Silence is Gold: Strategic Interference Mitigation Using Tokens in Heterogeneous Small Cell Networks

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***Abstract*—Electronic tokens have been successfully used as incentive mechanisms to stimulate self-interested network nodes to relay other nodes' traffic. In other words, tokens are paid to *buy transmission* (relaying) services. In this work, we propose a novel distributed token exchange framework which can be used in heterogeneous small cell networks to successfully mitigate interference among the self-interested users. Contrary to the traditional role of buying transmission, tokens are exchanged between users to *buy silence*. Heterogeneity poses unique challenges for interference mitigation, which are difficult to handle with previous solutions but can be effectively tackled with the proposed token design. This paper focuses on the rigorous design of the optimal token scheme that minimizes the system outage probability. We first analyze the optimal strategies of individual users, which only consider their own utility maximization and do not care about the system-wise performance. We prove that under some mild conditions the optimal strategy has a simple threshold structure. We then analytically derive the optimal token supply that minimizes the network outage probability. Analysis shows that even if each user adopts the optimal strategy that only maximizes its own utility, a careful token system design can lead to a significant overall network performance improvement. Simulation results show that not only does the proposed token system design greatly improve the network outage probability, it also improves the overall network QoS, particularly when the deployment density is high.**

***Index Terms*—Electronic Token, Heterogeneous Network (HetNet), Small Cell, Interference Mitigation.**

I. Introduction

A. Motivation

The demand for high-speed wireless data traffic is exploding at an astounding pace, which poses great challenges for the operators to provide sufficient network capacity and enhanced coverage. Dense deployment of distributed low-power low-cost small cells has been viewed as one of the most promising solutions to address this challenge [1]. Small cells are attractive because they can not only extend the service coverage but also boost the network capacity by shortening the access distance (cell splitting gain) and offloading traffic from the macro network (offloading gain). Extremely dense deployment of small cells has been hailed as a key solution to the 1000x problem [1].

As the density of small cells increases and the wireless architectures become more decentralized, *interference* has become the bottleneck of the overall system performance, which has motivated a lot of research in recent years. On the physical layer, advanced technologies such as interference cancellation [2], interference alignment [3], multi-user MIMO [4], coordinated multi-point transmission/reception [5], and large scale antenna system [6] have been extensively studied, both in academia and in industry. In Medium Access Control (MAC) and Radio Resource Control (RRC) layers, novel techniques such as inter-cell interference coordination and cancellation [2], power control [7], and fractional frequency reuse (FFR) [8] have been pursued. It is worth mentioning that these techniques can also be categorized into centralized or distributed solutions. Although performance of the distributed solution is typically inferior to the centralized counterpart, they enjoy the benefit of easier implementation and better scalability.

While these advanced techniques provide promising solutions for interference mitigation in cellular networks, recent development and industrial trends have imposed unique challenges that are yet to be well addressed. First of all, small cell networks are expected to be *heterogeneous* in terms of device capabilities, and *interoperability* is a significant challenge. Different categories of small cells (e.g., femtocells, picocells, microcells, etc.) need to co-exist, where these heterogeneous cells may have different power levels, provide different capacity, and support different users. Such heterogeneity applies to mobiles as well, which may support different cellular standard releases, have different types of services (video, voice, web browsing), etc. On the contrary, majority of existing interference mitigation studies assume that all devices in the network use the same techniques. Such implicit assumption of homogeneous capabilities may not be true and there lacks research of the performance impact of heterogeneity on these solutions.

Secondly, small cells are expected to be more dynamic and have much higher density than traditional base stations, as they are likely deployed by end users and can be moved geographically and powered on and off frequently. Static or semi-static solutions cannot handle such dynamics well. Plus, most distributed interference management techniques

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still require extensive coordination or information exchange between cells/users, which can congest the already-limited backhaul network.

The third challenge comes from the evolving cognitive nature of small cell users which makes them more intelligent [9]. Such intelligence also renders the network more heterogeneous. Majority of the existing research on interference problems focuses on optimizing the performance of either individual users or the overall system, assuming all users follow the established system-level solution. This is effective when the network is consisted of users that are dummy devices, and the design can achieve globally optimal performance. However, existing approaches may not work well when users become increasingly cognitive and self-interested [10], [11]. In certain scenarios, if users can improve their individual performance by unilaterally deviating from the prescribed actions, previous approaches may fail to provide efficient coordination among users to mitigate interference. A well-known example is the so-called *power racing* problem [12]: a self-interested user can increase its utility (e.g., capacity) by transmitting at higher power, which in turn creates larger interference to a neighboring user. This neighboring user then has to increase its transmit power to compensate for the increased interference, which in turn creates larger interference to the first user. This interaction continues until both cognitive users reach their maximum transmit power at which point both the overall system and the individual users suffer from significant interference. This type of problems cannot be solved by utilizing the existing solutions in which the incentive issues of autonomous users are disregarded.

Due to these challenges, an efficient interference mitigation method for the considered dense heterogeneous small cell networks requires that it be simple, scale easily with the volume of small cells and end users, handle system dynamics, involve minimal information exchange, and work with heterogeneous cognitive and self-interested devices that can be from different vendors and have different capabilities. We address these design challenges for the dense small cell interference problem and propose a novel framework using *tokens* to stimulate heterogeneous user cooperation by exploiting the long-term nature of the system states. We will show that it is possible to design effective token schemes for interference mitigation in a distributed and large-scale heterogeneous small cell network.

B. Prior Work on Incentives and Pricing

Using tokens, or other incentive schemes based on similar virtual currencies, is not a new concept. Many have been proposed in the literature and some were implemented in practice. Two types of systems have been extensively studied: relay networks [13]–[15] and peer-to-peer networks [16]–[19]. Monetary pricing schemes are proposed in [13] to stimulate relay cooperation in wireless networks. In [14], tokens are used to provide incentive for the self-interested transceivers to provide relay services. *Nuglet* is introduced in [15] as a virtual currency for relay transmissions. For peer-to-peer systems, a general economic framework for avoiding free-riders is established in [16], using a single scalar value called *KARMA*. Payment-based incentives are proposed in [17] and [18], also targeting the free-riders. Analytical study of such system to understand users' strategic behavior and the overall system efficiency is done in [19].

On the other hand, two different types of pricing have been presented in the literature. The first type uses price as a control signal, which often appears in the form of Lagrange multiplier, to enable decentralized coordination among users to solve, e.g., power control problems [20], [21]. In such application, the strategic and self-interested nature of users is not captured or utilized, and thus pricing is not used as an incentive device. Furthermore, these works only study the static solutions that can achieve the optimal network utility in a given network state. The long-term characteristic of user behavior which can stimulate collaboration is not exploited. The second type is monetary pricing [13], [22], which boosts cooperation among strategic and self-interested users but again focuses on static networks where the interacting users are fixed and myopic. Also, reliable and centralized financial accounting is needed to manage the payment for distributed users. Moreover, these methods do not scale easily to the large-scale dense network.

To the best of the authors' knowledge, this paper is the first to introduce tokens in a distributed small cell network. It is also the first to utilize tokens to stimulate long-term collaboration for the purpose of interference mitigation. Moreover, our work differs from the previous incentive schemes in several important aspects. Firstly, the functionality of tokens in our work is different from the previous research, where tokens are paid to the recipient to "buy transmission" [13], [14]. In this work, however, tokens are introduced to "buy silence". This incurs a much less severe cost to the recipient comparing to the "buy transmission" application, where the recipient both sacrifices its own transmission and actively spends its resources for others. Secondly, we study user's strategic behavior using a novel repeated game formalism to model the token exchange, which captures its long term characteristic. More importantly, the analytical results are general and can apply to an arbitrary number of neighbors. Thirdly, the recipient does not need to know the payer's transmission to receive a token. The only required communication between the users is the token exchange, which makes the system autonomic, secure, and easy to implement thanks to the less intrusive nature of our design. Last but not the least, the proposed token scheme can be utilized in conjunction with other interference management schemes to achieve even better performance.

C. Main Contributions

In this paper, we first propose a distributed token framework to stimulate heterogeneous user cooperation for interference mitigation in a small cell network. In our design, users are assumed to be self-interested, meaning that they only aim to maximize their own utilities and do not care about the system-wise performance. Minimal information exchanges between users is required so that it can accommodate the heterogeneity among devices. For such systems, the fundamental problem that prevents users from cooperation is the lack of incentives for individual users to sacrifice their own utility for better interference mitigation of other users. A self-interested user will refuse to manage its interference to other users if this hurts its utility, as has been shown in the power racing problem. Unlike previous solutions, we attack this problem by utilizing the long term nature of the network states and providing assurance to the self-interested user that by temporarily sacrificing its own performance, it can achieve a better utility in the long term. This is accomplished by the introduction of tokens, which allows the users to exchange the current utility decrease for future utility increase with a minimal information exchange.

More importantly, we provide a rigorous analysis of the proposed token system and prove its optimality. We first analyze the self-interested user behavior, and prove that under some mild conditions the optimal strategy has a simple threshold structure. This is accomplished by using *repeated game theory* [23] to analyze the problem. In our model, UEs are players holding the tokens; the game being played is to buy and sell “silence” services for tokens; and the *repeated* game setting fully captures the long term characteristic of tokens that can stimulate cooperation even for selfish users. We then analytically derive the optimal token design that minimizes the network outage probability. We prove that the network outage minimization depends on issuing a proper amount of tokens into the system. Moreover, the analytical results hold for arbitrary number of neighbors. Finally, the analytical results are supported by numerical simulations, and it is observed that the performance gain increases with the network density. This makes the token scheme particularly suitable for ultra-dense heterogeneous small cell networks.

The rest of this paper is organized as follows. Section II presents the system model. In Section III we introduce tokens in the small cell network, and discuss the proposed scheme and the system design problem. The general theory that guides the system design is presented in Section IV. More specifically, user-level optimal strategy is developed in Section IV-C and system-level optimal strategy is derived in Section IV-D. Simulation results are presented in Section V. Finally, Section VI concludes the paper and points to some future research directions.

II. System Model

We study a heterogeneous small cell (SC) network with N cells. We consider a high-density deployment with small inter-site distance (ISD). It is assumed all the small cells operate on the same frequency, which is attractive to operators with limited spectrum resources. We assume a *dynamic* small cell network: user equipments (UE) are assumed to be mobile and can move across the coverage boundary and be handed over to the neighboring cell; SCs can be user deployed and dynamically turned on and off. When SC i is on, its transmit power is denoted as $P_i(t)$. A time slotted system is considered, where the network topology is fixed at one time slot and changes independently in the next (due to user movement, cell on/off, etc). We assume that SC i can support up to L_i UEs, which depends on its hardware and software capability. We further assume that each SC schedules at most one user with active traffic at a given time-frequency grid. Note that this captures the TDM/FDM nature of user scheduling which is commonly used in cellular standards such as LTE [24]. We thus denote the SC/UE association by assigning the same index to the SC and UE, e.g., UE i is associated with SC i as its serving cell, for a given time-frequency resource grid. Due to the randomness of user traffic, a SC does not always have an active user to serve. We capture the user random activity by γ_i, which is the probability that user i has active downlink traffic in a time slot. Lastly, we assume that the network has degree M, i.e., a user has M direct neighboring users on average. Note that M is determined by the network topology and user distributions.

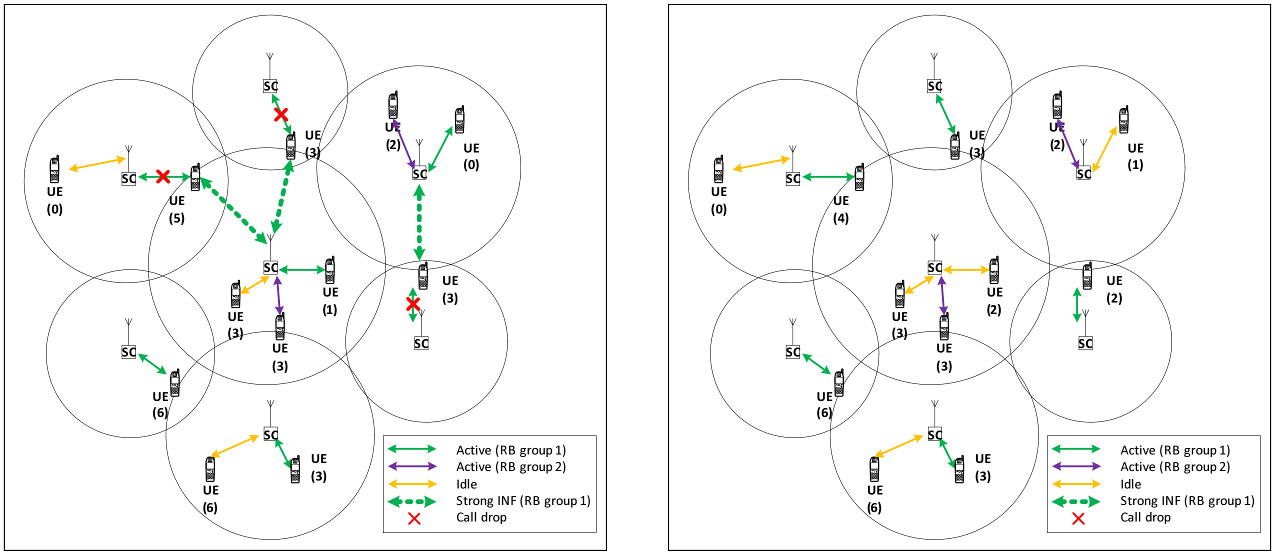
In such dense small cell deployment, typically thermal noise is not the performance-limiting factor. Instead, the interference from neighboring small cells (downlink interference) and their active users (uplink interference) plays a critical role in limiting the system capacity. We focus on the *downlink* interference problem in this work, and note that the extension to the uplink interference problem is straightforward. We use $g_{ij}(t)$ to denote the channel power between transmitter (SC) i and receiver (UE) j at time slot t. Note that $g_{ij}(t)$ takes into account both large-scale (pathloss and shadowing) and small-scale fading. The downlink interference problem occurs when a user is in the coverage intersection of more than one cell, where it will experience strong interference from the neighbor and hence its signal-to-interference-plus-noise ratio (SINR) may reduce significantly, resulting in low throughput and even radio link failure (RLF). Assuming SC/UE i experiences interference from SC/UE j at time slot t, its downlink SINR is:

$$SINR_i^{ON}(t) = \frac{P_i(t)g_{ii}(t)}{N_0 + P_j(t)g_{ji}(t)}, \tag{1}$$

where N_0 incorporates both the thermal noise and the residual interference besides SC/UE j. The superscript “ON” indicates that the interfering cell is transmitting. SINR reduction can be significant when the interference $P_t g_{ji}$ is large. This problem becomes worse when the user is receiving high-priority packets, since the interference causes more significant performance impact than the less important packets.

In Fig. 1, we illustrate the downlink interference problem in a heterogeneous small cell network. Particularly, Fig. 1(a) shows a snapshot of the SC/UE geometry and highlights the outage of three UEs due to strong interference from neighboring users on the same frequency. With the proposed token scheme which will be described extensively in Section III, Fig. 1(b) illustrates the same network geometry where token exchanges have resolved the outage issues and improved the network quality of service (QoS). This can be verified by noting the change of number of tokens in each UE’s possession, as well as the change of interference situations, in both figures.

A few remarks are in order regarding the considered model. Firstly, only the *downlink* interference problem is considered in this paper. Hence strictly speaking, *UE* is the victim of neighbor interferences. We however do not make such explicit distinction and generally refer to a SC/UE pair as the entity that potentially suffers from interference and takes actions. The same applies to the SC/UE that causes interference to other users. Secondly, it is assumed that an active user has the



(a) Before token exchanges

(b) After token exchanges

Fig. 1. Snapshot of a heterogeneous small cell network. The downlink interference problem is illustrated, (a) before and (b) after applying the token scheme. The number in the parentheses beneath "UE" indicates the number of tokens in its possession.

knowledge of which neighboring SC/UE causes the interference. In practice, this is accomplished by the UE performing cell measurements to detect the existence of strong co-channel neighboring cells [24]. Last but not the least, our focus is on the one-sided interference problem, i.e., SC/UE i gets dominant interference from SC/UE j but not vice versa. Due to the independent movement of users, probability of the two-sided problem, which happens when both users are simultaneously in the same coverage intersection area, is much smaller than the one-sided problem and hence will not be considered explicitly in this paper. However, the proposed token scheme also works for the two-sided interference problem[1].

III. Token Design for Interference Mitigation

A. Token scheme

We focus on the dominant downlink interferer to illustrate the token design. The performance degradation is typically dominated by the neighboring cell who causes the most interference [25], and hence solving this problem has the most significant gain[2]. For the interference scenario in Section II, an efficient and simple solution is for SC/UE j to shut down transmission to eliminate its interference to UE i. If the interference can be eliminated, the SINR becomes

$$SINR_i^{OFF}(t) = \frac{P_i(t) g_{ii}(t)}{N_0}, \tag{2}$$

where the superscript "OFF" indicates that the interfering cell is not transmitting. Note that (2) can be a significant improvement over (1) in an interference-limited network, due to the fact that small cells typically serve only a small area (e.g., inside an apartment) where the access distance is very short and SNR is high. However, shutting down the transmission of SC/UE j may lead to its performance loss, and hence an intelligent user may refuse to do so without any incentives.

We now formally model the user interactions as a game. Suppose that at time slot t, SC i is actively serving UE i. A neighboring SC j also actively serves UE j, but due to its proximity to UE i it causes significant interference. In this setting, SC/UE j can take one action from the binary action space $\mathcal{A} = \{0, 1\}$, where action $a_j(t) = 1$ means that SC/UE j decides to power off downlink transmission at time t, and action $a_j(t) = 0$ means otherwise. If SC/UE j decides to power off transmission to eliminate its interference to UE i, UE i enjoys a benefit $b_i(t)$, which depends on the decrease of the interference. UE j, on the other hand, incurs a cost $c_j(t)$ in order to accommodate UE i's transmission. This cost can be a lost opportunity for its own transmission, or other performance measures such as delay and QoS reduction. Formally, UE i and j are playing an *interference elimination game* $G = \langle \{i, j\}, \mathcal{A}, \{u_i, u_j\} \rangle$. In this game, the players are UE i and UE j. The requester UE i has no action; the interferer UE j can choose an action a_j from $\mathcal{A}$. The utilities of both players depend on UE j's action, i.e.,

$$u_i(a_j, t) = a_j(t) b_i(t), \tag{3}$$

$$u_j(a_j, t) = -a_j(t) c_j(t). \tag{4}$$

It is easy to see that the dominant strategy of UE j is $a_j(t) = 0, \forall t$, since powering off its own transmission only brings a cost but no immediate benefit.

Clearly, in a distributed and autonomous small cell network where nodes care about their own utilities, if the *instantaneous* benefit is the only metric for the users, UE j will refuse to help UE i even if it is aware of the interference problem, since this incurs a performance degradation to UE j but

[1]In this case, the token exchange will be initiated by one of the users (say SC/UE i). If SC/UE j accepts the token from SC/UE i and turns off its transmission, the problem is resolved. Otherwise, SC/UE j rejects the token exchange and subsequently offers a token to SC/UE i to turn off its transmission. More details of the token exchange mechanism can be found in Section III.

[2]As will be evident in Section III-B, the token design can be extended for multiple interferers.

provides no reward. In this work, we propose a small cell interference mitigation design with token exchanges, which provides incentives to shut down downlink transmission to eliminate interference. We denote the total amount of tokens circulating in the entire network by W. These tokens are equally assigned to all the users when they first enter the network, i.e., each user gets W/N tokens initially from the operator, and they can be exchanged among users to "buy" and "sell" silence services. In the aforementioned game involving two UEs i and j, UE i needs to decide whether to initiate a token exchange with UE j, based on the interference level and its QoS requirement. If UE i decides to "buy silence" and UE j accepts the proposal, then UE i sends one token (possibly via its serving cell SC i) to UE j (possibly via its serving cell SC j). UE i enjoys a benefit $b_i(t)$ while losing one token; UE j incurs a cost $c_j(t)$ but gains one token which can be used in the future for better utility. In other words, proper incentives are provided to force SC/UE take into account the future benefit of tokens when making the current "silence" decision. Detail protocol design for token exchanges is presented in Section IV-E.

We capture the need for silence services by λ_i, which is the probability that user i suffers from severe interference that it requires the dominant interfering SC/UE to power off its downlink transmission. For example, if SINR is the performance metric, then λ_i can be derived from (1) as

$$\lambda_i = \Pr\left\{ \frac{P_i g_{ii}}{N_0 + \sum_{j=i_1,\cdots,i_{M_i}} I_j P_j g_{ji}} < \mathsf{SINR_{th}} \right\} \quad (5)$$

where $j = i_1, \cdots, i_{M_i}$ are the M_i neighbors of user i with $\mathbb{E}[M_i] = M$, I_j is a random variable which takes value 1 with probability γ_j and 0 otherwise, and $\mathsf{SINR_{th}}$ is a threshold below which the link quality becomes intolerable, e.g., the SINR decoding threshold.

B. Remarks

Although the token design in Sec. III-A is introduced assuming a single active UE on a given time-frequency grid, it can be extended to account for multiple UEs per time-frequency grid (e.g., multi-user MIMO user scheduling) with some small modifications. If the intra-cell UEs experience dominant out-of-cell interference from different neighbors, they can each invoke the token design in Sec. III-A independently, with no coordination needed. On the other hand, if more than one UE experiences interference from the same neighbor, these UEs need to coordinate, possibly through the serving SC, to send one token to this neighbor to invoke the token exchange process. For example, each user can contribute a fraction of token depending on the severity of interference to its QoS.

The proposed token design can be easily extended to handling multiple strong interferers as follows. For SC/UE i, assume there are M_i strong interferers SC/UE $j(1), \cdots, j(M_i)$. Without loss of generality, we can further assume that these SC/UE's are ordered in decreasing order of their interference levels to SC/UE i. Now, SC/UE i can start invoking the token exchange protocol with each of the interferers $j(m), m = 1, \cdots, M_i$ in a sequential way. If SC/UE $j(m)$ accepts the token and powers off its transmission, SC/UE i checks if the residual interference still causes QoS failure, if so it moves on to the next strongest interferer and plays the interference elimination game with SC/UE $j(m+1)$. This procedure continues until either some neighboring SC/UE rejects the token proposal, or SC/UE i no longer needs to play the interference elimination game. Effectiveness of the extended token design will be shown in Section V. However, the rigorous analytical study and optimal token system design for multiple interferers is difficult and will not be addressed in this paper. The difficulty mainly comes from the fact that user strategy can change the token request rate λ, and hence makes the analysis intractable.

The token system enables a simple implementation of interference avoidance for self-interested nodes. It has many advantages:

1) The only exchange that happens between neighboring SC/UE pairs is the token. This greatly simplifies the interface requirement and can be used in any heterogeneous network that supports such interface. There already exist standard interfaces between LTE cells, e.g., X2 connection [26], that can be adapted for token exchanges. The proposed scheme can be implemented with minimal changes of the existing protocol. See Section IV-E for more details on the protocol design.
2) One token in the small cell network provides one unit opportunity for a neighboring SC/UE to "silence" its transmission. Such token has no value outside the network, which is important as it avoids financial problems (such as fraud) that are associated with monetary schemes. Also, no personal information or data forwarding is required with the token exchange. Hence the system is anonymous and secure. Plus, one can leverage the existing distributed secure electronic token transactions [15] in the proposed design.
3) The token scheme can be used together with other advanced interference mitigation schemes. For example, the token design can be used to eliminate the strongest source of interference which are otherwise difficult to handle, and then other advanced solutions can be used to remove the residual interference, and the combination of both technologies will lead to even better system performance.

Finally, some discussions on the benefit $b_i(t)$ and cost $c_i(t)$ are in order. There are two possible policies regarding the proposed token scheme, and the choice of benefit and cost depends on the adopted policy. A *static* policy is a policy in which the user strategy σ_i is set prior to the over-the-air system operation, where off-line computation of the optimal solution is performed and no run-time benefit/cost adjustment is done. In this case, even though the actual run-time benefit and cost may vary, the policy is determined using the expected benefit and cost which are computed using the empirical distributions derived from long-term observations. A *dynamic* policy is one in which instantaneous benefit and cost values are decided at run-time and used in the decision making. For example, dynamic policy can be implemented by periodically adapting the static policy, using the updated knowledge on benefit and cost based on its recent observations. In this paper, we only consider the static policy design, and leave dynamic policy design as a future research topic. Hence, while the instantaneous benefit $b_i(t)$ and cost $c_i(t)$ determines the actual performance, the proposed system design in Section IV will use the expected benefit $b_i = \mathbb{E}[b_i(t)]$ and $c_i = \mathbb{E}[c_i(t)]$. Moreover, we will keep b_i and c_i abstract: when applied to a particular system, b_i and c_i can be any Key Performance Indicator (KPI) that is of interest to the system designer, e.g., data rate, power, voice/video quality, latency, etc.

IV. Optimal Token System Design

A. Repeated Games

The key observation that motivates the proposed approach is that even with the system dynamics, users are active in the network for a long period of time, and proper incentives can be provided to them so that they are willing to take a relatively small loss at the moment to eliminate the interference to other cells, in exchange for getting the same treatment in the future for a relatively large benefit. Correspondingly, we model the proposed token system in a small cell network using *repeated games* [23] in which the one-shot "interference elimination" game defined in Section IV-A is repeatedly played by the users. Repeated game models the long-term nature of the token system, and captures the essence that each player needs to take into account the impact of the current decision on the *future* actions of players. In a small cell network, each intelligent user acts as a player, while the game being played is to exchange tokens for silence services. This game can be repeated since the users stay in the network for a long period of time. We further assume that the users discount the future utility at a constant rate $\beta \in (0, 1]$. For example, this can be interpreted as the probability a user does not leave the network. It should be noted that discounting is a main method to model the preference relation in an infinitely repeated game [23].

In the repeated interference elimination game, different users may choose different strategies. We thus denote user i's strategy as $\sigma_i : \mathcal{S}_i \rightarrow \mathcal{A}$, which is a mapping from the system state space $\mathcal{S}_i$ to the user action space $\mathcal{A}$. Each system state $s \in \mathcal{S}_i$ captures a combination of channel states and token holding that is known to user i. Table I summarizes the knowledge assumptions that apply to both players, assuming user i and j are playing the interference elimination game. It should be emphasized again that the RF knowledge assumptions are commonly available in most cellular standards, while the token and strategy assumptions only require self knowledge – other users' tokens and strategies are unknown.

For the considered repeated game problem, we denote $V_i(s|\sigma_i)$ as the long-term utility of user i when it is in a system state $s \in \mathcal{S}_i$ and adopts strategy σ_i. It should be emphasized that user i only knows the system state s of itself, including channel states to user i and its token holding. Since we consider static policies in this paper, the impact of dynamic channel states to the utility of user i is reflected by the silence demand rate λ_i, and by fixing the expected cost c_i and benefit b_i, the utility function only depends on the token holding k,

TABLE I
User knowledge in the repeated game with player i and j.

	User i	User j
Channel state to serving SC	Yes	Yes
Channel state to interferer SC	Yes	Yes
SINR	Yes	Yes
Token holding of user i	Yes	No
Token holding of user j	No	Yes
Strategy of user i	Yes	No
Strategy of user j	No	Yes

and the state equations that define $V_i(s|\sigma_i)$ can be written as

$$\begin{aligned} V_i(0|\sigma_i) &= (1-\lambda_i(1-\rho_0)\sigma_i(0))\beta V_i(0|\sigma_i) \\ &+\lambda_i(1-\rho_0)\sigma_i(0)(-c_i+\beta_i V_i(1|\sigma_i)); \end{aligned} \tag{6}$$

$$\begin{aligned} V_i(k|\sigma_i) &= \underbrace{\lambda_i(1-\rho_r)(b_i+\beta V_i(k-1|\sigma_i))}_{(A)} \\ + \quad & \underbrace{\lambda_i(1-\rho_0)\sigma_i(k)(-c_i+\beta V_i(k+1|\sigma_i))}_{(B)} \\ + \quad & \underbrace{(1-\lambda_i(1-\rho_r)-\lambda_i(1-\rho_0)\sigma_i(k))\beta V_i(k|\sigma_i)}_{(C)}. \end{aligned} \tag{7}$$

where ρ_0 is the probability that a different user is in need of the silence service from the user but does not request and ρ_r is the probability that a different user rejects the silence service when requested from the user. The first item (A) in (7) comes from when the user needs to send a silence request. In this case, it will spend one token and receive a benefit b_i. (B) and part of (C) correspond to when the user is at the receiving end of a silence request. Depending on the strategy $\sigma_i(k)$, it can either obtain one token by accepting the request, or remain with the same amount of tokens by rejecting the request. The remaining of (C) corresponds to when the user neither needs to send a silence request nor receives one.

In the above utility functions (6) and (7), a user's utility depends on other users' strategies through the terms ρ_0 and ρ_r. In practice, a user may not exactly know other users' strategies but need to learn them over time. In such case, ρ_0 and ρ_r are *beliefs* of the user about other users' strategies. These beliefs will be updated over time by incorporating the new observations of the user, and become statistically reliable with the user's long-term experience and averaging. In most of our analysis below, we study users' incentives with fixed ρ_0 and ρ_r. Detailed analysis on ρ_0 and ρ_r is deferred until Section IV-D.

B. Problem formulation

Ultimately the system designer is interested in maximizing the overall system performance, subject to all users acting in their own best interests by adopting the optimal user strategies. In order to achieve this goal, we need to understand the following hierarchical problems.

- **User-level Problem**. Designer needs to understand users' incentives and what their optimal strategies are, i.e., the strategies that maximize utilities at individual users. This problem is solved at each user. The user-level problem is to find the optimal strategy σ_i such that $\forall s \in \mathcal{S}$, we have $V_i(s|\sigma_i) \geq V_i(s|\sigma_i^{'})$, if $\sigma_i^{'} \neq \sigma_i$.

- **System-level Problem**. Assuming that each user adopts the strategy that maximizes its own utility, the system-level problem seeks to maximize the overall system performance by issuing an optimal amount of tokens into the system. This problem is solved by the system designer, e.g., Home eNodeB Management System (HeMS). It is important to note that the system-level problem can be solved only after understanding the behavior of individual users, which are self-interested but rational.

There can be various system metrics applied to the above system-level problem. In this paper, we focus on minimizing the *network outage probability*, which is the probability that an active user cannot meet the requested QoS due to interference. This is a useful performance metric as it captures the impact of inter-cell interference in the entire network, and also isolates the interference problem from others, e.g., scheduling or physical layer issues. We denote the network outage probability as

$$\bar{\mathsf{P}}_{\mathsf{out}} \doteq \mathbb{E}_{\mathcal{S}} \left\{ \mathsf{P}_{\mathsf{out}} \left(\{\sigma_i\}_{i=1}^N, s|W \right) \right\} \tag{8}$$

where $\mathsf{P}_{\mathsf{out}} \left(\{\sigma_i\}_{i=1}^N, s|W \right)$ is the network outage probability when the overall system state is s and UE i uses strategy σ_i. Hence, by issuing an optimal amount of tokens to the system, the system designer can optimize the outage probability from the network perspective, with an incentive compatibility constraint that individual users choose the optimal strategies that maximize their own utilities. We can formally cast the design problem as:

$$\begin{aligned} \underset{W}{\text{minimize}} \quad & \mathbb{E}_{\mathcal{S}} \left\{ \mathsf{P}_{\mathsf{out}} \left(\{\sigma_i\}_{i=1}^N, s|W \right) \right\} \\ \text{subject to} \quad & V_i\left(s|\sigma_i\right) \geq V_i\left(s|\sigma_i^{'}\right) \text{ if } \sigma_i^{'} \neq \sigma_i, \\ & \qquad \forall s \in \mathcal{S}, i = 1, \cdots, N. \end{aligned} \tag{9}$$

In the following section, a general solution to Problem (9) is developed, where we first solve the *user-level problem* by presenting a threshold-based strategy for individual users and proving its optimality, and then solve the ultimate *system-level problem* that minimizes the network outage probability.

C. Optimal User Strategy

Let us consider a representative SC/UE i that has been asked to silence its downlink transmission to help a neighboring user. Suppose user i already has k tokens. If it decides to power off its transmission, it will gain one more token to have a total of $k+1$ tokens in the next time slot; otherwise, it rejects the silence request and remains with k tokens in the next time slot. Since powering off its own transmission incurs a cost c_i, the user needs to compare the utility improvement from accumulating one more token with the cost c_i to make a utility maximization decision. In fact, using the one-shot deviation principle [27], we have the following result.

Lemma 1: A user strategy σ_i is optimal for user i (i.e., satisfying the incentive compatibility constraint in (9)) if and only if $\forall k \in \mathbb{N}$, σ_i satisfies:

$$V_i\left(k+1|\sigma_i\right) - V_i\left(k|\sigma_i\right) \geq \frac{c_i}{\beta}, \text{if } \sigma_i(k) = 1 \tag{10}$$

$$V_i\left(k+1|\sigma_i\right) - V_i\left(k|\sigma_i\right) < \frac{c_i}{\beta}, \text{if } \sigma_i(k) = 0 \tag{11}$$

Proof: See Appendix A. ■

This lemma tells us that the optimal user strategies always achieve a non-negative payoff for all possible token holdings. Finding the optimal strategy is difficult given that the number of all feasible strategies is large. It is also clear from Lemma 1 that the optimal strategy depends on the choice of the utility function at the user. In Proposition 1, we study whether a strategy σ is optimal for a particular type of utility functions. To simplify notation we write $V_i\left(k\right)$ instead of $V_i\left(k|\sigma\right)$ for the remainder of this paper.

Proposition 1: If the long-term utility function $V_i\left(k\right)$ is monotonically increasing and concave in k, then an optimal user strategy σ_i is a threshold strategy, i.e., there exists a threshold $K_{th,i}$ for user i such that

$$\sigma_i(k) = 1, \text{if } k \leq K_{th,i} \tag{12}$$

$$\sigma_i(k) = 0, \text{if } k > K_{th,i} \tag{13}$$

Proof: See Appendix B. ■

It should be emphasized that the conditions on the long-term utility function $V_i\left(k\right)$ are very mild, and have been widely used [28]. The first condition simply requires that getting more tokens would not reduce the utility, which is intuitively reasonable. The second part indicates that as a user earns more tokens, the marginal utility of having an additional token decreases. This is especially the case when the discount factor β, which was defined in Section IV-A, is taken into account in $V_i\left(k\right)$. The incentive of holding a token is that in the future when the user encounters the same downlink interference problem, it can use the token to ask the interfering SC/UE to shut down its transmission. However, keeping tokens has inherent risks due to the cell dynamics (e.g., user leaving the network) or other factors, which is modeled by β that "inflates" the value of tokens in the future.

D. Optimal Token System Design

In the previous section, we have shown that with proper incentives, users are willing to provide silence service to neighbors in exchange for tokens that can be used in the future. We also proved that intelligent users will not cooperate all the time, because they have incentives to stop accumulating tokens after a certain threshold. This suggests that if there are a lot of tokens in the network, with high probability users will not cooperate since they already have many tokens. On the other hand, it is intuitive that if there are too few tokens in the network, silence requests are rare because very few users have tokens to use. Therefore, there should be an optimal amount of total tokens which can maximize the overall system performance.

In this section, we study the optimal token supply that minimizes the network outage probability, which is defined in (8), assuming that all users follow their optimal strategies as discussed in Section IV-C. We focus on QoS failures that are caused by inter-cell interference, which captures the dominant outage event in small cell networks as they are interference-limited. Let us use $\rho_k(i)$ to denote the percentage of users that possess k tokens and the optimal user strategy is a threshold strategy with threshold $K_{th} = i$, as in Proposition 1. Since N is large, we use a continuous model to approximate the discrete distribution. The percentage of the users who would reject a silence request can be calculated as

$$\rho_{\mathsf{r}} = \sum_{i=0}^{\infty} \sum_{k=i+1}^{\infty} \rho_k(i). \tag{14}$$

Note that ρ_{r} represents the probability that a user's silence request to an interfering neighbor is rejected. As a result, the user will suffer from the strong interference and have an outage.

On the other hand, an outage can also happen when a user would like to pay one token to the interfering neighbor, but has no token in its possession to do so. This is captured by calculating the percentage of the small cells who have no token and hence they cannot request silence service when needed:

$$\rho_0 = \sum_{i=0}^{\infty} \rho_0(i). \tag{15}$$

The network outage probability can be expressed as

$$\begin{aligned} \bar{\mathsf{P}}_{\mathsf{out}} = & \Pr\{\text{User with no token needs to use a token}\} \\ & + \Pr\{\text{User receives and rejects a token request (REQ) }\}. \end{aligned} \tag{16}$$

For the most general case, the benefit, cost and demand rate are heterogeneous, meaning that each UE can have different values. This results in different threshold $K_{th,i}$ for different SC/UE i, as well as different activities for silence requests, which make (16) difficult to evaluate. In the system-level problem which tries to determine the optimal amount of tokens issued in the system, we will focus on homogeneous cost c and demand rate λ, which suggests that all the users will use the same threshold strategy $K_{th,i} = K$, i.e., no UE will hold more than K tokens. This is a reasonable assumption since the system designer does not have the complete knowledge of benefits and costs of all users. Hence, we can then drop index i in $\rho_k(i)$ and only consider ρ_k, which is the percentage of UEs that possess k tokens. Obviously we have $\sum_{k=0}^{K} \rho_k = 1$. Moreover, we have $\rho_{\mathsf{r}} = \rho_K$. With all these conditions, the network outage probability can be computed as

$$\begin{aligned} \bar{\mathsf{P}}_{\mathsf{out}} = & \Pr\{\text{UE needs to use a token}\} \Pr\{\text{UE has 0 token}\} \\ & + \Pr\{\text{UE receives a REQ}\} \Pr\{\text{UE has K tokens}\} \\ = & \lambda\rho_0 + \rho_K \left(1 - \Pr\{\text{No neighbor sends REQ}\}\right) \\ = & \lambda\rho_0 + \left(1 - \left(1 - \frac{\lambda}{M}\right)^M\right) \rho_K. \end{aligned} \tag{17}$$

We are ready to present the main result for the optimal token supply.

Proposition 2: If all users in the network follow the equilibrium strategy in Proposition 1, and use the same threshold K, then the optimal token supply that minimizes the network outage probability $\bar{\mathsf{P}}_{\mathsf{out}}$ in (17) is: if $M = 1$ then

$$\frac{W^*}{N} = \frac{K}{2}; \tag{18}$$

otherwise,

$$\frac{W^*}{N} = \frac{\left((K-1)\rho_K + \rho_0\right)(1-\rho_0)(1-\rho_K) - K\rho_K(1-\rho_K)^2}{\left(\rho_0 - \rho_K\right)^2} \tag{19}$$

where $\{\rho_0, \rho_K\}$ satisfies:

$$\begin{cases} 0 \le \rho_K \le \dfrac{1}{K+1} \le \rho_0 \le 1 \\ \dfrac{\rho_0(1-\rho_0)\left(1-(K+1)\rho_K\right)}{\rho_K(1-\rho_K)\left(1-(K+1)\rho_0\right)} = \dfrac{(1-\lambda)^M - 1}{\lambda} \\ \rho_0(1-\rho_0)^K = \rho_K(1-\rho_K)^K \end{cases} \tag{20}$$

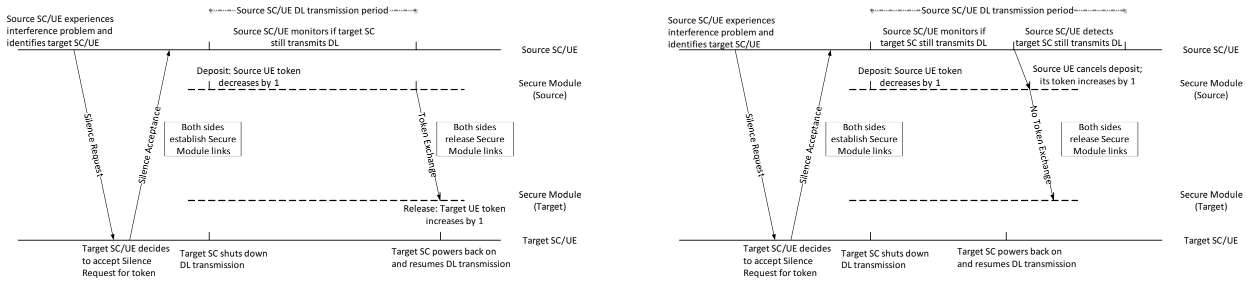
Proof: See Appendix C. ∎

A couple of observations can be made from Proposition 2. Firstly, the optimal token amount for the small cell network is $\frac{KN}{2}$ if on average each small cell has only one neighbor. This is the same conclusion as in [14] for the relay scenario, which has only one "neighbor" per transmission. However, Proposition 2 is more general for $M > 1$, as it solves the optimal W^* for an arbitrary M. Secondly, as we will also see in the simulation results, the optimal token supply per UE is neither too close to K to reduce the incentive for the SC/UE to grant silence requests, nor too close to 0 such that when the SC/UE needs help, it cannot afford to pay tokens. Thirdly, the complexity associated with the optimal token design is low thanks to the close-form expressions (18) to (20). Also, the required knowledge such as neighbor information and token distribution can be readily estimated using the existing system mechanism, statistics, and KPI framework that are used by most of the real-world small cell deployments.

E. Implementation consideration: token exchange protocol

In this section, we discuss how to design the token exchange protocol from a practical point of view. When a UE registers with the operator, a certain amount of tokens are assigned to it based on the unique user identity, such as its International Mobile Subscriber Identity (IMSI). These tokens can be used by the user while in the operator's network, to eliminate a neighboring user's downlink interference. Such process typically involves two stages: *target user identification*, and *token exchange*. In the following, we will describe the details regarding these two stages, using LTE protocols [24] as an example to highlight the implementation potential of the proposed token system.

In the target user identification stage, the user constantly monitors the downlink SINR. If the user detects that SINR (or other performance metrics associated with it) has become too low to maintain the requested QoS, it can invoke the token exchange scheme to potentially eliminate such interference. Unlike the other token applications in which identifying the "helper" node is non-trivial [14], finding the dominant source of interference is relatively easy in most cellular standards.



(a) Successful Token Passing

(b) Failed Token Passing

Fig. 2. Secure token passing process. Note that message exchanges between the source and target SC can be implemented leveraging the LTE X2 interface.

This is because users constantly detect the existence of co-channel neighboring cells and measure their signal strength, such as RSRP (Reference Signal Received Power) and RSRQ (Reference Signal Received Quality) in LTE, for the purpose of mobility. Such mechanism can be leveraged to identify the target SC/UE who causes the interference problem. Furthermore, a user can potentially decide *before the token exchange* how much performance improvement it can expect, by computing a new SINR without the contribution of this target neighboring cell. It should be noted that all these procedures are standard LTE operations and have already been implemented.

After the user has identified the target SC/UE, it sends a silence request message to the target. When the target SC/UE receives the request, it makes the decision on whether or not to provide the silence service. If the target SC/UE accepts the request, it sends back a silence acceptance message, and both sides enter a secure token passing process. If the target SC/UE denies the request, it sends back a silence decline message. In such scenario, no token exchange will happen. Noted again that the message exchange can be implemented by modifying the existing X2 interface [26] between neighboring LTE cells, or utilizing LTE D2D [29].

It is clear that a secure and efficient token passing process is crucial, especially for anonymous and self-interested SC/UE pairs. There are several existing solutions for token exchanges in e-commerce [30]–[32]. Most of these protocols are centralized with a Trusted Third Party (TTP), which does not fit in a distributed small cell network. Fortunately, a distributed protocol was proposed in [30] which does not require a centralized TTP, and hence is chosen for our proposed token exchange system. Once both SC/UE pairs enter a secure token passing process, a secure communication channel is established between them. Again, this can be an enhanced X2 interface between neighboring cells with authentication encryption. The purpose is to protect the token passing process from outside attacks. Moreover, both sides are equipped with independent tamper-proof secure modules (SMs). The purpose of SM is to create an equivalent of TTP whose data cannot be accessed by the players. Upon the target SC/UE deciding to accept the silence request from the source SC/UE, the token passing process works as follows. Firstly, the source SM reduces the user token holding by 1. At the same time the target SC/UE ceases downlink transmission while source SC/UE continues its own downlink transmission. Source user also continues to monitor downlink signals (via UE measurement), in order to confirm the target SC/UE indeed ceases transmission during the entire period. If this is true, at the end of the source SC/UE transmission period, the source SM sends an encrypted message to the target SM which moves one token from source SM to target SM. Otherwise, if during any period of the source SC/UE transmission period, source user discovers that target SC/UE resumes transmission, the source SM cancels the token passing process and informs the target SM. The deposited token is returned to the source user. Fig. 2 illustrates the described token passing process, where Fig. 2(a) depicts the success case while Fig. 2(b) shows the failure case.

V. Simulation Results

A. Setup

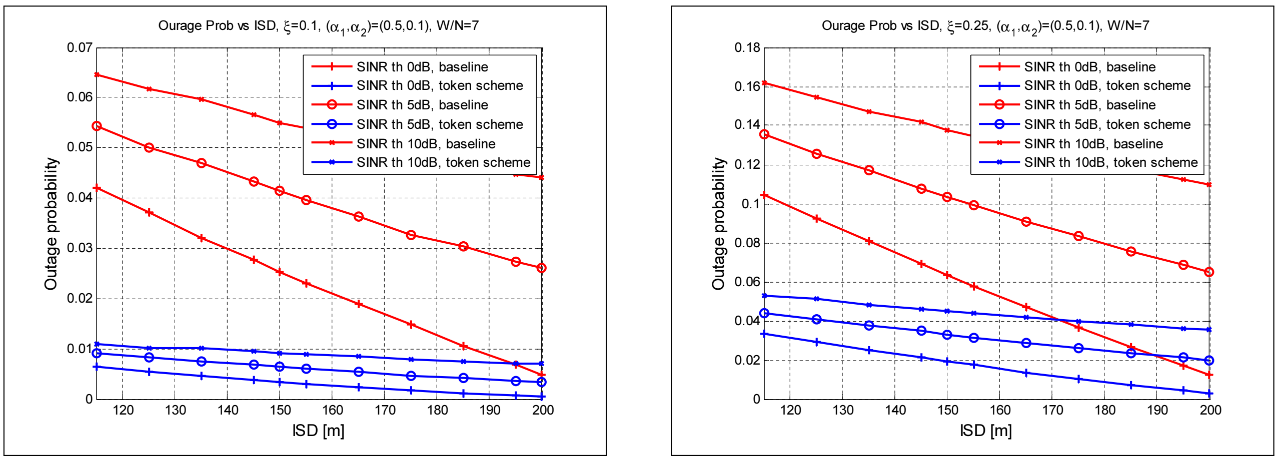
TABLE II
SIMULATION PARAMETERS

Parameters	**Value**
SCs	2500
UEs	2500
UEs per cell	0, 1, 2
Number of QoS classes	2
SC transmit power	10dBm or 20dBm
Pathloss model	3GPP in-to-out [33]
ISD	115m to 200m
Thermal noise density	-174dBm/Hz
Bandwidth	20MHz
Carrier frequency	2.1GHz
UE noise figure	5.5dB
Penetration loss (L_{ow})	10dB
d_0	1m

In order to verify the general theory that guides the optimal token system design, we resort to numerical simulations to demonstrate the effectiveness of the proposed design. In particular, an LTE-based system level simulator was developed in which the geometry of UEs and SCs is explicitly taken into account. We consider a large square area in which 2500 small cells and 2500 UEs are dropped. The entire area is divided into 50×50 small squares. At the center of each small square there is a SC base station whose location does not change over time. UEs, on the other hand, can move freely from time to time and be served by different SCs. The UE movement follows the random waypoint mobility model [34].

TABLE III
AVERAGE NUMBER OF OUTAGE UES, BASELINE VERSUS TOKEN EXCHANGE. $W/N = 7$, $K_{th} = 15$, AND $\xi = 0.25$.

ISD	125 meters			165 meters			200 meters		
λ	0.09	0.13	0.15	0.04	0.09	0.13	0.02	0.07	0.11
Baseline	224	303	373	117	224	321	30	162	274
Token	74	102	129	35	71	103	10	52	91
Performance Improvement	67%	66%	65%	70%	68%	68%	67%	68%	67%



(a) $\xi = 0.10$

(b) $\xi = 0.25$

Fig. 3. System outage probability versus SC/UE density (measured by ISD) with different SINR thresholds and different ξ. $(\alpha_1, \alpha_2) = (0.5, 0.1)$. Each UE starts with 7 tokens and the threshold is set to 15.

We consider a heterogeneous SC network with both high-power (20dBm) and low-power (10dBm) nodes, with a ratio of 1:5. UE association is performed at each time slot, in which high-power nodes are allowed to serve up to two active UEs while the low-power nodes can only have at most a single active UE. As has been discussed in Section II, multiple UEs will be scheduled to orthogonal but equally divided RB groups. Various values of the ISD are considered, ranging from 115 meters to 200 meters, where the former represents an ultra-dense small cell network while the latter corresponds to a relatively sparse deployment. Some other system simulation parameters are summarized in Table II.

We consider the 3GPP pathloss model that is recommended for system simulations of small cells and heterogeneous networks. Particularly, we consider the pathloss model suggested in [33]:

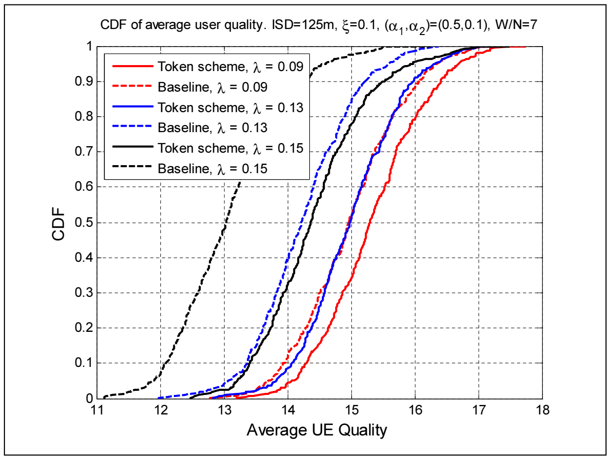
$$PL(d)[dB] = 15.3 + 37.6 \times \log_{10}(d) + L_{ow}, d > d_0. \quad (21)$$

Shadowing is not explicitly considered mainly for simplicity, as in this case the outage event is entirely decided by the system geometry and user QoS requirement.

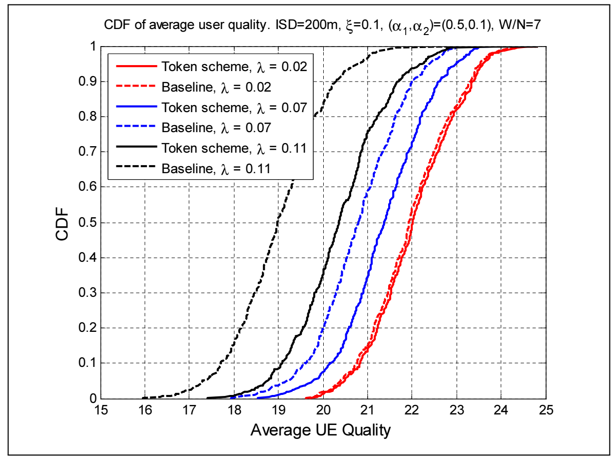
The theory proposed in Section IV uses the general expressions of benefit b, cost c, and utility $V(k)$. In the simulations, we study a specific system design by considering both *average QoS* and *outage probability* as the design objectives, to concretely illustrate the effectiveness of the proposed design. Particularly, we consider a downlink small cell system with two different QoS classes: *QoS class 1* which is a high-priority class, and *QoS class 2* which is a low-priority one. Note that this simplified two-level QoS model captures the essence of different packet priorities associated with different services. In practice, nevertheless, there are more QoS classes to accommodate various types of services, including voice, video streaming, real-time gaming, etc [24, Chapter 2]. The simplistic two-level QoS model can be considered as a coarse approximation of those sophisticated models. We adopt a linear QoS-to-Rate model for both classes: for $i = 1, 2$

$$Q_i(R_i) = \begin{cases} 0, & R_i < R_{i,min} \\ Q_{i,min} + \alpha_i (R_i - R_{i,min}), & R_i \geq R_{i,min} \end{cases} \quad (22)$$

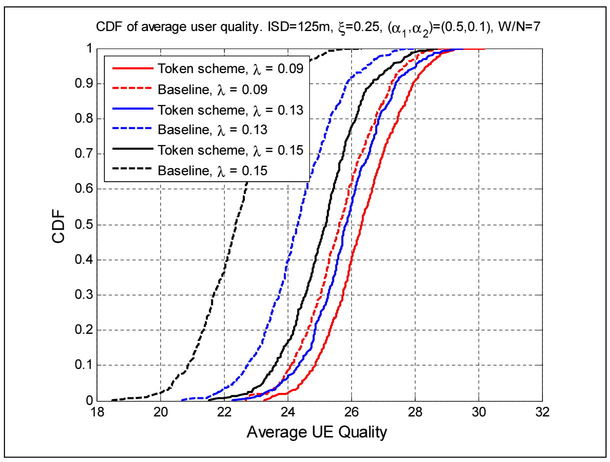
where $\alpha_1 > \alpha_2$ reflecting the higher priority of QoS class 1. It is worth mentioning that linear approximation of the QoS-to-Rate relationship is widely used in operational models [35], [36]. We use ξ to denote the probability of QoS class 1 packets when UE is active, and $(1 - \xi)$ to denote the probability of QoS class 2 packets when UE is active. We define system outage as the decoding failure of QoS class 1 due to strong interference, which is captured by the received SINR of packets associated with QoS class 1 falls below the SINR threshold $\mathsf{SINR_{th}}$ that corresponds to $R_{1,min}$. We consider average QoS improvement as the static benefit b for the token-payer, and the average QoS loss as the static cost c for the token-recipient. Such static benefit and cost are necessary for the *offline design*, in which strategies are derived based on first-order statistics of users and no on-the-fly adaptation exists. Furthermore, we assume full-buffer traffic for all UEs, i.e., $\gamma = 1$. Finally, a fixed token threshold 15 is used throughout all simulations. This is mainly due to the computational difficulty in solving for the utility function characterized in (6) and (7). In addition, using a fixed threshold over a variety of simulation parameters would also provide some insight into the robustness of the proposed token design.



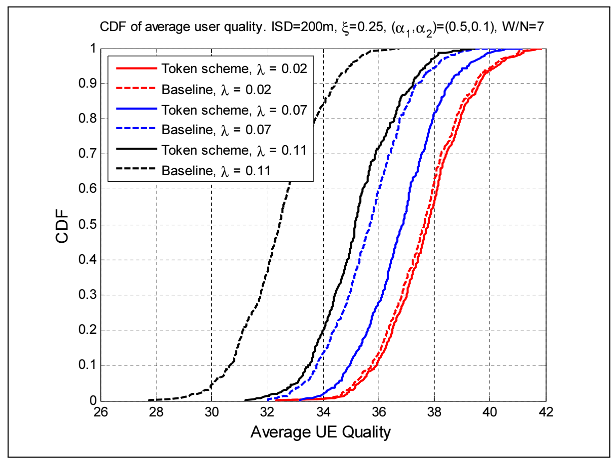
(a) ISD = 125 meters, $\xi = 0.10$



(b) ISD = 200 meters, $\xi = 0.10$



(c) ISD = 125 meters, $\xi = 0.25$



(d) ISD = 200 meters, $\xi = 0.25$

Fig. 4. CDF of user quality in different scenarios. Each sub-plot shows a different deployment density and ξ. $(\alpha_1, \alpha_2) = (0.5, 0.1)$. Each UE starts with 7 tokens and the threshold is set to 15.

Token design presented in Section III and IV is implemented for the aforementioned heterogeneous small sell network, including the extension to handle multiple interferers in Section III-B. In addition, we consider a baseline solution of static FFR [8] with a reuse factor of 2. It should be noted that FFR requires to identify and partition resources between cell-center and cell-edge UEs, which are very difficult in irregular heterogeneous networks and no optimal solution exists. On the other hand, centralized optimization in this simulation scenario is almost computational infeasible due to exhaustive search over a large number of devices, and hence will not be considered.

B. Performance

At initialization, we assign an equal amount of tokens to each UE, and then simulate the overall network for a total of 500 time slots. We first investigate whether the proposed token exchange schedule can improve the overall system performance by reducing the outage probability. Table III compares the number of UEs that are in outage with different solutions. The numbers of outage UEs in this table are obtained by averaging over 500 time slots. Different deployment densities are simulated, which are represented by the ISD between the neighboring cells. We can see from Table III that the system outage probability with the proposed token exchange scheme is much lower than the baseline system performance, which allows all self-interested users to maximize the individual outage probability without any incentive (tokens) to stimulate cooperations. Table III shows that the outage probability improvement can be up to 70%, which is a significant improvement. Furthermore, although the outage event becomes less as the ISD increases (due to the smaller percentage of boundary areas), the performance improvement in Table III is quite consistent throughout the considered ISD range.

Fig. 3 further plots the system outage probability versus ISD with different SINR thresholds and different ξ. The outage performance improvement can be observed for various ISD values, which demonstrates the effectiveness of the proposed token scheme. Moreover, the performance gain decreases as the ISD increases. The reason is that it is less likely to have significant inter-cell interference when the ISD is large. When ISD becomes very large, the neighboring cells are essentially isolated and hence no inter-cell interference exists. The proposed token system has no benefit in such scenario.

In addition to studying the system outage performance, we also compare the UE QoS of both schemes, using the same simulation parameters and geometry. Fig. 4 plots the CDF of UE quality in two different deployment densities, where the statistics are taken over all 500 time slots. More specifically, Fig. 4(a) and 4(c) show the CDFs of the average UE quality for $ISD = 125$ meters (ultra-dense deployment), while Fig. 4(b) and 4(d) illustrate the same for $ISD = 200$ meters (sparse deployment). From the CDF curves, it is clear that the token scheme can also improve the average user quality. This is mainly due to the imbalance between QoS class 1 and class 2, and the gain comes largely from saving QoS class 1 failures due to strong interference at the price of sacrificing neighboring user's class 2 transmission, made possible by the token exchanges. Such benefit does not exist in the baseline algorithm. We also note that the user quality gain increases as we increase λ. This is because λ is proportional to the SINR threshold, and thus we will have a better SINR distribution with larger λ. Finally, we can see that the quality gain is more significant in the ultra-dense deployment than in the normal or sparse deployment. This observation, combined with the conclusion from Fig. 3, proves the importance of the proposed token scheme in the ultra-dense small cell deployment, which has been hailed as a key option for 5G wireless systems.

Finally, we study the impact of token supply and demonstrate the importance of optimal token system design. We vary the initial token supply to each UE and repeat the simulation to compute the system outage probability averaged over 500 time slots. In Fig. 5 we compare the simulated average system outage performance with different initial token supplies. We can see that as we increase the initial token supply, the system outage probability first reduces, which is due to the fact that more tokens will stimulate more UEs to request silence services and hence improve the outage performance. However, as we continue increasing the token supply, the system outage probability increases. This is due to the fact that most of the UEs have too many tokens (more than K_{th}), and hence are unwilling to provide silence service to their neighbors. We also numerically compute the optimal token supply based on (19), and plot on the figure for comparison. It should be noted that in our given setup the edge UEs will see less neighboring cells than the middle UEs. We thus use an approximate M, which was computed as the average number of neighbors in the layout, in (19). It is clear from Fig. 5 that the analytical solution (19) matches the numerical solution very well.

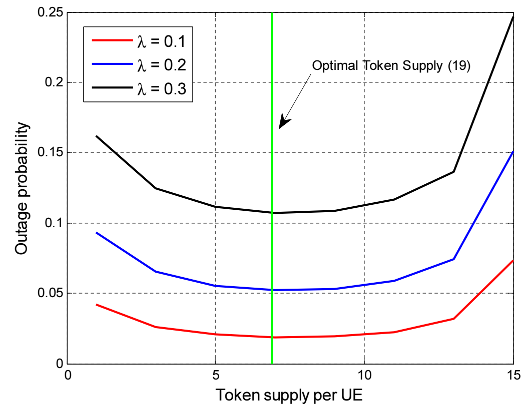


Fig. 5. Outage probability versus token supplies. $\xi = 0.25$.

VI. Conclusions

We have proposed a novel token framework to address the downlink interference problem in a heterogeneous small cell network with self-interested users. Heterogeneity poses several unique challenges that were not addressed by the previous solutions. However, as we have proved in this paper, a simple yet optimal token scheme can be used to incentivize self-interested users to cooperate for interference mitigation in a heterogeneous network. Optimal token system design that minimizes the network outage probability was developed. We first provided a complete solution to the optimal user strategy that only aims at individual utility maximization. We proved that with some mild conditions, the optimal user strategy has a simple threshold structure. We then derived the optimal token supply that minimizes the network outage probability, assuming each user adopts its utility-optimal strategy. Numerical results were provided to prove the effectiveness of the token system. The proposed token design can compliment the existing interference management techniques, scale well with the volume of users, require minimal information exchange, work with cognitive and self-interested devices, and can be implemented with some simple enhancements to the existing LTE protocol. All these advantages render the token design a strong candidate for the deployment of ultra-dense heterogeneous small cell networks.

There are some interesting problems that have not been addressed in this work, which are the subjects of potential future work. For example, we have only considered a on-off power strategy for tokens. A more refined approach would be to increase the dimension of the action space, and allow the transmit power to be reduced only low enough to meet the neighboring user's target QoS, instead of completely powering off. Such scenario also raises the possibility for multiple tokens to be exchanged, and the amount of tokens can be proportional to the requested power decrease. Another important question is how robust the optimal token system design derived in Section IV-C and IV-D is, with respect to the inaccuracies associated with the parameters such as average benefit/cost, number of neighbors, etc. Thirdly, although the analysis can be easily extended to multiple UEs per cell, a careful study of the high-user-density scenario is valuable, particularly comparing the performance of the proposed token design with other distributed interference mitigation mechanisms such as user scheduling. Finally, ensuring the security and reliability of token exchanges represents a crucial aspect of the proposed design and represents an important topic for further investigation.

Appendix A
Proof of Lemma 1

We drop the user index i for notation convenience. Recall that a user strategy is *optimal* if and only if $\forall k \in \mathbb{N}$, we have $V(k|\sigma) \geq V(k|\sigma^{'})$, if $\sigma^{'} \neq \sigma$. The user utility maximization problem satisfies the consistency and continuity conditions for the one-shot deviation principle [37]. So we only need to prove that the strategy σ is *unimprovable* [37].

Note that a strategy is *unimprovable* if we cannot find a $k^{'} \in \mathbb{N}$ such that, there exists another strategy $\sigma^{'}$ which is identical to σ for all values except $k^{'}$. Note that by definition, both σ and $\sigma^{'}$ result in the same marginal utility $V(k+1)-V(k)$ for all values except $k^{'}$. Hence we only need to prove that for $k^{'}$, it is not possible to construct a different strategy $\sigma^{'}$ that achieves better payoff. Since $\sigma^{'} \neq \sigma$, it has to be one of the following two cases.

Case 1: $\sigma^{'}$ chooses to be 0 for some $\beta(V(k^{'}+1)-V(k^{'})) \geq c$, where β is applied since the marginal utility is achieved in the future time slot. Note that in this case, strategy $\sigma^{'}$ achieves a payoff 0 (since it chooses no action), while strategy σ achieves a payoff $\beta(V(k^{'}+1)-V(k^{'}))-c \geq 0$. Hence σ has a better payoff.

Case 2: $\sigma^{'}$ chooses to be 1 for some $\beta(V(k^{'}+1)-V(k^{'})) < c$. Note that in this case, strategy $\sigma^{'}$ achieves a payoff $\beta(V(k^{'}+1)-V(k^{'}))-c < 0$, while strategy σ achieves a payoff 0. Hence σ has a better payoff.

Thus $\sigma^{'}$ cannot have a better payoff than σ at the value $k^{'}$. This completes the proof.

APPENDIX B
PROOF OF PROPOSITION 1

We drop the user index i for notation convenience. Since $V(k)$ is monotonically increasing, we have $V(k+1)-V(k) > 0$. Using the definition of concavity, $V(k)$ must satisfy

$$V(\alpha k_1 + (1-\alpha)k_2) \geq \alpha V(k_1) + (1-\alpha)V(k_2), \qquad (23)$$

$\forall k = 0, \cdots, W$. If we choose $\alpha = 1/2$, $k_1 = k$, $k_2 = k+2$, we have

$$V(k+1) - V(k) \geq V(k+2) - V(k+1). \qquad (24)$$

From inequality (24) we can prove that

Case 1: If $\exists k \in \mathbb{N}$ such that $V(k+1) - V(k) \geq \frac{c}{\beta}$ and hence $\sigma(k) = 1$, then $\forall k^{'} \leq k$, we have

$$V(k^{'}+1) - V(k^{'}) \geq V(k+1) - V(k) \geq \frac{c}{\beta}. \qquad (25)$$

Thus $\sigma(k^{'}) = 1, \forall k^{'} \leq k$.

Case 2: If $\exists k \in \mathbb{N}$ such that $V(k+1) - V(k) < \frac{c}{\beta}$ and hence $\sigma(k) = 0$, then $\forall k^{'} \geq k$, we have

$$V(k^{'}+1) - V(k^{'}) \leq V(k+1) - V(k) < \frac{c}{\beta}. \qquad (26)$$

Thus $\sigma(k^{'}) = 0, \forall k^{'} \geq k$.

Putting both cases together proves Proposition 1.

APPENDIX C
PROOF OF PROPOSITION 2

We already have an expression of $\bar{\mathsf{P}}_{\mathsf{out}}$ in (17):

$$\bar{\mathsf{P}}_{\mathsf{out}} = \lambda\rho_0 + \left(1 - \left(1 - \frac{\lambda}{M}\right)^M\right)\rho_K. \qquad (27)$$

If we denote $\alpha_1 \doteq \lambda$ and $\alpha_2 \doteq 1 - \left(1 - \frac{\lambda}{M}\right)^M$, the objective function becomes

$$\bar{\mathsf{P}}_{\mathsf{out}} = \alpha_1\rho_0 + \alpha_2\rho_K, \qquad (28)$$

with $\alpha_1 \leq \alpha_2$. Applying Proposition 4 in [14], we have that $\forall k = 0, 1, \cdots, K$,

$$\rho_k = \left(\frac{1-\rho_0}{1-\rho_K}\right)^k \rho_0. \qquad (29)$$

In particular,

$$\rho_K (1-\rho_K)^K = \rho_0 (1-\rho_0)^K. \qquad (30)$$

A. A related optimization problem

In order to prove Proposition 2, we first need to solve the following optimization problem

$$\begin{array}{ll} \underset{\{\mathbf{x_1}, \mathbf{x_2}\}}{\text{minimize}} & \alpha_1 x_1 + \alpha_2 x_2 \\ \text{subject to} & x_1 (1-x_1)^K = x_2 (1-x_2)^K \\ & 0 \leq x_1, x_2 \leq 1 \end{array} \qquad (31)$$

Since $\alpha_1 \leq \alpha_2$, it is easy to show that the optimal solution must satisfy $x_2^* \leq x_1^*$, because otherwise we can switch x_1^* and x_2^* and get a smaller objective.

Next, consider $g(x) = x(1-x)^K$. A simple check of the derivative $g'(x)$ shows that $g(x)$ is increasing in $[0, \frac{1}{K+1}]$ and decreasing in $[\frac{1}{K+1}, 1]$. Hence the optimal solution satisfies $0 \leq x_2^* \leq \frac{1}{K+1} \leq x_1^* \leq 1$. Define $f(x_1, x_2) = \alpha_1 x_1 + \alpha_2 x_2$. We have

$$\begin{aligned} & f(x_1 + \partial x_1, x_2 + \partial x_2) \\ = \ & \alpha_1(x_1 + \partial x_1) + \alpha_2(x_2 + \partial x_2) && (32) \\ = \ & f(x_1, x_2) + (\alpha_1 \partial x_1 + \alpha_2 \partial x_2) && (33) \\ = \ & f(x_1, x_2) + \partial x_2 (\alpha_1 \frac{\partial x_1}{\partial x_2} + \alpha_2). && (34) \end{aligned}$$

Using the expression of $g(x)$, we have

$$\frac{\partial x_1}{\partial x_2} = \frac{(1-x_2)^{K-1}(1-(K+1)x_2)}{(1-x_1)^{K-1}(1-(K+1)x_1)}. \qquad (35)$$

Because $0 \leq x_2^* \leq \frac{1}{K+1} \leq x_1^* \leq 1$, we have:

$$\frac{\partial x_1}{\partial x_2} \leq 0. \qquad (36)$$

A straightforward calculus exercise also shows that

$$\left|\frac{\partial x_1}{\partial x_2}\right| \geq 1. \qquad (37)$$

Hence,

$$f(x_1 + \partial x_1, x_2 + \partial x_2) = f(x_1, x_2) + \partial x_2 (\alpha_2 - \alpha_1 \left|\frac{\partial x_1}{\partial x_2}\right|), \qquad (38)$$

where as $x_2 : 0 \to \frac{1}{K+1}$ and $x_1 : 1 \to \frac{1}{K+1}$ while satisfying $g(x_1) = g(x_2)$, $\alpha_2 - \alpha_1 \left|\frac{\partial x_1}{\partial x_2}\right|$ monotonically increases from negative to positive. Hence the minimal value of $f(x_1, x_2)$

happens when $\alpha_2 - \alpha_1 \left|\frac{\partial x_1}{\partial x_2}\right| = 0$. Finally, the optimal solution to (31) satisfies:

$$\begin{cases} 0 \le x_2^* \le \dfrac{1}{K+1} \le x_1^* \le 1 \\ \dfrac{(1-x_2^*)^{K-1}\,(1-(K+1)x_2^*)}{(1-x_1^*)^{K-1}\,(1-(K+1)x_1^*)} = -\dfrac{\alpha_2}{\alpha_1} \\ x_1^*(1-x_1^*)^K = x_2^*(1-x_2^*)^K \end{cases} \tag{39}$$

or equivalently,

$$\begin{cases} 0 \le x_2^* \le \dfrac{1}{K+1} \le x_1^* \le 1 \\ \dfrac{x_1^*(1-x_1^*)\,(1-(K+1)x_2^*)}{x_2^*(1-x_2^*)\,(1-(K+1)x_1^*)} = -\dfrac{\alpha_2}{\alpha_1} \\ x_1^*(1-x_1^*)^K = x_2^*(1-x_2^*)^K \end{cases} \tag{40}$$

B. Derivation of optimal token supply

We now have the conditions (40) the optimal solution $\{\rho_0^*, \rho_K^*\}$ must satisfy. Using (29), we have

$$\rho_k^* = \left(\frac{1-\rho_0^*}{1-\rho_K^*}\right)^k \rho_0^*. \tag{41}$$

The optimal token supply can be derived as

$$\frac{W^*}{N} = \sum_{k=0}^{K} k\rho_k^* \tag{42}$$

$$= \rho_0^* \sum_{k=0}^{K} k \left(\frac{1-\rho_0^*}{1-\rho_K^*}\right)^k. \tag{43}$$

(44)

Thus if $\alpha_1 = \alpha_2 = \frac{1}{2}$ we have

$$\frac{W^*}{N} = \frac{K}{2}; \tag{45}$$

otherwise we have

$$\frac{W^*}{N} = \frac{((K-1)\rho_K^* + \rho_0^*)\,(1-\rho_0^*)(1-\rho_K^*) - K\rho_K^*(1-\rho_K^*)^2}{(\rho_0^* - \rho_K^*)^2} \tag{46}$$

where $\{\rho_0^*, \rho_K^*\}$ satisfy:

$$\begin{cases} 0 \le \rho_K^* \le \dfrac{1}{K+1} \le \rho_0^* \le 1 \\ \dfrac{\rho_0^*(1-\rho_0^*)\,(1-(K+1)\rho_K^*)}{\rho_K^*(1-\rho_K^*)\,(1-(K+1)\rho_0^*)} = -\dfrac{\alpha_2}{\alpha_1} \\ \rho_0^*(1-\rho_0^*)^K = \rho_K^*(1-\rho_K^*)^K \end{cases} \tag{47}$$

Note that $\alpha_1 = \alpha_2 = \frac{1}{2}$ is equivalent to $M = 1$, which completes the proof.

References

[1] A. Ghosh *et al.*, "Heterogeneous cellular networks: From theory to practice," *IEEE Commun. Mag.*, vol. 50, no. 6, pp. 54–64, June 2012.

[2] J. G. Andrews, "Interference cancellation for cellular systems: a contemporary overview," *IEEE Commun. Mag.*, vol. 12, no. 2, pp. 19–29, Apr. 2005.

[3] V. Cadambe and S. Jafar, "Interference alignment and degrees of freedom of the k-user interference channel," *IEEE Trans. Info. Theory*, vol. 54, no. 8, pp. 3425–3441, Aug. 2008.

[4] D. Gesbert, M. Kountouris, R. Heath, C.-B. Chae, and T. Salzer, "Shifting the MIMO paradigm," *IEEE Signal Processing Magazine*, vol. 24, no. 5, pp. 36–46, Sept. 2007.

[5] D. Lee *et al.*, "Coordinated multipoint transmission and reception in LTE-Advanced: deployment scenarios and operational challenges," *IEEE Commun. Mag.*, vol. 50, no. 2, pp. 148–155, Feb. 2012.

[6] E. Larsson, O. Edfors, F. Tufvesson, and T. Marzetta, "Massive MIMO for next generation wireless systems," *IEEE Commun. Mag.*, vol. 52, no. 2, pp. 186–195, Feb. 2014.

[7] M. Pischella and J. C. Belfiore, "Power control in distributed cooperative OFDMA cellular networks," *IEEE Trans. Wireless Commun.*, vol. 7, no. 5, pp. 1900–1906, May 2008.

[8] A. Simonsson, "Frequency reuse and intercell interference co-ordination in E-UTRA," in *Proc. IEEE VTC 2007 Spring*, pp. 3091–3095.

[9] T. Zahir, K. Arshad, A. Nakata, and K. Moessner, "Interference management in femtocells," *IEEE Communications Surveys Tutorials*, vol. 15, no. 1, pp. 293–311, First 2013.

[10] S.-M. Cheng, W.-C. Ao, and K.-C. Chen, "Downlink capacity of two-tier cognitive femto networks," in *Proceedings of the IEEE 21st PIMRC*, Sept. 2010, pp. 1303–1308.

[11] J. Torregoza, R. Enkhbat, and W.-J. Hwang, "Joint power control, base station assignment, and channel assignment in cognitive femtocell networks," *EURASIP Journal on Wireless Communications and Networking*, vol. 1, May 2010.

[12] Y. Miyamoto, "Downlink power control method and CDMA communication system incorporating the control method," US Patent 20 020 001 292.A1, Jan. 3, 2002.

[13] Y. Xi and E. Yeh, "Pricing, competition, and routing in relay networks," in *47th Annual Allerton Conference on Communication, Control, and Computing*, 2009, pp. 507–514.

[14] J. Xu and M. van der Schaar, "Token system design for autonomic wireless relay networks," *IEEE Trans. Commun.*, vol. 61, no. 7, pp. 2924–2935, July 2013.

[15] L. Buttyan and J.-P. Hubaux, "Nuglets: a virtual currency to stimulate cooperation in self-organized mobile ad hoc networks," EPFL-DI-ICA, Tech. Rep. DSC/2001/001, Jan. 2001.

[16] V. Vishnumurthy, S. Chandrakumar, and E. G. Sirer, "KARMA: A secure economic framework for peer-to-peer resource sharing," in *Workshop on the Economics of Peer-to-Peer Systems*, Berkeley, California, June 2003.

[17] D. R. Figueiredo, J. K. Shapiro, and D. Towsley, "Payment-based incentives for anonymous peer-to-peer systems," University of Massachusetts Computer Science, Tech. Rep. 04-62, July 2004.

[18] G. Tan and S. A. Jarvis, "A payment-based incentive and service differentiation mechanism for peer-to-peer streaming broadcast," in *Proc. 14th IEEE International Workshop on Quality of Service (IWQoS 2006)*, June 2006, pp. 41–50.

[19] E. J. Friedman, J. Y. Halpern, and I. Kash, "Efficiency and Nash equilibria in a scrip system for P2P networks," in *Proc. the 7th ACM Conference on Electronic Commerce*, 2006, pp. 140–149.

[20] M. Xiao, N. Shroff, and E. K. P. Chong, "A utility-based power-control scheme in wireless cellular systems," *IEEE/ACM Trans. Networking*, vol. 11, no. 2, pp. 210–221, Apr 2003.

[21] M. Rasti, A.-R. Sharafat, and B. Seyfe, "Pareto-efficient and goal-driven power control in wireless networks: A game-theoretic approach with a novel pricing scheme," *IEEE/ACM Trans. Networking*, vol. 17, no. 2, pp. 556–569, April 2009.

[22] J. Huang, Z. Han, M. Chiang, and H. Poor, "Auction-based resource allocation for cooperative communications," *IEEE J. Select. Areas Commun.*, vol. 26, no. 7, pp. 1226–1237, Sept. 2008.

[23] G. Mailath and L. Samuelson, Eds., *Repeated games and reputations: long-run relationships*. New York, USA: Oxford University Press, 2006.

[24] S. Sesia, I. Toufik, and M. Baker, *LTE - The UMTS Long Term Evolution: From Theory to Practice*. Wiley, Sept. 2011.

[25] D. Lopez-Perez *et al.*, "Coordinated multipoint transmission and reception in LTE-Advanced: deployment scenarios and operational challenges," *IEEE Wireless Communications*, vol. 18, no. 3, pp. 22–30, June 2011.

[26] 3GPP, "Evolved Universal Terrestrial Radio Access Network (E-UTRAN); X2 general aspects and principles," TS 36.420, 2012.

[27] D. Blackwell, "Discounted dynamic programming," *Ann. Math. Stat.*, vol. 36, no. 1, pp. 226–235, Feb. 1965.

[28] H. R. Varian, Ed., *Microeconomic Analysis*. W. W. Norton and Company, 1992.

[29] 3GPP, "Feasibility study for proximity services (ProSe) (Release 12)," TR 22.803, 2012.

[30] G. Avoine and S. Vaudenay, "Fair exchange with guardian angels," in *Information Security Applications*, vol. 2908, 2004, pp. 188–202.

[31] M. K. Franklin and M. K. Reiter, "Fair exchange with a semi-trusted third party," in *Proceedings of the 4th ACM conference on Computer and Communications Security*, Zurich, Switzerland, 1997, pp. 1–5.

[32] F. Bao, R.-H. Deng, and W. Mao, "Efficient and practical fair exchange protocols with off-line TTP," in *Proceedings of 1998 IEEE Symposium on Security and Privacy*, Zurich, Switzerland, 1998, pp. 77–85.

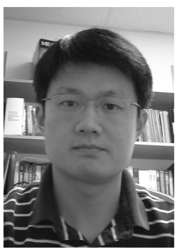
[33] 3GPP, "Simulation assumptions and parameters for FDD HeNB RF requirements," Tech. Rep. R4-092042, 2009.

[34] T. Camp, J. Boleng, and V. Davies, "A survey of mobility models for ad hoc network research," *Wireless Communications and Mobile Computing*, vol. 2, no. 5, pp. 483–502, 2002.

[35] M. van der Schaar and P. Chou, Eds., *Multimedia over IP and Wireless Networks: Compression, Networking, and Systems*. Elsevier, Mar. 2007.

[36] C. Shen and M. van der Schaar, "Optimal resource allocation for multimedia applications over multiaccess fading channels," *IEEE Trans. Wireless Commun.*, vol. 7, no. 9, pp. 3546–3557, September 2008.

[37] R. J. Aumann and S. Hart, Eds., *Handbook of Game Theory with Economic Applications, Vol. 2*. North-Holland, 1994.



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