with intervention rules based on aggregate power when $|\tilde{\mathcal{N}}(\mathbf{p}^\star)| \geq 3$.

V. Intervention Based on Estimation With Errors

Recall that Y denotes the signal space and that $\rho(\mathbf{p})$ denotes the distribution of signals given a power profile $\mathbf{p}$. We denote expectation with respect to the distribution $\rho(\mathbf{p})$ by $E_{\mathbf{p}}$. Suppose that the users maximize their own average SINRs.[13] Then the condition for an intervention rule f to sustain a target power profile $\mathbf{p}^\star$ is given by

$$E_{\mathbf{p}^\star}[\gamma_i(f(y), \mathbf{p}^\star)] \geq E_{(p_i, \mathbf{p}_{-i}^\star)}[\gamma_i(f(y), p_i, \mathbf{p}_{-i}^\star)] \tag{40}$$

for all i and $p_i \in \mathcal{P}_i$. Suppose that the objective of the manager is to minimize the average power level of the intervention device while sustaining the target power profile $\mathbf{p}^\star$. Then the design problem can be written as

$$\min_f \quad E_{\mathbf{p}^\star}[f(y)] \tag{41}$$

$$s.t. \quad E_{\mathbf{p}^\star}[\gamma_i(f(y), \mathbf{p}^\star)] \geq E_{(p_i, \mathbf{p}_{-i}^\star)}[\gamma_i(f(y), p_i, \mathbf{p}_{-i}^\star)], \ \forall p_i \in \mathcal{P}_i, \forall i. \tag{42}$$

Note that there may exist no intervention rule that sustains $\mathbf{p}^\star$ if signals are too noisy (i.e., a deviation from $\mathbf{p}^\star$ is not easily detected) or the maximum power of the intervention device is too low (i.e., no strong punishment is available).

We assume the intervention device estimates the aggregate transmit power $\sum_j h_{0j} p_j$ with an additive error. That is, $y = \sum_j h_{0j} p_j + \theta$, where θ is a random variable with a cumulative distribution function denoted by Φ. We restrict attention to a simple class of intervention rules that use a threshold rule: the intervention device puts no power if $y \leq \bar{y}$ and some positive power $\hat{p}_0 \leq P_0$ otherwise. Such an intervention rule can be expressed as

$$f^{th}(y) = \begin{cases} 0 & \text{if } y \leq \bar{y}, \\ \hat{p}_0 & \text{otherwise.} \end{cases} \tag{43}$$

[13]We use this assumption to dispense with knowledge about the utility functions of the users for the designer. Alternatively, we can assume that the users maximize their own expected utilities from SINRs, which requires for the designer to know the utility functions of the users in order to check whether an intervention rule sustains a target power profile.

The probability that intervention is not triggered when the users choose a power profile $\mathbf{p}$ is given by $Pr(y \leq \bar{y}) = \Phi(\bar{y} - \sum_{i=1}^{N} h_{0i} p_i)$. Hence, the design problem is given by

$$\min_{(\bar{y},\hat{p}_0)} \quad \Phi\left(\bar{y} - \sum_{j=1}^{N} h_{0j} p_j^{\star}\right) P_0 \tag{44}$$

$$s.t. \quad \Phi\left(\bar{y} - \sum_{j=1}^{N} h_{0j} p_j^{\star}\right) \gamma_i(0, \mathbf{p}^{\star}) + \left[1 - \Phi\left(\bar{y} - \sum_{j=1}^{N} h_{0j} p_j^{\star}\right)\right] \gamma_i(\hat{p}_0, \mathbf{p}^{\star}) \tag{45}$$

$$\geq \Phi\left(\bar{y} - \sum_{j \neq i} h_{0j} p_j^{\star} - h_{0i} p_i\right) \gamma_i(0, p_i, \mathbf{p}_{-i}^{\star}) \tag{46}$$

$$+ \left[1 - \Phi\left(\bar{y} - \sum_{j \neq i} h_{0j} p_j^{\star} - h_{0i} p_i\right)\right] \gamma_i(\hat{p}_0, p_i, \mathbf{p}_{-i}^{\star}), \ \forall p_i \in \mathcal{P}_i, \forall i. \tag{47}$$

To obtain illustrative results, we assume that the estimation error θ is uniformly distributed in $[-\theta_0, \theta_0]$ [23] [24]. Given a target power profile $\mathbf{p}^{\star}$, we are interested in when $\mathbf{p}^{\star}$ can be sustained without performance loss (i.e. the expected transmit power of the intervention device is 0) and how to set $\bar{y}$ and $\hat{p}_0$ to sustain $\mathbf{p}^{\star}$ without performance loss. These issues are addressed in Theorem 8.

Theorem 8: Given the target power profile $\mathbf{p}^{\star}$ and the uniformly distributed error θ, we can sustain $\mathbf{p}^{\star}$ without performance loss ($E_{\mathbf{p}^{\star}}[f(y)] = 0$), if and only if

$$\theta_0 < \frac{1}{2} \min_{i \in \tilde{\mathcal{N}}(\mathbf{p}^{\star})} h_{0i} p_i^{\star}. \tag{48}$$

To sustain $\mathbf{p}^{\star}$ without performance loss, we set the threshold as

$$\bar{y} = \sum_{i=1}^{N} h_{0i} p_i + \theta_0, \tag{49}$$

and the transmit power as

$$\hat{p}_0 \geq \max\left\{\frac{2\theta_0}{h_{0i} p_i^{\star} - 2\theta_0}, \ \frac{P_i - p_i^{\star}}{p_i^{\star}}\right\} \cdot \frac{\sum_{j \neq i} h_{ij} p_j^{\star} + n_i}{h_{i0}}, \ \forall \ i \in \tilde{\mathcal{N}}(\mathbf{p}^{\star}). \tag{50}$$

Proof: See Appendix J. ■

From Theorem 8, we can see that there will be no performance loss when the error is small. Conversely, if we know the error bound θ_0, we can get a lower bound on the target power profile that can be sustained without performance loss. Note that usually the designer can choose the intervention device whose intervention capability P_0 is not smaller than $\hat{p}_0$. Hence, the lower bound on $\hat{p}_0$ is also the lower bound for P_0. When the error is small, the intervention device does not need higher intervention capability P_0 compared to the case with perfect monitoring. The intervention device needs higher intervention capability only when there exists i such that

$$\theta_0 > \frac{P_i - p_i^{\star}}{2P_i} \cdot h_{0i} p_i^{\star}. \tag{51}$$

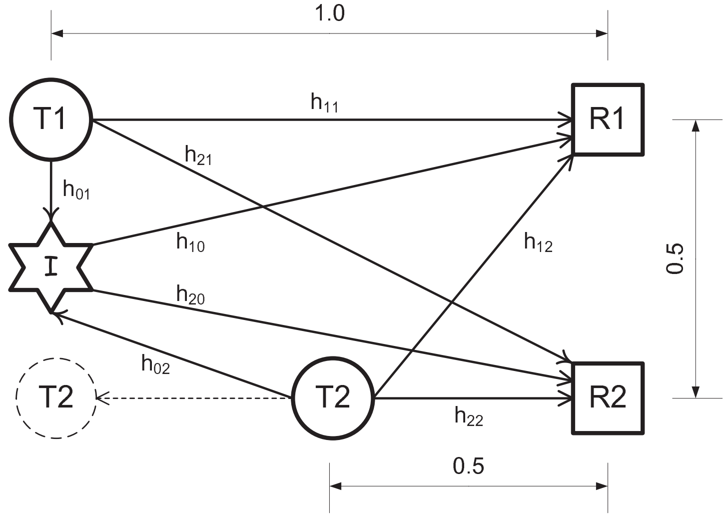


Fig. 1. An example wireless ad-hoc network with two users.

Similar to the case with perfect monitoring, the intervention rule based on aggregate receive power usually cannot strongly sustain a target power profile in the case with imperfect monitoring. Actually, any target power profile $\mathbf{p}^\star$ that lies in the set

$$\left\{\mathbf{p}^\star : p_i^\star > \frac{2\theta_0}{h_{0i}},\ i = 1, \ldots, N\right\} \tag{52}$$

and satisfies (49) and (50) can be sustained by the same $\bar{y}$ and $\hat{p}_0$.

VI. Simulation Results

We consider a two-user network shown in Fig. 1. User 2's transmitter is near to user 1's receiver, causing significant interference to user 1. The distance from user 1's transmitter to its receiver is normalized to 1. Originally, the distance from user 2's transmitter to its receiver is 0.5. The distance between the two users' transmitters and that between the two users' receivers are both 0.5. Without specific notice, we assume that the positions of the transmitters and receivers of both users remain the same. In the simulation of Fig. 2, we let user 2's transmitter move away from its receiver as shown by the dashed left arrow, resulting in less interference to user 1. We assume that the channel gain h is reciprocal to the distance d with the path loss exponent $a = 3$, namely $h = d^{-a}$. The noise powers at the receivers of both users are 0.2. The power budgets of both users are 10.

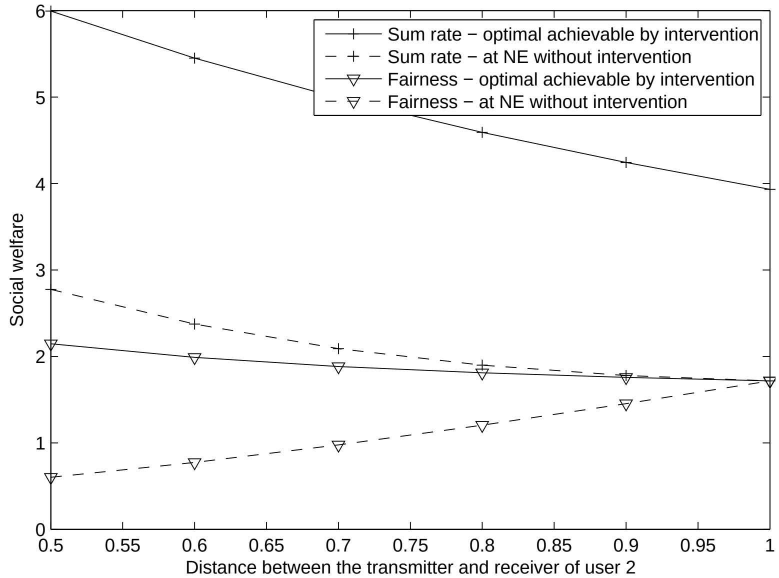


Fig. 2. The optimal social welfare achievable by intervention and the social welfare at NE without intervention, when user 2's transmitter moves away from its receiver.

A. Performance Improvement By Intervention

Now we examine the performance improvement by using intervention mechanisms. We let user 2's transmitter moves away from its receiver. In Fig. 2, we show the performance achieved by intervention and that at the NE without intervention, under two criteria for social welfare. The sum rate is define by

$$\log\left(1+\gamma_1\right)+\log\left(1+\gamma_2\right), \tag{53}$$

and the fairness is defined by

$$\log\left(1+\min\left\{\gamma_1,\gamma_2\right\}\right). \tag{54}$$

As we can see from Fig. 2, the sum rate achievable by intervention doubles that at the NE without intervention in all the cases. The fairness achievable by intervention is much larger than that at the NE without intervention in most cases. When the distance from user 2's transmitter to its receiver is 1.0, the network is symmetric. Only at this point is the NE without intervention optimal in terms of fairness.

B. Minimum Power Budget

Now we show the power budget requirement for different intervention rules. In Fig. 3, we show the contour of the minimum power budget for different intervention rules under different target power profiles.

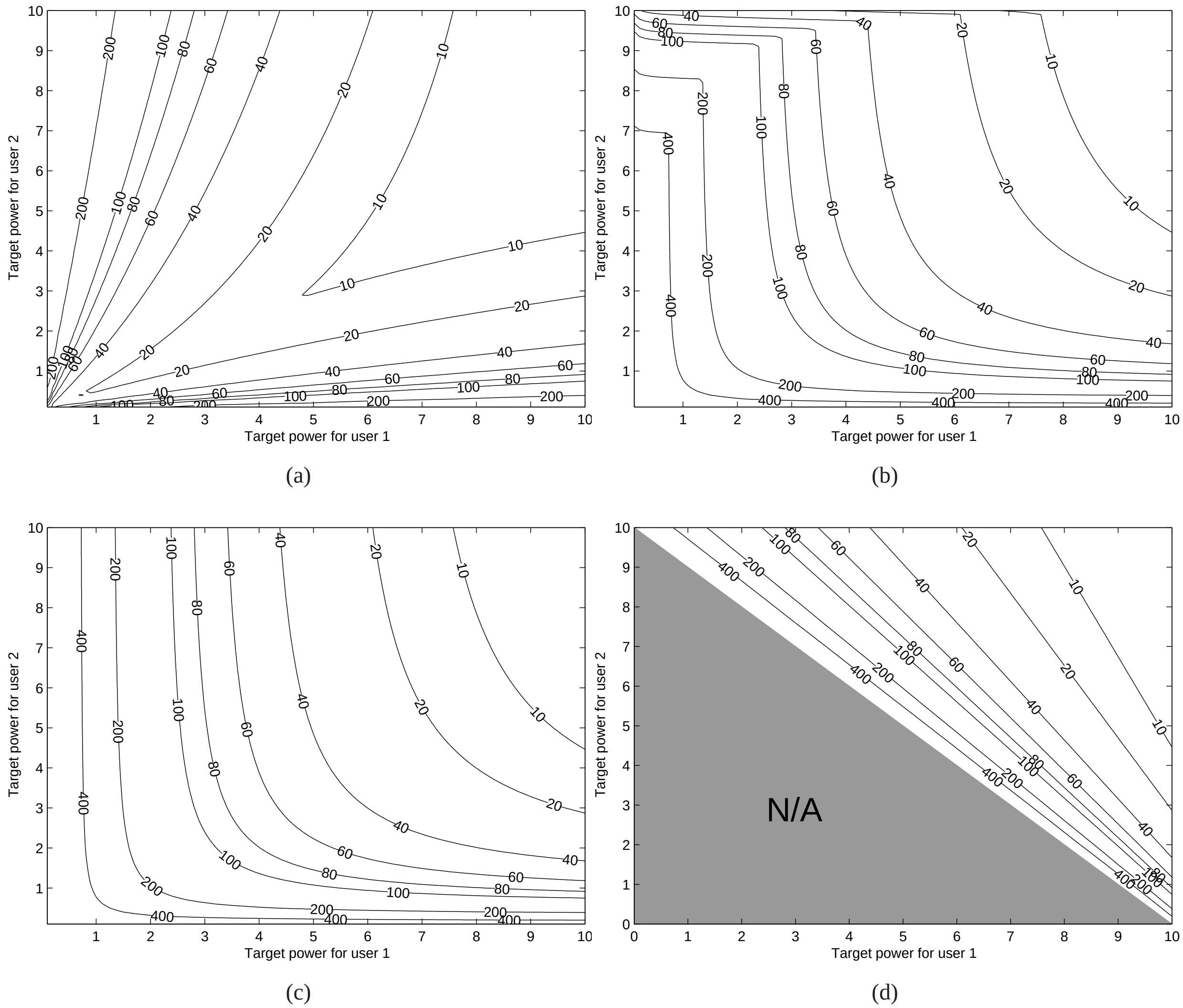
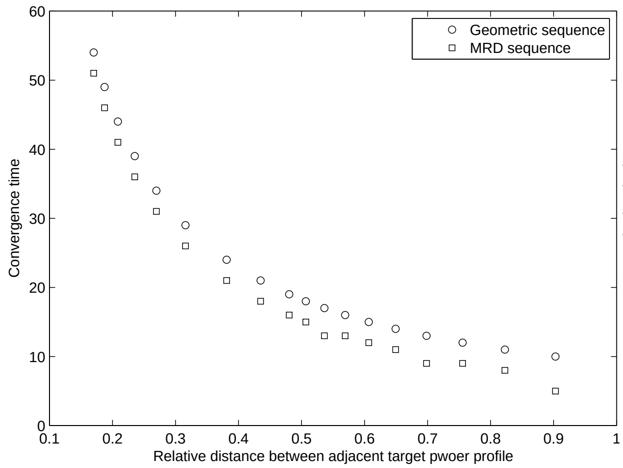
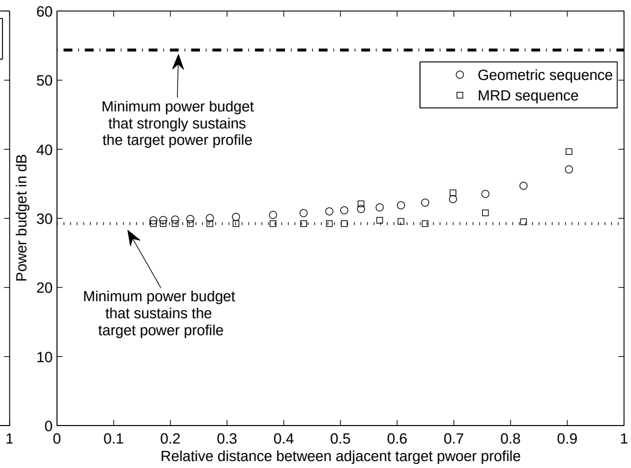


Fig. 3. Contour of the minimum power budget of first-order intervention under different target power profiles. (a): minimum power budget to sustain a target power profile obtained by Theorem 1 and Theorem 6; (b): minimum power budget to strongly sustain a target power profile obtained by simulation; (c): upper bound on the minimum power budget to strongly sustain a target power profile obtained by Theorem 2; (d): upper bound on the minimum power budget for strong sustainment and fast convergence obtained by Theorem 3.

Fig. 3(a) shows minimum power budget to sustain a target power profile using first-order intervention based on individual transmit powers obtained by Theorem 1 and that using first-order intervention based on aggregate receive power obtained by Theorem 6. Since the power budget requirements are the same for these two intervention rules, we show them in the same figure. Fig. 3(b) shows the minimum power budget to strongly sustain a target power profile obtained by simulation. As we expect, the power budget



(a)



(b)

Fig. 4. Given the relative distance between adjacent target power profiles, the convergence time and the power budget requirement of different sequences of intermediate target power profiles in a five-user network. The relative distance between the maximum power profile and the target power profile is $\sum_{i=1}^{5} \frac{P_i - p_i^\star}{P_i} = 3.6 > 1$. (a): convergence time; (b): power budget requirement.

requirement for strongly sustainment is higher. Fig. 3(c) shows the upper bound on the minimum power budget to strongly sustain a target power profile obtained by Theorem 2. We can see that the result in Theorem 2 serves as a good upper bound. Finally, Fig. 3(d) shows the upper bound on the minimum power budget for strong sustainment and fast convergence obtained by Theorem 3. In this case, the system reaches NE in at most two time slots. To achieve this fast convergence, the intervention device needs a much higher power budget. In addition, not all the target power profiles can be sustained. The target power profiles that cannot be sustained lie in the shadow area in the figure.

C. Power Budget and Convergence Time Tradeoff In Dynamic Adjustment Process

Now we study the tradeoff between the power budget and the convergence time in the dynamic adjustment process. To better illustrate the tradeoff, we use a five-user network in the simulation for Fig. 4 and Fig. 5. The channel gains and noise powers used in the simulation are one realization of the random variables. Since different realizations result in similar tradeoff, we only show the results for one realization. The target power profile is $p_1^\star = P_1$ and $p_i^\star = 0.1 P_i$ for $i > 1$. The target power profile is $p_1^\star = P_1$ and $p_i^\star = 0.1 P_i$ for $i = 2, \ldots, 5$. Since the relative distance between the maximum power profile and the target power profile is $\sum_{i=1}^{5} \frac{P_i - p_i^\star}{P_i} = 3.6 > 1$, we cannot reach the target power profile from the maximum power profile directly using Theorem 3. Instead, we need a sequence of intermediate target

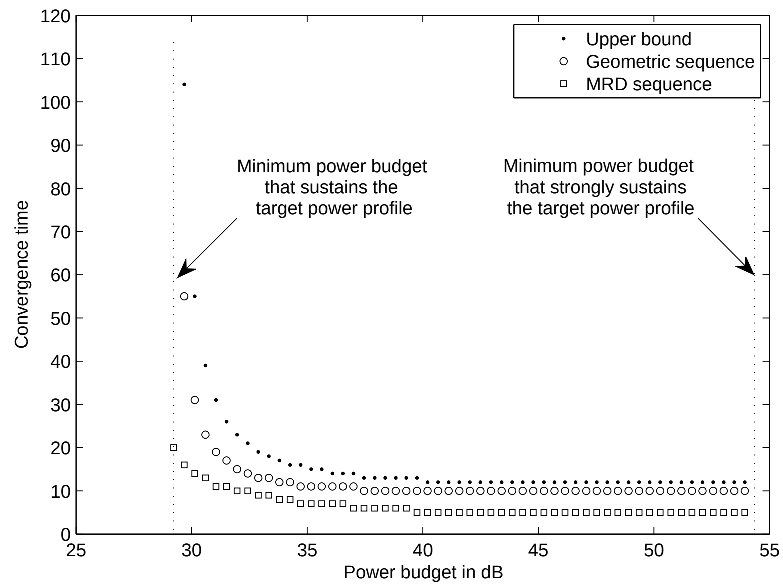


Fig. 5. Given power budget requirements, the convergence time of different sequences of intermediate target power profiles in a five-user network. The relative distance between the maximum power profile and the target power profile is $\sum_{i=1}^{5}\frac{P_i-p_i^\star}{P_i} = 3.6 > 1$.

power profile before we reach the final target power profile.

First, suppose that there is no power budget requirement. Without power budget limit, our goal is to reach the final target power profile in as few time slots as possible. Since it is not easy to construct a sequence of intermediate target power profiles given a desired convergence time, we construct the sequence according to the desired relative distance between adjacent intermediate target power profiles, which is an indicator for the convergence time. In Fig. 4, we show the convergence time and the power budget requirement of the MRD sequence generated by Algorithm 2 and the geometric sequence under different relative distances. We can see from Fig 4(a) that a larger relative distance results in a faster convergence for both sequences. Thus, we can use the relative distance, a metric amenable for the construction of the sequence, to control the convergence speed of the adjustment process. In particular, when the relative distance is $\delta = 0.9 = 1 - \min_i p_i^\star P_i$, the MRD sequence converges in $N' + 1 = 5$ steps as predicted by Theorem 4. In Fig 4(b), we can see that for both sequences, the power budget requirement is decreasing with the relative distances in most cases. The power budget is lower for the MRD sequence. For both sequences, it requires much less power by the dynamic adjustment process than by the strong sustainment condition in Theorem 2.

Second, suppose that there is a power budget requirement. Given different power budget requirements,

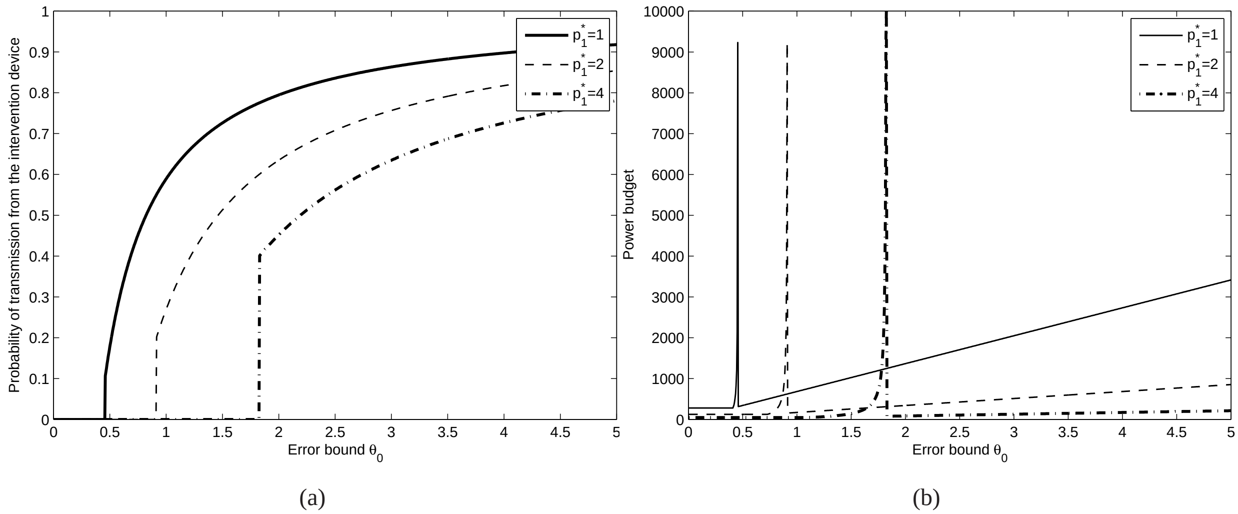


Fig. 6. Given the error bounds θ_0, the transmission probability and the power budget requirement of the intervention device, under different target power profiles in the two-user network in Fig. 1. The target power for user 1 is $p_1^\star = 1, 2, 4$ and the target power for user 2 is $p_2^\star = 10 = P_2$. (a): transmission probability; (b): power budget requirement.

we show the convergence time of MRD and geometric sequences and the upper bound on the convergence time in Fig. 5. We can see from the figure that under most power budget requirements, the convergence time of MRD sequence is roughly half of that of the naive geometric sequence. When the power budget is small (near to the minimum power budget that sustains the target power profile), the fast convergence of MRD sequence is even more significant compared to that of the geometric sequence. When the power budget approaches the minimum requirement that strongly sustains the target power profile, the convergence time of MRD sequence is 5, which is half of that of the geometric sequence.

D. Impact of Imperfect Monitoring

Now we consider the two-user network in Fig. 1. We show in Fig 6 the transmission probability and the power budget requirement under different error bounds θ_0 and different target power profiles $\mathbf{p}^\star$. Generally, both the transmission probability and the power budget requirement are lower when the error bound is smaller. There can be no transmission,thus no punishment, if the error bound θ_0 is smaller than $\frac{h_{01}p_1^\star}{2}$. When the error bound is increasing, a lower target power will trigger the punishment earlier, as can be seen from Fig. 6(a). From Fig. 6(b), we can see that although the transmission probability is 0 when the error bound θ_0 is smaller than $\frac{h_{01}p_1^\star}{2}$, the power budget requirement is increasing in θ_0. When θ_0 approaches $\frac{h_{01}p_1^\star}{2}$, the power budget requirement is extremely large because $\frac{2\theta_0}{h_{01}p_1^\star - 2\theta_0}$ is the dominant factor in the lower bound on the power budget requirement. The interpretation is that to avoid punishment and performance loss, the in power budget as a threat should be larger when the error bound is larger. When θ_0 exceeds $\frac{h_{01}p_1^\star}{2}$, the power budget requirement is not as large as that when θ_0 approaches $\frac{h_{01}p_1^\star}{2}$, because the inevitable punishment is allowed.

VII. Conclusion

In this paper, we proposed intervention for efficient power control in wireless ad hoc networks with selfish users. To achieve efficient outcomes, we proposed intervention schemes to induce selfish users to transmit at desired power levels. Different from other incentive mechanisms such as pricing and auctions, intervention punishes the users by directly decreasing their SINR's, thus provides an credible threat to regulate the users' behaviors. We proposed different intervention rules with different monitoring technologies. We analyzed the minimum power budget requirement of the proposed intervention rules and provided the design principles. We also studied the possible performance loss in imperfect monitoring scenarios. Interesting future works could be the performance analysis on the higher-order intervention rules, performance analysis for general intervention rules and error distributions in imperfect monitoring scenarios, and so on.

Appendix A
Estimation of Channel Gains and Individual Transmit Powers

In order to determine the intervention coefficients and the power budget requirement, the system designer need to know the normalized cross channel gains $\{\frac{h_{ij}}{h_{i0}}\}_{j=1,j\neq i}^{N}\ \forall\ i$ and the normalized noise powers $\{\frac{n_i}{h_{i0}}\}_{j=1,j\neq i}^{N}\ \forall\ i$. It is difficult to collect this information, even if the users are willing to estimate local information, namely $\{h_{ij}\}_{j=1}^{N}$ (as in [4]), h_{i0}, and n_i for user i, and report their local information to the system designer. This requires each user to estimate the cross channel gains by the pilot signals from its neighbors and the intervention device, which has a large signaling overhead and is difficult to synchronize. Moreover, when the users are selfish, they have no incentives to report their local information, or even to estimate $\{h_{ij}\}_{j=1}^{N}$, because knowing the aggregate interference and noise power is enough for them to make the decision. As we will see below, by using intervention, the designer can estimate the normalized cross channel gains and the normalized noise powers without any user cooperation.

The system designer estimates the normalized cross channel gains and the normalized noise powers by adjusting the intervention coefficients and observing the reaction of the users. The designer can take N rounds of measurements to estimate them. One round of measurements is performed as follows.

There are N steps in each round. At the beginning of each round, all the intervention coefficients are set to 0. In round 1, the system designer adjusts the intervention coefficients one by one, starting from α_1. In step 1, $\alpha_1 = 0$ initially. At this time, user 1 transmits at the maximum power P_1 and the total received power at the intervention device is

$$\sum_{j=1}^{N} h_{0j} P_j. \tag{55}$$

Then the designer can find the α_1 such that user 1 transmits at the reference power level $\tilde{p}_i$, resulting in a reduced received power

$$\sum_{j=2}^{n} h_{0j} P_j + h_{01} \tilde{p}_1. \tag{56}$$

The search for α_1 can be done efficiently by bisection methods, since user 1's transmit power and thus the total received power is decreasing in α_1. During the search, the designer can measure the received power right after α_1 is changed, because user 1 reacts immediately and there is no convergence process here.

According to the two received power levels, we can obtain h_{01}. In addition, α_1 should satisfy the following condition to convert user 1 from transmitting P_1 to transmitting $\tilde{p}_1$:

$$h_{10}\alpha_1 = \sum_{j \neq 1} \alpha_j (P_j - \tilde{p}_j) + \sum_{j \neq 1} h_{1j} P_j + n_1 = \sum_{j \neq 1} h_{1j} P_j + n_1, \tag{57}$$

from which we can obtain

$$\frac{\sum_{j \neq 1} h_{1j} P_j + n_1}{h_{i0}} \tag{58}$$

as the first measurement in round 1.

Similarly, at step i, the designer finds the proper α_i such that user i is converted from transmitting P_i to transmitting $\tilde{p}_i$. The designer can obtain h_{0i} and the ith measurement of round 1:

$$\frac{\sum_{j<i} h_{ij} \tilde{p}_j + \sum_{j>i} h_{ij} P_j + n_i}{h_{i0}}. \tag{59}$$

Hence, after round 1, we can get the following vector of measurement $\mathbf{m}_1$:

$$\mathbf{m}_1 = \left[\frac{\sum_{j>1} h_{1j} P_j + n_1}{h_{10}}, \ \ldots, \ \frac{\sum_{j<i} h_{ij} \tilde{p}_j + \sum_{j>i} h_{ij} P_j + n_i}{h_{i0}}, \ \ldots, \ \frac{\sum_{j<N} h_{Nj} \tilde{p}_j + n_N}{h_{N0}} \right]. \tag{60}$$

In round n, the designer uses the similar adjustment for the intervention coefficients, but in a different order. Specifically, the designer starts from α_n, and adjusts α_1 to α_{n-1} after α_N. The vector of measurement obtained in round n is

$$\mathbf{m}_n = \left[m_{n1}, \ \ldots, \ m_{ni}, \ \ldots, \ m_{nN} \right], \tag{61}$$

where

$$m_{ni} = \begin{cases} \frac{\sum_{j=1}^{i-1} h_{ij}\tilde{p}_j + \sum_{j=i+1}^{n-1} h_{ij}P_j + \sum_{j=n}^{N} h_{ij}\tilde{p}_j + n_i}{h_{i0}}, & i < n \\ \frac{\sum_{j=1}^{n-1} h_{ij}P_j + \sum_{j=n}^{i-1} h_{ij}\tilde{p}_j + \sum_{j=i+1}^{N} h_{ij}P_j + n_i}{h_{i0}}, & i \geq n \end{cases}. \tag{62}$$

Note that we obtain m_{nn} first, and m_{n1} to $m_{n,n-1}$ after m_{nN}. For exposition purpose, we write $\mathbf{m}_n$ in the order of the index of users associated with the measurements.

Given the measurements $\{\mathbf{m}_n\}_{n=1}^N$, we can easily obtain the normalized cross channel gains and the normalized noise powers. From $\mathbf{m}_n$ and $\mathbf{m}_{n+1}$ $\forall\ n < N$, we have

$$m_{n+1,i} - m_{ni} = \frac{h_{in}P_n - h_{in}\tilde{p}_n}{h_{i0}},\ \forall\ i \neq n, \tag{63}$$

from which we can get $\frac{h_{in}}{h_{i0}}$ $\forall\ i \neq n$.

To sum up, we can get $\{\frac{h_{in}}{h_{i0}}\}_{i\neq n}$ according to $\mathbf{m}_n$ and $\mathbf{m}_{n+1}$ for all $n < N$, and get $\{\frac{h_{iN}}{h_{i0}}\}_{i\neq N}$ according to $\mathbf{m}_N$ and $\mathbf{m}_1$. Now that we know all the normalized channel gains, we can get the normalized noise powers $\frac{n_i}{h_{i0}}$ easily.

From the above discussion, we know that in each round of the measurements, the system designer can estimate the channel gains from the users to the intervention device $\{h_{0i}\}_{i=1}^N$. If the intervention device has N receivers at different locations, it can estimate the individual transmit powers in the following way. Denote the channel gains from the users to the nth receiver of the intervention device as $\{h_{0i}^n\}_{i=1}^N$. At each time slot, the aggregate received power at the nth receiver of the intervention device

$$\sum_{j=1}^{N} h_{0j}^n p_j + n_0^n, \tag{64}$$

where n_0^n is the noise power known to the intervention device.

Since the designer knows the values of N different linear combinations of the N individual transmit powers, it can solve the group of N linear equations to obtain the N individual transmit powers.

APPENDIX B
PROOF OF THEOREM 1

We define the best response correspondence of user i by

$$BR_i(f, \mathbf{p}_{-i}) = \arg\max_{p_i \in \mathcal{P}_i} \gamma_i(f, p_i, \mathbf{p}_{-i}). \tag{65}$$

We derive the best response correspondence given a first-order intervention rule f_1^I, $BR_i(f_1^I, \mathbf{p}_{-i})$. Since p_i such that $p_i < p_i^\star$ is strictly dominated by $p_i^\star$, we can restrict attention to $p_i \in [p_i^\star, P_i]$ when computing $BR_i(f_1^I, \mathbf{p}_{-i})$. First, consider user $i \notin \tilde{\mathcal{N}}(\mathbf{p}^\star)$. Since p_i such that $p_i < p_i^\star = P_i$ is strictly dominated, we have $\{p_i^\star\} = BR_i(f_1^I, \mathbf{p}_{-i})$ for all $(f_1^I, \mathbf{p}_{-i})$. Next, consider user $i \in \tilde{\mathcal{N}}(\mathbf{p}^\star)$. We consider two cases depending on $\mathbf{p}_{-i}$ and P_0.

Case 1: $P_0 \leq \sum_{j\neq i} \alpha_j |p_j - p_j^\star|$.

In this case, $f_1^I(p_i, \mathbf{p}_{-i}) = P_0$ for all $p_i \in \mathcal{P}_i$, and thus $BR_i(f_1^I, \mathbf{p}_{-i}) = \{P_i\}$.

Case 2: $P_0 > \sum_{j\neq i} \alpha_j |p_j - p_j^\star|$.

Let

$$\bar{p}_i = p_i^\star + \frac{P_0 - \sum_{j\neq i} \alpha_j |p_j - p_j^\star|}{\alpha_i}. \tag{66}$$

Then $f_1^I(p_i, \mathbf{p}_{-i}) = \sum_{i=1}^N \alpha_i |p_i - p_i^\star|$ if $p_i \leq \bar{p}_i$ and $f_1^I(p_i, \mathbf{p}_{-i}) = P_0$ otherwise.

Case 2-1: $\bar{p}_i \geq P_i$.

In this case, for $p_i \in [p_i^\star, P_i]$ we have

$$\begin{aligned} \gamma_i(f_1^I, \mathbf{p}) &= \frac{h_{ii} p_i}{h_{i0}\left(\sum_{j=1}^N \alpha_j |p_j - p_j^\star|\right) + \sum_{j\neq i} h_{ij} p_j + n_i} &(67)\\ &= \frac{h_{ii} p_i}{\alpha_i p_i h_{i0} + c_i}, &(68)\end{aligned}$$

where

$$c_i = h_{i0}\left(\sum_{j\neq i} \alpha_j |p_j - p_j^\star|\right) + \sum_{j\neq i} h_{ij} p_j + n_i - \alpha_i p_i^\star h_{i0}. \tag{69}$$

Thus, we have

$$BR_i(f_1^I, \mathbf{p}_{-i}) = \begin{cases} \{p_i^\star\}, & \text{if } c_i < 0, \\ [p_i^\star, P_i], & \text{if } c_i = 0, \\ \{P_i\}, & \text{if } c_i > 0. \end{cases} \tag{70}$$

Case 2-2: $\bar{p}_i < P_i$.

In this case, we can show that $\gamma_i(f_1^I, p_i^\star, \mathbf{p}_{-i}) \geq \gamma_i(f_1^I, P_i, \mathbf{p}_{-i})$ if and only if

$$P_0 \geq d_i \triangleq \frac{P_i}{p_i^\star} \sum_{j\neq i} \alpha_j |p_j - p_j^\star| + \frac{(P_i - p_i^\star)(\sum_{j\neq i} h_{ij} p_j + n_i)}{p_i^\star h_{i0}}. \tag{71}$$

Thus, we have

$$BR_i(f_1^I, \mathbf{p}_{-i}) = \begin{cases} \{p_i^\star\}, & \text{if } c_i < 0 \text{ and } d_i < P_0, \\ \{p_i^\star, P_i\}, & \text{if } c_i < 0 \text{ and } d_i = P_0, \\ \{P_i\}, & \text{if } c_i < 0 \text{ and } d_i > P_0, \text{ or } c_i \geq 0. \end{cases} \tag{72}$$

Set $\mathbf{p}_{-i} = \mathbf{p}_{-i}^\star$. Since $P_0 > 0$, we always have Case 2. Thus, $p_i^\star \in BR_i(f_1^I, \mathbf{p}_{-i}^\star)$ if and only if (i) $\bar{p}_i \geq P_i$ and $c_i \leq 0$, or (ii) $\bar{p}_i < P_i$, $c_i < 0$, and $d_i \leq P_0$. With $\mathbf{p}_{-i} = \mathbf{p}_{-i}^\star$, (i) can be rewritten as $P_0 \geq \alpha_i(P_i - p_i^\star)$ and $\alpha_i \geq (\sum_{j\neq i} h_{ij}p_j^\star + n_i)/(p_i^\star h_{i0})$, while (ii) as $P_0 < \alpha_i(P_i - p_i^\star)$, $\alpha_i > (\sum_{j\neq i} h_{ij}p_j^\star + n_i)/(p_i^\star h_{i0})$, and $P_0 \geq (P_i - p_i^\star)(\sum_{j\neq i} h_{ij}p_j^\star + n_i)/(p_i^\star h_{i0})$. These conditions can be written more compactly as $\alpha_i \geq (\sum_{j\neq i} h_{ij}p_j^\star + n_i)/(p_i^\star h_{i0})$ and $P_0 \geq (P_i - p_i^\star)(\sum_{j\neq i} h_{ij}p_j^\star + n_i)/(p_i^\star h_{i0})$, yielding Theorem 1.

APPENDIX C
PROOF OF THEOREM 2

By the proof of Theorem 1, for $i \notin \tilde{\mathcal{N}}(\mathbf{p}^\star)$, we have $\{p_i^\star\} = BR_i(f_1^I, \mathbf{p}_{-i})$ for all $(f_1^I, \mathbf{p}_{-i})$. Also, for $i \in \tilde{\mathcal{N}}(\mathbf{p}^\star)$, $\{p_i^\star\} = BR_i(f_1^I, \mathbf{p}_{-i})$ if and only if (i) $P_0 > \sum_{j\neq i} \alpha_j|p_j - p_j^\star|$, $\bar{p}_i \geq P_i$ and $c_i < 0$, or (ii) $P_0 > \sum_{j\neq i} \alpha_j|p_j - p_j^\star|$, $\bar{p}_i < P_i$, $c_i < 0$, and $d_i < P_0$. These conditions can be written more compactly as $c_i < 0$ and $d_i < P_0$, or equivalently,

$$\alpha_i > \frac{1}{p_i^\star}\sum_{j\neq i} \alpha_j|p_j - p_j^\star| + \frac{\sum_{j\neq i} h_{ij}p_j + n_i}{p_i^\star h_{i0}} \tag{73}$$

and

$$P_0 > \frac{P_i}{p_i^\star}\sum_{j\neq i} \alpha_j|p_j - p_j^\star| + \frac{(P_i - p_i^\star)(\sum_{j\neq i} h_{ij}p_j + n_i)}{p_i^\star h_{i0}}. \tag{74}$$

Since $p_i < p_i^\star$ is strictly dominated for all i, we can restrict attention to power profiles in $\prod_i [p_i^\star, P_i]$ when finding NE. Note that in this region the right-hand sides of (73) and (74) are both increasing in p_j, for $j \neq i$.

Equations (13) and (14) in Theorem 2 mean that (73) and (74) hold for each $i = 1, \ldots, N'$ at $\mathbf{p}_{-i} = (p_1^\star, \ldots, p_{i-1}^\star, P_{i+1}, \ldots, P_N)$. For user 1, (73) and (74) hold at $\mathbf{p}_{-1} = (P_2, \ldots, P_N)$, which implies that $\{p_1^\star\} = BR_1(f_1^I, \mathbf{p}_{-1})$ for all $\mathbf{p}_{-1} \in \prod_{i\neq 1}[p_i^\star, P_i]$. This in turn implies that there cannot exist a NE such that $p_1 \neq p_1^\star$. Applying this argument sequentially to users 2 through N', we obtain that $\{p_i^\star\} = BR_1(f_1^I, \mathbf{p}_{-i}^\star)$ for all i, and that there cannot exist any other NE than $\mathbf{p}^\star$.

APPENDIX D
PROOF OF THEOREM 3

From the proofs of Theorems 1 and 2, we know that, for $i \in \tilde{\mathcal{N}}(\mathbf{p}^\star)$, (19) and (20) imply $\{p_i^\star\} = BR_i(f_1^s, \mathbf{p}_{-i})$ for all $\mathbf{p}_{-i} \in \prod_{j\neq i}[p_j^\star, P_j]$. Since a NE of $\Gamma_{f_1^s}$ must lie in $\prod_i [p_i^\star, P_i]$, we obtain $\{\mathbf{p}^\star\} = \mathcal{E}(f_1^s)$. Also, since $p_i < p_i^\star$ is strictly dominated, we have $p_i^1 \in BR_i(f_1^s, \mathbf{p}_{-i}^0) \subset [p_i^\star, P_i]$ for any $\mathbf{p}^0$. Then $\mathbf{p}^1 \in \prod_i [p_i^\star, P_i]$, and thus $\mathbf{p}^2 = \mathbf{p}^\star$. Also, if $\mathbf{p}^0 \in \prod_i [p_i^\star, P_i]$, we have $\mathbf{p}^1 = \mathbf{p}^\star$. Hence, it remains to show that, for any $\mathbf{p}^\star \in \prod_i (0, P_i]$ such that $\sum_{i=1}^N (P_i - p_i^\star)/P_i < 1$, there exists $(\alpha_1, \ldots, \alpha_N) \in \mathbb{R}_+^N$ satisfying (19) for all $i \in \tilde{\mathcal{N}}(\mathbf{p}^\star)$. Since the result is trivial if $N' \triangleq |\tilde{\mathcal{N}}(\mathbf{p}^\star)| \leq 1$, we assume that $N' \geq 2$.

Note that (19) can be rewritten as

$$p_i^\star \alpha_i - \sum_{j\neq i}(P_j - p_j^\star)\alpha_j > \frac{\sum_{j\neq i} h_{ij} P_j + n_i}{h_{i0}}. \tag{75}$$

Since $p_i^\star = P_i$ for all $i \notin \tilde{\mathcal{N}}(\mathbf{p}^\star)$, the choice of α_i for $i \notin \tilde{\mathcal{N}}(\mathbf{p}^\star)$ is irrelevant to (75), and the inequality (75) for all $i \in \tilde{\mathcal{N}}(\mathbf{p}^\star)$ can be expressed in a matrix form

$$(\mathbf{\Lambda} - \mathbf{1q})\,\alpha > \mathbf{b}, \tag{76}$$

where

$$\mathbf{\Lambda} = \operatorname{diag}(P_1, \ldots, P_{N'}) \in \mathbb{R}^{N'\times N'}, \tag{77}$$

$$\mathbf{1} = \begin{bmatrix} 1 & \cdots & 1 \end{bmatrix}^T \in \mathbb{R}^{N'\times 1}, \tag{78}$$

$$\mathbf{q} = \begin{bmatrix} (P_1 - p_1^\star) & (P_2 - p_2^\star) & \cdots & (P_{N'} - p_{N'}^\star) \end{bmatrix} \in \mathbb{R}^{1\times N'}, \tag{79}$$

$$\alpha = \begin{bmatrix} \alpha_1 & \cdots & \alpha_{N'} \end{bmatrix}^T \in \mathbb{R}^{N'\times 1}, \text{ and} \tag{80}$$

$$\mathbf{b} = \begin{bmatrix} \frac{\sum_{j\neq 1} h_{1j}P_j + n_1}{h_{10}} & \cdots & \frac{\sum_{j\neq N'} h_{N'j}P_j + n_{N'}}{h_{N'0}} \end{bmatrix}^T \in \mathbb{R}^{N'\times 1}. \tag{81}$$

(We use $\mathbf{a} > \mathbf{b}$ to mean $a_i > b_i$ for all i, and $\mathbf{a} \geq \mathbf{b}$ to mean $a_i \geq b_i$ for all i.) We are interested in whether there exists $\alpha \geq 0$ satisfying (76). Since the domain of α is unbounded above and $\mathbf{b} > 0$, there exists $\alpha \geq 0$ satisfying (76) if and only if there exist $\alpha \geq 0$ and $\beta = [\beta_1 \ \cdots \ \beta_{N'}]^T > 0$ satisfying $(\mathbf{\Lambda} - \mathbf{1q})\,\alpha = \beta$.

By the Matrix Inversion Lemma [27], we have

$$(\mathbf{\Lambda} - \mathbf{1q})^{-1} = \mathbf{\Lambda}^{-1} + \mathbf{\Lambda}^{-1}\mathbf{1}(1 - \mathbf{q\Lambda}^{-1}\mathbf{1})^{-1}\mathbf{q\Lambda}^{-1}. \tag{82}$$

Since

$$\mathbf{q\Lambda}^{-1}\mathbf{1} = \sum_{i=1}^{N'} \frac{P_i - p_i^\star}{P_i}, \tag{83}$$

$\mathbf{\Lambda} - \mathbf{1q}$ is invertible if and only if

$$\sum_{i=1}^{N} \frac{P_i - p_i^\star}{P_i} \neq 1. \tag{84}$$

Suppose that (84) holds. Then $\alpha = (\mathbf{\Lambda} - \mathbf{1q})^{-1}\,\beta$ can be expressed as

$$\alpha_i = \frac{\sum_{j=1}^{N} \frac{P_j - p_j^\star}{P_j}\beta_j}{P_i(1 - \sum_{j=1}^{N} \frac{P_j - p_j^\star}{P_j})} + \frac{\beta_i}{P_i} \tag{85}$$

for all $i = 1, \ldots, N'$. If $\sum_{j=1}^{N}(P_j - p_j^\star)/P_j < 1$, then we have $\alpha \geq 0$ (in fact, $\alpha > 0$) for any choice of $\beta > 0$. On the contrary, if $\sum_{j=1}^{N}(P_j - p_j^\star)/P_j > 1$, then for any choice of $\beta > 0$ we cannot find $\alpha \geq 0$ satisfying $(\mathbf{\Lambda} - \mathbf{1q})\,\alpha = \beta$. To see this, let $i' \in \arg\min_i \beta_i$. Then we have

$$\alpha_{i'} = \frac{\sum_{j=1}^{N} \frac{P_j - p_j^\star}{P_j}\beta_j}{P_{i'}(1 - \sum_{j=1}^{N} \frac{P_j - p_j^\star}{P_j})} + \frac{\beta_{i'}}{P_{i'}} = \frac{\frac{\beta_{i'}}{P_{i'}} + \sum_{j=1}^{N} \frac{P_j - \tilde{p}_j}{P_j} \cdot \frac{\beta_j - \beta_{i'}}{P_{i'}}}{1 - \sum_{j=1}^{N} \frac{P_j - p_j^\star}{P_j}} < 0. \tag{86}$$

APPENDIX E

PROOF OF LEMMA 1

Using the proof of Theorem 2, we have $\{\tilde{p}_i^t\} = BR_i(f^t, \tilde{\mathbf{p}}_{-i}^{t-1})$ for $i \in \mathcal{N}(\tilde{\mathbf{p}}^t)$, where $\tilde{\mathbf{p}}_{-i}^{t-1} \in \prod_{j \neq i}[p_j^\star, P_j]$ and

$$f_t(\mathbf{p}) = \left[\sum_{i=1}^{N} \alpha_i^t |p_i - \tilde{p}_i^t|\right]_0^{P_0^t}, \tag{87}$$

if and only if

$$\alpha_i^t > \frac{1}{\tilde{p}_i^t} \sum_{j \neq i} \alpha_j^t (\tilde{p}_j^{t-1} - \tilde{p}_j^t) + \frac{\sum_{j \neq i} h_{ij} \tilde{p}_j^{t-1} + n_i}{\tilde{p}_i^t h_{i0}} \tag{88}$$

and

$$P_0^t > \frac{P_i}{\tilde{p}_i^t} \sum_{j \neq i} \alpha_j^t (\tilde{p}_j^{t-1} - \tilde{p}_j^t) + \frac{(P_i - \tilde{p}_i^t)(\sum_{j \neq i} h_{ij} \tilde{p}_j^{t-1} + n_i)}{\tilde{p}_i^t h_{i0}}. \tag{89}$$

Note that $i \notin \mathcal{N}(\tilde{\mathbf{p}}^t)$ implies $\tilde{p}_i^{t-1} = \tilde{p}_i^t = P_i$. Using the proof of Theorem 3, we can show that the constraint $d(\tilde{\mathbf{p}}^{t-1}, \tilde{\mathbf{p}}^t) = \delta$ guarantees the existence of $\alpha^t = (\alpha_1^t, \ldots, \alpha_N^t) \geq 0$ satisfying (88) for all $i \in \mathcal{N}(\tilde{\mathbf{p}}^t)$. The infimum intervention rate for user i is given by

$$\frac{\sum_{j=1}^{N} \frac{\tilde{p}_j^{t-1} - \tilde{p}_j^t}{\tilde{p}_j^{t-1}} b_j^{t-1}}{\tilde{p}_i^{t-1}\left(1 - \sum_{j=1}^{N} \frac{\tilde{p}_j^{t-1} - \tilde{p}_j^t}{\tilde{p}_j^{t-1}}\right)} + \frac{b_i^{t-1}}{\tilde{p}_i^{t-1}}, \tag{90}$$

where $b_i^{t-1} = (\sum_{j \neq i} h_{ij} \tilde{p}_j^{t-1} + n_i)/h_{i0}$ for all i. Thus, the minimum power budget to achieve $\mathbf{p}^t = \tilde{\mathbf{p}}^t$ is given by

$$\max_{i \in \mathcal{N}(\tilde{\mathbf{p}}^t)} \frac{P_i}{\tilde{p}_i^t} \sum_{j \neq i} \alpha_j (\tilde{p}_j^{t-1} - \tilde{p}_j^t) + \frac{P_i - \tilde{p}_i^t}{\tilde{p}_i^t} b_i^{t-1} \tag{91}$$

$$= \max_{i \in \mathcal{N}(\tilde{\mathbf{p}}^t)} \frac{P_i}{\tilde{p}_i^t} \sum_{j \neq i} \left(\frac{\tilde{p}_j^{t-1} - \tilde{p}_j^t}{\tilde{p}_j^{t-1}} \frac{\sum_{n=1}^{N} \frac{\tilde{p}_n^{t-1} - \tilde{p}_n^t}{\tilde{p}_n^{t-1}} b_n^{t-1}}{1 - \sum_{n=1}^{N} \frac{\tilde{p}_n^{t-1} - \tilde{p}_n^t}{\tilde{p}_n^{t-1}}} + \frac{\tilde{p}_j^{t-1} - \tilde{p}_j^t}{\tilde{p}_j^{t-1}} b_j^{t-1} \right) + \frac{P_i - \tilde{p}_i^t}{\tilde{p}_i^t} b_i^{t-1} \tag{92}$$

The problem to find $\tilde{\mathbf{p}}^t$ can be written as

$$\min_{\tilde{\mathbf{p}}^t} \max_{i\in\mathcal{N}(\tilde{\mathbf{p}}^t)} \frac{P_i}{\tilde{p}_i^t} \sum_{j\neq i} \left(\frac{\tilde{p}_j^{t-1} - \tilde{p}_j^t}{\tilde{p}_j^{t-1}} \frac{\sum_{n=1}^N \frac{\tilde{p}_n^{t-1}-\tilde{p}_n^t}{\tilde{p}_n^{t-1}} b_n^{t-1}}{1 - \sum_{n=1}^N \frac{\tilde{p}_n^{t-1}-\tilde{p}_n^t}{\tilde{p}_n^{t-1}}} + \frac{\tilde{p}_j^{t-1} - \tilde{p}_j^t}{\tilde{p}_j^{t-1}} b_j^{t-1} \right) + \frac{P_i - \tilde{p}_i^t}{\tilde{p}_i^t} b_i^{t-1} \tag{93}$$

$$\text{subject to } \sum_{i=1}^N \frac{\tilde{p}_i^{t-1} - \tilde{p}_i^t}{\tilde{p}_i^{t-1}} = \delta, \tag{94}$$

$$\mathbf{p}^\star \leq \tilde{\mathbf{p}}^t \leq \tilde{\mathbf{p}}^{t-1}. \tag{95}$$

Let $\mu_i = \tilde{p}_i^t / \tilde{p}_i^{t-1}$ for all i. Imposing the constraint (94), the objective function in (93) can be simplified to

$$\max_{i\in\mathcal{N}(\tilde{\mathbf{p}}^t)} \frac{P_i}{\tilde{p}_i^{t-1}} \frac{\sum_{j=1}^N (1-\mu_j) b_j^{t-1}}{1-\delta} + \left(\frac{P_i - \tilde{p}_i^{t-1}}{\tilde{p}_i^{t-1}} \right) b_i^{t-1} \tag{96}$$

Since $\tilde{p}_i^{t-1} = P_i$ for all $i \notin \mathcal{N}(\tilde{\mathbf{p}}^t)$, we have

$$\max_{i\in\mathcal{N}(\tilde{\mathbf{p}}^t)} \frac{P_i}{\tilde{p}_i^{t-1}} \frac{\sum_{j=1}^N (1-\mu_j) b_j^{t-1}}{1-\delta} + \left(\frac{P_i - \tilde{p}_i^{t-1}}{\tilde{p}_i^{t-1}} \right) b_i^{t-1} \tag{97}$$

$$= \max_{i\in\mathcal{N}} \frac{P_i}{\tilde{p}_i^{t-1}} \frac{\sum_{j=1}^N (1-\mu_j) b_j^{t-1}}{1-\delta} + \left(\frac{P_i - \tilde{p}_i^{t-1}}{\tilde{p}_i^{t-1}} \right) b_i^{t-1}. \tag{98}$$

Hence, the problem (93)–(95) is equivalent to the LP problem (23)–(25), and the result follows.

Appendix F
Proof of Theorem 4

Suppose that $p_i^\star = P_i$. Since $\tilde{p}_i^1 = P_i$, the constraint $p_i^\star \leq \tilde{p}_i^t \leq \tilde{p}_i^{t-1}$ implies $\tilde{p}_i^t = P_i$ for all $t \geq 1$. Suppose that Algorithm 2 moves on to step $t \geq 2$. This implies that there exists i such that $\tilde{p}_i^{t-1} > p_i^\star$. Let $\mathcal{N}^{t-1} = \{i \in \mathcal{N} : \tilde{p}_i^{t-1} > p_i^\star\}$. Suppose that there is no $i \in \mathcal{N}^{t-1}$ such that $\tilde{p}_i^t = p_i^\star$. Then by construction, we have $\mu_i > p_i^\star / \tilde{p}_i^{t-1}$ for some i and $\mu_i = 1$ for all other i. Then $\sum_{i=1}^N \mu_i > N - 1 + p_i^\star/\tilde{p}_i^{t-1} \geq N - 1 + \min_i (p_i^\star / P_i) \geq N - \delta$, which contradicts the constraint $\sum_{i=1}^N \mu_i = N - \delta$. Hence, we have $|\mathcal{N}^t| \leq |\mathcal{N}^{t-1}| - 1$. Since $|\mathcal{N}^1| = N'$, Algorithm 2 must terminate at step $N'+1$ or before.

Choose any $\mathbf{p}^0 \in \mathcal{P}$. Choose f^1 such that $f^1(\mathbf{p}) = 0$ for all $\mathbf{p}$. Then $\{P_i\} = BR_i(f^1, \mathbf{p}_{-i}^0)$ for all i, and thus we obtain $\mathbf{p}^1 = \mathbf{P} = \tilde{\mathbf{p}}^1$. Now consider $t = 2, \ldots, T-1$. By the proof of Lemma 1, we can find α^t and P_0^t in f^t that yield $\{\tilde{p}_i^t\} = BR_i(f^t, \tilde{\mathbf{p}}_{-i}^{t-1})$. Since $\mathbf{p}^1 = \tilde{\mathbf{p}}^1$, we have $\mathbf{p}^t = \tilde{\mathbf{p}}^t$ for all $t = 2, \ldots, T-1$. Lastly, for $t = T$, we have

$$\sum_{i=1}^N \frac{p_i^{T-1} - \tilde{p}_i^T}{p_i^{T-1}} = \sum_{i=1}^N \frac{\tilde{p}_i^{T-1} - p_i^\star}{\tilde{p}_i^{T-1}} < 1, \tag{99}$$

which leads to $\mathbf{p}^T = \tilde{\mathbf{p}}^T$.

APPENDIX G

PROOF OF THEOREM 5

We construct a geometric sequence of T intermediate target power profiles as follows

$$\tilde{p}_i^t = (\eta_i)^{t-1} P_i,\ \forall\ i,\ t = 1, \ldots, T, \tag{100}$$

where $\eta_i = (p_i^\star / P_i)^{\frac{1}{T-1}},\ i = 1, \ldots, N$.

According to Theorem 3, we know that the minimum power budget to reach the final target power profile in T time slots and to sustain all the $\{\tilde{\mathbf{p}}^t\}_{t=1}^T$ as the best response can be written as

$$P_{0,T} = \max_{1 \le t \le T} \max_i \frac{P_i}{\tilde{p}_i^{t-1}} \frac{\sum_{j=1}^N \frac{\tilde{p}_j^{t-1} - \tilde{p}_j^t}{\tilde{p}_j^{t-1}} b_j^{t-1}}{1 - \sum_{j=1}^N \frac{\tilde{p}_j^{t-1} - \tilde{p}_j^t}{\tilde{p}_j^{t-1}}} + \frac{P_i - \tilde{p}_i^{t-1}}{\tilde{p}_i^{t-1}} b_i^{t-1} \tag{101}$$

Given the power budget P_0, the minimum convergence time T should satisfy

$$P_{0,T} \le P_0 < P_{0,T-1}. \tag{102}$$

Hence, we consider the geometric sequence with $T-1$ intermediate target power profiles

$$\tilde{p}_i^t = (\eta_i)^{t-1} P_i,\ \forall\ i,\ t = 1, \ldots, T-1, \tag{103}$$

where $\eta_i = (p_i^\star / P_i)^{\frac{1}{T-2}},\ i = 1, \ldots, N$. We have

$$P_0 < P_{0,T-1} = \max_{1 \le t \le T-1} \max_i \frac{P_i}{\tilde{p}_i^{t-1}} \frac{\sum_{j=1}^N \frac{\tilde{p}_j^{t-1} - \tilde{p}_j^t}{\tilde{p}_j^{t-1}} b_j^{t-1}}{1 - \sum_{j=1}^N \frac{\tilde{p}_j^{t-1} - \tilde{p}_j^t}{\tilde{p}_j^{t-1}}} + \frac{P_i - \tilde{p}_i^{t-1}}{\tilde{p}_i^{t-1}} b_i^{t-1} \tag{104}$$

$$= \max_{1 \le t \le T-1} \max_i \frac{1}{(\eta_i)^{t-1}} \frac{\sum_{j=1}^N (1-\eta_j) b_j^{t-1}}{1 - \sum_{j=1}^N (1-\eta_j)} + \left(\frac{1}{(\eta_i)^{t-1}} - 1\right) b_i^{t-1} \tag{105}$$

$$\le \max_{1 \le t \le T-1} \max_i \frac{1}{(\min_n \eta_n)^{t-1}} \frac{\sum_{j=1}^N (1-\eta_j) b_j^{t-1}}{1 - \sum_{j=1}^N (1-\eta_j)} + \left(\frac{1}{(\min_n \eta_n)^{t-1}} - 1\right) b_i^{t-1} \tag{106}$$

Since

$$\max_i \frac{1}{(\min_n \eta_n)^{t-1}} \frac{\sum_{j=1}^N (1-\eta_j) b_j^{t-1}}{1 - \sum_{j=1}^N (1-\eta_j)} + \left(\frac{1}{(\min_n \eta_n)^{t-1}} - 1\right) b_i^{t-1} \tag{107}$$

$$= \max_i \frac{1}{(\min_n \eta_n)^t} \frac{\sum_{j=1}^N (1-\eta_j) \left(b_j^{t-1} \cdot \min_n \eta_n\right)}{1 - \sum_{j=1}^N (1-\eta_j)} + \left(\frac{1}{(\min_n \eta_n)^t} - \frac{1}{(\min_n \eta_n)}\right) \left(b_i^{t-1} \cdot \min_n \eta_n\right)$$

$$< \max_i \frac{1}{(\min_n \eta_n)^t} \frac{\sum_{j=1}^N (1-\eta_j) b_j^t}{1 - \sum_{j=1}^N (1-\eta_j)} + \left(\frac{1}{(\min_n \eta_n)^t} - 1\right) b_i^t \tag{108}$$

$$\tag{109}$$

we have

$$P_0 < \max_i \frac{1}{(\min_n \eta_n)^{T-2}} \frac{\sum_{j=1}^N (1-\eta_j) b_j^{T-2}}{1-\sum_{j=1}^N (1-\eta_j)} + \left(\frac{1}{(\min_n \eta_n)^{T-2}} - 1 \right) b_i^{T-2} \tag{110}$$

$$= \left(\max_n \frac{P_n}{p_n^\star} \right) \frac{\sum_{j=1}^N (1-\eta_j) b_j^{T-2}}{1-\sum_{j=1}^N (1-\eta_j)} + \left(\max_n \frac{P_n}{p_n^\star} - 1 \right) \left(\max_i b_i^{T-2} \right) \tag{111}$$

$$< \left(\max_n \frac{P_n}{p_n^\star} \right) \frac{\sum_{j=1}^N (1-\eta_j)}{1-\sum_{j=1}^N (1-\eta_j)} \left(\max_i b_i^{T-2} \right) + \left(\max_n \frac{P_n}{p_n^\star} - 1 \right) \left(\max_i b_i^{T-2} \right) \tag{112}$$

$$= \left(\max_n \frac{P_n}{p_n^\star} \right) \left(\max_i \frac{\sum_{j \neq i} h_{ij} p_j^\star + n_i}{h_{i0}} \right) \left(\frac{1}{1-\sum_{j=1}^N (1-\eta_j)} - \frac{1}{\max_n \frac{P_n}{p_n^\star}} \right). \tag{113}$$

Solving $\sum_{i=1}^N \left(\frac{p_i^\star}{P_i} \right)^{\frac{1}{T-2}} = \sum_{i=1}^N \eta_i$ in the above inequality, we can get the upper bound on T in Theorem 5.

Since we need $\frac{1}{C} < 1$, the power budget in the upper bound should satisfy

$$P_0 > \left(\max_i \frac{P_i}{p_i^\star} - 1 \right) \max_i \frac{\sum_{j \neq i} h_{ij} p_j^\star + n_i}{h_{i0}}. \tag{114}$$

Appendix H

Proof of Theorem 6

Note that f_1^A in (37) can be rewritten as

$$f_1^A(\mathbf{p}) = \left[\left| \sum_{i=1}^N \alpha_0 h_{0i} (p_i - p_i^\star) \right| \right]_0^{P_0}. \tag{115}$$

First, consider user $i \notin \tilde{\mathcal{N}}(\mathbf{p}^\star)$. Since $f_1^A(p_i, \mathbf{p}_{-i}^\star)$ is weakly increasing in p_i, regardless of α_0 and P_0, we have $\{p_i^\star\} = BR_i(f_1^A, \mathbf{p}_{-i}^\star)$ for all f_1^A. Next, consider user $i \in \tilde{\mathcal{N}}(\mathbf{p}^\star)$. Using a similar argument to that in the proof of Theorem 1, we can show that $p_i^\star \in BR_i(f_1^A, \mathbf{p}_{-i}^\star)$ if and only if

$$\alpha_0 h_{0i} \geq \frac{\sum_{j \neq i} h_{ij} p_j^\star + n_i}{p_i^\star h_{i0}} \tag{116}$$

and

$$P_0 \geq \frac{(P_i - p_i^\star)(\sum_{j \neq i} h_{ij} p_j^\star + n_i)}{p_i^\star h_{i0}}, \tag{117}$$

which yields Theorem 6.

Appendix I

Proof of Theorem 7

Let $i, j \in \tilde{\mathcal{N}}(\mathbf{p}^\star)$ be the indexes of users that satisfy the condition in Theorem 7. Define $\tilde{\mathbf{p}}$ by $\tilde{p}_i = p_i^\star + \delta$, $\tilde{p}_j = p_j^\star - (h_{0i}/h_{0j})\delta$, and $\tilde{p}_k = p_k^\star$ for all $k \neq i, j$, for some $\delta > 0$. This choice of $\tilde{\mathbf{p}}$ yields

$\sum_{i=1}^{N} h_{0i}p_i^\star = \sum_{i=1}^{N} h_{0i}\tilde{p}_i$. Note that, for all $i \in \tilde{\mathcal{N}}(\mathbf{p}^\star)$, both α_0^i and P_0^i, evaluated at $\mathbf{p}^\star$, are strictly decreasing in $p_i^\star$ and strictly increasing in $p_j^\star$ for all $j \neq i$. Thus, α_0^i and P_0^i are smaller while α_0^j and P_0^j are larger at $\tilde{\mathbf{p}}$ than at $\mathbf{p}^\star$. Also, α_0^k and P_0^k, for $k \in \tilde{\mathcal{N}}(\mathbf{p}^\star) \setminus \{i, j\}$, have the same values at $\tilde{\mathbf{p}}$ and at $\mathbf{p}^\star$. We can choose δ sufficiently small so that $\tilde{p}_i < P_i$, $\tilde{p}_j > 0$, and $\bar{\alpha}_0$ and $\bar{P}_0$ are no larger at $\tilde{\mathbf{p}}$ than at $\mathbf{p}^\star$. Hence, if f_1^A sustains $\mathbf{p}^\star$, it must sustain $\tilde{\mathbf{p}}$ as well. Lastly, for a given $\epsilon > 0$, we can choose δ sufficiently small so that $|\tilde{\mathbf{p}} - \mathbf{p}^\star| < \epsilon$.

APPENDIX J
PROOF OF THEOREM 8

To prove this theorem, we first derive the best response of the users, and then derive the conditions under which the best response is the target power profile and under which there is no performance loss by the transmission from the device.

The expected utility of user i is

$$\Phi\left(\bar{y} - \sum_{j\neq i} h_{0j}p_j^\star - h_{0i}p_i\right)\gamma_i(0, p_i, \mathbf{p}_{-i}^\star) + \left[1 - \Phi\left(\bar{y} - \sum_{j\neq i} h_{0j}p_j^\star - h_{0i}p_i\right)\right]\gamma_i(\hat{p}_0, p_i, \mathbf{p}_{-i}^\star). \quad (118)$$

To simplify the notation, we define

$$a_i = \bar{y} - \sum_{j\neq i} h_{0j}p_j^\star \quad (119)$$

and

$$C_{i1} = \frac{h_{ii}}{\sum_{j\neq i} h_{ij}p_j^\star + n_i},\ C_{i2} = \frac{h_{ii}}{h_{i0}\hat{p}_0 + \sum_{j\neq i} h_{ij}p_j^\star + n_i}. \quad (120)$$

Combined with the fact that the cumulative distribution function of a uniformly distributed random variable in $[-\theta_0, \theta_0]$ is $\Phi(x) = \left[\frac{x}{\theta_0} + \frac{1}{2}\right]_0^1$, we can write expected utility of user i as

$$\left[\frac{a_i - h_{0i}p_i}{\theta_0} + \frac{1}{2}\right]_0^1 \cdot C_{i1} \cdot p_i + \left(1 - \left[\frac{a_i - h_{0i}p_i}{\theta_0} + \frac{1}{2}\right]_0^1\right) \cdot C_{i2} \cdot p_i. \quad (121)$$

When $a_i - h_{0i}p_i < -\theta_0$, namely $p_i > \frac{a_i+\theta_0}{h_{0i}}$, the expected utility is $C_{i2} \cdot p_i$, which is linearly increasing in p_i with slope C_{i2}. When $-\theta_0 \leq a_i - h_{0i}p_i \leq \theta_0$, namely $\frac{a_i-\theta_0}{h_{0i}} \leq p_i \leq \frac{a_i+\theta_0}{h_{0i}}$, the expected utility is

$$\frac{h_{0i}(C_{i2} - C_{i1})}{2\theta_0} \cdot p_i^2 + \left[\frac{C_{i1} + C_{i2}}{2} + \frac{a_i(C_{i1} - C_{i2})}{2\theta_0}\right] \cdot p_i, \quad (122)$$

which is a concave parabola with vertex at $p_i = \frac{a_i}{2h_{0i}} + \frac{\theta_0}{2h_{0i}} \cdot \frac{C_{i1}+C_{i2}}{C_{i1}-C_{i2}}$. When $a_i - h_{0i}p_i > \theta_0$, namely $p_i < \frac{a_i-\theta_0}{h_{0i}}$, the expected utility is $C_{i1} \cdot p_i$, which is linearly increasing in p_i with slope C_{i1}.

Depending on which region the vertex of the parabola lies in, we discuss the best response of user i in three cases.

Case 1: $\frac{a_i}{2h_{0i}} + \frac{\theta_0}{2h_{0i}} \cdot \frac{C_{i1}+C_{i2}}{C_{i1}-C_{i2}} \leq \frac{a_i-\theta_0}{h_{0i}}$.

In this case, the utility increases linearly with slope C_{i1} when $p_i < \frac{a_i-\theta_0}{h_{0i}}$, decreases on the parabola when $\frac{a_i-\theta_0}{h_{0i}} \leq p_i \leq \frac{a_i+\theta_0}{h_{0i}}$, and increases with slope C_{i2} when $p_i > \frac{a_i+\theta_0}{h_{0i}}$. Hence, the best response of user i is $\frac{a_i-\theta_0}{h_{0i}}$ or P_i.

Case 2: $\frac{a_i-\theta_0}{h_{0i}} < \frac{a_i}{2h_{0i}} + \frac{\theta_0}{2h_{0i}} \cdot \frac{C_{i1}+C_{i2}}{C_{i1}-C_{i2}} < \frac{a_i+\theta_0}{h_{0i}}$.

In this case, the utility increases linearly with slope C_{i1} when $p_i < \frac{a_i-\theta_0}{h_{0i}}$, increases on the parabola on the left of the axis of symmetry, then decreases on it on the right, and increases with slope C_{i2} when $p_i > \frac{a_i+\theta_0}{h_{0i}}$. Hence, the best response of user i is $\frac{a_i}{2h_{0i}} + \frac{\theta_0}{2h_{0i}} \cdot \frac{C_{i1}+C_{i2}}{C_{i1}-C_{i2}}$ or P_i.

Case 3: $\frac{a_i}{2h_{0i}} + \frac{\theta_0}{2h_{0i}} \cdot \frac{C_{i1}+C_{i2}}{C_{i1}-C_{i2}} \geq \frac{a_i+\theta_0}{h_{0i}}$.

In this case, the utility increases linearly with slope C_{i1} when $p_i < \frac{a_i-\theta_0}{h_{0i}}$, increases on the parabola when $\frac{a_i-\theta_0}{h_{0i}} \leq p_i \leq \frac{a_i+\theta_0}{h_{0i}}$, and increases with slope C_{i2} when $p_i > \frac{a_i+\theta_0}{h_{0i}}$. Hence, the best response of user i is P_i.

From the discussion on the three cases, we know that for user $i \in \tilde{\mathcal{N}}(\mathbf{p}^\star)$, the best response can be $p_i^\star$ only in case 1 and case 2. However, in case 2, suppose the best response is $p_i^\star = \frac{a_i}{2h_{0i}} + \frac{\theta_0}{2h_{0i}} \cdot \frac{C_{i1}+C_{i2}}{C_{i1}-C_{i2}}$. Then we have $\frac{a_i-\theta_0}{h_{0i}} < p_i^\star < \frac{a_i+\theta_0}{h_{0i}}$, which gives us

$$-\theta_0 < a_i - h_{0i}p_i^\star < \theta_0. \tag{123}$$

Hence, in case 2, if the best response of user $i \in \tilde{\mathcal{N}}(\mathbf{p}^\star)$ is $p_i^\star$, the probability of transmission from the intervention device $\Phi(a_i - h_{0i}p_i)$ is larger than 0.

The only case in which the best response of user i can be $p_i^\star < P_i$ and the transmission probability can be 0 is case 1. The sufficient and necessary condition for the best response to be $p_i^\star = \frac{a_i-\theta_0}{h_{0i}}$ is

$$C_{i1}p_i^\star = C_{i1}\frac{a_i-\theta_0}{h_{0i}} > C_{i2}P_i, \tag{124}$$

which gives us

$$\hat{p}_0 > \frac{P_i - p_i^\star}{p_i^\star} \cdot \frac{\sum_{j\neq i} h_{ij}p_j^\star + n_i}{h_{i0}}. \tag{125}$$

The constraint that $\frac{a_i}{2h_{0i}} + \frac{\theta_0}{2h_{0i}} \cdot \frac{C_{i1}+C_{i2}}{C_{i1}-C_{i2}} \leq \frac{a_i-\theta_0}{h_{0i}}$ gives us

$$(a_i - 3\theta_0) \cdot C_{i1} \geq (a_i - \theta_0) \cdot C_{i2} \tag{126}$$

$$\Rightarrow \quad a_i > 3\theta_0 \text{ and } (a_i - 3\theta_0) \cdot \left(h_{i0}\hat{p}_0 + \sum_{j\neq i} h_{ij}p_j^\star + n_i \right) \geq (a_i - \theta_0) \cdot \left(\sum_{j\neq i} h_{ij}p_j^\star + n_i \right). \tag{127}$$

To conclude, the sufficient and necessary condition for the best response of user $i \in \tilde{\mathcal{N}}(\mathbf{p}^\star)$ to be $p_i^\star$ is

$$a_i > 3\theta_0 \Rightarrow h_{0i}p_i^\star = a_i - \theta_0 > 2\theta_0 \tag{128}$$

and

$$\hat{p}_0 \geq \max\left\{\frac{2\theta_0}{h_{0i}p_i^\star - 2\theta_0},\ \frac{P_i - p_i^\star}{p_i^\star}\right\} \cdot \frac{\sum_{j\neq i} h_{ij}p_j^\star + n_i}{h_{i0}}. \tag{129}$$

For user $j \in \mathcal{N} \setminus \tilde{\mathcal{N}}(\mathbf{p}^\star)$, if the above conditions hold, user j's expected utility will be

$$\frac{h_{jj}p_j}{\sum_{k\neq j} h_{jk}p_k^\star + n_j}. \tag{130}$$

Hence, the best response of user j will be P_j.

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