A Game Theoretic Analysis of Incentives in Content Production and Sharing Over Peer-to-Peer Networks

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***Abstract*—Peer-to-peer (P2P) networks can be easily deployed to distribute user-generated content at a low cost, but the free-rider problem hinders the efficient utilization of P2P networks. Using game theory, we investigate incentive schemes to overcome the free-rider problem in content production and sharing. We build a basic model and obtain two benchmark outcomes: 1) the non-cooperative outcome without any incentive scheme and 2) the cooperative outcome. We then propose and examine three incentive schemes based on pricing, reciprocation, and intervention. We also study a brute-force scheme that enforces full sharing of produced content. We find that 1) cooperative peers share all produced content while non-cooperative peers do not share at all without an incentive scheme; 2) by utilizing the P2P network efficiently, the cooperative outcome achieves higher social welfare than the non-cooperative outcome does; 3) a cooperative outcome can be achieved among non-cooperative peers by introducing an incentive scheme based on pricing, reciprocation, or intervention; and 4) enforced full sharing has ambiguous welfare effects on peers. In addition to describing the solutions of different formulations, we discuss enforcement and informational requirements to implement each solution, aiming to offer a guideline for protocol design for P2P networks.**

***Index Terms*—Game theory, incentives, network economics, peer-to-peer (P2P) networks, pricing.**

I. Introduction

RECENT developments in technology have significantly reduced the cost of producing and distributing content in various forms such as images, sounds, videos, and text. Once produced only by companies with a large capital, content can now be produced by end-users. In today's Internet-based social communities, peer-to-peer (P2P) networks offer a cost effective and easily deployable framework for sharing user-generated content [1]. While P2P networks have many advantages such as scalability, resilience, and effectiveness in coping with dynamics and heterogeneity [2], they are vulnerable to intrinsic incentive problems in that the transfer of content incurs costs to both uploading and downloading peers while benefiting only downloading peers. Since the social cost of transfer (the sum of upload and download costs) exceeds the private cost of transfer (download costs), peers tend to download excessively as in the tragedy of the commons problem. On the other hand, since upload incurs costs to uploading peers without giving them direct benefit, peers tend to upload too little. The incentive problem stating that peers desire to benefit from P2P networks while not contributing to them is referred to as the free-rider (or free-loader) problem.

Various incentive schemes to mitigate the free-rider problem have been proposed and analyzed in the literature. Cooperative schemes (e.g., [3], [4]) utilize helpers that download files on behalf of a peer in the same collaborative group. Helpers can improve the download performance of P2P networks by sharing their spare upload capacities. However, forming and sustaining collaborative groups in a distributed system poses a main challenge to cooperative schemes. Pricing schemes (e.g., [5], [6]) use virtual currency or micropayment to reward upload and charge download. Pricing schemes have a solid theoretical foundation as they are based on economic models. However, they are often regarded impractical because they require an accounting infrastructure to track the transactions of peers [7]. Differential service schemes (e.g., [8], [9]) treat peers differentially depending on the ratings of peers. Since a peer with a good reputation is treated preferentially, differential service schemes provide incentives for peers to contribute in order to build and maintain a good reputation. However, differential service schemes require large communication overheads to determine and announce the ratings of peers. The rating of a peer is determined by its past actions, which are observed by different peers, and the rating has to be known to other peers or the system.

Game theory [10] offers a useful framework to model multiuser interaction and has been applied to analyze the behavior of peers in P2P networks. Incentive schemes such as pricing schemes and differential service schemes have been investigated using non-cooperative game theory. Pricing schemes can be easily incorporated in static game models as in [11], while differential service schemes have been studied in the context of different game models. [7] uses a static game model to analyze the steady-state outcome of learning dynamics under a differential service scheme. [12] simulates an evolutionary game model to examine the performance of a differential service scheme based on peer reciprocation. [13] uses a repeated game model to construct a differential service scheme based on the idea of social norms [14]. [15] and [16] apply the mechanism design approach to build optimal incentive-compatible differential service schemes. [17] uses both repeated game and mechanism design approaches to propose cheat-proof and attack-resistant differential service schemes. Cooperative game theory has also been used to investigate coalition formation among peers [18], [19].

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TABLE I
COMPARISONS OF THE APPROACHES DISCUSSED IN THE PAPER

Section	Scheme	Model	Performance (Pareto efficiency)	Requirements for a protocol designer	
				Enforcement on peers	Knowledge about the system
III	None	Non-cooperative static game	Inefficiency	None	None
IV	Control scheme (full control)	Cooperative game	Efficiency	Sharing levels	Complete knowledge
V	Incentive scheme (pricing)	Non-cooperative static game	Can achieve efficiency	Payments	Complete knowledge (can be replaced with learning)
VI.A	Incentive scheme (reciprocation)	Non-cooperative repeated game	Can achieve efficiency	Repeated game strategy (self-enforcing)	Complete knowledge
VI.B	Incentive scheme (intervention)	Non-cooperative static game	Can achieve efficiency	Intervention by the system	Complete knowledge
VII	Control scheme (limited control)	Hybrid of cooperative and non-cooperative game	Inefficiency	Full sharing	None

In this paper, we investigate incentives schemes for content production and sharing over P2P networks using game theory. Unlike existing game-theoretic works on P2P networks, which focus on a particular game model to construct incentive schemes, we build a basic model and use it as a unified framework based on which different incentive schemes are examined applying various game theoretic models. Specifically, we analyze the basic model as a non-cooperative game and examine five schemes—cooperative, pricing, reciprocation, intervention, and enforced full sharing—using different game formulations derived from the basic model, as summarized in Table I. Hence, instead of arguing for a particular incentive scheme and a modeling approach, we show that alternative incentive schemes can provide incentives for sharing in P2P networks from a neutral perspective. As can be seen from Table I, different schemes and the corresponding game models have different requirements for implementation. Since the characteristics of P2P networks vary depending on the architecture, peers, and content, the effectiveness of an incentive scheme will depend on the network environment. Thus, our analysis in this paper can serve as a guideline for a protocol designer when modeling, comparing, and selecting incentive schemes.

Another distinctive feature of our modeling is that we allow peers to make production decisions whereas most existing works assume that peers are endowed with a certain amount of content (see, for example, [11] and [16]). When produced content and downloaded content are substitutable in consumption, the amount of content a peer produces is affected by the amount of content available in a P2P network. By endogenizing the amount of content that peers produce, we can capture the strategic link between producing and downloading content. In addition, we consider scenarios where peers can be bound to share all produced content while they make production and download decisions in a non-cooperative manner. These scenarios can be formulated as games with partial cooperation in which the strategies of players can be enforced only in some stages. The concepts developed in the discussion of enforced full sharing can be applied to other scenarios that can be modeled as multistage games.

The rest of this paper is organized as follows. In Section II, we formulate the basic model that describes a scenario of content production and sharing. In Section III, we analyze the basic model as a non-cooperative game and identify the free-rider problem. In Section IV, we investigate cooperative schemes by deriving a coalitional game based on the basic model. In Section V, we augment the basic model with a pricing scheme to induce non-cooperative peers to achieve cooperative outcomes. In Section VI, we study differential service schemes based on reciprocation and intervention. In Section VII, we analyze a partially cooperative scenario where peers behave non-cooperatively while full sharing is enforced. In Section VIII, we provide numerical illustration. In Section IX, we conclude and discuss future directions. Proofs of propositions are provided either following propositions or in the Appendix.

II. MODEL

We consider a fully connected P2P network of N peers as in [7], [11]. Peers produce[1] content (e.g., photos, videos, news, and customer reviews) and use the P2P network to distribute produced content. Following [20], we model the content production and sharing scenario as a sequential game consisting of three stages, which is called the content production and sharing (CPS) game.

- *Stage One (Production)*: Each peer determines its level of production. $x_i \in \mathbb{R}_+$ represents the amount of content produced by peer i and is known only to peer i.
- *Stage Two (Sharing)*: Each peer specifies its level of sharing. $y_i \in [0, x_i]$ represents the amount of content that peer i makes available to other peers. $(y_1, \ldots, y_N)$ is known to all peers at the end of stage two.
- *Stage Three (Transfer)*: Each peer determines the amounts of content that it downloads from other peers. Peer i serves all the requests it receives from any other peer up to y_i. $z_{ij} \in [0, y_j]$ represents the amount of content that peer i

[1]We use the term production in a broad sense to mean any method of obtaining content other than download in the P2P network.

downloads from peer $j \neq i$, or equivalently peer j uploads to peer i.

Let $\mathcal{N} \triangleq \{1, \ldots, N\}$ be the set of peers in the P2P network. For notations, we define $\mathbf{x} \triangleq (x_1, \ldots, x_N)$, $\mathbf{y} \triangleq (y_1, \ldots, y_N)$, and $\mathbf{Z} \triangleq [z_{ij}]_{i,j \in \mathcal{N}}$, an N-by-N matrix whose (i,j)-entry is given by z_{ij}, where we set $z_{ii} = 0$ for all $i \in \mathcal{N}$. The download profile of peer i is given by the ith row of $\mathbf{Z}$, denoted by $\mathbf{z}_i \triangleq (z_{i1}, \ldots, z_{iN})$. Similarly, the upload profile of peer i is given by the ith column of $\mathbf{Z}$, denoted by $\mathbf{z}^i \triangleq (z_{1i}, \ldots, z_{Ni})$. Given the elements of $\mathbf{Z}$, we can compute the download volume of peer i by $d_i(\mathbf{z}_i) \triangleq \sum_{j=1}^N z_{ij}$ and its upload volume by $u_i(\mathbf{z}^i) \triangleq \sum_{j=1}^N z_{ji}$. For notational convenience, we suppress the dependence of d_i and u_i on $\mathbf{Z}$ and write d_i and u_i instead of $d_i(\mathbf{z}_i)$ and $u_i(\mathbf{z}^i)$, respectively. Also, we define $w(\mathbf{Z})$ to be the total transfer volume of the P2P network given $\mathbf{Z}$, i.e., $w(\mathbf{Z}) \triangleq \sum_{i=1}^N \sum_{j=1}^N z_{ij} = \sum_{i=1}^N d_i = \sum_{i=1}^N u_i$, which can be considered as a measure of the utilization of the P2P network.

We assume that peers produce nonidentical content of homogeneous quality, which allows us to focus on the quantity of content. The total amount of content that peer i has at the end of the CPS game, which we call the consumption of peer i, is given by the sum of the amounts it produces and downloads, $x_i + d_i$. The utility of peer i is given by the benefit of consumption minus the costs of production, download, and upload

$$v_i(\mathbf{x}, \mathbf{y}, \mathbf{Z}) = f(x_i + d_i) - \kappa x_i - \delta d_i - \sigma u_i.$$

We analyze the case of homogeneous peers in that f, κ, δ, and σ are the same for all peers. The benefit of consumption is measured by a concave function $f : \mathbb{R}_+ \to \mathbb{R}_+$ as in [21]. We assume that f is twice continuously differentiable and satisfies $f(0) = 0$, $f' > 0$ and $f'' < 0$ on $\mathbb{R}_{++}$. We also assume that $f'(0)$ is finite,[2] $f'(0) > \kappa$, and $\lim_{x\to\infty} f'(x) = 0$ so that for every $\alpha \in (0, f'(0)]$ there exists a unique $\hat{x}_\alpha \geq 0$ that satisfies $f'(\hat{x}_\alpha) = \alpha$, as illustrated in Fig. 1. We define $f^*(\alpha) = \sup_{x \geq 0}\{f(x) - \alpha x\}$ for $\alpha \in \mathbb{R}$ as the conjugate of f [22].[3] We use linear cost functions as widely adopted in the literature (see, for example, [7], [23]). The cost of producing the amount of content x_i is given by κx_i, where $\kappa > 0$ is the marginal cost of production. Download and upload create costs in terms of bandwidth usage, and transferring the amount of content z_{ij} from peer j to peer i induces a cost of δz_{ij} to peer i (the downloader) and σz_{ij} to peer j (the uploader), where $\delta > 0$ and $\sigma > 0$ are the marginal costs of download and upload, respectively. The P2P network has a positive social value only if obtaining a unit of content through the P2P network costs less to peers than producing it privately. Hence, we assume that $\kappa > \delta + \sigma$ to ensure that the P2P network is socially valuable.

III. Non-Cooperative Analysis

We first study the non-cooperative outcome of the CPS game without any incentive scheme. Noncooperative peers choose their strategies to maximize their own utilities given others' strategies. Thus, peers' strategies should be self-enforcing at non-cooperative equilibrium in that no peer can gain by choosing a different strategy unilaterally. A strategy for peer i in the CPS game is its complete contingent plan over the three stages and is denoted by $(x_i, y_i(x_i), \mathbf{z}_i(x_i, \mathbf{y}))$. A stage-one strategy for peer i is represented by $x_i \in \mathbb{R}_+$, a stage-two strategy by a function $y_i : \mathbb{R}_+ \to \mathbb{R}_+$ such that $y_i(x_i) \leq x_i$ for all $x_i \in \mathbb{R}_+$, and a stage-three strategy by a function $\mathbf{z}_i : \mathcal{I}_3 \to \mathbb{R}_+^N$ such that $z_{ij}(x_i, \mathbf{y}) \leq y_j$ for all $j \neq i$ and $z_{ij}(x_i, \mathbf{y}) = 0$ for $j = i$, where $\mathcal{I}_3 \triangleq \{(x_i, \mathbf{y}) | x_i \in \mathbb{R}_+, y_i \in [0, x_i], y_j \in \mathbb{R}_+, \forall j \neq i\}$ is the set of possible information sets at the beginning of stage three.

Nash equilibrium (NE) of the CPS game is defined as a strategy profile such that no peer can improve its utility by a unilateral deviation. The play on the equilibrium path at an NE is called an NE outcome of the CPS game. A refinement of NE for sequential games is subgame-perfect equilibrium (SPE), which requires that players choose NE strategies in any subgame, thereby eliminating incredible threats. Subgame perfection provides robustness in equilibrium strategies in that deviation is unprofitable not only at the beginning of the game but also at any stage of the game. However, formally there is no subgame of the CPS game starting from stage two or three because the stage-one choice of a peer is not revealed to other peers. Hence, SPE fails to provide a refinement of NE in the CPS game.

In order to extend the spirit of subgame perfection to non-singleton information sets, we can use sequential rationality, which postulates that players behave optimally in each information set for a given belief system [10]. Sequential rationality is required by the solution concepts of perfect Bayesian equilibrium (PBE) and sequential equilibrium (SE).[4] The difference between these two solution concepts disappears in the CPS game because the consistent belief of peer i on $\mathbf{x}_{-i} \triangleq (x_1, \ldots, x_{i-1}, x_{i+1}, \ldots, x_N)$ should be the correct $\mathbf{x}_{-i}$ in both solution concepts. Hence, we use SE to refer to a solution concept requiring sequential rationality and specify only the strategy profile to describe an SE suppressing the

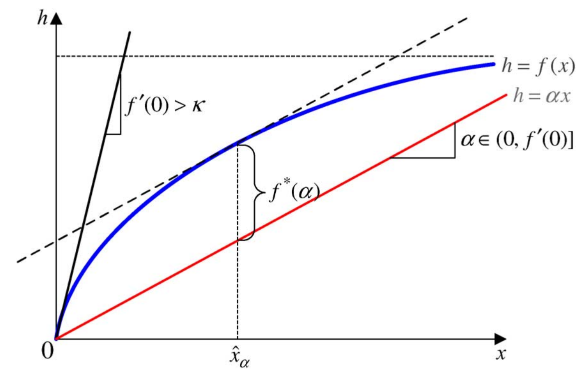

Fig. 1. Illustration of the benefit function.

[2]We use $f'(0)$ to represent the right derivative of f at 0.

[3]Note that the definition of a conjugate is adjusted as f is a concave function.

[4]A strategy profile and a belief system constitute a PBE if the strategies are sequentially rational given the belief system and the beliefs are updated using Bayes' rule, wherever possible, given the strategy profile. SE is a refinement of PBE in that SE perturbs the strategy profile to make Bayes' rule applicable in every information set. See [10] for the formal definitions of PBE and SE.

belief system with an implicit premise that peers hold correct beliefs.[5] An SE strategy profile of the CPS game can be found applying a backward induction argument, which is described in detail in [20]. As with NE, the play on the equilibrium path at an SE is called an SE outcome of the CPS game.

Proposition 1: At the unique SE outcome of the CPS game, we have $x_i = \hat{x}_\kappa, y_i = 0, \mathbf{z}_i = (0, \ldots, 0)$ for all $i \in \mathcal{N}$. Thus, $w(\mathbf{Z}) = 0$ at SE.

Proof: A formal proof can be found in [20, Prop. 1]. Since sharing can incur the cost of upload while it gives no benefit to the sharing peer, it is never optimal for a peer to share a positive amount. Expecting no sharing, each peer produces the autarkic optimal amount of content, $\hat{x}_\kappa$, which maximizes $f(x) - \kappa x$. ■

Proposition 1 shows that, without any incentive scheme, non-cooperative peers do not utilize the socially valuable P2P network because they are not compensated for their upload. This result explains the free-riding behavior of peers in file sharing P2P networks such as Napster and Gnutella, as reported in [25], [26]. Using a similar argument as in the formal proof of Proposition 1, we can show that the NE outcome of the CPS game is the same as the SE outcome. NE may prescribe suboptimal strategies off the equilibrium path, but there cannot be a positive amount shared on the equilibrium path.[6] Individual utility and total utility at non-cooperative equilibrium are $f^*(\kappa)$ and $\Pi^{NC} = Nf^*(\kappa)$, respectively.

IV. Cooperative Schemes

We consider cooperative schemes in the CPS game, which allow peers to form collaborative groups and maximize their joint welfare. In order to prevent peers from behaving non-cooperatively, cooperative schemes need to enforce the actions of peers by a contract or a protocol. The protocol designer can implement a cooperative scheme if he knows the utility functions of all peers in order to determine a desired operating point and can enforce the operating point. An example of P2P networks to which a cooperative scheme can be applied is a camera network, where cameras in different locations capture the images of an object from various angles. A property that a desired operating point should possess is Pareto efficiency (PE), which is satisfied when there is no other operating point that makes some peers better off without making other peers worse off. We define social welfare by the sum of the utilities of peers, i.e., $\Pi(\mathbf{x}, \mathbf{y}, \mathbf{Z}) \triangleq \sum_{i=1}^N v_i(\mathbf{x}, \mathbf{y}, \mathbf{Z})$. Then an allocation is Pareto efficient (PE) if it maximizes social welfare among feasible allocations.[7]

Proposition 2: Let $\beta \triangleq (1/N)\kappa + ((N-1)/N)(\delta + \sigma)$. At PE, we have $\sum_{i=1}^N x_i = \hat{x}_\beta$ and $x_i = y_i = z_{ji}$ for all $j \neq i$, for all $i \in \mathcal{N}$. Thus, $w(\mathbf{Z}) = (N-1)\hat{x}_\beta$ at PE.

Proof: A formal proof can be found in [20, Prop. 2]. Since the transfer of content is more cost efficient than production, PE can occur only when peers share all produced content and download all shared content. Then the social welfare maximization problem can be written as

$$\max_{\mathbf{x} \geq 0} Nf\left(\sum_{i=1}^N x_i\right) - [\kappa + (N-1)(\delta + \sigma)] \sum_{i=1}^N x_i.$$

The first-order optimality condition for $X \triangleq \sum_{i=1}^N x_i$ is $f'(X) = \beta$. Note that β is the per capita marginal cost of obtaining one unit of content when N peers share all produced content. Thus, at PE, the level of total production is chosen to equate the marginal benefit and the marginal cost of supplying content to every peer in the P2P network. ■

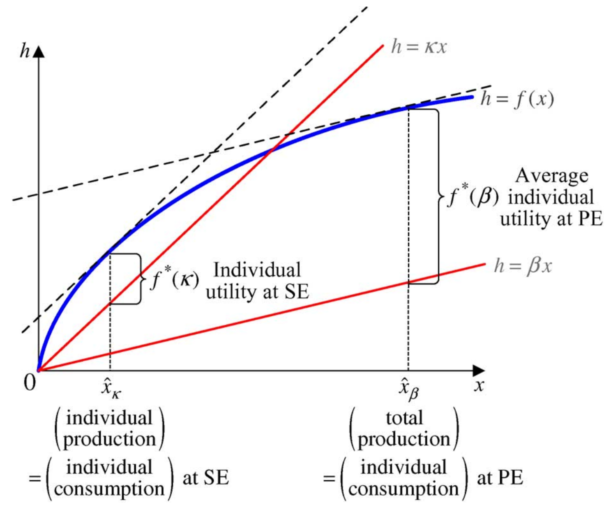

Fig. 2. Comparison of SE and PE.

At PE, peers jointly produce $\hat{x}_\beta$ and share all produced content so that each peer consumes the total amount produced. The utility of peer i producing x_i^o at a PE allocation $(\mathbf{x}^o, \mathbf{y}^o, \mathbf{Z}^o)$ is given by

$$v_i(\mathbf{x}^o, \mathbf{y}^o, \mathbf{Z}^o) = f(\hat{x}_\beta) - \delta\hat{x}_\beta - [\kappa + (N-1)\sigma - \delta]\, x_i^o. \quad (1)$$

Note that the utility of a peer is decreasing in its production level given that the total amount of production is fixed and that all produced content is shared. Total utility at PE is given by $\Pi^{PE} = Nf^*(\beta)$. Since $\beta < \kappa$ for $N \geq 2$, the consumption of a peer and total utility are smaller at non-cooperative equilibrium than at PE, i.e., $\hat{x}_\kappa < \hat{x}_\beta$ and $\Pi^{NC} < \Pi^{PE}$, as depicted in Fig. 2.

In order to derive a coalitional game [27] based on the CPS game, we need to compute the maximum total utility that a subset of peers can achieve. Define $\tilde{\beta}(n)$ by

$$\tilde{\beta}(n) = \frac{1}{n}\kappa + \frac{n-1}{n}(\delta + \sigma)$$

for $n = 1, 2, \ldots$. Note that $\tilde{\beta}(1) = \kappa$, $\tilde{\beta}(N) = \beta$, and $\tilde{\beta}(n) \to \delta + \sigma$ as $n \to \infty$. $\tilde{\beta}(n)$ can be interpreted as the per capita marginal cost of obtaining one unit of content when n peers share all produced content. The maximum total utility achievable with n peers is given by $G(n) \triangleq nf^*(\tilde{\beta}(n))$, and the maximum average individual utility achievable with n peers by

[5] This requirement can be relaxed using the notion of self-confirming equilibrium (SCE) [24], which requires only observational consistency in beliefs.

[6] A similar remark holds for SCE. As soon as a peer shares its content, it learns that others request its content, and thus it will choose not to share at all in order to avoid upload costs.

[7] An allocation $(\mathbf{x}, \mathbf{y}, \mathbf{Z})$ is feasible if $x_i \geq 0$, $0 \leq y_i \leq x_i$, $z_{ii} = 0$, and $0 \leq z_{ij} \leq y_j$ for all $j \neq i$, for all $i \in \mathcal{N}$.

$g(n) \triangleq G(n)/n = f^*(\tilde{\beta}(n))$. Marginal product (MP) measures an increment in the maximum total utility when the nth peer joins the P2P network, i.e., $MP(1) \triangleq G(1)$ and $MP(n) \triangleq G(n) - G(n-1) = nf^*(\tilde{\beta}(n)) - (n-1)f^*(\tilde{\beta}(n-1))$ for $n \geq 2$. The following proposition gives some properties of the functions g and MP.

Proposition 3:

i) $g(n)$ is increasing in n, and $\lim_{n\to\infty} g(n) = f^*(\delta + \sigma)$.
ii) $MP(n)$ is increasing in n, $MP(n) > g(n)$ for all $n \geq 2$, and $\lim_{n\to\infty}[MP(n) - g(n)] = 0$.

Since $g(n)$ is increasing in n, there are increasing returns to scale when inputs and outputs are taken to be peers and total utility, respectively. As there are more peers in the P2P network, the cost efficient P2P network can be utilized more extensively, which results in an increase in the maximum average individual utility.[8] Proposition 3(i) in addition states that the maximum average individual utility is bounded above. Proposition 3(ii) shows that the no-surplus condition in the sense of [29] is satisfied only in the limiting case with infinitely many peers. This implies that the distribution of total utility to peers according to their MP, which is proposed by the marginal productivity theory of distribution of neoclassical economics, is not feasible unless there are infinitely many peers. Thus, we rely on cooperative game theory as an alternative theory of distribution.

Let $\mathcal{S}$ with $\mathcal{S} \neq \emptyset$ and $\mathcal{S} \subseteq \mathcal{N}$ be a coalition of peers. The characteristic function $v : 2^{\mathcal{N}} \to \mathbb{R}$, which assigns each coalition the maximum total utility it can create, is given by

$$v(\mathcal{S}) = |\mathcal{S}| f^* \left(\tilde{\beta}\left(|\mathcal{S}|\right)\right) \quad (2)$$

where $|\mathcal{S}|$ denotes the number of peers in coalition $\mathcal{S}$. We set $v(\emptyset) = 0$. A coalitional game is described by the characteristic function v, and we consider two solution concepts for coalitional games, the core and the Shapley value. A utility profile $(v_1^c, \ldots, v_N^c)$ is in the core of the coalitional game v if

$$\sum_{i\in\mathcal{N}} v_i^c = v(\mathcal{N}) \quad \text{and} \quad \sum_{i\in\mathcal{S}} v_i^c \geq v(\mathcal{S}), \quad \forall\, \mathcal{S} \subseteq \mathcal{N}.$$

The first condition states that the maximum total utility with the grand coalition $\mathcal{N}$ is distributed to peers (i.e., a PE allocation is chosen) while the second condition states that no coalition can improve the utilities of its members from the current distribution. Hence, the core describes the stable distributions of total utility in that no coalition of peers can improve their utilities by separating from the grand coalition. This implies that, when the protocol designer enforces an allocation that achieves a utility profile in the core, no coalition of peers can object the allocation credibly by threatening to leave the P2P network. The Shapley value, whose expression can be found in [27], is a distribution of total utility, $v(\mathcal{N})$, that satisfies a certain set of axioms. The Shapley value can be considered as a fair distribution of utility as it takes into account the MP of peers in all possible orders of arrival in the P2P network.

[8]In our model, there are no congestion effects in that the marginal costs of upload and download are independent of the number of peers in the network. If we generalize our model so that the marginal costs of upload and download are increasing in the number of peers, then additional peers will have not only positive externalities but also negative externalities on the existing peers and there may exist an optimal network size that maximizes total utility as in [28].

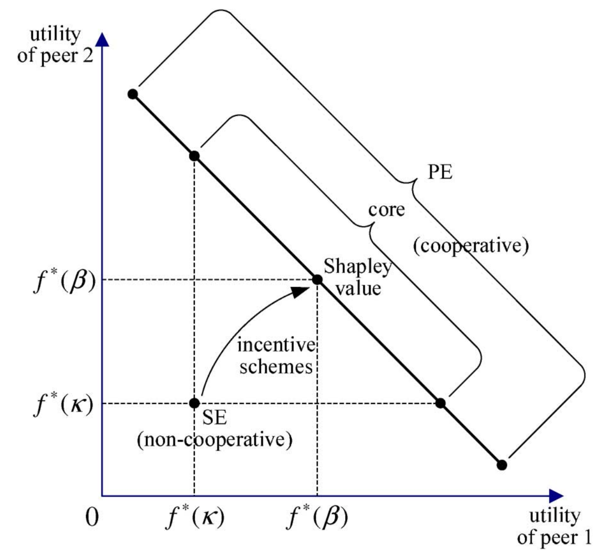

Fig. 3. Two-peer illustration of utility profiles achieved at non-cooperative and cooperative solution concepts. Incentive schemes can induce non-cooperative peers to achieve a cooperative outcome.

Proposition 4:

i) The core of the coalitional game v is a nonempty convex set whose vertices are given by $(MP(1), MP(2), \ldots, MP(N))$ and all of its permutations. At the core, we have $\sum_{i=1}^N x_i = \hat{x}_\beta$, $x_i = y_i = z_{ji}$ for all $j \neq i$, for all $i \in \mathcal{N}$, and
$$\sum_{i\in\mathcal{S}} x_i \leq |\mathcal{S}| \frac{f(\hat{x}_\beta) - \delta\hat{x}_\beta - f^*\left(\tilde{\beta}\left(|\mathcal{S}|\right)\right)}{\kappa + (N-1)\sigma - \delta} \quad (3)$$
for all $\mathcal{S} \subseteq \mathcal{N}$.
ii) The Shapley value of the coalitional game v is $v_i = f^*(\beta)$ for all $i \in \mathcal{N}$, which is attained at the symmetric PE allocation, $x_i = y_i = z_{ji} = \hat{x}_\beta/N$ for all $j \neq i$, for all $i \in \mathcal{N}$.

Fig. 3 illustrates the results in Propositions 1, 2, and 4 with two peers. Since $\Pi^{NC} < \Pi^{PE}$, PE allocations achieve a higher total utility than the SE allocation. Also, since the core imposes additional constraints on PE, the core is a subset of PE utility profiles. Proposition 3(ii) implies that the coalitional game v is convex [30], and thus the results in Proposition 4 can be considered as the corollaries of theorems in [30]. In particular, the core is nonempty and coincides with the unique stable set in the sense of [31]. Also, the Shapley value is the center of gravity of the core, which is consistent with the illustration in Fig. 3. Hence, by prescribing the allocation that yields the Shapley value, the protocol designer can obtain the stability property of the core and the fairness property of the Shapley value at the same time.

The maximum utility that a peer can obtain by itself is $f^*(\kappa)$, which can be considered as a reservation utility. An allocation $(\mathbf{x}, \mathbf{y}, \mathbf{Z})$ satisfies the participation (or individual rationality) constraint for peer i if $v_i(\mathbf{x}, \mathbf{y}, \mathbf{Z}) \geq f^*(\kappa)$. An allocation $(\mathbf{x}, \mathbf{y}, \mathbf{Z})$ is participation-efficient if it is PE and satisfies the participation constraint for every peer. Among PE allocations, condition (3) for a singleton coalition $\mathcal{S} = \{i\}$

$$x_i \le \frac{f(\hat{x}_\beta) - \delta \hat{x}_\beta - f^*(\kappa)}{\kappa + (N-1)\sigma - \delta}$$

is required for the participation constraint for peer i. Since the utility of a peer decreases in its production level among PE allocations as shown in (1), the participation constraint puts an upper bound on the individual production level to prevent a peer from leaving the P2P network. The core is a stronger concept than participation-efficiency in that the core prevents not only a single peer from leaving the P2P network but also a subset of peers from forming their own P2P network.

Peers choose actions $\mathbf{x}$, $\mathbf{y}$, and $\mathbf{Z}$ over the three stages of the CPS game. Suppose that the protocol designer can enforce the sharing levels of peers in stage two while he cannot enforce the choices in stages one and three. Then the CPS game is reduced to the CPS game with enforced sharing levels $\mathbf{y}^e = (y_1^e, \dots, y_N^e)$, where the stage-two choice of peers is fixed at some $\mathbf{y}^e$.

Proposition 5: Suppose that $\hat{x}_\kappa \le \sum_{i=1}^N y_i^e \le \hat{x}_\delta$. At the SE outcome of the CPS game with enforced sharing levels $\mathbf{y}^e$, we have $x_i = y_i^e$ and $z_{ij} = y_j^e$ for all $j \ne i$, for all $i \in \mathcal{N}$.

Proposition 5 shows that when peers are required to share $\mathbf{y}^e$ that satisfies $\hat{x}_\kappa \le \sum_{i=1}^N y_i^e \le \hat{x}_\delta$, they produce exactly the enforced sharing levels and download all shared content in their self-interest. Since $\hat{x}_\kappa \le \hat{x}_\beta \le \hat{x}_\delta$, the protocol designer can implement a PE allocation by enforcing only the sharing levels $\mathbf{y}^e$ such that $\sum_{i=1}^N y_i^e = \hat{x}_\beta$, leaving peers to choose the production and download levels non-cooperatively.

V. Pricing Schemes

A cooperative scheme is often impractical in a distributed system because it requires a central authority with a complete knowledge about the system. As an alternative to cooperative schemes, incentive schemes can be used to achieve a cooperative outcome while preserving the non-cooperative decision making of peers. Pricing is an extensively studied form of incentives to achieve an efficient use of network resources [32]. Pricing schemes have been used in P2P-based web services such as MojoNation in the forms of tokens and credits. We say that a pricing scheme is optimal if it achieves PE at non-cooperative equilibrium. In order to determine an optimal pricing scheme, the protocol designer needs to know the utility functions of peers. An optimal pricing scheme can be implemented when the protocol designer can enforce payments and each peer knows the pricing rule applied to it.

In this section, we examine a class of pricing schemes under which the payment to a peer is increasing in its upload volume and decreasing in its download volume at the same constant rate.[9] We call such a pricing scheme a linear pricing scheme, and the payment to peer i under a linear pricing scheme with price $p > 0$ is given by $t_i(\mathbf{Z}) = p(u_i - d_i)$. Note that a linear pricing scheme with any price satisfies budget balance since it simply transfers payments from a downloading peer to an uploading peer, i.e., $\sum_{i=1}^N t_i(\mathbf{Z}) = p(\sum_{i=1}^N u_i - \sum_{i=1}^N d_i) = 0$ for all $\mathbf{Z}$ and p. The payoff of peer i in the CPS game with the linear pricing scheme with price p is given by

$$\begin{aligned}\pi_i(\mathbf{x}, \mathbf{y}, \mathbf{Z}) &= v_i(\mathbf{x}, \mathbf{y}, \mathbf{Z}) + t_i(\mathbf{Z}) \\ &= f(x_i + d_i) - \kappa x_i - (p + \delta) d_i + (p - \sigma) u_i.\end{aligned}$$

In effect, the linear pricing scheme with price p increases the cost of download from δ to $p + \delta$ and decreases the cost of upload from σ to $\sigma - p$. If the reward for upload exceeds the cost of upload, i.e., $p > \sigma$, then peers receive a net benefit from uploading, which provides them with an incentive for sharing. The following proposition shows that there exists an optimal pricing scheme in the class of linear pricing schemes. Moreover, the optimal linear pricing scheme transfers the utilities of peers so that the equilibrium payoff profile coincides with the Shapley value for any PE allocation chosen by peers.

Proposition 6: Let $p^* = [\kappa + (N-1)\sigma - \delta]/N$. At the SE outcome of the CPS game with the linear pricing scheme with price p^*, we have $\sum_{i=1}^N x_i = \hat{x}_\beta$ and $x_i = y_i = z_{ji}$ for all $j \ne i$, for all $i \in \mathcal{N}$. The payoff of each peer at SE is given by $f^*(\beta)$.

Proof: Proposition 6 follows from Lemma 1 and Proposition 7 of [20]. ■

A notable feature of the optimal price p^* is that peers are indifferent between the two alternative methods of obtaining data, production and download, when they face the optimal price. At PE without pricing schemes, peers prefer download to production because the marginal cost of download, δ, is smaller than that of production and upload, $\kappa + (N-1)\sigma$ [see the coefficient of the term x_i^o in (1)]. The optimal price is chosen such that it equates the effective marginal cost of download $p^* + \delta$ with that of production and upload $\kappa - (N-1)(p^* - \sigma)$. As a result, at SE with the optimal linear pricing scheme, peers obtain the same payoff regardless of their production levels. With linear cost functions and linear prices, this is a necessary property of any nondiscriminatory pricing scheme that induces non-cooperative homogeneous peers to produce a positive bounded amount of content in aggregate. If the payoff to a peer is increasing in its production level, then the peer would produce and upload as much as it can (i.e., the peer is overcompensated for its production and the supply of content is unbounded). On the other hand, if the payoff to a peer is decreasing in its production level, then the peer would not produce at all (i.e., the peer is undercompensated for its production and the supply of content is zero). The optimal price can be considered as an equilibrium price of content in that the compensation is enough to provide incentives for production and upload but prevents overproduction so that supply equals demand.

Imposing that $y_i = x_i$ and $u_i = (N-1)y_i$ for all $i \in \mathcal{N}$, the social welfare maximization problem can be written as

$$\begin{aligned}&\max_{\mathbf{x},\mathbf{d}} \sum_{i=1}^N \{f(x_i + d_i) - [\kappa + (N-1)\sigma] x_i - \delta d_i\} \\ &\text{subject to } x_i \ge 0,\ 0 \le d_i \le \sum_{j \ne i} x_j, \text{ for all } i = 1, \dots, N. \quad (4)\end{aligned}$$

Let $p_i \ge 0$ be a Lagrange multiplier on the constraint $d_i \le \sum_{j \ne i} x_j$ for each $i \in \mathcal{N}$, which can be interpreted as the price

[9] Another class of pricing schemes, called MP pricing schemes, is proposed and analyzed in [20]. Under an MP pricing scheme, payments are determined based on the sharing levels of peers.

that peer i pays for its download. Then the Lagrangian function can be written as

$$\mathcal{L}(\mathbf{x},\mathbf{d};\mathbf{p}) = \sum_{i=1}^{N}\left\{ f(x_i+d_i) - [\kappa+(N-1)\sigma]\,x_i - \delta d_i + p_i\left(\sum_{j\neq i} x_j - d_i\right)\right\}$$

and the first-order optimality conditions for x_i and d_i are given by

$$f'(x_i+d_i)-\kappa-(N-1)\sigma+\sum_{j\neq i} p_j \leq 0 \text{ (with equality if } x_i>0)$$

and

$$f'(x_i+d_i)-\delta-p_i \leq 0 \text{ (with equality if } d_i>0).$$

At PE, we have $x_i+d_i=\hat{x}_\beta$, which yields the Lagrange multiplier $p_i=p^*$ for all $i\in\mathcal{N}$. The dual decomposition of (4) can be written as

$$\max_{x_i,d_i\geq 0} f(x_i+d_i) - \left[\kappa+(N-1)\sigma-\sum_{j\neq i} p_j\right] x_i - (p_i+\delta)d_i \quad (5)$$

for each i. The solution to (5) is given by

$$x_i=0 \quad\text{and}\quad d_i=\hat{x}_{(p_i+\delta)} \quad \text{if } \sum_{j=1}^{N} p_j < \kappa+(N-1)\sigma-\delta, \quad (6)$$

$$x_i+d_i=\hat{x}_{(p_i+\delta)}=\hat{x}_{\left[\kappa+(N-1)\sigma-\sum_{j\neq i} p_j\right]} \quad \text{if } \sum_{j=1}^{N} p_j = \kappa+(N-1)\sigma-\delta, \quad\text{and} \quad (7)$$

$$x_i=\hat{x}_{\left[\kappa+(N-1)\sigma-\sum_{j\neq i} p_j\right]} \text{ and } d_i=0 \quad \text{if } \sum_{j=1}^{N} p_j > \kappa+(N-1)\sigma-\delta.^{10} \quad (8)$$

Thus, the maximum value of (5) is given by $h_i(\mathbf{p}) \triangleq f^*(\min\{p_i+\delta, \kappa+(N-1)\sigma-\sum_{j\neq i} p_j\})$, and $p_i=p^*$ for all i is the solution of the dual problem, $\min_{\mathbf{p}\geq 0}\sum_{i=1}^{N} h_i(\mathbf{p})$. Thus, a uniform linear pricing scheme suffices to obtain PE allocations. The problem (4) is more general than the resource allocation problem in [33] in the following aspect. In our problem, peers can choose to become either a seller or a buyer (or both), and the amount of resources (i.e., content) supplied in the P2P network is chosen by peers. On the contrary, in [33], buyers and sellers are predetermined, and sellers hold a fixed supply of resources.

If the protocol designer knows the utility functions of peers, he can compute the optimal price p^* using the expression in Proposition 6. At the optimal price, equilibrium requires that peers produce $\hat{x}_\beta$ in aggregate. Peers can coordinate to achieve total production $\hat{x}_\beta$ using the following quantity adjustment process. Initially, each peer i chooses arbitrary production and download levels (x_i,d_i) that satisfy $x_i+d_i=\hat{x}_\beta$. Peers share their production fully throughout the process, and thus they can observe the production levels of other peers indirectly. If $\sum_{i=1}^{N} x_i > \hat{x}_\beta$ (respectively, $\sum_{i=1}^{N} x_i < \hat{x}_\beta$), then $d_i < \sum_{j\neq i} x_j$ (respectively, $d_i > \sum_{j\neq i} x_j$) for all i. Hence, if each peer i adjusts its production and download levels by

$$\frac{dx_i}{dt} = -\frac{dd_i}{dt} = \eta_i\left(d_i - \sum_{j\neq i} x_j\right) \quad (9)$$

for some constant $\eta_i>0$,[11] then the allocation will converge to an equilibrium allocation, which satisfies $\sum_{i=1}^{N} x_i=\hat{x}_\beta$.

Suppose instead that the protocol designer does not know the utility functions of peers. In this case, the protocol designer can still find the optimal price by using a price adjustment process similar to that in [34]. That is, a price adjustment process can substitute knowledge about the utility functions of peers. In the proposed price adjustment process, the protocol designer announces a price p, which applies to every peer. Given the price, each peer i chooses (x_i,d_i) by solving (5) and reports its choice to the protocol designer. We assume that peers can coordinate their choices to satisfy $d_i=\sum_{j\neq i} x_j$ for all $i\in\mathcal{N}$ whenever possible, for example, by using the quantity adjustment process (9). The total demand for content at price p is denoted by $D(p)$ and can be computed as $\sum_{i=1}^{N} d_i$. Similarly, the total supply of content at price p is denoted by $S(p)$ and can be computed as $\sum_{i=1}^{N} u_i=(N-1)\sum_{i=1}^{N} x_i$, as a peer can upload its production up to $(N-1)$ times. Using (6)–(8), we obtain

$$D(p)=\begin{cases} N\hat{x}_{(p+\delta)}, & \text{if } p<p^* \\ (N-1)\hat{x}_\beta, & \text{if } p=p^* \\ 0, & \text{if } p>p^* \end{cases}$$

and

$$S(p)=\begin{cases} 0, & \text{if } p<p^* \\ (N-1)\hat{x}_\beta, & \text{if } p=p^* \\ N(N-1)\hat{x}_{[\kappa-(N-1)(p-\sigma)]}, & \text{if } p>p^* \end{cases}$$

as depicted in Fig. 4. We define the excess demand at price p by $ED(p)\triangleq D(p)-S(p)$. The protocol designer adjusts the price following the process

$$\frac{dp}{dt}=\eta ED(p)$$

for some constant $\eta>0$. Since $ED(p)>0$ for $p<p^*$ and $ED(p)<0$ for $p>p^*$, the price will converge to the optimal price p^* starting from any initial price. We compare the above price adjustment process with that in [34]. In [34], there are multiple resources with fixed supply, and each resource manager adjusts the price of his resource so that aggregate demand for the resource equals the supply of the resource. In our formulation, by focusing on a uniform linear pricing scheme, we treat resources provided by different peers as a single resource. Hence, the protocol designer needs to aggregate demand and supply by

[10]We set $\hat{x}_\alpha=+\infty$ when $\alpha\leq 0$.

[11]Since x_i cannot be negative, we assume that peer i stops adjusting its quantity when $x_i=0$ and $d_i<\sum_{j\neq i} x_j$.

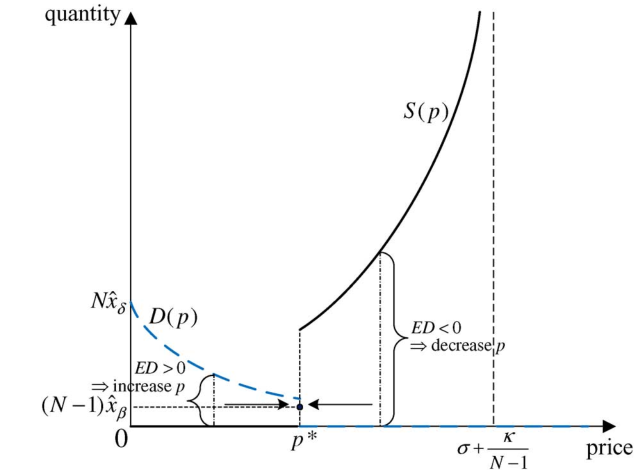

Fig. 4. Equilibrium interpretation of the optimal price p^*. $D(p)$ represents the total demand for download and $S(p)$ the total supply of upload at price p. The equilibrium price equates demand and supply, i.e., $D(p^*) = S(p^*)$. The protocol designer can reach the equilibrium price by adjusting the price depending on the excess demand ED.

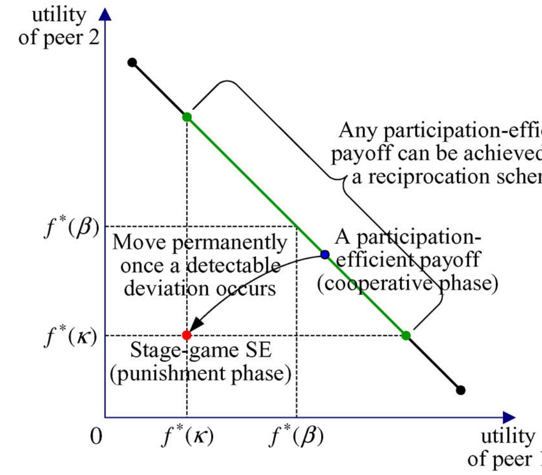

Fig. 5. Two-peer illustration of the operation of a reciprocation scheme.

all peers and adjust the price of content to eliminate excess demand or supply.

We have assumed that peers report their demand and supply truthfully in the price adjustment process. Suppose instead that peers know the price adjustment process used by the protocol designer and can engage in strategic misrepresentation as in [35]. We find that, unlike in [35], no peer can gain from influencing the equilibrium price by misreporting its demand or supply provided that other peers report truthfully. In [35], a user, acting as a buyer, can benefit from a lower price of a resource by underreporting its demand. On the contrary, in our model, a peer is both a buyer and a seller, and thus it can lower the price only by increasing its supply, which hurts it as a seller. Peer i can make the price adjustment process stop at $p' < p^*$ by reporting $x_i = \hat{x}_{(p'+\delta)}$ and $d_i = 0$. Since $p' + \delta < \beta < \kappa - (N-1)(p' - \sigma)$, the payoff of peer i is

$$\begin{aligned}\pi_i &= f\left(\hat{x}_{(p'+\delta)}\right) - \left[\kappa - (N-1)(p' - \sigma)\right]\hat{x}_{(p'+\delta)}\\ &< f\left(\hat{x}_{(p'+\delta)}\right) - \beta\hat{x}_{(p'+\delta)} < f^*(\beta),\end{aligned}$$

and thus it obtains a lower payoff by manipulating the equilibrium price at p'. For $p > p^*$, the optimal production and download levels for peer j are $x_j = \hat{x}_{[\kappa-(N-1)(p-\sigma)]}$ and $d_j = 0$. Since $d_i \leq \sum_{j \neq i} x_j$, peer i alone cannot induce $ED(p) = 0$, or $\sum_{i=1}^N d_i = (N-1)\sum_{i=1}^N x_i$, for some $p > p^*$, if $N > 2$. When $N = 2$, peer i can make the price adjustment process stop at $p'' > p^*$ by reporting $x_i = 0$ and $d_i = \hat{x}_{[\kappa-(N-1)(p''-\sigma)]}$. Since $\kappa - (N-1)(p'' - \sigma) < \beta < p'' + \delta$, the payoff of peer i is

$$\begin{aligned}\pi_i &= f\left(\hat{x}_{[\kappa-(N-1)(p''-\sigma)]}\right) - (p'' + \delta)\hat{x}_{[\kappa-(N-1)(p''-\sigma)]}\\ &< f\left(\hat{x}_{[\kappa-(N-1)(p''-\sigma)]}\right) - \beta\hat{x}_{[\kappa-(N-1)(p''-\sigma)]} < f^*(\beta).\end{aligned}$$

Again, peer i cannot gain from misreporting.

VI. Differential Service Schemes

Another form of incentives that encourage sharing by non-cooperative peers is differential service, in which peers obtain different qualities of service depending on their contribution levels. Differential service schemes are widely adopted in file sharing P2P networks such as BitTorrent [36] and KaZaA, in the forms of tit-for-tat and reputation. In this section, we capture the differential service in the CPS game using two approaches based on reciprocation and intervention. In a reciprocation scheme, peers can reciprocate service to each other based on private or public history. In an intervention scheme, the system treats peers differentially based on their contribution to the system.

A. Reciprocation Schemes

Suppose that peers interact repeatedly over time in the P2P network. A reciprocation scheme can support a cooperative outcome among non-cooperative peers by providing rewards and punishments depending on the past behavior of peers. The repeated CPS game is a supergame in which the CPS game is played repeatedly. We use the limit of means criterion [37] to evaluate the utility of a peer in the repeated CPS game to obtain the following result.[12]

Proposition 7: Any participation-efficient allocation can be supported as a non-cooperative equilibrium of the repeated CPS game.

By Proposition 5, when a PE allocation is prescribed, any deviation that is profitable in the current CPS game involves a deviation in sharing levels, which can be publicly observed by peers. Hence, the protocol designer can deter peers from free-riding in the P2P network by making peers play the SE of the one-shot CPS game in all subsequent CPS games whenever a peer does not share its required amount of content. Fig. 5 illustrates the repeated game strategy used in a reciprocation scheme.

We have assumed that peers serve all the transfer requests they receive in stage three. Suppose instead that a peer can choose whether to upload or not to another peer that requests its content. Then the punishment following a deviation in sharing levels can be asymmetric by prescribing peers not to upload to a peer that has ever deviated, which effectively excludes the deviating peer from the P2P network. Similarly, refusing a download request from a peer that has not deviated can also be deterred

[12]A similar result can be obtained with the discounting criterion, in which case Proposition 7 is restated as "Any strictly participation-efficient allocation can be supported as a non-cooperative equilibrium of the repeated CPS game when peers are sufficiently patient."

by using private retaliation (i.e., a non-deviating peer whose request was refused does the same thing in return to the peer that has refused its request) in all subsequent CPS games.

B. Intervention Schemes

Intervention [38] refers to the system directly influencing the quality of service for users depending on their behavior. We consider a particular form of intervention applicable to P2P networks. Suppose that the P2P network can reduce the download rate of a peer depending on its rating, where the rating of peer i is defined by its upload to download ratio, i.e., $r_i = u_i/d_i$. Then an intervention scheme can be described by an intervention function $q : \mathbb{R}_+ \rightarrow \mathbb{R}_+$, which represents an increase in the marginal cost of download. That is, when peer i has rating r_i, its marginal cost of download after intervention is given by $\delta + q(r_i)$. Note that the range of q is constrained to be nonnegative since we assume that the system can only decrease download rates. This imposes a restriction in incentive design compared to a pricing scheme, where it is usually assumed that a payment function can take any positive or negative real number. However, intervention schemes have advantages in implementation over pricing and reciprocation schemes. Unlike a pricing scheme, there is no need for transactions in an intervention scheme since intervention affects peers directly through the system. Also, intervention can be considered as a substitute of the punishment strategy in repeated games, but it requires neither repeated interaction among peers nor the maintenance of history by peers since punishment is executed by the system rather than by peers.[13] We say that an intervention scheme is optimal if it achieves a PE allocation with zero intervention level at non-cooperative equilibrium. Since any positive level of intervention results in performance degradation, it is desirable to have intervention only as a threat, which is called for when misbehavior occurs.

Proposition 8: Define an intervention function q^* by $q^*(r_i) = p^*[1 - r_i]^+$, where $p^* = [\kappa + (N-1)\sigma - \delta]/N$ and $[r]^+ = \max\{r, 0\}$. At the SE outcome of the CPS game with the intervention scheme q^*, we have $x_i = y_i = z_{ji} = \hat{x}_\beta/N$ for all $j \neq i$, for all $i \in \mathcal{N}$. Moreover, $q^*(r_i) = 0$ at SE.

Proof: As long as $r_i \leq 1$, or $u_i \leq d_i$, for all i, the intervention scheme q^* is equivalent to the optimal linear pricing scheme p^*. Since increasing u_i beyond d_i can only increase the cost of upload without affecting the level of intervention, we must have $u_i \leq d_i$ for all i at SE. Among SE with the optimal pricing scheme p^* given in Proposition 6, $u_i \leq d_i$ is satisfied for each i only when the total production $\hat{x}_\beta$ is split equally to all peers. At this allocation, $u_i = d_i = (N-1)\hat{x}_\beta/N$, and thus $r_i = 1$ and $q^*(r_i) = 0$ at SE. ■

Proposition 8 shows that an optimal intervention scheme can be constructed to achieve the symmetric PE allocation without intervening at equilibrium. Under the optimal intervention scheme q^*, a peer experiences a reduced download rate whenever it downloads more than it uploads. Since every peer downloads and uploads the same amount at the symmetric PE allocation, reduced download rates act only as a threat at equilibrium, deterring peers from deviation. The model of [7] can be considered as using another form of intervention, where the system determines the proportion of shared content that a peer is allowed to download as a function of the contribution of the peer. The model of [23] can also be interpreted as using an intervention scheme, where the system no longer serves a peer when its cumulative average rating falls below a threshold level. [38] applies an intervention scheme to a multiuser access network, where the system can jam packets randomly with a probability that depends on the transmission probabilities of users. In [38], intervention affects all users in the system to the same degree, thus represented by a function that depends on the actions of all users. On the contrary, intervention considered in this paper, [7], and [23] influences a peer depending only on its own action, thus allowing the differential service to peers.

VII. Full Sharing Scheme

We have seen in Proposition 5 that the protocol designer can achieve a PE allocation by enforcing the sharing levels of peers. As an alternative scenario, suppose that the protocol designer can enforce full sharing among peers, but not sharing levels.[14] The resulting CPS game is called the CPS game with enforced full sharing. Formally, the CPS game with enforced full sharing is a restricted version of the CPS game where the stage-two choice of each peer i is fixed as $y_i = x_i$. Note that enforced sharing levels constrain the production decisions of peers in that peers need to produce at least the required sharing levels. On the contrary, under enforced full sharing, peers can choose any levels of production in stage one. The following proposition characterizes allocations at the SE of the CPS game with enforced full sharing.

Proposition 9: Let $\gamma \triangleq \kappa + (N-1)\sigma$. Define $\tilde{x}_\gamma$ by $\tilde{x}_\gamma = \hat{x}_\gamma$ if $\gamma \leq f'(0)$ and $\tilde{x}_\gamma = 0$, otherwise (i.e., $\tilde{x}_\gamma = \arg\max_{x \geq 0}\{f(x) - \gamma x\}$). At the SE outcome of the CPS game with enforced full sharing, we have $\sum_{i=1}^N x_i = \tilde{x}_\gamma$ and $x_i = y_i = z_{ji}$ for all $j \neq i$, for all $i \in \mathcal{N}$. Thus, $w(\mathbf{Z}) = (N-1)\tilde{x}_\gamma$ at SE with enforced full sharing.

Proof: A formal proof can be found in [20, Prop. 3]. Since peers download all shared content at SE, enforced full sharing increases the effective marginal cost of production from κ to $\kappa + (N-1)\sigma$, which includes the marginal cost of upload to $(N-1)$ peers. The stage-one problem for peer i can be written as

$$\max_{x_i \geq 0} f\left(\sum_{i=1}^N x_i\right) - [\kappa + (N-1)\sigma]\, x_i - \delta \sum_{j \neq i} x_j$$

given $\mathbf{x}_{-i}$, and the result follows. ■

As peers face effectively a higher cost of production with enforced full sharing, non-cooperative peers reduce their production when full sharing is enforced, i.e., $\tilde{x}_\gamma < \hat{x}_\kappa$. Total utility at SE with enforced full sharing is given by $\Pi^{FS} = N[f(\tilde{x}_\gamma) - \beta\tilde{x}_\gamma]$. SE with enforced full sharing in the case of $\gamma \leq f'(0)$ is illustrated in Fig. 6.

[13]For example, the considered type of intervention can be implemented in a distributed way by requiring peers to use a certain program to download and upload files, which can adjust the download rate of a peer automatically based on its past usage.

[14]For example, full sharing can be enforced when there exists an indispensable technology for production and peers have access to it under the condition of sharing the produced content.

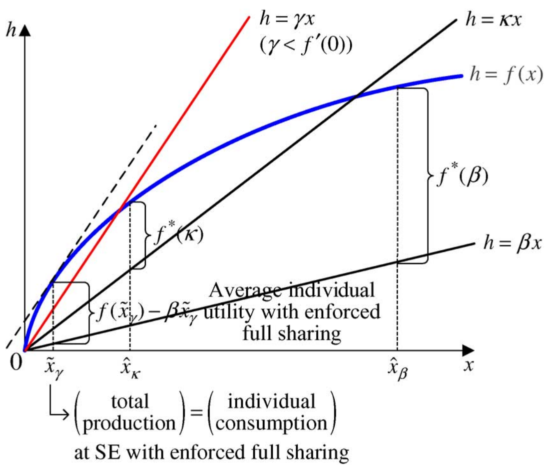

Fig. 6. Illustration of SE with enforced full sharing.

To make welfare comparisons, we first consider a scenario in which the number of peers in the P2P network is fixed as N. The price of anarchy (PoA)[15] is defined to be the ratio of social welfare at the worst non-cooperative equilibrium to that at PE

$$PoA \triangleq \frac{\Pi^{NC}}{\Pi^{PE}} = \frac{f^*(\kappa)}{f^*(\beta)}. \quad (10)$$

The price of no sharing (PoNS) compares social welfare at SE with and without enforced full sharing

$$PoNS \triangleq \frac{\Pi^{NC}}{\Pi^{FS}} = \frac{f^*(\kappa)}{f(\tilde{x}_\gamma) - \beta\tilde{x}_\gamma}(= +\infty \text{ if } \tilde{x}_\gamma = 0).$$

Finally, the price of underproduction (PoU) compares social welfare at SE with enforced full sharing and at PE

$$PoU \triangleq \frac{\Pi^{FS}}{\Pi^{PE}} = \frac{f(\tilde{x}_\gamma) - \beta\tilde{x}_\gamma}{f^*(\beta)}. \quad (11)$$

When $\tilde{x}_\gamma > 0$, the PoA can be decomposed as the product of the PoNS and the PoU, i.e., $PoA = PoNS \times PoU$. The PoA is a widely used measure of the inefficiency of non-cooperative equilibria. The PoNS measures the welfare implication of enforced full sharing on self-interested peers, and thus it can be used to analyze the value of a technology that enables enforced full sharing. The PoU measures inefficiency due to underproduction caused by the selfish behavior of peers assuming that full sharing is enforced. The following proposition examines the range of values that each measure of inefficiency can take when we vary the benefit and cost functions f, κ, δ, and σ.

Proposition 10: For a fixed size $N \geq 2$ of the P2P network, $PoA \in (0, 1)$, $PoNS \in (0, \infty]$, and $PoU \in [0, 1)$. These bounds are tight.

Proof: A formal proof can be found in [20, Prop. 4]. ■

Since $\beta < \kappa < \gamma$ for $N \geq 2$, it follows immediately from (10) and (11) that $PoA, PoU < 1$, which shows that selfish behavior results in efficiency losses regardless of whether full sharing is enforced or not. The relative size of Π^{NC} and Π^{FS} is ambiguous, which implies that the enforcement of full sharing may make peers worse off. This is because enforced full sharing has two offsetting effects on social welfare. On one hand, full sharing has a positive effect on welfare by reducing the cost of obtaining one unit of content to β, compared to κ in the case of no sharing. On the other hand, full sharing has a negative effect by increasing the effective cost of producing one unit of content from κ to γ. Therefore, the overall welfare implication of enforced full sharing is determined by the stronger of the two effects.

[15]Since the non-cooperative outcome of the CPS game is unique, the price of anarchy and the price of stability coincide for the CPS game.

Next we consider a scenario in which the number of peers in a P2P network is endogenously determined by peers. There are total N peers that are connected to each other, and they can form groups to share their content within a group. The maximum average individual utility increases with the number of peers in a group as shown in Proposition 3(i). Thus, when peers can use a cooperative scheme, peers will form a P2P network with all the N peers if they accept a new peer as long as the inclusion of an additional peer benefits existing peers assuming that peers split total utility equally. When peers behave non-cooperatively without any incentive scheme, peers do not share content at all, and thus their utilities do not depend on the number of peers. Hence, the previous results that $\Pi^{PE} = Nf^*(\beta)$ and $\Pi^{NC} = Nf^*(\kappa)$ are still valid with endogenous network formation. To analyze a scenario with enforced full sharing, define $\tilde{\gamma}(n)$ by $\tilde{\gamma}(n) = \kappa + (n-1)\sigma$ for $n = 1, 2, \ldots$. Then the average individual utility of a peer in a P2P network of size n is given by

$$g^{FS}(n) = f\left(\tilde{x}_{\tilde{\gamma}(n)}\right) - \tilde{\beta}(n)\tilde{x}_{\tilde{\gamma}(n)}.$$

Increasing the size of a P2P network has two opposing effects on average individual utility. On one hand, increasing the number of peers benefits peers by reducing the effective marginal cost of obtaining content as represented by $\tilde{\beta}$, which decreases with n. On the other hand, increasing the number of peers does harm to peers by increasing the effective marginal cost of producing content as represented by $\tilde{\gamma}$, which increases with n. Hence, we can expect that there exists an optimal size of a P2P network that balances these positive and negative effects.

Since $g^{FS}(1) = f^*(\kappa) > 0$ and $g^{FS}(n) = 0$ for all $n \geq (f'(0) - \kappa)/\sigma + 1$, there must exist a maximizer of $g^{FS}(n)$ among $n = 1, \ldots, \lfloor (f'(0) - \kappa)/\sigma + 1 \rfloor$, denoted by N^*, where $\lfloor \alpha \rfloor$ is the largest integer smaller than or equal to α. We assume that N^* is unique, which will hold for a generic specification of the utility function. When N peers form P2P networks endogenously to maximize their individual utilities, they will form $\lfloor N/N^* \rfloor$ networks of size N^* and one network of residual peers. Hence, total utility that a coalition $\mathcal{S}$ can create is given by

$$v^{FS}(\mathcal{S}) = \left\lfloor \frac{|\mathcal{S}|}{N^*} \right\rfloor N^* g^{FS}(N^*) + \left(|\mathcal{S}| - \left\lfloor \frac{|\mathcal{S}|}{N^*} \right\rfloor\right) g^{FS}\left(|\mathcal{S}| - \left\lfloor \frac{|\mathcal{S}|}{N^*} \right\rfloor\right).$$

In order to examine the stability property of endogenous network formation, we characterize the core of the coalitional game v^{FS}.

Proposition 11: Suppose that $N^* < N$. If N is a multiple of N^*, then the core of the coalitional game v^{FS} consists of a unique element $v_i = g^{FS}(N^*)$ for all $i \in \mathcal{N}$. Otherwise, the core is empty.

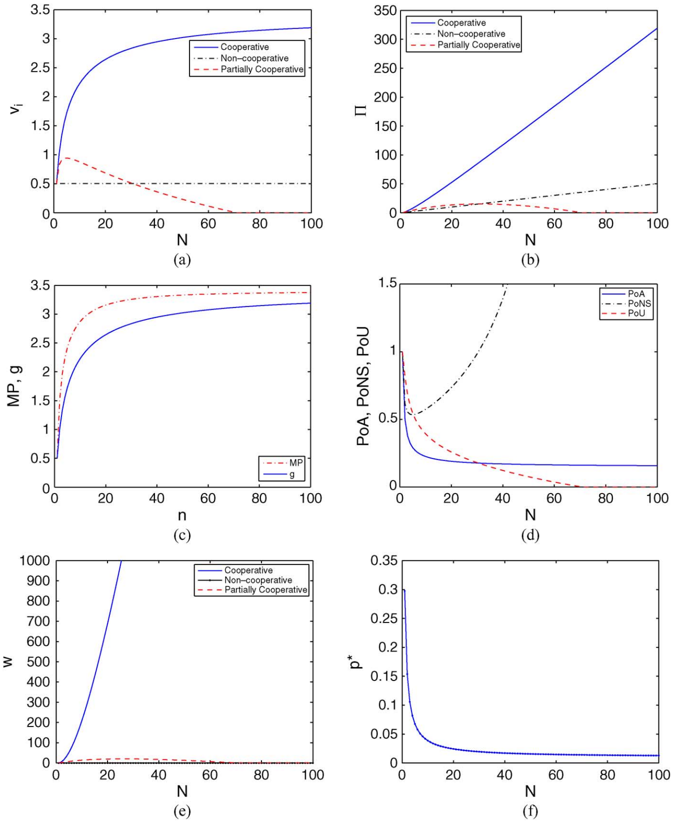

Fig. 7. Numerical results with $f(x) = log(1+x)$, $\kappa = 0.3$, $\delta = 0.0025$, $\sigma = 0.01$, and varying N from 1 to 100. (a) Average individual utility, (b) total utility, (c) marginal product and the maximum average individual utility, (d) inefficiency measures, (e) utilization of the P2P network, and (f) optimal linear prices.

Note that we necessarily have $N^* < N$ when $N \geq (f'(0) - \kappa)/\sigma + 1$. When N is not a multiple of N^*, there is a residual network, whose size is smaller than N^*. A peer in the residual network can bid a utility smaller than $g^{FS}(N^*)$ to form a network of size N^* including itself, yielding instability for the networks of size N^*. Suppose that N is a multiple of N^* so that the core is nonempty. The utility profile in the core is achieved by peers forming N/N^* networks, producing $x_i = \hat{x}_{\tilde{\gamma}(N^*)}/N^*$ for all i, and sharing all produced content within a network. Social welfare at the allocation with the core property is $\Pi^{FS} = Ng^{FS}(N^*)$. Since $g^{FS}(N^*) \geq g^{FS}(1) = f^*(\kappa)$, we have $PoNS \leq 1$ when peers can form P2P networks of the optimal size. That is, with endogenous network formation, enforced full sharing can only improve the welfare of peers because peers are given the option of operating in an autarkic manner.

VIII. Numerical Illustration

In this section, we provide illustrative results using a particular utility specification and varying the number of peers. For the utility function of peers, we use $f(x) = \log(1+x)$, $\kappa = 0.3$, $\delta = 0.0025$, and $\sigma = 0.01$.[16] We consider the (exogenous) number of peers in the P2P network, N, from 1 to 100. Fig. 7(a) shows average individual utility in the three scenarios: $f^*(\beta)$ in the cooperative case, $f^*(\kappa)$ in the non-cooperative case (without any incentive scheme), and $f(\tilde{x}_\gamma) - \beta\tilde{x}_\gamma$ in the partially cooperative case (i.e., enforced full sharing). It can be seen that $f^*(\beta)$ is increasing in N, verifying Proposition 3(i), that $f^*(\kappa)$ is independent of N, and that $f(\tilde{x}_\gamma) - \beta\tilde{x}_\gamma$ reaches a peak at $N = 5$ and is zero for all $N \geq 71$. Fig. 7(b) plots total utility in the three scenarios: Π^{PE} in the cooperative case, Π^{NC} in the non-cooperative case, and Π^{FS} in the partially cooperative case.

Fig. 7(c) compares the MP of the nth peer, $MP(n)$, with the maximum average individual utility that n peers can achieve, $g(n)$, verifying Proposition 3(ii). Fig. 7(d) plots the three inefficiency measures defined in Section VII. Since $f^*(\kappa)$ is independent of N, we can see that the PoA and the PoNS change with N in the opposite way that $f^*(\beta)$ and $f(\tilde{x}_\gamma) - \beta\tilde{x}_\gamma$ change, respectively. Since $f^*(\beta)$ converges to $f^*(\delta + \sigma) = 3.3945$, the PoA converges to $f^*(\kappa)/f^*(\delta + \sigma) = 0.1485$ as N goes to infinity. Fig. 7(e) shows the utilization of the P2P network in the three scenarios: $(N-1)\hat{x}_\beta$ in the cooperative case, 0 in the non-cooperative case, and $(N-1)\hat{x}_\gamma$ in the partially cooperative case. We can see no utilization in the non-cooperative case and underutilization (and no utilization for $N \geq 71$) in the partially cooperative case compared to the cooperative case, which exhibits high utilization of the P2P network. Finally, Fig. 7(f) plots the optimal linear price p^* as a function of N. As can be seen from its expression in Proposition 6, p^* decreases with N and converges to $\sigma = 0.01$ as N goes to infinity.

IX. Conclusion and Future Directions

In this paper, we have investigated incentive issues in content production and sharing using various game theoretic approaches. We have characterized the non-cooperative and cooperative outcomes of the CPS game and have shown that incentive schemes such as pricing schemes and differential service schemes can induce non-cooperative peers to achieve a cooperative outcome. We have also discussed enforcement and informational requirements for the protocol designer to implement different schemes.[17] For a rigorous comparison of schemes, we need to develop a method to quantify implementation overheads. A related issue is to study learning processes for the protocol designer to find an optimal incentive scheme without knowing the utility functions of peers.

Our model can be extended into several directions. First, we can generalize the CPS game by incorporating nonlinear production cost functions, connectivity topologies, and peer heterogeneity in terms of benefit, production, and upload and download costs. All the concepts in this paper can easily extend to a general model although it is more difficult to obtain closed-form expressions for solutions. Second, we can consider a scenario where prices are set by supplying peers or by the protocol designer in order to maximize the payoff of the price-setting entity. In this paper, we have focused on optimal prices that achieve PE allocations. However, when there is imperfect competition (i.e., the price-setting entity has some degree of monopoly power), the resulting prices may differ from the optimal ones. Lastly, we can formulate a dynamic version of the CPS game in which

[16]The authors of [21] use the same benefit function for their illustrative examples.

[17]In this paper, we have not addressed the issues of the complexity of computing different game theoretic solutions. Some complexity issues can be found in [39].

peers can transfer not only content they produced but also content they downloaded. A dynamic analysis will allow us to study the spread and the long-run distribution of content over the network depending on the connectivity and the cost parameters of peers.

APPENDIX
PROOFS OF PROPOSITIONS

A. Proof of Proposition 3

Proof: i) Note that $g(n) = f^*(\tilde{\beta}(n)) = f^* \circ \tilde{\beta}(n)$. Also, $f^*(\alpha) = f(\hat{x}_\alpha) - \alpha\hat{x}_\alpha$, where $f'(\hat{x}_\alpha) = \alpha$, for $\alpha \in (0, f'(0)]$. That is, $\hat{x}_\alpha$ is the unique maximizer of $f(x) - \alpha x$ on $\mathbb{R}_+$. Choose $\alpha_1, \alpha_2 \in (0, f'(0)]$ such that $\alpha_1 < \alpha_2$. Then $f^*(\alpha_2) = f(\hat{x}_{\alpha_2}) - \alpha_2\hat{x}_{\alpha_2} < f(\hat{x}_{\alpha_2}) - \alpha_1\hat{x}_{\alpha_2} < f(\hat{x}_{\alpha_1}) - \alpha_1\hat{x}_{\alpha_1} = f^*(\alpha_1)$. Hence, f^* is decreasing on $(0, f'(0)]$. Since $\tilde{\beta}(n) = (\kappa - \delta - \sigma)/n + \delta + \sigma$, $\tilde{\beta}$ is decreasing in n and its range lies in $(\delta + \sigma, \kappa] \subset (0, f'(0)]$. Since g is a composite function of two decreasing functions, it is increasing.

Since f is closed and strictly concave on $\mathbb{R}_+$, f^* is differentiable on $\mathbb{R}_{++}$ [22]. Then f^* is continuous on $\mathbb{R}_{++}$, and thus $\lim_{n\to\infty} g(n) = \lim_{n\to\infty} f^*(\tilde{\beta}(n)) = f^*(\lim_{n\to\infty} \tilde{\beta}(n)) = f^*(\delta + \sigma)$.

ii) To prove that MP is increasing in n, it suffices to show the strict convexity of G, taking the domain of $\tilde{\beta}$ and G as $\mathbb{R}_{++}$ instead of $\{1, 2, \ldots\}$. Since $G(n) = nf^*(\tilde{\beta}(n))$ and f^* and $\tilde{\beta}$ are differentiable on $\mathbb{R}_{++}$, by the chain rule G is differentiable and

$$G'(n) = f^*\left(\tilde{\beta}(n)\right) + n(f^*)'\left(\tilde{\beta}(n)\right)\tilde{\beta}'(n).$$

Note that $(f^*)' = -(f')^{-1}$ on $(0, f'(0))$, f' is continuously differentiable on $\mathbb{R}_{++}$, and $f''(x) \neq 0$ for all $x \in \mathbb{R}_{++}$. By the inverse function theorem, f^* is twice continuously differentiable on $(0, f'(0))$, and we have

$$G''(n) = (f^*)'\left(\tilde{\beta}(n)\right)\left[2\tilde{\beta}'(n) + n\tilde{\beta}''(n)\right] + n(f^*)''\left(\tilde{\beta}(n)\right)\left(\tilde{\beta}'(n)\right)^2.$$

Since $2\tilde{\beta}'(n) + n\tilde{\beta}''(n) = 0$ and f^* is strictly convex on $(0, f'(0))$, we have $G''(n) = n(f^*)''(\tilde{\beta}(n))(\tilde{\beta}'(n))^2 > 0$ for all $n \in \mathbb{R}_{++}$. Thus, $MP(n)$ is increasing in n.

Note that $MP(n) - g(n) = (n-1)[g(n) - g(n-1)]$ for $n \geq 2$. Since g is increasing, we have $MP(n) - g(n) > 0$ for all $n \geq 2$.

Since f^* is convex and differentiable, we have

$$\begin{aligned} f^*\left(\tilde{\beta}(n)\right) - f^*\left(\tilde{\beta}(n-1)\right) &\leq -(f^*)'\left(\tilde{\beta}(n)\right)\left[\tilde{\beta}(n-1) - \tilde{\beta}(n)\right] \\ &= \hat{x}_{\tilde{\beta}(n)}\left[\tilde{\beta}(n-1) - \tilde{\beta}(n)\right] \\ &< \hat{x}_{(\delta+\sigma)}\left[\tilde{\beta}(n-1) - \tilde{\beta}(n)\right]. \end{aligned}$$

Note that $(n-1)[\tilde{\beta}(n-1) - \tilde{\beta}(n)] = (\kappa - \delta - \sigma)/n$. Hence,

$$0 < (n-1)\left[g(n) - g(n-1)\right] < \hat{x}_{(\delta+\sigma)}\frac{1}{n}(\kappa - \delta - \sigma)$$

and taking limits as $n \to \infty$ yields the desired result. ■

B. Proof of Proposition 4

Proof: i) Proposition 3(ii) implies that the coalitional game v is convex. Hence, the first sentence follows from theorems 3 and 4 of [30]. The first condition for the core requires that a PE allocation be chosen, i.e., $\sum_{i=1}^N x_i = \hat{x}_\beta$, $x_i = y_i = z_{ji}$ for all $j \neq i$, for all $i \in \mathcal{N}$ as shown in Proposition 2. Choose an arbitrary coalition $\mathcal{S}$. For a PE allocation $(\mathbf{x}, \mathbf{y}, \mathbf{Z})$, we have

$$\sum_{i\in\mathcal{S}} v_i(\mathbf{x}, \mathbf{y}, \mathbf{Z}) = f(\hat{x}_\beta) - \delta\hat{x}_\beta - [\kappa + (N-1)\sigma - \delta]\sum_{i\in\mathcal{S}} x_i.$$

Hence, using (2), we can show that (3) is needed to satisfy the second condition for the core.

ii) Let $(v_1, \ldots, v_N)$ be the Shapley value of the coalitional game v. By the efficiency property of the Shapley value, we have $\sum_{i=1}^N v_i = v(\mathcal{N})$. Also, by the symmetry axiom, we have $v_i = v_j$ for all $i, j \in \mathcal{N}$. Combining these two yields $v_i = v(\mathcal{N})/N = f^*(\beta)$ for all $i \in \mathcal{N}$. Using (1), we can see that $x_i = \hat{x}_\beta/N$ is necessary to obtain the Shapley value. ■

C. Proof of Proposition 5

Proof: Suppose that $d_i < \sum_{j\neq i} y_j^e$ for some $i \in \mathcal{N}$ at SE. Then it must be the case that $x_i + \sum_{j\neq i} y_j^e > \hat{x}_\delta$. Since $\sum_{j\neq i} y_j^e \leq \hat{x}_\delta$, we have $x_i > 0$. Then we obtain a contradiction to SE because peer i can improve its utility by reducing x_i and increasing d_i by the same amount. Thus, at SE $z_{ij} = y_j^e$ for all $j \neq i$, for all $i \in \mathcal{N}$.

The requirement for peer i that $y_i = y_i^e$ in stage two restricts its stage-one choice with $x_i \geq y_i^e$. Suppose that $x_i > y_i^e$ for some $i \in \mathcal{N}$ at SE. Since $d_i = \sum_{j\neq i} y_j^e$ at SE, the first-order effect of increasing x_i on v_i is given by $\partial v_i/\partial x_i = f'(x_i + \sum_{j\neq i} y_j^e) - \kappa$. Since $x_i > y_i^e$ implies $x_i + \sum_{j\neq i} y_j^e > \hat{x}_\kappa$, we have $\partial v_i/\partial x_i < 0$ for $x_i > y_i^e$, and thus peer i becomes worse off by choosing $x_i > y_i^e$, contradicting SE. ■

D. Proof of Proposition 7

Proof: Let $(\mathbf{x}^o, \mathbf{y}^o, \mathbf{Z}^o)$ be a participation-efficient allocation. Consider the following repeated game strategy for peer i: start with a cooperative strategy in the CPS game $x_i = x_i^o$, $y_i(x_i) = x_i$, and $\mathbf{z}_i(x_i, \mathbf{y}) = \mathbf{z}_i^*(x_i, \mathbf{y})$, where $\mathbf{z}_i^*(x_i, \mathbf{y})$ is the optimal download profile of peer i given $(x_i, \mathbf{y})$, play the cooperative strategy if $\mathbf{y} = \mathbf{y}^o$ in all the previous CPS games, and play the SE strategy of the one-shot CPS game, i.e., $x_i = \hat{x}_\kappa$, $y_i(x_i) = 0$, and $\mathbf{z}_i(x_i, \mathbf{y}) = \mathbf{z}_i^*(x_i, \mathbf{y})$, if $\mathbf{y} \neq \mathbf{y}^o$ in at least one of the previous CPS games. Proposition 5 implies that peer i cannot gain in the current CPS game by deviating to $x_i > x_i^o$ or $z_{ij} < y_j^o$ for some $j \neq i$. Hence, a profitable deviation involves either $x_i < x_i^o$ in stage one or $y_i < y_i^o$ in stage two (or both). Either case results in a reduction in the sharing level from y_i^o. Since sharing levels are publicly observed, any profitable deviation is detectable and punishment will be triggered. Since the gain from deviation in the current CPS game is bounded above, it will be erased by the punishment in the long run. In other words, peer i receives $v_i^o \triangleq v_i(\mathbf{x}^o, \mathbf{y}^o, \mathbf{Z}^o)$ on average if it follows the described repeated game strategy and $f^*(\kappa)$ if it deviates in a way that the deviation increases the current utility. Since $(\mathbf{x}^o, \mathbf{y}^o, \mathbf{Z}^o)$ is participation-efficient, we have $v_i^o \geq f^*(\kappa)$ for all $i \in \mathcal{N}$. Hence, the described repeated game strategy, which realizes the allocation $(\mathbf{x}^o, \mathbf{y}^o, \mathbf{Z}^o)$ in every CPS game, is a non-cooperative equilibrium of the repeated CPS game. ∎

E. Proof of Proposition 11

Proof: Assume that the core is nonempty and choose a utility profile $\mathbf{v} = (v_1, \ldots, v_N)$ in the core. Suppose that there exists a peer i with $v_i < g^{FS}(N^*)$. Consider a coalition $\mathcal{S}$ of size N^* that do not include peer i, which must exist since $N > N^*$. Then $\sum_{j \in \mathcal{S}} v_j = N^* g^{FS}(N^*)$, and thus $\sum_{j \in \mathcal{S} \setminus \{k\}} v_j \leq (N^* - 1) g^{FS}(N^*)$, where peer k is the one that receives the highest utility among peers in $\mathcal{S}$. Then $\sum_{j \in (\{i\} \bigcup \mathcal{S} \setminus \{k\})} v_j < N^* g^{FS}(N^*)$, and thus peer i and peers in $\mathcal{S} \setminus \{k\}$ can block the utility profile $\mathbf{v}$. Hence, we need to have $v_i \geq g^{FS}(N^*)$ for all $i \in \mathcal{N}$. This is possible, with equality, only if N is a multiple of N^*. We can confirm that the core is nonempty since the utility profile $v_i = g^{FS}(N^*)$ for all $i \in \mathcal{N}$ satisfies the definition of the core. ∎

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