On the Correlated Equilibrium Selection for Two-User Channel Access Games

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***Abstract*—In this letter, we consider a simple two-user channel access game and investigate how to improve the performance of Nash equilibrium (NE) by employing a correlated device that coordinates the users' actions and leads them to play a higher efficiency correlated equilibrium (CE). Unlike existing papers, we discuss both the public and private CEs and quantify their performances in various simple, but illustrative scenarios. Moreover, we propose a simple procedure for the correlated device to select the CE leading to the highest payoff, without the need for the users to report their private utility functions.**

***Index Terms*—Channel access game, equilibrium selection.**

I. Introduction

THE channel access problem has long been an important research topic for multiuser wireless communication networks. Most media-access-control (MAC) protocols in existing networks need to address this problem to let multiple users share the same physical medium. Game theory [1], especially noncooperative game theory, has been extensively used as a tool for modeling and analyzing this problem in different settings, e.g., ALOHA systems [2], [3], and general back-off-based random access MAC protocols [4]. A plethora of research works have been devoted to designing channel access algorithms to achieve high throughput in decentralized multiuser wireless communication environments, using game theoretic solutions. Most of these works focused on the concept of Nash equilibrium (NE) and provide solutions showing the existence of this equilibrium in specific channel access games. However, it is well known that the NE does not always lead to the best performances for the users. Hence, other equilibrium concepts and the information requirements to reach such equilibriums need to be investigated.

In this letter, we will focus on studying the performance and implementation of the correlated equilibrium (CE) [5] for a simple two-user channel access game. Even though the set-up is simple, this study can provide important insights for equilibrium selection and implementation in more complicated multiuser communication scenarios. A CE is a randomized play from which no user has the incentive to deviate. It is a more general equilibrium concept than the NE and has been recently used in characterizing several wireless networking problems [6], [7]. In this letter, motivated by these works, our focus will be on discussing the information availability which is required to reach CEs. We also quantify the performances of different CEs in various simple, but illustrative channel access scenarios. Also, unlike prior work, we discuss both public and private CEs and propose a solution for *selecting* the CE with the highest payoffs.

In the following parts of this letter, we will first give a game theoretic formulation of the channel access problem in Section II; then in Section III, we illustrate how users can achieve a higher performance by using CE than that attained at the mixed NE. We also quantify the performance of different CEs in various channel access scenarios. In addition, we discuss in Section IV how the correlated device can select the CE that leads to the highest payoff. We also show how the correlated device can be implemented in networks for coordinating among users, which enables them to reach CEs, in Section V. Finally, conclusions are drawn in Section VI.

II. Model for Two-User Channel Access Games

A. Considered Network Scenario

We consider a simple channel access scenario, where there are two users trying to access a time-slotted network. During a single time slot, each user can either transmit a packet or wait. If two users transmit simultaneously, collision will occur and both transmissions will fail. To reduce the probability of contention, back-off-based random access MAC protocol is usually implemented as a coordination solution in such networks. For example, in a slotted ALOHA system [8], when a collision occurs, the user will transmit again only after a random period of time, which is randomly chosen from the range of 0 to its maximum waiting time. However, if users are competing for the channel in such a back-off-based random access network, a self-interested user will intend to apply a shorter maximum waiting time in order to increase its transmission opportunity. If both users behave selfishly, this behavior will actually decrease both users' probabilities of successful transmission. In the remaining part of this letter, we will use game theory to model, analyze, and characterize the behaviors of self-interested users in a simple, but illustrative random access communication scenario.

B. Game Theoretic Formulation

Many research works (e.g., [3]) on random access networks model the strategic interactions among the users as a repeated game, with each time slot of the network being a stage of the game. In this letter, however, we will characterize the interaction between users using a one-shot game, by assuming that the network has reached the steady state in the duration of the game.

The one-shot channel access game can be abstracted as a matrix game $\Omega = (\mathcal{P}, \mathcal{A}, \mathcal{U})$, where $\mathcal{P} = \{P_0, P_1\}$ is the set of players, and $\mathcal{A}$ is the action space of the users. We define the actions of the user to be a set of parameters in the random access protocol. For example, they may be the parameters (e.g., the

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TABLE I
UTILITIES OF THE CHANNEL ACCESS GAME

User P_1 / User P_0	A_L	A_H
A_L	$(u^\star, u^\star)$	(u_0, u_1)
A_H	(u_1, u_0)	$(0, 0)$

maximum waiting time or retransmission probability in a slotted ALOHA protocol) representing different levels of aggressiveness. The mixed strategy of P_i is a probability distribution $\mathcal{S}_i$ over its action space $\mathcal{A}$, and every entry of $\mathcal{S}(a)$, with $a \in \mathcal{A}$, is the probability that user will play action a. To simplify our analysis, we further assume that each user has two possible actions, i.e., $\mathcal{A} = \{0, 1\} = \{A_L, A_H\}$, with A_H being the more aggressive random access protocol (e.g., having a shorter maximum waiting time or higher retransmission probability).

We let a_i be the action of P_i and a_{-i} be the action of his opponent. The utility vector function $\mathcal{U}$ is a mapping from the two users' joint action to a two-dimensional real vector, of which each element is the utility of a user under this joint action, i.e., $\mathcal{U} : \mathcal{A} \times \mathcal{A} \mapsto \mathbb{R}_+^2$. The users are assumed to be always saturated, i.e., they always have packets to send. Since we suppose the network has reached the steady state, the utility of the user is the probability of his successful transmission in the steady state. In this letter, we assume that the users play the game without knowing the utility of the other user.

Hence, we can derive the utilities of the users in this game from the steady-state analysis of the random access network. Generally, the utilities of the users are as in Table I, with $0 < u_0 < u_1$ and $0 < u^\star < u_1$. (An example of networks with utilities in this form can be found in [9].) Note that in order to make the analysis in Section III simple, we assume that the payoff when both users take action A_H is zero. However, if this payoff is $0 < \epsilon < u_0$, we can still apply a similar analysis, as long as $\epsilon < u_0 < u_1$ and $\epsilon < u^\star < u_1$ hold.

III. CHARACTERIZING DIFFERENT STRATEGIES IN THE CHANNEL ACCESS GAME

The channel access game in Table I has three NEs: two pure NEs and one mixed NE. We will first characterize these NEs under different information availabilities. Then, we will present a new solution based on CE and quantify its performance under different channel access scenarios.

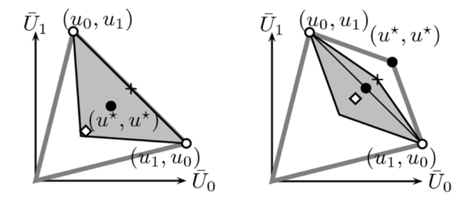
A. Nash Equilibriums of the Channel Access Game

1) No Available Information About the Other User:

- *Strategy*: A reasonable strategy for the user without any information about the other user is to maximize his minimal expected payoff. For example, $\mathcal{S}_0 = \arg\max_{\mathcal{S}} \min \mathrm{E}_{\mathcal{S}}(U_0)$ gives the max-min utility for P_0, which leads to

$$\mathcal{S}_0 = (S_0(0), S_0(1)) = \left(\frac{u_0}{u_0 + u_1 - u^\star}, \frac{u_1 - u^\star}{u_0 + u_1 - u^\star}\right). \tag{1}$$

- *Payoff*: If both users play based on this strategy, it will result in the mixed NE of the game [1]. The average payoffs for both users are the same, which is $(u_0 u_1 / u_0 + u_1 - u^\star)$. We note that the average payoff of the mixed NE may even be lower than u_0, if $u^\star < u_0$. Thus, in the mixed NE, although no information exchange is required between the users, the user may get an expected payoff less than what he could get from the pure NEs, which are discussed next.



(a) $u^\star \le \frac{1}{2}(u_0 + u_1)$ (b) $u^\star > \frac{1}{2}(u_0 + u_1)$

Fig. 1. Utilities of various equilibriums of the channel access game under different payoff functions. Pure NE: ($\circ$). Mixed NE: ($\diamond$). CEs: (gray polygon). The symmetric CEs at utility frontier: ($+$).

2) Explicit Information Exchange Between Users:

- *Strategy*: When there is explicit information exchange between users, they can both know exactly what action the other user is now choosing, and each user chooses his strategy as the best response to the other user's action. In this case, the best-response strategy is $\mathcal{S}_i(a_i) = 1$ if $a_i \neq a_{-i}$, and $\mathcal{S}_i(a_i) = 0$, otherwise.
- *Payoff*: If both users employ this strategy in the channel access game, they will end up with any one of the two pure NEs [1], if they start in either one of them; or they will oscillate between the two non-equilibrium points. We also note that in either pure NE, one user always has more opportunities to access to the network, because he uses the more aggressive protocol. The users' performances at both pure and mixed NEs are illustrated in Fig. 1.

Hence, a channel access algorithm which was designed only for "reaching these various NEs" is not desirable for the following reasons. First, it is because the mixed NE yields an expected payoff which may be even smaller than u_0 if $u^\star < u_0$ and, thus, not efficient compared to either of the pure NEs. On the other hand, the pure NEs will give individual users' higher payoffs, but neither of them is fair to the user who always uses the less aggressive action. Therefore, we next consider alternative solutions, beyond the mixed and pure NEs.

B. Channel Access Game Using a Correlated Device

Another solution for the simple channel access game described in the previous section can be developed using the concept of correlated equilibrium [5]. For the general form of channel access game as in Table I and utility vector function $\mathcal{U} = (\mathcal{U}_0, \mathcal{U}_1)$, where $\mathcal{U}_k(i, j)$ is P_k's payoff when the joint action is (i, j), $i, j, k \in \{0, 1\}$, we let $p_{i,j}$ denote the joint probability distribution of P_0 choosing action i and P_1 choosing action j. A CE $p_{i,j}$ is a joint probability distribution which satisfies

$$\begin{aligned} \sum_{i,j} p_{i,j} \mathcal{U}_0(i,j) &\ge \sum_{i,j} p_{i,j} \mathcal{U}_0(1-i,j) \\ \sum_{i,j} p_{i,j} \mathcal{U}_1(i,j) &\ge \sum_{i,j} p_{i,j} \mathcal{U}_1(i,1-j). \end{aligned} \tag{2}$$

All the CEs of the game form a convex set. For the specific game of Table I, it is easy to verify that the joint distribution of $p_{0,1} = p_{1,0} = (1/2)$ and $p_{0,0} = p_{1,1} = 0$ is a CE, and both users will receive an average payoff of $(u_0 + u_1)/2$, which may be higher than their expected payoff from the mixed NE.

TABLE II
INFORMATION AND THE ASSOCIATED EQUILIBRIUM

Equilibriums	Information availability
Mixed Nash equilibrium	no information
Pure Nash equilibrium	explicit information
Correlated equilibrium	via the correlated device

To achieve CEs usually requires some correlated device who generates a randomized output and sends "suggestion" signals to both users according to his randomized output. Given a specific suggestion from this device, neither user will have the incentive to deviate from it, because any such deviation will result in a lower payoff. We now first characterize the information availability required to reach the CE with $p_{0,1} = p_{1,0} = (1/2)$ and $p_{0,0} = p_{1,1} = 0$.

- *Information exchange via a correlated device*: To enable the two users to play the CE, we can let o_r be the randomized output of the correlated device, with probability distribution of $p(o_r = 0) = p(o_r = 1) = 1/2$. We define the suggestion signal P_i gets as o_i, and the belief of P_i to be $b_i = P(a_{-i}|o_i)$, i.e., the probability of the other user's action conditioned on the suggestion he gets. Suppose both users get the public suggestion, which means $o_0 = o_1 = o_r$. We assume P_0 has the following belief about P_1: $b_0(a_1 = A_L|o_0 = 0) = 1$, $b_0(a_1 = A_H|o_0 = 1) = 1$, and P_1 has following belief about P_0: $b_1(a_0 = A_H|o_1 = 0) = 1$, $b_1(a_0 = A_L|o_1 = 1) = 1$.
- *Strategy*: With these beliefs, each user makes his best response according to the public suggestion signal from the correlated device. When $o_r = 0$, P_0 believes P_1 will choose the less aggressive action; therefore, he will decide to be more aggressive, while P_1 will choose to be less aggressive because his belief tells that P_0 will choose the more aggressive action.
- *Payoff*: After observing the output of the correlated device and assuming that the other user will play according to the suggestion given by the randomized output (which are also their beliefs), both users will have no incentive to deviate from the suggested action because this would cause more collisions and give them lower payoff. Thus, they will both receive an average payoff of $(u_0 + u_1)/2$. The channel access game with different information availabilities and resulted equilibriums is summarized in Table II.

C. Correlated Equilibriums Under Different Utility Functions

The choice of CE and NE is highly dependent on the network conditions (e.g., different maximum waiting time or retransmission probability the users are choosing), which is represented by the utility function or payoff table. We will consider how the CE should be chosen for different payoff tables, especially when $u^\star$ varies.

1) Low Payoff When Simultaneously Choosing the Less Aggressive Action [$u^\star \leq (1/2)(u_0 + u_1)$, Fig. 1(a)]: In this scenario, the CE achieved in Section III-B is Pareto-optimal and has a higher payoff than the mixed NE, as shown in Fig. 1(a).

2) High Payoff When Simultaneously Choosing the Less Aggressive Action [$u^\star > (1/2)(u_0 + u_1)$, Fig. 1(b)]: If $u^\star > (1/2)(u_0 + u_1)$, all the pure and mixed NEs still remain the same, but the CE achieved in Section III-B, which has an average payoff of $(1/2)(u_0 + u_1)$, will no longer be Pareto-optimal. Actually, it may be even inferior to the average payoff of mixed NE, when $u^\star > \left(u_0^2 + u_1^2/(u_0 + u_1)\right)$. However, we notice that the symmetric CE (i.e., $p_{0,1} = p_{1,0}$) with the highest payoffs will have the following form:

$$p_{0,1} = p_{1,0} = \alpha, \quad p_{0,0} = 1 - 2\alpha, \quad p_{1,1} = 0. \tag{3}$$

The optimal α can be calculated by solving

$$\begin{aligned} \max_{\alpha \in [0,(1/2)]} \quad & \alpha(u_0 + u_1) + (1 - 2\alpha)u^\star \\ \text{s.t.} \quad & (1 - 2\alpha)u^\star + \alpha u_0 \geq (1 - 2\alpha)u_1 \\ & \alpha u_1 \geq \alpha u^\star. \end{aligned} \tag{4}$$

The two constraints are from the definitions of CE. With $\alpha = (u_1 - u^\star/u_0 + 2\,(u_1 - u^\star))$, the CE will have the highest average payoff of $(u_1\,(u_0 + u_1 - u^\star)/u_0 + 2\,(u_1 - u^\star))$, which is always larger than the average payoff of the mixed NE.

In order to reach this equilibrium, the output of the correlated device will have a distribution of

$$p(o_r = 0) = 1 - 2\alpha, \quad p(o_r = 1) = p(o_r = 2) = \alpha. \tag{5}$$

It then sends the following private suggestions to the users (o_0 to P_0 and o_1 to P_1):

$$\begin{aligned} o_0 &= 0, \quad o_1 = 0, \text{ if } o_r = 0 \\ o_0 &= 0, \quad o_1 = 1, \text{ if } o_r = 1 \\ o_0 &= 1, \quad o_1 = 0, \text{ if } o_r = 2. \end{aligned} \tag{6}$$

Unlike what we discussed in Section III-B, the suggestions users get in (6) are not always the same, i.e., o_0 can be different from o_1. In fact, if o_r in (5) is used as the public signal, the users would be able to deviate from the suggestion to benefit himself. Hence, we name this equilibrium *private CE*, because the suggestion from the correlated device is private to the user, and name the previously discussed one *public CE*. The utility gain of the private CE (U_{priv}) over the public CE (U_{pub}) is illustrated in Fig. 2.

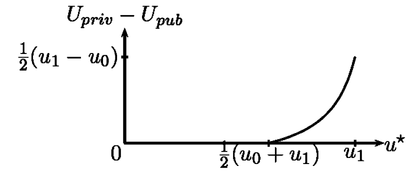


Fig. 2. Average utility gain of private CE over public CE.

Given the suggestion from the correlated device, P_0 will have the following beliefs:

$$\begin{aligned} b_0(a_1 = A_L|o_0 = 0) &= \frac{1 - 2\alpha}{1 - \alpha} \\ b_0(a_1 = A_L|o_0 = 1) &= 1 \\ b_0(a_1 = A_H|o_0 = 0) &= \frac{\alpha}{1 - \alpha} \\ b_0(a_1 = A_H|o_0 = 1) &= 0. \end{aligned} \tag{7}$$

It can be easily verified that given this belief, P_0 has no incentive to deviate from the correlated device's suggestion. The same conclusion can also be drawn for P_1. However, we also note that having these correct beliefs is not a necessary condition for the users to reach the desired private CE. For example, if the users both have beliefs as in the public CE (see Section III-B) and the private suggestions are generated according to (5) and

(6), they will still reach the private CE in (3). Hence, the users can reach the private CE even without knowing the value of α.

IV. Correlated Equilibrium Selection

In Section III-C, we have shown that the best achievable symmetric CE depends on the utilities of the users. Therefore, the correlated device needs to know each user's utility function to enable the users to play the CE with the highest payoffs. Unlike a network moderator who collects the utility functions from each user, which requires explicit information exchange, the correlated device can infer users' utilities by observing their strategies under the mixed NE. This can be done by first configuring the network to find out whether a private CE outperforms the public CE and then calculating the optimal private CE if necessary.

To do this, first the correlated device will let the users play the mixed NE by refraining to send the suggestion signals to them and observe their strategies. As shown in (1), a mixed strategy of $(\lambda, 1-\lambda)_{\lambda\in[0,1]}$ will give $u^\star = u_0 + u_1 - \lambda^{-1}u_0$. Subsequently, the correlated device will intentionally let the user get a payoff of u_0 when they both choose the less aggressive action, instead of $u^\star$ in Table I. This can be done by manipulating the handshake signal or sending a jamming packet to cause a collision, such that each user gets the same probability of successful transmission as if the other user were choosing the more aggressive action. Assuming that P_0's strategy under the new mixed NE is $(\lambda', 1-\lambda')_{\lambda'\in[0,1]}$, we have $u_0 = u_0 + u_1 - \lambda'^{-1}u_0$. If $1+\lambda'^{-1} > \lambda^{-1} \iff u^\star > (u_0 + u_1/2)$, then the correlated device knows that the best private CE outperforms the public CE [Fig. 1(b)] and can calculate the optimal private CE as in Section III-C, which is given by $\alpha = (u_1 - u^\star/u_0 + 2(u_1 - u^\star)) = (1 - \lambda/2 - \lambda)$. If $1 + \lambda'^{-1} \leq \lambda^{-1}$, the correlated device will let the users play the public CE [Fig. 1(a)].

This procedure involves only limited complexity for the correlated device, and it requires no reports of utility functions from the users, while it can effectively help the correlated device to find the CE with the highest payoffs for the users.

V. Implementation of the Correlated Device

For our two-user two-channel case, there are various ways to implement the correlated device to enable the two users to play the CEs. For example, the correlated device can be a certain arithmetic operation on the absolute time, which is always accessible to both users. Suppose the absolute time is represented by a binary vector $t_N = \{t_n\}_{0\leq n\leq N-1}$, and $r_N = \{r_n\}_{0\leq n\leq N-1}$ is an m-sequence of length N, and let

$$o_r = \oplus(t_N + r_N) \tag{8}$$

where $+$ is the addition with module 2 (which is equivalent to the bit-wise XOR), and $\oplus(x_N) = \bmod\left(\sum_{i=0}^{N-1} x(i), 2\right)$. This operation is implemented by the protocol of both users with the same m-sequence. Thus, o_r will be a binary random variable with equal probability of being 0 or 1. By implementing such a protocol, the two users will play the public CE without additional communication overhead, because they can both implement the correlated device in (8) locally.

To enable the two users to play the private CE as in (3), a correlated device needs to be implemented that has three possible outputs and sends private suggestions to both users according to (6). Therefore, we use an operation of $f(t_N)$ to generate the random signal, i.e., $o_r = f(t_N)$. The output of the operation $f(t_N)$ should satisfy (5). This can still be implemented by a simple modular operation with a larger divisor. However, unlike in the public CE, where the suggestion signals to the two users are identical, to achieve the private CE, different private signals are sent to the two users. Hence, playing private CEs requires more information than the public CE.

The use of a correlated device introduces a new MAC solution that is not *fully-distributed* or *fully-centralized*, as often adopted by most existing protocols. There are numerous differences between the correlated device which we used in this letter and a centralized moderator. For instance, in the presence of correlated device and its suggestion signals, the self-interested user is able to make his decision autonomously to get the best possible payoff for himself, while a centralized moderator may enforce the user to operate at some point which is not his best interest. Moreover, a correlated device just generates random signals, which requires much less computation and communication overheads than a centralized moderator.

VI. Conclusion

We formulated the channel access problem as a noncooperative game and illustrated how the different information availabilities that users possess influence their strategies and the equilibriums of the game. It is also shown that CE is a better solution (in terms of the average payoff) than mixed NEs in different scenarios. Also, we discussed how the correlated device can select the CE which leads to the highest payoffs for the users and how the correlated device can be implemented to enable the users to play the desirable CE.

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