

## MOTIVATION AND GOAL

## Motivation

- Screening helps timely diagnosis (e.g., cancers)
- For every 1000 mammograms, 500 false positives, and 200 unnecessary biopsy
- Clinical guidelines are **not personalized!**
  - Yearly after the age between 45-54 in USA, Biennial after the age of 40 in Canada, Japan
- Disease models are complex, MISCAN-COLON
- Screening works use simplistic disease models

## Goal

- Develop a **general framework** for screening
  - Personalized to patient feature and history
  - Applicable to many diseases

## STEPS TO DPSCREEN

**Idea 1. Define belief over entire path and not current state.**  $\mathbf{b}(\vec{s}, l)$ ,  $l = 1/0 \implies$  diagnosed/not

• **Bayesian belief update.**

$$\hat{\mathbf{b}}(\vec{s}, l) = Pr(\vec{s}, l | \mathbf{b}, \underbrace{\tau}_{\text{Scheduled visit}}, \underbrace{[y, z, \tilde{\tau}]}_{\text{Observation}}, x)$$

• **Bellman equation.**

$$V(\mathbf{b}, t) = \max_{\tau} \left[ \sum_{\vec{s}, z, l} \mathbf{b}(\vec{s}, l) Pr(z | \vec{s}, l) \tilde{C}(\vec{s}, l, t, z) + \sum_{z, y, \tilde{\tau}} Pr(z, y, \tilde{\tau} | \tau, \mathbf{b}) V(\hat{\mathbf{b}}, t + \tilde{\tau}) \right]$$

$\tilde{C}$ : cost for screening and cost of delay per epoch

• **Result. Value function is piecewise linear**

$$V(\mathbf{b}, t) = \max_{\alpha \in \Gamma(t)} \alpha^t \mathbf{b}$$

- Optimal  $\alpha$  vector decides the optimal policy
- If  $\Gamma(t+1)$  is known, then a recursion derived from Bellman equation can determine  $\Gamma(t)$

$$\Gamma(t) = \mathcal{R}^{exact}[\Gamma(t+1)] \quad (2)$$

$|\Gamma(t)|$  exponential in  $t \implies \mathcal{R}^{exact}$  **intractable!**

• Point-based value iteration **PBVI**

$$\Gamma(t) = \mathcal{R}^{PBVI}[\Gamma(t+1)] \quad (3)$$

$|\mathbf{b}|$  large  $\implies \mathcal{R}^{PBVI}$  **intractable!**

## RELATED WORKS

## Screening in Operations Research and Statistics

- Partially Observable Markov Decision Process POMDP (Ayer et.al.)
- **Bayesian Optimal Design** (Rizopoulos et.al.)
  - **Pros:** Principled search for optimal policies
  - **Cons:** Markov/Semi-Markov, Stationary

## Screening in Medical Literature

- **Stochastic simulation** based (Frazie et.al.)
  - **Pros:** No assumptions on disease models
  - **Cons:** Not personalized, Compare a fixed set of policies chosen by experts

**This work** No assumptions on disease models! Principled search for optimal policies!

## PROJECTED PBVI

## Idea 2. Dimensionality reduction:

- Sample  $K$  i.i.d. paths from disease model
- Project the beliefs over the sampled subset

## Idea 3. Basis set of policies:

- Random exploration
- Clinical guidelines/policies from existing works

**Intuition:** Ensure better performance than basis

• **Belief set construction:**

- Use stochastic simulations to generate outcomes using basis set
- Construct belief set  $\bar{B}$ : Bayesian update conditional on the sampled subset

• **Projected PBVI:** One  $\alpha$  vector per point in  $\bar{B}$

$$\hat{\Gamma}(t) = \mathcal{R}_{\bar{B}}^{PPBVI}[\hat{\Gamma}(t+1)] \quad (4)$$

$$\hat{V}(\mathbf{b}, t) = \max_{\alpha \in \hat{\Gamma}(t)} \alpha^t \mathbf{b} \quad (5)$$

- **Approximation error:**  $|\hat{V}(\mathbf{b}, t) - V(\mathbf{b}, t)| \leq \Omega(\bar{B})$ ,  $\Omega(\bar{B})$  worst case sampling density
- **Computational complexity:**  $\mathcal{O}(T^3 |\bar{B}|^2 K |\mathcal{Y}| |\mathcal{Z}|)$
- **Robustness:** Errors in model estimation? Optimal policy is locally constant over the space of models, i.e small errors don't matter!
- Optimize hyperparameters, i.e, sampling policy and the basis set, is future work

## MODEL AND PROBLEM FORMULATION

- **Disease model:** Finite state stochastic process, one absorbing disease state,  $\vec{S}$  is path of the stochastic process,  $Pr(\vec{S} = \vec{s} | \mathbf{x})$  is probability of the path,  $\mathbf{x} \in \mathcal{X}$  feature vector (age, gender, etc.), states are hidden
- **Diagnostic test:**  $Z(t)$ : test outcome at time  $t$ ,  $Pr(Z(t) = z | \vec{s}(t), \mathbf{x})$ : probability of test outcome
- **External information:**  $Y(t)$ : observation by the patient,  $Pr(Y(t) = y | \vec{s}(t), \mathbf{x})$ : probability of the observation, if  $Y(t) \in \mathcal{Y}$  patient goes to the clinician
- **Screening policy:**  $\pi : \mathcal{H} \times \mathcal{X} \rightarrow \{1, ., T\}$ : map from observation history, features to next arrival time
- **Costs:** Cost of screening : 1 (normalized), Cost of delay in detection:  $C(t_d - t_D; t_D)$ ,  $t_D/t_d$  is the time of incidence/detection
- **Optimal screening policy:** Minimizer of the weighted sum of the aggregate discounted screening costs and the delay costs, with weight  $w$ , discount factor  $\delta$ , and the set of arrival times  $\mathcal{T}_s$

$$\pi^* = \arg \min \left( \overbrace{(1-w)E\left[\sum_{t \in \mathcal{T}_s} \delta^t\right]}^{\text{Screening costs}} + \overbrace{wE\left[C(t_d - t_D; t_D)\right]}^{\text{Delay costs}} \right) \quad (1)$$

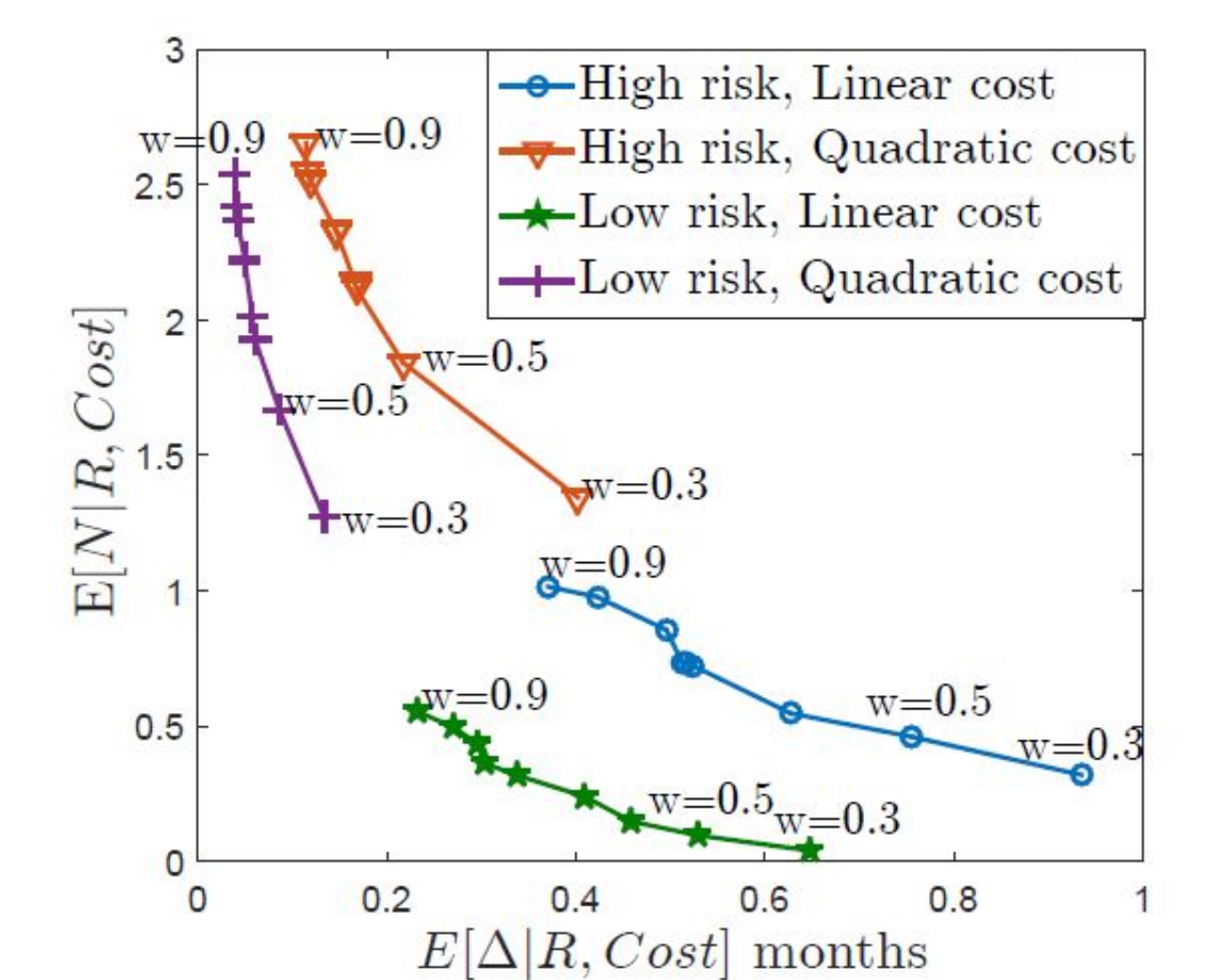
• **Challenges:** Standard POMDP and POSMDP cannot be used

- Disease model is not Markov/Semi-Markov
- Time between decision epochs depends on scheduled date, external information and the state path

## ILLUSTRATIVE EXPERIMENTS

- **Dataset:** Deidentified breast cancer dataset of 45,000 women. At least one mammogram/woman.
- **Features:** Number of family members with breast cancer history, age, bmi, menopause
- **Disease model: Pre-incidence.** Two state Markov model for the onset of breast cancer. **Post-incidence.** Universal tumor growth law. Tumor growth  $\implies$  lumps develop  $\implies$  self-arrivals
- **Model Estimation:** Parameters of the disease model estimated using standard MCMC methods.
- **Benchmarks:** Annual and Biennial screening
- **Metrics:**  $E[\Delta|R]$ ,  $E[\Delta|R, D]$   $E[N|R]$ : Expected delay (months) given risk, Expected delay given risk and disease, expected number of screenings given risk,

Risk	Metrics	DPSCREEN with self-exam	DPSCREEN w/o self-exam	Annual
Low	$E[N R]$	0.32,	0.55	1
	$E[\Delta R]$	0.23	0.23	0.24
	$E[\Delta R, D]$	9.2	9.2	9.4
High	$E[N R]$	0.43	0.72	1
	$E[\Delta R]$	0.50	0.52	0.52
	$E[\Delta R, D]$	6.7	7.07	7.07



## CONCLUSION

- Developed a general framework for screening
- Extended PBVI to address the challenges imposed by screening
- Gains  $> 30\%$  on breast cancer screening dataset
- Potential impact beyond screening in stopping time problems