Congestion, Information, and Secret Information in Flow Networks

Khoa Tran Phan, Mihaela van der Schaar, *Fellow, IEEE*, and William R. Zame

***Abstract*—Some users of a communications network may have more information about traffic on the network than others do—and this fact may be secret. Such secret information would allow the possessor to tailor its own traffic to the traffic of others; this would help the secret information possessor or informed user and (might) harm other uninformed users. To quantitatively study the impact of secret information, we formulate a flow control game with incomplete information where users choose their flows in order to maximize their (expected) utilities given the distribution of the actions of others. In this environment, the natural baseline notion is Bayesian Nash Equilibrium (BNE); we establish the existence of BNE. Next, we assume that there is a user who knows the *realized* congestion created by other users, but that the presence of this informed user is *not* known by other uninformed users; thus, the informed user has *secret information*. For this environment, we define a new equilibrium concept: the Bayesian Nash Equilibrium with Secret Information (BNE-SI) and establish its existence. We establish rigorous estimates for the benefit (to the informed user) and harm (to the uninformed users) that result from secret information; both the benefit and the harm become *smaller* for large networks. Interestingly, simulations demonstrate that secret information may in fact benefit *all* users. Secret information may also harm uninformed users in particular scenarios. This analysis can be used as a starting point for securing communications networks, both from the network manager and the user's perspectives.**

***Index Terms*—Bayesian game, communication games, secret information.**

I. Introduction

The smooth functioning of many communication networks depends on the information users have about the network and about other users. In such settings, it is of concern that some users (whom we call *informed users*) might acquire "illicit" information, and that this illicit information might aid informed users and—perhaps more importantly—harm others (whom we call *uninformed users*). This potential for harm might reduce the willingness of uninformed users to pay for network services; if the potential for harm is great enough, it might deter uninformed users from joining the network at all. Thus the designer/manager of a network has an important incentive to secure the network so that no users can obtain illicit information. Because such security might be costly, it is important to know the extent to which illicit information would be useful to an informed user and harmful to uninformed users.

Many kinds of illicit information might be relevant; in this paper we focus our attention on illicit information about the behavior of other users. We set our study in the context of flow control. We consider a network of users of which only some may be online at a given moment. Users are distinguished by their utility functions, which we think of as their *types*. Each of the users chooses a flow to send to the network and derives a utility that depends on its own flow and on network congestion, which we proxy by the ratio of total flow to the capacity of the network. In our baseline scenario, the distribution of characteristics of users is commonly known but the realization of characteristics of users who are online at a given moment is not. Hence users can work out—or learn—the distribution of congestion, but not the realized congestion. For this scenario, an appropriate solution notion is Bayesian Nash Equilibrium (BNE). Under appropriate assumptions, we show that BNE exist. To explore the impact of secret information, we depart from the baseline scenario by assuming that some (informed) user knows, not only its own type or utility function and the distribution of types of potential users, but also the *realized congestion created by other users*. Furthermore, other users do not know this; thus, the informed user has *secret information*.[1] For this scenario, an appropriate solution notion is what we call Bayesian Nash Equilibrium with Secret Information (BNE-SI); under the same assumptions as before, we show that BNE-SI exist.

To see the distinction in a simple setting, suppose there are two players whose utilities depend on the actions of both players. Player 1 chooses an action first, player 2 chooses an action second. It is clear that the "correct" action for player 1 depends not only on whether or not player 2 sees the action of player 1 before choosing his own action, but also on whether or not player 1 knows whether player 2 sees his action. The standard Harsanyi framework [3], [21] would incorporate a probability p that player 2 sees this action and assume that p is common knowledge. Hence, player 1 believes with probability p that player 2 sees player 1's action, and player 2 believes that player 1 believes with probability p that player 2 sees player 1's action, and so forth. In our framework, player 1 believes (with probability 1) that player 2 does not see, and player 2 knows this. In the Harsanyi framework, the equilibrium behavior of

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[1]We assume here that the informed user learns *all* the relevant information—in this case, the realized congestion—at no cost. A more elaborate model ought to take account of the amount of information that might be acquired and the cost of acquiring it, and the steps that a network manager might undertake to prevent users from acquiring such information.

player 1 must take into account that player 2 might see the action of player 1; in our framework, the equilibrium behavior of player 1 does not take this into account. Hence, player 2's knowledge is secret. In our model, player 2 is the "informed" player and player 1 does not know that player 2 is informed.

Back to the flow control game, information matters to the informed user because a user who knows the congestion in the network can choose to send a low flow when the network is congested and a high flow when it is not. The behavior of the informed user matters to the uninformed users because by sending a high flow when the network is not congested the informed user appropriates the benefits of low congestion that would otherwise accrue to the uninformed users. The fact that information is secret matters because it prevents uninformed users from countering the behavior of the informed user of this information. Secret information *always* confers a benefit to the informed user.[2] The behavior of the informed user is beneficial to other users when it reduces congestion and detrimental to others when it increases congestion. Both of these effects are attenuated when there are many users in the network, most obviously because the impact of *any* one user is attenuated when the network is large, more subtly because the Law of Large Numbers reduces the usefulness of secret information, and more subtly still because the latter effect feeds back into the behavior of an informed user. To make this point, we establish upper bounds in terms of the number of potential users and the capacity of the network on the possible gain to an informed user and the possible harm to uninformed users. These bounds show that, perhaps paradoxically, secret information may be less important in larger networks than in smaller networks. Simulations confirm our theoretical results; indeed they suggest that the bounds we provide are quite conservative.

A. *Related Literature*

Game-theoretic tools have been applied to analyze the behavior of users and their performance in communications networks; for example see [1] and [2] and references therein. Particularly, there is by now a substantial literature that uses Bayesian games [3] to model the interactions among selfish users with incomplete information who compete for access to network resources (e.g., power and bandwidth). In these models, action spaces typically represent power levels, transmission probabilities, or expenditures on resources; user types when considered typically represent channel gains. Much of this literature asks about existence and uniqueness of Bayesian Nash equilibrium and system performance at equilibrium. For instance, [4] uses a Bayesian game to model the interaction among wireless users who must have information about channel conditions and must choose between frequency division multiplexing or full bandwidth spreading. [5] uses a Bayesian game to study the power allocation problem in fading multiple access channels, where users selfishly maximize their ergodic capacity with incomplete information about the fading channel gains. In [6], Bayesian games are used to model the power allocation problem in multicarrier interference networks. In [7], random-access protocols are formulated as Bayesian games and their performance at equilibrium is analyzed. [8] uses Bayesian games to design distributed resource allocation strategies in TDMA networks. [9], [10] use Bayesian games to capture the effects of information availability and asymmetry on the problem faced by a profit-maximizing manager. [11] studies a routing game with incomplete information, using a potential function approach.

A literature that might seem parallel to ours but is actually quite distinct considers the problem of malicious users: users whose objective is to damage the network and/or increase the cost incurred by other users; see for instance [12]–[14]. Our informed users seek only to maximize their own utility; their behavior may harm others, but this is a side consequence of their own selfish maximizing behavior; it is *not* malicious.

B. *Plan of the Paper*

Following this Introduction, Section II formalizes our environment, defines Bayesian Nash equilibrium (BNE), establishes that BNE exists, and solves (numerically) for BNE in some simple settings. Section III introduces secret information, defines Bayesian Nash Equilibrium with Secret Information (BNE-SI), establishes that it exists, and solves (numerically) for BNE-SI in the same settings. Section IV defines the gain and harm resulting from secret information, provides some discussion and rigorous estimates for the gain and harm, and simulations in (yet again) the same settings. Section V concludes. All proofs are collected in the Appendix.

II. System Model

We consider a network that admits at most $N+1$ users at a given moment and has nominal flow capacity C. The potential users in the network are of types θ where $\theta \in \Theta$, the type space; for convenience we assume the type space $\Theta \subset \mathbb{R}^K$ is compact (closed, bounded, and non-empty). A user of type θ is distinguished by a utility function $U(x, c, \theta)$ that depends on the flow x the user sends to the network and on network congestion $c = \Phi/C$, where Φ is the total flow sent by all users. Throughout we assume the following:

(A1) Flow choices x lie in some compact non-empty interval action space $\mathbb{A} \subset \mathbb{R}_+$.

(A2) Utility U is twice differentiable in x, c and derivatives are continuous in x, c, θ.[3]

(A3) For each c, θ: $U(0, c, \theta) = 0$.

(A4) For each x, θ: $U(x, c, \theta)$ is (weakly) decreasing in the network congestion c.

(A5) For each θ: $U(x, c, \theta)$ is strictly concave in own flow x.[4]

Assumption (A3) says that a user receives zero utility if not sending anything to the network regardless of its type and network congestion. Assumption (A4) means that the utility for a user decreases when the network congestion increases which happens when other users send larger flows or the network capacity decreases. It is a practical assumption that captures the

[2]By contrast, information that an informed user is known to possess need *not* confer a benefit on the informed user, and might actually be harmful.

[3]Continuity in θ guarantees equicontinuity in x, c.

[4]Keep in mind that $U(x, c, \theta)$ depends on own flow x directly (in the first argument) and indirectly (in the second argument via c), hence strict concavity with x means that

$$\frac{\partial^2 U}{\partial x^2} = U_{11}(x, c, \theta) + \left(\frac{2}{C}\right) U_{12}(x, c, \theta) + \left(\frac{1}{C^2}\right) U_{22}(x, c, \theta) < 0.$$

congestion effect. Assumption (A5) is commonly considered for analysis tractability. Also, without any loss of generality (w.l.o.g.), U is assumed to be increasing with θ. Some of the utility functions we have in mind have the form $U(x,c,\theta) = b(x,\theta) - xh(c)$, where b (benefit) is strictly concave increasing in x, and h (harm) is strictly convex increasing in c. We interpret $b(x,\theta)$ as the *benefit* derived by a user with type θ who sends flow x and $h(c)$ is the corresponding *per-unit cost* when the network congestion is c. We should note that such utility functions exhibit negative externalities which is typical scenario in flow control games in communications networks [16], [17]. Some frequently studied benefit and cost functions are such as those listed below; see [9], [10], [15]–[19] and references therein.[5]

- Benefit functions $b(x,\theta)$
  - —Quadratic functions: $b(x,\theta) = \theta\Big(-(1/2)x^2 + \gamma x\Big)$ for some positive constant γ.
  - —Logarithm functions: $b(x,\theta) = \theta \log(\gamma + x)$ for some nonnegative constant γ and $\gamma = 0$ gives proportional fair utility function.
  - —α-fair functions: $b(x,\theta) = \theta(1-\alpha)^{-1}x^{1-\alpha}$ for $\alpha \in (0,1) \cup (1,\infty)$.
- Cost functions $h(c)$
  - —Delay functions: $h(c) = 1/(C - Cc)$ or logarithm of delay functions: $h(c) = -\log(C - Cc)$ where C is the nominal flow capacity.
  - —Linear cost function: $h(c) = c$.

The total flow Φ is the sum of own flow x and flow sent by others Φ_0, i.e., $\Phi = x + \Phi_0$ and hence, network congestion is the sum of own congestion and congestion caused by others $c = \Phi/C = (x/C) + (\Phi_0/C)$. If $\xi = \Phi_0/C$ is the congestion caused by others we define

$$V(x,\xi,\theta) = U\left(x, \left(\frac{x}{C}\right) + \xi, \theta\right)$$

for utility as function of own flow x, type θ and congestion caused by others ξ. We write $V^*(\xi,\theta)$ for the maximum function

$$V^*(\xi,\theta) = \max_{x\in\mathbb{A}} V(x,\xi,\theta).$$

The types of users who are online at a given moment is random; user types are drawn independently from a distribution ν on Θ. To allow for the possibility that the fewer than $N+1$ users are online at a given moment, we adjoin a "dummy" type $\hat{0}$ to Θ: $\widehat{\Theta} = \Theta \cup \{\hat{0}\}$. The dummy type $\hat{0}$—the *offline type*—always sends a flow of 0 and derives utility of 0: $U(0,c,\hat{0}) = 0$. We assume that the distribution of the number of users online at a given moment is binomial, so the distribution of types in $\widehat{\Theta}$ is

$$\nu = q\nu + (1-q)\delta_{\hat{0}}.$$

That is, if a user is drawn at random from $\widehat{\Theta}$ the probability the user is online is p and the probability the user is offline is $(1-p)$; if the user is online its type is drawn from Θ according to the distribution ν. Throughout we assume $0 < p \le 1$; if $p = 1$ then the maximal number of users are online at all times.

[5]The literature does not usually consider the capacity of the network in an explicit way, and therefore uses total flow directly as a proxy for congestion. We highlight capacity explicitly and use the ratio of total flow to capacity as a proxy for congestion because it facilitates analysis of growing networks and comparisons across networks.

III. Some Basic Results on the Bayesian–Nash Equilibrium

A. Existence of BNE

We assume for the moment that all of the above is *common knowledge*; that is, each user knows the description of the environment and his own type; each user knows that all other users have the same knowledge; each user knows that all other users know that all other users have the same knowledge, etc. (We deviate from the common knowledge assumption in the following section when we introduce secret information.) In this context, a *strategy* is a (measurable) function $X : \widehat{\Theta} \to \mathbb{A}$ such that $X(\hat{0}) = 0$ (offline users send 0 flow). Given a strategy X, consider a user of type θ who sends the flow x. If the profile of other users is $\lambda = (\theta_1,\ldots,\theta_N)$ then the total flow of other users is $X(\lambda) = \sum_{i=1}^{N} X(\theta_i)$, the congestion caused by other users is $\xi(\lambda) = X(\lambda)/C$, and the utility of user with type θ is $V(x,\xi(\lambda),\theta)$. Hence, expected utility of a user of type θ who sends the flow x, assuming others send flows according to X is

$$E_{\theta_1,\ldots,\theta_N}\big[U(x|\theta,X)\big] = \int_{\widehat{\Theta}^N} V(x,\xi(\lambda),\theta)\,d\nu(\lambda) \qquad (1)$$

where $d\nu(\lambda)$ denotes $d\nu(\theta_1)\ldots d\nu(\theta_N)$. X is a *Bayesian Nash Equilibrium (BNE)* if for (almost) every $\theta \in \Theta$, the flow $X(\theta)$ maximizes expected utility, assuming others send flows according to X; that is

$$E_{\theta_1,\ldots,\theta_N}\big[U(X(\theta)|\theta,X)\big] \ge E_{\theta_1,\ldots,\theta_N}\big[U(x|\theta,X)\big]$$

for every $x \in \mathbb{A}$. Several points should be noted.

1) Only pure strategies are considered. In fact, only pure strategies are relevant since the best responses maximizing $E_{\theta_1,\ldots,\theta_N}\big[U(x|\theta,X)\big]$ are in pure strategies and are unique due to strict concavity of U in own flow x.

2) The definition requires that $X(\theta)$ be optimal given the distribution of flow choices of others, but in this context this reduces to the requirement that $X(\theta)$ be optimal given the distribution of *congestion* caused by others. In a long-standing network, the users of the network might simply have *learned* the distribution of congestion—either because congestion was observed or announced directly by the protocol designer or because it was inferred from own utility over time. In particular, there is no need to assume that users can solve for the strategies employed by others.

Theorem 1: Under assumptions A1)–A5), a Bayesian Nash Equilibrium exists.

We caution the reader that BNE need not to be unique (although it will be so in some circumstances), so that to assume users follow a particular BNE equilibrium requires either that they have learned to coordinate or that the network manager suggests a particular BNE protocol and users follow that protocol.

B. Calculating BNE

To illustrate the nature of BNE and in particular the influence of the number of users and the capacity, we offer a simple example. A preliminary remark may be useful. Fix a BNE X and a type $\theta \in \Theta$. By definition, $X(\theta)$ solves the following optimization problem:

$$X(\theta) = \arg\max_{x \in \mathbb{A}} \; E_{\theta_1,\ldots,\theta_N}\big[U(x|\theta, X)\big]. \tag{2}$$

Strict concavity guarantees that the solution is determined by the first order condition (possibly at the end points):

$$\frac{\partial E_{\theta_1,\ldots,\theta_N}\big[U(x|\theta, X)\big]}{\partial x}\Big|_{x=X(\theta)} = 0. \tag{3}$$

Substituting the definition of $E_{\theta_1,\ldots,\theta_N}\big[U(x|\theta, X)\big]$ in (1) into (3) provides a functional equation for the BNE. However, this functional equation can be intractable and difficult or impossible to solve in closed form—even if the utility function U is relatively simple and $q = 1$ (so the number of users online is not random). However, the following simple Example illustrates how it can be solved—albeit not quite in closed form—in a special case. (In the following sections, we extend this simple example to allow for secret information and use it as the basis of simulations.)

1) Example 1: There are $N + 1$ users. Utility has the form

$$U(x, c, \theta) = \theta\left(x - \frac{\gamma}{2}x^2\right) - xc$$

for some $\gamma \in (0, 1]$. Users choose flows in $\mathbb{A} = [0, 1]$; types belong to $\Theta = [1, 2]$ and are uniformly distributed. The online probability $p = 1$. The quadratic utility function has also been used in [10] and [19] and many others.

To solve for BNE, assume for the moment that optimal flow is interior. The functional equation for a type θ becomes

$$\theta - \gamma x - \left(\frac{2}{C}\right)x - \overline{\xi} = 0 \tag{4}$$

where $\overline{\xi}$ is the expected congestion caused by others. If we write

$$A = \int_1^2 X(\theta) d\nu(\theta) \tag{5}$$

for expected individual flow, then $\overline{\xi} = NA/C$ and we can solve for $X(\theta)$ in terms of A. Allowing for the possibility that optimal flow is *not* interior (i.e., that optimal flow might be 0 or 1) leads to

$$X(\theta) = \begin{cases} 0 & 1 \le \theta < \overline{\xi} \\ \frac{\theta - \overline{\xi}}{\theta\gamma + \frac{2}{C}} & \overline{\xi} \le \theta \le \frac{1}{1-\gamma}\left(\overline{\xi} + \frac{2}{C}\right) \\ 1 & \frac{1}{1-\gamma}\left(\overline{\xi} + \frac{2}{C}\right) < \theta \le 2 \end{cases}. \tag{6}$$

Thus, we can solve for the entire BNE in terms of expected individual flow A. In turn, A is determined from X by (5). Note that $X(\theta)$ is increasing in A, so there is a *unique* A for which (5) is satisfied; hence, BNE is also unique. Also, it can be seen that the BNE X is monotone increasing with θ.

Expected (or equilibrium) utilities at BNE are

$$v(\theta) = \theta\left(X(\theta) - \frac{\gamma}{2}X(\theta)^2\right) - \frac{1}{C}X(\theta)\big(X(\theta) + NA\big). \tag{7}$$

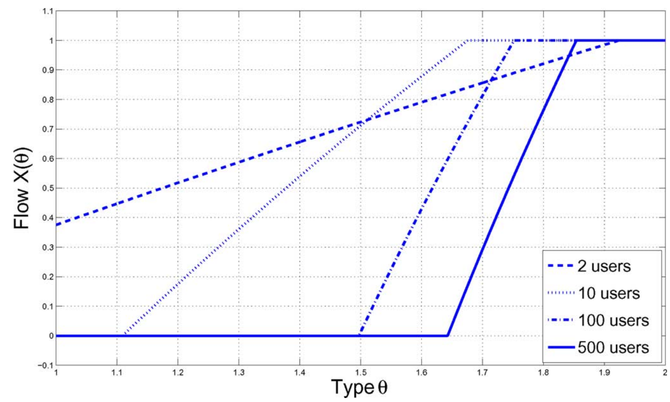

Fig. 1. BNE flow $X(\theta)$: $\beta = 0.7$.

C. Simulations

In Fig. 1, we calculate and display BNE flow $X(\theta)$, fixing $\gamma = 0.1$ and $p = 1$. We show BNE flow and expected utility as functions of type for number of users $N + 1 = 2, 10, 100, 500$; anticipating our analysis of secret information below, we take capacity $C = (N + 1)^\beta$ for $\beta = .7, .8, .9, 1.0$ which can be interpreted as capacity scaling factor. We caution the reader that flows are *not* piecewise linear, although the curvature is not noticeable in the figures. We can observe the effect of the scaling factor β. When $\beta = 1$, i.e., the capacity grows at the same rate as the number of users in the system, given a particular type, users tend to send larger flows when there are more users. In contradiction, for smaller values of β, i.e., the network capacity grows slower than the number of users grows, users tend to send smaller flows when there are more users. Particularly, when $\beta = .7$, users send zero flow for a wide range of types $\theta \in [1, 1.65]$. This is because when the capacity of the system grows slower than the number of users does, sending smaller flows helps to create smaller congestion, especially when there are large number of users. Note that the congestion c is inversely proportional to the capacity C. On the other hands, when the capacity of the system grows fast enough, users tend to send larger flows at equilibrium in the presence of more users.

Fig. 2 shows the expected utility at equilibrium $v(\theta)$. We can see that the utility is increasing with type θ and is smaller when there are more users in the network, especially when for small values of β. For $\beta = 1$, the utility of users is not so much dependent on the number of users in the network since we proxy the congestion as the ratio between the total flow and network capacity.

A useful way to understand the results of these calculations is to think about what happens if $C = (N + 1)$ and $N \to \infty$. In that case, we might imagine a "limit network" with a continuum of users, for which each user's contribution to total flow—and hence to congestion—is negligible. Assuming that the Law of Large Numbers holds exactly in the continuum limit, a user of type θ maximizes $\theta\big(x - (\gamma/2)x^2\big) - xA$, where A is average flow (and is independent of x). Solving yields

$$X^\infty(\theta) = \begin{cases} \frac{\theta - A}{\gamma\theta}, & 1 \le \theta \le \frac{A}{1-\gamma} \\ 1, & \frac{A}{1-\gamma} < \theta \le 2 \end{cases} \tag{8}$$

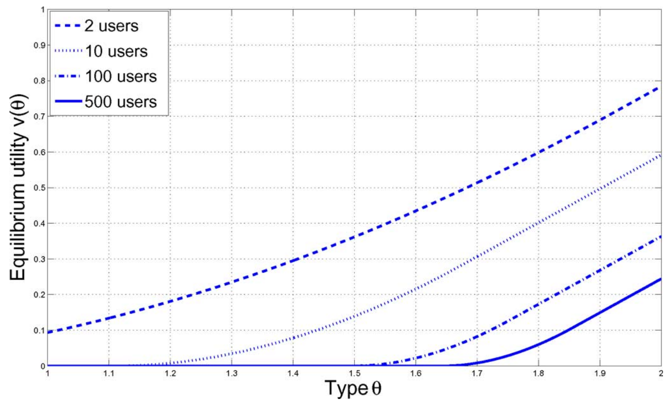

Fig. 2. BNE utility $v(\theta)$: $\beta = 0.7$.

where, as before, A is expected individual flow: $A = \int_1^2 X^\infty(\eta)d\eta$. For the above example, $0 \leq A \leq 1$, and thus, for small γ, $X^\infty(\theta)$ is approximately equal to 1 for all values of $\theta \in [1,2]$. □

IV. Secret Information: Equilibrium

To this point we have assumed that each user knows the distribution of other number and types of users who are online and the strategies they follow, hence the distribution of congestion, but not the realized number of users or the realized congestion. We now consider a setting in which one user—the informed user—has additional information.[6] We focus on the starkest scenario in which the informed user knows realized congestion caused by others. It would be more than enough for user 0 to observe the types of other users, and hence, given a particular BNE, to infer their flow choices. However, it seems much more natural to assume, as we do here, that user 0 observes congestion directly, perhaps because it is able to observe network information that is improperly secured. We assume that the presence of an informed user is *not* common knowledge; rather other users have the same beliefs as above, and hence use the same strategies—and the informed user knows this. Thus, the informed user has *secret information*.[7] In this environment, *Bayesian Nash Equilibrium with Secret Information* BNE-SI consists of a strategy $X : \Theta \to \mathbb{A}$ and a strategy $F : [0,\infty) \times \Theta \to \mathbb{A}$ for the informed user 0 such that:

- for an uninformed user of type θ^*, the flow $x = X(\theta^*)$ maximizes $E_{\theta_1,\ldots,\theta_N}\left[U(x|\theta^*,X)\right]$ where $E_{\theta_1,\ldots,\theta_N}\left[U(x|\theta^*,X)\right]$ is defined as in (1);
- for the informed user of type θ, the flow $x = F(\xi,\theta)$ maximizes $V(x,\xi,\theta)$.

Again, uninformed users behave according to the BNE X (as in Section II) but the informed user optimizes given the *realized congestion created other uninformed users* in the network. We emphasize that at BNE-SI equilibrium, the informed user conditions her behavior on her own type and on the realized congestion, while other users believe (wrongly) that all other users condition only on their own type (and follow the strategy X). Our informed user basically has the available information and acts like the leader in Stackelberg Bayesian games [9], [10]. However, in Stackelberg games, the presence of the leader is common knowledge.

Our approach to secret information departs from the usual approach in the economics literature, which (almost) always assumes that all details of the environment are common knowledge; see [3], [21], and [22] for instance. The usual approach in the economics literature would be to posit a commonly known probability $p > 0$ that some user is informed, and employ a notion of equilibrium in which uninformed users take account of the probability that some user is informed. Our approach seems more appropriate to the problem at hand and more tractable as well. Moreover, we can interpret that at BNE, users take actions (choose flows) simultaneously without knowing the realized actions of other users. In order to optimize its expected utility, users need to know the distribution of congestion (but not the realized congestion). In the setting where one user is informed, it is very reasonable to ask how that user becomes informed. One possibility is that he acts last and covertly observes the congestion on the network before sending his own flow. Another possibility is that he has learned the BNE from experience, covertly observes the types of other users, and calculates the flows they will send. More generally, the informed user might secretly know *something* that is relevant—some signal of realized congestion—even if he might not know *everything* that is relevant. Here, we are treating the extreme case where he knows *everything* that is relevant.

Our assumptions guarantee that the informed user's optimization problem always has a unique solution, so the assumptions of the previous section guarantee the existence of a BNE-SI.

Corollary 1: Bayesian Nash Equilibrium with Secret Information exists.

Note that optimal behavior of the informed user is completely determined by the BNE strategy used by the uninformed users, so there are exactly as many BNE-SI as there are BNE. To give a little additional insight, we continue the example of Section II to record the strategy of the informed user.

1) Example 2: Everything is in Example 1 except that one user is informed. It is easily checked that if the informed user has type θ and congestion caused by others is ξ, the optimal flow choice for the informed user is

$$F(\xi,\theta) = \begin{cases} 0, & 1 \leq \theta < \xi \\ \frac{\theta-\xi}{\theta\gamma+\frac{2}{C}}, & \xi \leq \theta \leq \frac{1}{1-\gamma}\left(\xi+\frac{2}{C}\right) \\ 1, & \frac{1}{1-\gamma}\left(\xi+\frac{2}{C}\right) < \theta \leq 2 \end{cases}. \quad (9)$$

Note that given a fixed ξ, the flow of the informed user is still monotone increasing in its type θ. We can see that the informed user adapts its flow according to network congestion, sending a larger flow $F(\xi,\theta)$ when the congestion caused by others ξ is small and a smaller flow when ξ is large. Also, compared to (6), it can be seen that $F(\xi,\theta) \leq X(\theta)$ if and only if $\xi \geq \overline{\xi}$, i.e., the informed user will send smaller flow than the flow if it were uninformed when the realized congestion caused by others is larger than the average congestion and vice versa.

[6] A different advantage of information in wireless systems has been treated in [20] where the authors showed that a user could improve its performance if it had more information about the strategy of the competing user.

[7] If the presence of an informed user were common knowledge, our model would reduce to a conventional Bayesian game with asymmetrically informed agents.

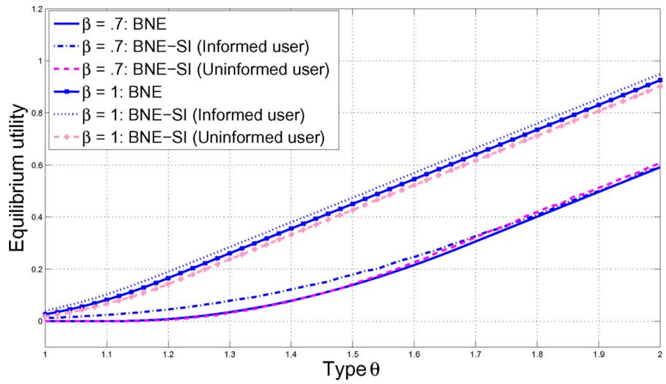

Fig. 3. Equilibrium utility at BNE, and at BNE-SI for an informed user and an uninformed user.

For the above example, Fig. 3 plots the equilibrium utility with respect to type for a user at BNE, and for both an uninformed user and an informed user at BNE-SI for a network with 10 users for $\beta = 1$ and .7. We can observe that while secret information always (unsurprisingly) confers a benefit to the informed user, it could be beneficial or harmful to the uninformed user(s). The magnitudes of the gain or harm also depend on the users' types, the characteristic of the network, i.e., the parameter β etc. More careful analysis follows next.

V. Secret Information: Benefit and Harm

A. Some Analysis

In addition we make technical assumptions about the derivatives of U for flow choices, congestion and types for which utility is non-negative. Set

$$\Gamma = \Big\{(x, c, \theta) \in \mathbb{A} \times [0, \infty) \times \Theta : U(x, c, \theta) \geq 0\Big\}.$$

Note that (A3) guarantees that optimal choices will never yield utility less than 0 so only tuples in Γ are relevant for optimization.

(A6) Utility U and its first and second partials are uniformly bounded on Γ.

(A7) The second derivative of U with respect to own flow is bounded away from 0 on Γ.

Assumption (A7) requires that the curvature of U is bounded away from 0 on $\mathbb{A}$: $-\partial^2 U(x, c, \theta)/\partial x^2 \geq 1/u_l > 0$ for all $x \in \mathbb{A}$ and some u_l. We could have assumed that the curvature of U was bounded away from 0 everywhere which would be a consequence of strict concavity if the relevant portion of the domain of U is compact. Assumption (A7) is weaker and is commonly assumed in existing works [23], [24].

The benefit that information confers on an informed user is the difference between the utility the informed user obtains when uninformed users follow a given BNE but the informed user conditions on its own type *and* on the realized congestion of others, and the utility the informed user obtains when it and the uninformed users follow the given BNE. Because the informed user might have different incentives to acquire secret information (which might be costly) depending on its type, we fix the type of the informed user, and calculate expectations over the types of the uninformed users. Hence, the gain to an informed user of type $\theta \in \Theta$ is

$$G_{N,C}(\theta) = \int_{\widehat{\Theta}^N} V\Big(F(\xi(\lambda), \theta), \xi(\lambda), \theta\Big) d\nu(\lambda) - \int_{\widehat{\Theta}^N} V\Big(X(\theta), \xi(\lambda), \theta\Big) d\nu(\lambda) \quad (10)$$

where $\xi(\lambda)$ is the congestion created by uninformed users for type profile $\lambda = (\theta_1, \ldots, \theta_N) \in \widehat{\Theta}^N$. (We retain the subscripts N, C to emphasize that the gain to an informed user depends on the size and capacity of the network.)

Because the informed user could always disregard his information and others do not know he has it, the informed user must do at least as well in a BNE-SI as in the corresponding BNE, and will do strictly better except in degenerate scenarios. That is, secret information always has positive value to the user who possesses it: $G_{N,C}(\theta) > 0$. The magnitude of this value will of course depend on the particular environment, but we can bound it above by an expression that depends on the parameters of the network.

Theorem 2: There is a constant K_1 that depends only on the bounds [given in Assumptions (A6), (A7)] on U and its derivatives on the set Γ such that for every $\theta \in \Theta$ and every $\alpha \in (0, 1)$ we have

$$G_{N,C}(\theta) \leq K_1 \left(N^{-1+2\alpha} + \left(\frac{1}{C}\right) N^{1-\alpha}\right). \quad (11)$$

In particular, if capacity $C > N^{\beta}$ for some $\beta \in (1/2, 1]$ then

$$G_{N,C}(\theta) \leq 2K_1 \left(N^{(1-2\beta)/3}\right). \quad (12)$$

To see the importance of Theorem 2 consider two thought experiments.

- *Increase both the number of subscribers N and the capacity C.* Because $\beta > 1/2$ the quantity $N^{(1-2\beta)/3} \to 0$ as $N \to \infty$; hence, if number of subscribers and capacity grow in such a way that $C > N^{\beta}$ with $\beta > 1/2$, then

  $$G_{N,C}(\theta) \to 0 \text{ as } N \to \infty \ \ (\text{uniformly for } \theta \in \Theta).$$

  Slightly imprecisely, if the number of users grows and the capacity grows faster the *square root* of the number of users, then the benefit an informed user derives from its secret information tends to 0.
- *Increase the number of subscribers N keeping the capacity C fixed.* Suppose the capacity is C and the initial number of subscribers is N, where $C = N^{\beta}$, $\beta < 1/2$. If the number of subscribers increases k-fold to kN, while capacity remains at C, the relationship between capacity and number of subscribers will change to $C = (kN)^{\beta'}$ where

  $$\beta' = \frac{\beta \log N + \log k}{\log N + \log k}.$$

  Hence, when the capacity remains unchanged, it will require a very substantial expansion of the network such that (secret) information is of little benefit to the informed user.

The harm inflicted on an uninformed user by the behavior of an informed user is the difference between the (expected) utility of an uninformed user when *all* users are uninformed and the (expected) utility of an uninformed user when one user is informed. The former utility depends on the type of the uninformed user; the latter utility depends on the types of both the uninformed user and the informed user. To define the latter utility, fix a type θ^* of the uninformed user, a type θ of the informed user, and a profile $\hat{\lambda} = (\theta_1, \ldots, \theta_{N-1}) \in \widehat{\Theta}^{N-1}$ of other uninformed users. As before, write $\xi(\hat{\lambda})$ for the congestion caused by users $\theta_1, \ldots, \theta_{N-1}$; the congestion caused by the uninformed user θ^* is $X(\theta^*)/C$ so the total congestion caused by the uninformed users is $\xi^*(\hat{\lambda}) = \xi(\hat{\lambda}) + X(\theta^*)/C$. Hence the informed user will send the flow $F(\xi^*(\hat{\lambda}), \theta)$ and the utility obtained by the uninformed user θ^* will be $V\left(X(\theta^*), [\xi(\hat{\lambda}) + (1/C)F(\xi^*(\hat{\lambda}), \theta)], \theta^*\right)$. Taking expectations over profiles $\hat{\lambda}$ yields

$$\begin{aligned} &E_{\theta_1,\ldots,\theta_{N-1}}\left[U(X(\theta^*)\,|\,\theta^*, \theta, X)\right] \\ &= \int_{\widehat{\Theta}^{N-1}} V\left(X(\theta^*), \left[\xi(\hat{\lambda}) + \left(\frac{1}{C}\right) F(\xi^*(\hat{\lambda}), \theta)\right], \theta^*\right) d\nu(\hat{\lambda}) \end{aligned} \tag{13}$$

where $d\nu(\hat{\lambda})$ denotes $d\nu(\theta_1) \ldots d\nu(\theta_{N-1})$ and the harm inflicted on the uninformed user is

$$\begin{aligned} &H_{N,C}(\theta^*, \theta) \\ &= \int_{\widehat{\Theta}^{N-1}} V\left(X(\theta^*), \left[\xi(\hat{\lambda}) + \left(\frac{1}{C}\right) X(\theta)\right], \theta^*\right) d\nu(\hat{\lambda}) \\ &\quad - E_{\theta_1,\ldots,\theta_{N-1}}\left[U(X(\theta^*)\,|\,\theta^*, \theta, X)\right]. \end{aligned} \tag{14}$$

As we have noted, secret information is always of value to the informed user, but the impact on *uninformed* users is not obvious. Consider the flow sent by the informed user in comparison to the flow that would be sent if it were uninformed. If the realized total flow of other users is high, the informed user will tend to send a lower flow than if it were uninformed; if the realized total flow of other users is low, the informed user will tend to send a higher flow than if it were uninformed. Because congestion is harmful to everyone, the actions of the informed user will benefit everyone when the realized total flow of uninformed users is high and harm the uninformed users when the realized total flow of uninformed users is low. In particular, the behavior of the informed user with type θ will *benefit* an uninformed user with type θ^* for at least *some* type realizations $\hat{\lambda} \in \widehat{\Theta}^{N-1}$ of other uninformed users which can be shown to be in the following set:

$$\widehat{\Theta}_B^{N-1}(\theta^*, \theta) = \left\{\hat{\lambda} \in \widehat{\Theta}^{N-1} \mid F(\xi^*(\hat{\lambda}), \theta) < X(\theta)\right\}. \tag{15}$$

Whether the actions of the informed user will benefit the uninformed user *on average* depends on the elements of the environment. As an example, let us examine the case of linear per-unit cost function, i.e., $U(x, c, \theta) = b(x, \theta) - xc$ with BNE X. The harm inflicted on the uninformed user is

$$H_{N,C}(\theta^*, \theta) = \frac{1}{C} \int_{\widehat{\Theta}^{N-1}} X(\theta^*)\Big(F(\xi^*(\hat{\lambda}), \theta) - X(\theta)\Big) d\nu(\hat{\lambda}). \tag{16}$$

An immediate result is that $H_{N,C}(\theta^*, \theta)$ is negative if $\int F(\xi^*(\hat{\lambda}), \theta) d\nu(\hat{\lambda}) < X(\theta)$. In other words, the actions of the informed user benefit uninformed user if its expected flow is less than its flow if it were uninformed. Otherwise, the effect of the informed user on the uninformed users cannot be assessed. Our simulations (discussed below in Section IV-2 and also in Fig. 3) suggest that settings in which the benefit dominates harm and settings in which harm dominates benefit are both occurring. In any event, we can bound the harm above by an expression that depends on the parameters of the network.

Theorem 3: There is a constant K_2 that depends only on the bounds on derivatives of U on the set Γ [see Assumption (A6)] such that if $C > N^\beta$ for some $\beta > 1/2$ then

$$|H_{N,C}(\theta^*, \theta)| \leq K_2 N^{(1-8\beta)/3}. \tag{17}$$

which gives $|H_{N,C}(\theta^*, \theta)| \to 0$ as $N \to \infty$. In particular, if $C > N^\beta$ with $\beta > 7/8$ then

$$N \int_\Theta H_{N,C}(\theta^*, \theta) d\nu(\theta^*) \to 0 \text{ as } N \to \infty, \quad (\text{uniformly for } \theta \in \Theta). \tag{18}$$

The harm for each uninformed user tends to 0 as the network becomes large provided the capacity grows faster the *square root* of the number of users. By definition, $N \int_\Theta H_{N,C}(\theta^*, \theta) d\nu(\theta^*)$ is the expected *total* harm to uninformed users, so the second conclusion is that expected *total* harm to uninformed users tends to 0 as the network becomes large—provided the capacity of the network does not grow too much more slowly than the number of users.

B. Simulations

To illustrate Theorems 2 and 3, we present simulations in Fig. 4 that show the maximum gain available to an informed user and the average harm inflicted on others by the behavior of an informed user. Utility is $U(x, c, \theta) = \theta(x - .05x^2) - xc$ and $C = (N+1)^\beta$; types are distributed uniformly on $[1, 2]$. In all cases, we present the average of 15 000 draws. These simulations suggest that the bounds presented in Theorems 2, 3 are crude: convergence of gain and harm appear to be faster than the bounds established in Theorems 2, 3.[8] Moreover, the convergence rate of the harm is faster than that of the benefit as analytically demonstrated in Theorems 2 and 3. The simulations also confirm that the behavior of the informed user has both positive and negative effects on the performance of the uninformed users, depending on the environment parameters. We can see that for $\beta = 1$, the behavior of the informed user is always harmful to the uninformed users. This can be explained as follows. When the network capacity grows fast, the informed user is able to take advantage of this capacity expansion and can send higher flows (to derive more benefit) without causing (much) more congestion. Again, note that congestion is defined as the ratio between total flow and network capacity. Such behavior is harmful to the uninformed users. However, for smaller values of β, the behavior of the informed user harms the uninformed users

[8]It seems likely that a better estimate for (expected) harm could be obtained if we were careful to account separately for realizations for which secret information leads to greater congestion and hence is harmful and realizations for which secret information leads to less congestion and hence is beneficial.

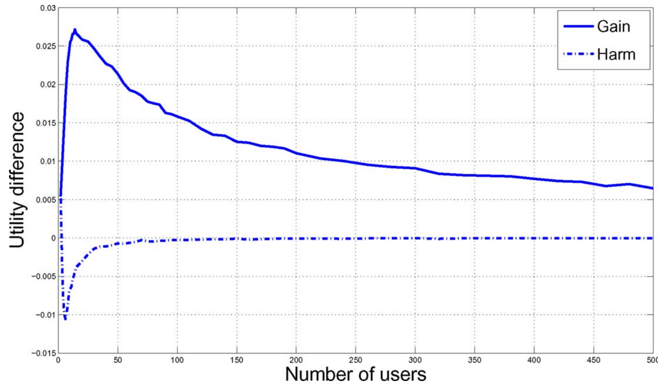

Fig. 4. Quadratic utility function; gain and harm: $\beta = 0.7$.

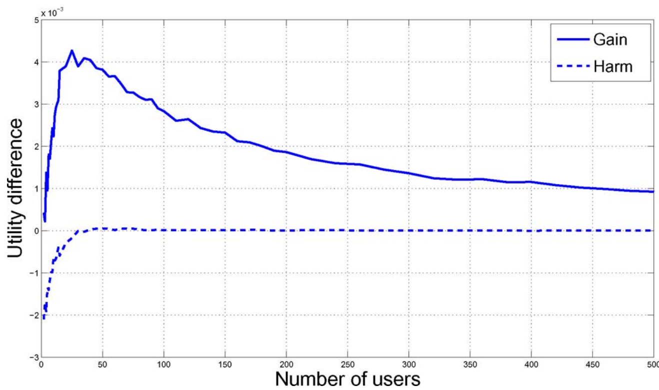

Fig. 5. Logarithm utility function; gain and harm: $\beta = 0.7$.

for small networks and benefits the uninformed users for larger networks. This is because when the capacity grows slower than the number of users, the informed user tends to send smaller flows to reduce the network congestion which is dominant.

To demonstrate the a different effect of secret information on the gain and harm, we consider another example using logarithm utility function $U(x,c,\theta) = \theta \log(x+1) - xc$. The type space is $\Theta = [1,2]$ with uniformly distributed types. The action space is $\mathbb{A} = [0,\infty]$. The nominal capacity is $C = (N+1)^{\beta}$ for $\beta = 1$ and .7. Different from the case of quadratic utility function, we can see from Fig. 5 that secret information is always beneficial to the uninformed users, for networks with any size. The convergence of the gain and harm to 0 is also verified.

C. Several Informed Users

Our analysis presumes there is a single informed user but it is also of interest to think about the possibility that there might be multiple informed users. To properly analyze such a setting one must first answer the question of what such users know about each other. One possibility would be to assume that such users are unaware of the existence of other informed users but still manage to observe the flows of all others, including the flows of other users with secret information. However, in that case there would seem to be a problem of deciding which user moves first. A second, more complicated, possibility would be to assume that the informed users are aware of each other and play equilibrium strategies in a game among themselves. A third, and still more complicated, possibility would be to assume that informed users are aware of the *possibility* of other informed users and play some equilibrium strategies in a Bayesian game. Inspecting the proof of Theorem 3 suggests that each of these possibilities would lead to an estimate of the gain $\widehat{G}_N(\theta)$ to each informed user that is of the same order as the estimate provided by Theorem 2, but with a different constant.

A crude estimate of the harm caused to others $\widehat{H}_{N,C}$ by the presence of k informed users can be obtained by noting that the harm to uninformed users is bounded above by the harm that would result by simply summing the flows chosen by each of the k informed users—ignoring strategic behavior among informed users. This leads to the crude estimate $|\widehat{H}_{N,C}| \leq kK_2N^{(1-2\beta)/6}$.

D. Large Informed User

In our analysis, we have presumed that the informed user is identical to other users except that for possessing additional information. In particular, the informed user faces the same flow constraints as the uninformed users. Informally: the informed user is "the same size" as uninformed users. However, in some circumstances it would seem natural to consider an informed user who is "somewhat bigger" than uninformed users, and hence faces weaker flow constraints—i.e., $x \in \mathbb{A}'$ where $\mathbb{A}' \supset \mathbb{A}$ is a compact interval—or perhaps "much bigger" than uninformed users, and hence faces no flow constraints at all—i.e, $x \in [0,\infty)$. In the first case , the conclusions of Theorems 2, 3 would remain unchanged, except that the constants K_1, K_2 would depend on upper bounds of U and its first and second partial derivatives on the larger interval $\mathbb{A}'$. In the second case, it would seem necessary to supplement the arguments used in the proofs of Theorems 2, 3 with careful estimates of the distribution of the optimal flow of the informed user. To obtain such careful estimates it would in turn seem necessary to know more about strategies used by uninformed users in BNE, which might in turn require additional assumptions on utility functions. Further research seems needed here.

VI. Conclusion: The Cost and Benefit of Secret Information

We have considered here a scenario in which a single user, otherwise no different from other users, has secret information, which is complete and acquired at no cost. As noted, these represent quite restrictive assumptions. More realistic scenarios would envisage the presence of some (perhaps a small number) of users who can acquire some information at some cost and who might be different (perhaps larger) than other users. In such a scenario, these users would face a tradeoff between the cost of acquiring information and the benefit conveyed by that information.

The analysis presented here has important implications for the design and operation of networks. On the one hand, preventing users on a network from obtaining information about (the usage of) others might be expensive—and would typically be more expensive for larger networks. On the other hand (as our analysis suggests), the benefit of such information to users who have it—and thus the incentives to acquire it—and the total harm done by the availability of such information would seem to be smaller for larger networks. Paradoxically, this suggests that security may be *less* of a concern for large networks than for small networks. Again, further research is needed.

APPENDIX
PROOFS

Proof of Theorem 1: [25] shows that there is an equilibrium in mixed strategies. The assumption that utility is strictly concave in own flow shows that best responses are unique, so an equilibrium in mixed strategies is necessarily in pure strategies. ■

Proof of Corollary 1: The existence of BNE-SI follows immediately from Theorem 1 and the existence of the informed user's best response. ■

Proof of Theorem 2: Fix a BNE X and the type $\theta \in \Theta$ of the informed user. Define

$$\overline{\xi} = \left(\frac{N}{C}\right) \int_{\widehat{\Theta}} X(\eta) d\nu(\eta).$$

This is the expected congestion caused by uninformed users. Recall that V^* is the maximum function

$$V^*(\xi, \theta) = \max_{x \in \mathbb{A}} V(x, \xi, \theta) = V(F(\xi, \theta), \xi, \theta).$$

(Keep in mind that θ is the fixed type of the informed user.) If $\lambda = (\lambda_1, \ldots, \lambda_N) \in \widehat{\Theta}^N$ is a profile of types of uninformed users, we continue to write $\xi(\lambda) = (1/C)\sum_{i=1}^N X(\theta_i)$ for the congestion caused by these users. The definition of Bayesian Nash equilibrium implies that $V^*(\overline{\xi}, \theta) \leq E_{\theta_1, \ldots, \theta_N}\left[U(X(\theta)|\theta, X)\right]$ so

$$G_{N,C}(\theta) \leq \int_{\widehat{\Theta}^N} V^*(\xi(\lambda), \theta) d\nu(\lambda) - V^*(\overline{\xi}, \theta) \tag{19}$$

$$= \int_{\widehat{\Theta}^N} \left[V^*(\xi(\lambda), \theta) - V^*(\overline{\xi}, \theta)\right] d\nu(\lambda). \tag{20}$$

For any $\varepsilon > 0$[9], define a partition $\widehat{\Theta}_1^N, \widehat{\Theta}_2^N$ of $\widehat{\Theta}^N$:

$$\widehat{\Theta}_1^N = \left\{\lambda \in \widehat{\Theta}^N : \left|\xi(\lambda) - \overline{\xi}\right| < \frac{N\varepsilon}{C}\right\} \tag{21}$$

$$\widehat{\Theta}_2^N = \left\{\lambda \in \widehat{\Theta}^N : \left|\xi(\lambda) - \overline{\xi}\right| \geq \frac{N\varepsilon}{C}\right\}. \tag{22}$$

Define $I(\lambda) = V^*(\xi(\lambda), \theta) - V^*(\overline{\xi}, \theta)$ and split the right-hand side of (20) as

$$\int_{\widehat{\Theta}^N} I(\lambda) d\nu(\lambda) = \int_{\widehat{\Theta}_1^N} I(\lambda) d\nu(\lambda) + \int_{\widehat{\Theta}_2^N} I(\lambda) d\nu(\lambda). \tag{23}$$

To estimate the integral over $\widehat{\Theta}_1^N$, we first use the Envelope Theorem [26] to see that

$$\frac{\partial V^*(\xi, \theta)}{\partial \xi} = \frac{\partial V(x, \xi, \theta)}{\partial \xi}\Big|_{x = F(\xi, \theta)}.$$

The assumptions guarantee that the partial derivatives of V at optimal choices are uniformly bounded, so $|\partial V^*(\xi, \theta)/\partial \xi| \leq L$ for some L. The Mean Value Theorem guarantees that

$$|I(\lambda)| = |V^*(\xi(\lambda), \theta) - V^*(\overline{\xi}, \theta)| \leq L|\xi(\lambda) - \overline{\xi}| \leq L\frac{N\varepsilon}{C} \tag{24}$$

for $\lambda \in \widehat{\Theta}_1^N$. Hence

$$\int_{\widehat{\Theta}_1^N} I(\lambda) d\nu(\lambda) \leq L\frac{N\varepsilon}{C}. \tag{25}$$

[9]Later, we will set $\varepsilon = N^{-\alpha}$ to obtain the required bounds.

To estimate the integral over $\widehat{\Theta}_2^N$, we note that

$$\left|\xi(\lambda) - \overline{\xi}\right| < \frac{N\varepsilon}{C} \iff \left|\frac{1}{N}\sum X(\theta_i) - E(X)\right| < \varepsilon.$$

Hence, Chebyshev's Inequality guarantees that $\int_{\widehat{\Theta}_2^N} d\nu(\lambda) < \mathrm{Var}(X)/N\varepsilon^2$. The assumptions guarantee that $V^*(\overline{\xi}, \theta) \geq 0$ and that $V^*(\xi(\lambda), \theta)$ is bounded; because X takes its values in the compact interval $\mathbb{A}$, $\mathrm{Var}(X) \leq \mathrm{length}(\mathbb{A})$. Hence there is a constant M (depending only on the upper bound of utility and the length of $\mathbb{A}$) such that

$$\int_{\widehat{\Theta}_2^N} I(\lambda) d\nu(\lambda) \leq M\frac{1}{N\varepsilon^2}. \tag{26}$$

To obtain the bounds in Theorem 2, we set $\varepsilon = N^{-\alpha}$, define $K_1 = \max\{L, M\}$, and combine the estimates for the integrals over $\widehat{\Theta}_1^N, \widehat{\Theta}_2^N$ to obtain the first conclusion:

$$G_{N,C}(\theta) \leq K_1\left(N^{-1+2\alpha} + \frac{N^{1-\alpha}}{C}\right).$$

The second conclusion follows from setting $\alpha = (2 - \beta)/3$. ■

Proof of Theorem 3: Fix a type θ of the informed user and a type θ^* of an uninformed user. By definition, $V(x, \xi, \theta)$ is maximized when $x = F(\xi, \theta)$ so strict concavity implies that

$$|V(F(\xi, \theta), \xi, \theta) - V(x, \xi, \theta)| \geq \left(\frac{\min|V_{11}|}{2}\right)|F(\xi, \theta) - x|^2 \tag{27}$$

for each $x \in \mathbb{A}$. Denote $\hat{\lambda} = (\lambda_1, \ldots, \lambda_{N-1}) \in \widehat{\Theta}^{N-1}$ a profile of types of other uninformed users and write $\xi(\hat{\lambda}) = (1/C)\sum_{i=1}^{N-1} X(\theta_i)$ for the congestion caused by these users and denote $\xi^*(\hat{\lambda}) = \xi(\hat{\lambda}) + (1/C)X(\theta^*)$. Starting with the definition, doing some algebra, applying the Cauchy–Schwarz inequality and using (27), we have

$$\begin{aligned}
&|H_{N,C}(\theta^*, \theta)| \\
&= \Big|\int_{\widehat{\Theta}^{N-1}}\Big[V\Big(X(\theta^*), \xi(\hat{\lambda}) + \left(\frac{1}{C}\right)X(\theta), \theta^*\Big) \\
&\quad - V\Big(X(\theta^*), \xi(\hat{\lambda}) + \left(\frac{1}{C}\right)F(\xi^*(\hat{\lambda}), \theta), \theta^*\Big)\Big] d\nu(\hat{\lambda})\Big| \\
&\leq \max|V_2|\int_{\widehat{\Theta}^{N-1}}\Big|\xi(\hat{\lambda}) + \left(\frac{1}{C}\right)X(\theta)) - \xi(\hat{\lambda}) \\
&\quad - \left(\frac{1}{C}\right)F(\xi^*(\hat{\lambda}), \theta)\Big| d\nu(\hat{\lambda}) \\
&= \left(\frac{1}{C}\right)\max|V_2|\int_{\widehat{\Theta}^{N-1}}|F(\xi^*(\hat{\lambda}), \theta) - X(\theta)| d\nu(\hat{\lambda}) \\
&\leq \left(\frac{1}{C}\right)\max|V_2|\left(\int_{\widehat{\Theta}^{N-1}}|F(\xi^*(\hat{\lambda}), \theta) - X(\theta)|^2 d\nu(\hat{\lambda})\right)^{1/2} \\
&\leq \frac{2\max|V_2|}{C\min|V_{11}|}\Big(\int_{\widehat{\Theta}^{N-1}}|V(F(\xi^*(\hat{\lambda}), \theta), \xi^*(\hat{\lambda}), \theta) \\
&\quad - V(X(\theta), \xi^*(\hat{\lambda}), \theta)| d\nu(\hat{\lambda})\Big)^{1/2}.
\end{aligned}$$

The last integral (inside the parentheses) is the utility gain to an informed user of type θ given that the uninformed user of type θ^* is online and taking expectations over the types of $N-1$ other users. Arguing exactly as in the proof of Theorem 2, this gain is bounded by $2K_1N^{(1-2\beta)/3}$. Both conclusions now follow immediately by algebraic substitution. ■

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