Conjectural Equilibrium in Multiuser Power Control Games

Yi Su, *Student Member, IEEE*, and Mihaela van der Schaar, *Senior Member, IEEE*

***Abstract*—This paper considers a noncooperative game in which competing users sharing a frequency-selective interference channel selfishly optimize their power allocation in order to improve their achievable rates. Previously, it was shown that a user having the knowledge of its opponents' channel state information can make foresighted decisions and substantially improve its performance compared with the case in which it deploys the conventional iterative water-filling algorithm, which does not exploit such knowledge. This paper discusses how a foresighted user can acquire this knowledge by modeling its experienced interference as a function of its own power allocation. To characterize the outcome of the multiuser interaction, the conjectural equilibrium is introduced, and the existence of this equilibrium for the investigated water-filling game is proven. Importantly, we show that both the Nash equilibrium and the Stackelberg equilibrium are special cases of the conjectural equilibrium. We also develop practical algorithms to form accurate beliefs and select desirable power allocation strategies. Numerical simulations indicate that a foresighted user without any a priori knowledge of its competitors' private information can effectively learn how the other users will respond to its actions, and induce the entire system to an operating point that improves both its own achievable rate as well as the rates of the other participants in the water-filling game.**

***Index Terms*—Interference channel, power control, noncooperative game, conjectural equilibrium.**

I. Introduction

MULTIUSER communication systems represent competitive environments, where devices built according to different standards and architectures compete for the limited available resources. These devices can differ greatly in terms of their channel conditions, user-defined utilities, action strategies, ability to sense the environment and gather information about competing users, and subsequently reason and adapt their transmission strategies based on the available information. Spectrum sharing among multiple competing devices in the interference-limited communication systems represents such an environment. In particular, the performance of each device depends not only on its own power allocation strategy, but also on that of the other devices. Individual devices may differ in both their knowledge of the system-wide channel state information (which is for instance constrained by their spectrum sensing abilities and/or information exchange overheads) as well as their decision making mechanism for choosing their optimal power allocation.

A. Literature Review

During the past two decades, multiuser power control in wireless networks has been extensively modeled and analyzed within the game theoretic framework. The existing literature includes solutions for both narrowband and wideband power control. These works can be further subdivided based on the deployed utility functions, e.g., rate maximization, power minimization, energy efficiency maximization, etc. For a comprehensive review, we refer the readers to [1]–[3] and the references therein.

This paper focuses on the multiuser interaction in frequency-selective Gaussian interference channels in which self-interested users are trying to maximize their achievable rates. Throughout this paper, we focus on a simple, yet practical approach, which minimizes the complexity of transceivers by treating interference as additive noise. From a particular user's perspective, it is well known that, for fixed interference power, the optimal power allocation is the so-called water-filling solution. Therefore, the spectrum sharing problem can also be regarded as a *water-filling game*. Specifically, the participants in the water-filling game are modeled as players with individual goals and strategies. They compete or cooperate with each other until they can agree on an acceptable resource allocation outcome. Existing research can be categorized into two types, *noncooperative* games and *cooperative* games [4]–[15].

First, the formulation of the multiuser environment as a noncooperative game has appeared in several recent works [4]–[11]. An iterative water-filling (IW) algorithm has been proposed to mitigate the mutual interference and optimize the performance without the need for a central controller [4]. At every decision stage, selfish users deploying this algorithm myopically maximize their achievable rates by water-filling across the entire frequency band until a Nash equilibrium (NE) is reached. Sufficient conditions under which the iterative water-filling algorithm converges to a unique NE are derived and the closed form solution to the water-filling problem is investigated for some special scenarios [5]–[8]. Alternatively, self-enforcing protocols are studied in the repeated game setting, where efficient, fair, and incentive compatible spectrum sharing is shown to be possible by punishing misbehavior and thereby, compelling users to cooperate [9].

Because the IW algorithm may lead to Pareto-inefficient solutions [10], i.e., selfishness is detrimental in the interference channel, there also have been a number of related works studying spectrum sharing in the setting of cooperative games [12]–[16]. Several (near-) optimal algorithms were proposed to attain different operating points, e.g., weighted sum-rate

Manuscript received September 08, 2008; accepted March 10, 2009. First published April 07, 2009; current version published August 12, 2009. The associate editor coordinating the review of this manuscript and approving it for publication was Prof. Subhrakanti Dey. This work was supported by NSF 0830556 and ONR.

The authors are with the Department of Electrical Engineering (EE), University of California, Los Angeles, CA 90095 USA (e-mail: yisu@ee.ucla.edu; mihaela@ee.ucla.edu).

Color versions of one or more of the figures in this paper are available online at http://ieeexplore.org.

Digital Object Identifier 10.1109/TSP.2009.2020395

maximization, proportional fairness, max-min fairness, etc., on the Pareto boundary of the achievable rate region. These works assume that users cooperatively maximize a common objective function and require explicit information exchanges among the users.

In short, most of the existing research mainly concentrates on studying the existence and performance of NE in noncooperative games or on developing efficient algorithms to approach the Pareto boundary in cooperative games. Our focus in this paper is on the noncooperative setting, which explicitly considers the self-interested and competitive nature of individual players. However, most of the existing works in the noncollaborative setting often neglect an important intrinsic dimension of the information-decentralized multiuser interaction. They assume homogeneous users acting only based on their own private information and disregarding their ability to acquire and process information about their opponents, thereby being able to improve their performance. The best response strategy of a selfish user that knows its myopic opponents' private information, including their channel state information and power constraints, was first investigated in [11] using the Stackelberg equilibrium (SE) formulation. It was shown in [11] that, surprisingly, a foresighted user playing the SE can improve both its performance as well as the performance of all the other users. These results highlight the significance of information availability in water-filling games. However, one key question remains unsolved: how should a foresighted user acquire its desired information and adapt its response?

B. Contributions

First, as opposed to our previous approach [11], which assumes a foresighted user having perfect knowledge of its competitors' responses to its actions, we discuss in this paper how the foresighted user without any such *a priori* knowledge can accumulate this knowledge and improve its performance when participating in the water-filling games. We propose that the foresighted user can explicitly model its competitors' response as a function of its power allocation by repeatedly interacting with the environment and observing the resulting interference.

Second, we introduce the concept of conjectural equilibrium (CE) to characterize the strategic behavior of a user that models the response of its myopic competing users, and the existence of this equilibrium in the water-filling game is proven. Some previously adopted solutions, including NE and SE, are shown to be special cases of the CE. The basic notion of CE was first proposed by Hahn in the context of a market model [19]. A general multi-agent framework is proposed in [20] to study the existence of and the convergence to CE in market interactions. Specifically, a strategic user is assumed to model the market price as a linear function of its desired demand. It is observed that it may be better or worse off than without modeling, depending on its initial belief. However, we note that using the linear model is purely heuristic in [20]. In contrast to this heuristic belief formation, we apply CE in the water-filling game, because it provides a practical solution concept to approach the performance bound of SE.

Finally, we show that deploying the linear model to form conjectures can suitably explore the problem structure of the water-filling game, and therefore, it can lead to a substantial performance improvement. Practical algorithms are developed to form accurate beliefs and select desirable power allocation strategies. It is shown that, a foresighted user without any *a priori* knowledge can effectively learn how the other users will respond to its actions and guide the system to an operating point having comparable performance to the algorithm in [11], where perfect a priori knowledge is assumed. More importantly, as opposed to the two-user algorithm in [11], the proposed algorithm in this paper can be applied in general multiuser communication scenarios, where more than two users exist.

The rest of the paper is organized as follows. Section II presents the noncooperative game model, reviews the existing related noncooperative solutions, and introduces the concept of CE. The existence of this CE in the water-filling game is proven in Section III. Section IV develops practical algorithms to form beliefs and approach CE. Numerical results are provided in Section V to show that a foresighted user can achieve substantial performance improvement if it models its competitors in the water-filling game. In Section VI, we summarize the connection and difference between our approach and related work on repeated games, learning solutions and Stackelberg formulations of power control problems, and discuss the possible extension to the communication scenarios with multiple foresighted users. Conclusions are drawn in Section VII.

II. System Model and Conjectural Equilibrium

In this section, we describe the mathematical model of the frequency-selective interference channel and formulate the noncooperative multiuser water-filling game. We summarize the existing noncooperative game theoretic solutions and introduce the conjecture equilibrium in the water-filling game context.

A. System Description and Existing Results

Fig. 1 illustrates a frequency-selective Gaussian interference channel model. There are K transmitters and K receivers in the system. Each transmitter and receiver pair can be viewed as a player (or user). The transfer function of the channel from transmitter i to receiver j is denoted as $H_{ij}(f)$, where $0 \leq f \leq F_s$. The noise power spectral density (PSD) that receiver k experiences is denoted as $N_k(f)$. Denote player k's transmit PSD as $P_k(f)$. For user k, the transmit PSD is subject to its power constraint

$$\int_0^{F_s} P_k(f)df \leq \mathbf{P}_\mathbf{k}^{\max}. \tag{1}$$

Indeed, the problem of finding the capacity region for this interference channel has been open for many years. This paper follows a suboptimal but practical approach that minimizes the complexity of the transceiver by treating interference as noise. For a fixed $P_k(f)$, user k can achieve the following data rate:

$$R_k = \int_0^{F_s} \ln\left(1 + \frac{P_k(f)}{\sigma_k(f) + \sum_{j\neq k} P_j(f)\alpha_{jk}(f)}\right) df \tag{2}$$

where $\sigma_k(f)$ and $\alpha_{jk}(f)$ are defined as $N_k(f)/|H_{kk}(f)|^2$ and $|H_{jk}(f)|^2/|H_{kk}(f)|^2$.

To fully capture the performance tradeoff in the system, the concept of a rate region is defined as

$$\mathcal{R} = \{(R_1,\ldots,R_K) : \exists(P_1(f), \ldots, P_K(f)) \text{ satisfying (1) and (2)}\}.$$

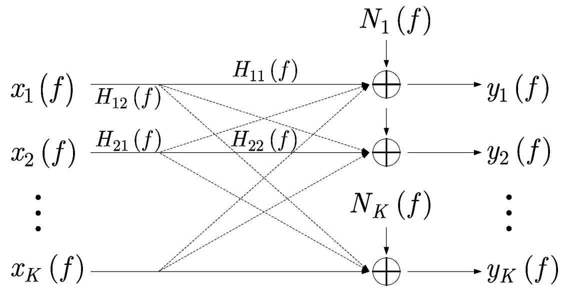

Fig. 1. Gaussian interference channel model.

The multiuser interaction in the interference channel can be modeled as a game. Let $\mathcal{G} = (\mathcal{K}, \mathcal{A}, U)$ denote a game with $\mathcal{K} = \{1, \ldots, K\}$ being the set of players, $\mathcal{A} = \times_{k\in\mathcal{K}}\mathcal{A}_k$ being the set of actions available to the users (in which $\mathcal{A}_k$ is the set of actions available to user k), and $U = \times_{k\in\mathcal{K}} U_k$ being the users' payoff functions (in which $U_k : \mathcal{A} \rightarrow \mathcal{R}$ is the user k's payoff function) [17]. In the water-filling game, the players' payoffs are the respective achievable data rates and their strategies are to determine their transmit PSDs satisfying the constraint in (1).

As aforementioned, existing research mainly focuses on two types of games, i.e., *cooperative* games and *noncooperative* games. Specifically, cooperative approaches aim to maximize the weighted sum or weighted product of data rates [12]–[16]. Because of the nonconvexity of the rate as a function of power allocations, the computational complexity of optimal solutions (e.g., exhaustive search) in finding the rate region is prohibitively high. Existing works aim to approach the Pareto boundary of this rate region and provide near-optimal performance. Moreover, it should be noted that cooperation among users is indispensable for the multiuser system to operate at the Pareto boundary. On the other hand, instead of solving the optimization problem globally, the IW algorithm models the users as myopic decision makers [4]. This means that they optimize their transmit PSD by water-filling and compete to increase their transmission rates with the sole objective of maximizing their own data rates in (2) regardless of the coupling among users. In other words, users are assumed to be myopic, i.e., they update actions shortsightedly, without considering the long-term impacts of taking these actions. The outcome of this noncooperative scenario is characterized by the concept of Nash equilibrium, which is defined to be any point $(a_1^*, \ldots, a_K^*)$ satisfying

$$U_k(a_k^*, a_{-k}^*) \geq U_k(a_k, a_{-k}^*), \quad \forall a_k \in \mathcal{A}_k \text{ and } k \in \mathcal{K}, \quad (3)$$

where $a_{-k}^* = (a_1^*, \ldots, a_{k-1}^*, a_{k+1}^*, \ldots, a_K^*)$ [17]. The existence and the uniqueness of NE are proven under a wide range of realistic conditions and can be obtained by the IW algorithm [5]–[7].

The recent approach in [11] demonstrates that the myopic behavior can be further improved. If a selfish user gets the private information about its competitors and knows how they react, it can foresightedly consider the coupling of players' actions. In this case, its best response strategy is to play the SE strategy. To define SE, we first define the action a_k^* to be a best response (BR) to actions a_{-k} if

$$U_k(a_k^*, a_{-k}) \geq U_k(a_k, a_{-k}), \forall a_k \in \mathcal{A}_k. \quad (4)$$

User k's best response to a_{-k} is denoted as $\mathrm{BR}_k(a_{-k})$. Let $\mathrm{NE}(a_k)$ be the Nash equilibrium strategy [1] of the remaining players if player k chooses to play a_k, i.e.

$$\mathrm{NE}(a_k) = a_{-k}, \text{if } a_i = \mathrm{BR}_i(a_{-i}), \forall i \neq k. \quad (5)$$

The strategy profile $(a_k^*, \mathrm{NE}(a_k^*))$ is a Stackelberg equilibrium with user k leading[2] if and only if [18]

$$U_k(a_k^*, \mathrm{NE}(a_k^*)) \geq U_k(a_k, \mathrm{NE}(a_k)), \forall a_k \in \mathcal{A}_k. \quad (6)$$

Specifically, to find the SE in the water-filling game, we need to solve the following bi-level programming problem [11], where user 1 is assumed to be the foresighted user

$$\begin{aligned}
&\max_{P_1(f)} \int_0^{F_s} \ln\left(1 + \frac{P_1(f)}{\sigma_1(f) + \sum_{k=2}^K \alpha_{k1}(f)P_k(f)}\right) df \\
&\text{s.t.} \int_0^{F_s} P_1(f)df \leq \mathrm{P}_1^{\max} \\
&\quad P_1(f) \geq 0 \\
&\quad P_k(f) = \arg\max_{P_k'(f)\in\mathcal{A}_k} \int_0^{F_s} \ln\Bigg(1 \\
&\qquad + \frac{P_k'(f)}{\sigma_k(f) + \sum_{i=1, i\neq k}^K \alpha_{ik}(f)P_i(f)}\Bigg) df \\
&\qquad\qquad k = 2, \ldots, K. \quad (7)
\end{aligned}$$

It should be pointed out that the foresighted user needs to know the private information $\sigma_k(f), \alpha_{ik}(f), \mathrm{P}_\mathrm{k}^{\max}$ of all its competitors in order to formulate the above optimization. The previous approach in [11] assumes that the foresighted user has the perfect knowledge of this private information. Importantly, it was shown in [11] that users' performance is substantially improved compared with that of IW algorithm if the foresighted user plays the SE strategy, even though the remaining users behave myopically. However, how such a foresighted user should accumulate this required information remains unsolved. In the remaining part of this paper, we will show that the foresighted user can obtain this information and improve its performance by forming conjectures over the behavior of its competitors through repeated interaction with the environment.

Before introducing the conjectural equilibrium, we define the discretized version of the water-filling game. In practice, instead of optimizing over continuous frequency variables, the frequency band is often divided into a total number of N small frequency bins [12]–[14], such that each frequency bin could be viewed as a flat fading channel and $\sigma_k(f), \alpha_{jk}(f)$ can be approximated as a constant within each small frequency bin. Denote $\sigma_k(f) = \sigma_k^n, \alpha_{jk}(f) = \alpha_{jk}^n$, and $P_k(f) = P_k^n$ in $(n-1)/(N)F_s < f < (n)/(N)F_s$ for any $n \in \{1, \ldots, N\}, j, k \in \mathcal{K}$. As a result, (2) and (7) can be reformulated correspondingly.

[1]For the cases where the equilibrium solution $\mathrm{NE}(a_k)$ is not unique for every a_k, the Stackelberg equilibrium needs to be reformulated [18]. Note that in this paper, $\mathrm{NE}(a_k)$ is always unique in the water-filling game.

[2]Note that we investigate the steady-state performance and the initial action order does not matter. The leader and the follower are differentiated based on the users' ability to forecast their competitors' responses.

B. Conjectural Equilibrium

In a game-theoretic setting, which equilibria will be played is determined based on the existing assumptions about the players' knowledge and beliefs. For example, the standard NE solution is a set of strategies where no player has a unilateral incentive to change its strategy. An implicit underlying assumption is that each Nash player takes the other players' actions as given. Therefore, it chooses to myopically maximize its own payoff [17]. Another example is that of a SE strategy, where the foresighted user needs to know the structure of the resulting $\mathrm{NE}(a_k)$ for any $a_k \in \mathcal{A}_k$ and believes that all the remaining players play the NE strategy. Summarizing, the players operating at equilibrium can be viewed as decision makers behaving optimally with respect to their *beliefs* about the policies adopted by the other players.

To rigorously define the CE, we need to include two new elements $\mathcal{S}$ and s and, based on this, reformulate the strategic game $\mathcal{G} = (\mathcal{K}, \mathcal{A}, U, \mathcal{S}, s)$ [20]. $\mathcal{S} = \times_{k\in\mathcal{K}}\mathcal{S}_k$ is the *state space*, where $\mathcal{S}_k$ is the part of the state relevant to the kth user. Specifically, the state in the water-filling game is defined as the interference that users experience. The utility function $U = \times_{k\in\mathcal{K}}U_k$ is a mapping from users' state space and actions to real numbers, $U_k : \mathcal{S}_k \times \mathcal{A}_k \rightarrow \mathcal{R}$. The *state determination function* $s = \times_{k\in\mathcal{K}}s_k$ maps joint actions to states for each component $s_k : \mathcal{A} \rightarrow \mathcal{S}_k$. Each user cannot directly observe the actions chosen by the others, and each user has some belief about the state that would result from performing its available actions. The *belief function* $\tilde{s} = \times_{k\in\mathcal{K}}\tilde{s}_k$ is defined to be $\tilde{s}_k : \mathcal{A}_k \rightarrow \mathcal{S}_k$ such that $\tilde{s}_k(a_k)$ represents the state that the player k believes that would result if it selects action a_k. Notice that the beliefs are not expressed in terms of other player's actions and preferences, and the multiuser coupling in these beliefs is captured directly by individual users forming conjectures of the effects of their own actions. In noncooperative scenarios, each user chooses the action $a_k \in \mathcal{A}_k$ if it believes this action maximizes its utility.

Definition 1 (Conjectural Equilibrium): In the game $\mathcal{G}$ defined above, a configuration of belief functions $(\tilde{s}_1^*, \ldots, \tilde{s}_K^*)$ and a joint action $a^* = (a_1^*, \ldots, a_K^*)$ constitute a conjectural equilibrium, if for each $k \in \mathcal{K}$

$$\begin{aligned} \tilde{s}_k^*(a_k^*) &= s_k(a_1^*, \ldots, a_K^*) \quad \text{and} \\ a_k^* &= \arg\max_{a_k\in\mathcal{A}_k} U_k(\tilde{s}_k^*(a_k), a_k). \end{aligned} \tag{8}$$

From the definition, we can see that, at CE, all users' expectations based on their beliefs are realized and each user behaves optimally according to its expectation. In other words, users' beliefs are consistent with the outcome of the play and they behave optimally with respect to their beliefs. CE considers the users' beliefs rather than their perfect knowledge $\mathrm{NE}(a_k)$ as in SE, which makes CE an appropriate solution concept when the perfect knowledge is not available. The key problem is how to configure the belief functions such that it leads to a CE having a satisfactory performance. Section III discusses this problem in water-filling games.

III. Conjectural Equilibrium in Water-Filling Games

In this section, we discuss how to configure a user's belief about its experienced interference as a linear function of its transmit power, and show that such CE exists and it is a relaxation of both NE and SE. We begin by stating several fundamental assumptions used throughout the investigation hereafter.

Assumption 1: There is only one foresighted user modeling its competitors' reaction as a function of its allocated power, and all the remaining users are myopic users that deploy the IW algorithm. Without loss of generality, we assume that this foresighted user is user 1. The discussion of the extensions to multiple foresighted users is addressed in Section VI.

Assumption 2: Every user is able to perfectly measure its experienced equivalent noise PSD σ_k^n and interference PSD $\sum_{i=1,i\neq k}^{K} \alpha_{ik}^n P_i^n$ in all frequency channels.

Assumption 3: Users $2, \ldots, K$ react to any small variation in their experienced interference by setting their power allocations according to the water-filling strategy.

Assumption 4: In the lower-level problem formed by user $2, \ldots, K$ in (7), there always exists a unique NE. It has been shown that under a wide range of realistic channels, the IW algorithm converges to a unique Nash equilibrium [5]–[7]. Define matrices G^f in which $G_{ii}^f = 0, i = 1, \ldots, K$ and $G_{ij}^f = \alpha_{ij}^f, i \neq j$, for $f = 1, \ldots, N$. In particular, according to a general sufficient condition developed recently that guarantee the uniqueness of NE [6], we consider the channels that satisfy $\|G^f\| < 1$ for $f = 1, \ldots, N$.

Next, we formally define the concept of stationary interference.

Definition 2 (Stationary Interference): The stationary interference that user 1 experiences in the nth channel is the accumulated interference $I_1^n = \sum_{i=2}^{K} \alpha_{i1}^n P_i^n$ when best-response users $2, \ldots, K$ reach their NE in the lower-level problem in (7). Note that I_1^n is in fact a function of user 1's power allocation $\boldsymbol{P}_1 = [P_1^1, \ldots, P_1^N]$ in the water-filling game and it can also be denoted as $I_1^n(\boldsymbol{P}_1)$.

A. Linear Belief of Stationary Interference

As discussed before, both the state space and belief functions need to be defined in order to investigate the existence of CE. In the market models for pure exchange economy [20], the market price is impacted by the other consumers' announced demand. Therefore, it is natural to define the state to be the market price in such scenarios. However, the proposed approach in [20] that models and updates the belief on the market price as a linear function of the excess demand is entirely heuristic. This is not the case in our setting, where forming linear conjectures fits the natural structure of the considered interference game.

In the water-filling game, we define state $\mathcal{S}_k$ to be the stationary interference caused to user k, because besides its own power allocation, its utility only depends on the interference that its competitors cause to it. Notice that the action available to user k is to choose the transmitted power allocations subjected to its maximum power constraint. By the definition of belief function in Section II-B, we need to express the stationary interference as a function of the transmitted power. As we will see later, it is natural to deploy linear belief models due to the linearity of the caused stationary interference in terms of the allocated power, and hence, forming such beliefs can lead to significant performance improvements because they capture the inherent characteristics of the actual interference coupling.

Define $\boldsymbol{P}_{\mathbf{1}}^{n+}, \boldsymbol{P}_{\mathbf{1}}^{n-}$ as $\boldsymbol{P}_{\mathbf{1}}^{n+} = [P_1^1, \ldots, P_1^n + \varepsilon, \ldots, P_1^N]$, $\boldsymbol{P}_{\mathbf{1}}^{n-} = [P_1^1, \ldots, P_1^n - \varepsilon, \ldots, P_1^N]$ for arbitrarily small positive variation in power ε. Given user 1's power allocation $\boldsymbol{P}_{\mathbf{1}}, \mathrm{NE}_n(\boldsymbol{P}_{\mathbf{1}}) = [P_2^n, \ldots, P_K^n]^T$ represents the power that user $2, \ldots, K$ allocate in the n th channel at equilibrium. Vector $\boldsymbol{\alpha}^{\boldsymbol{n}} = \{\alpha_{ij}^n : i \neq j\}$ contains channel gains in the nth frequency bin. Indicator function $y = \mathrm{sign}(x)$ is a mapping of $\mathcal{R}^{K-1} \rightarrow \{0,1\}^{K-1}$, which is defined to be: $y_i = 1$, if $x_i > 0$, and $y_i = 0$, otherwise. Based on these notations, the following proposition motivates us to develop linear belief functions of stationary interference.

Proposition 1 (Linearity of Stationary Interference): If the number of frequency bins N is sufficiently large, the first derivative of the stationary interference that user 1 experiences in the nth channel with respect to its allocated power in the mth channel satisfies

$$\frac{\partial I_1^n}{\partial P_1^n} = c(\boldsymbol{\alpha}^{\boldsymbol{n}}, \mathrm{sign}(\mathrm{NE}_n(\boldsymbol{P}_{\mathbf{1}}))) \quad \text{if there does not exist } k \in \{2, \ldots, K\} \text{ satisfying } P_k^n = 0 \text{ and } \lambda_k^n = 0;$$

$$\left.\frac{\partial I_1^n}{\partial P_1^n}\right|_{P_1^n \rightarrow P_1^{n+}} = c\left(\boldsymbol{\alpha}^{\boldsymbol{n}}, \mathrm{sign}\left(\mathrm{NE}_n\left(\boldsymbol{P}_{\mathbf{1}}^{n+}\right)\right)\right)$$

$$\left.\frac{\partial I_1^n}{\partial P_1^n}\right|_{P_1^n \rightarrow P_1^{n-}} = c\left(\boldsymbol{\alpha}^{\boldsymbol{n}}, \mathrm{sign}\left(\mathrm{NE}_n\left(\boldsymbol{P}_{\mathbf{1}}^{n-}\right)\right)\right), \text{ otherwise;}$$

$$\frac{\partial I_1^n}{\partial P_1^m} = 0, \text{ if } m \neq n$$

in which $\lambda_k^n (k \in \{2, \ldots, K\})$ is the Lagrange multiplier of $P_k^n \geq 0$ at the optimum of lower-level problem in (7). The function $sign(\cdot)$ is the indicator of which polyhedron the piece-wise affine water-filling function [6] lies in. $c(\boldsymbol{\alpha}^n, \boldsymbol{y})$ represents a constant determined by $\boldsymbol{\alpha}^{\boldsymbol{n}}$ and the nonzero elements of $\boldsymbol{y}$.

Proof: By the definition of I_1^n, we have $(\partial I_1^n)/(\partial P_1^m) = \sum_{i=2}^K \alpha_{i1}^n (\partial P_i^n)/(\partial P_1^m)$. We differentiate two different cases:

1) If there does not exist any $k \in \{2, \ldots, K\}$ satisfying $P_k^n = 0$ and $\lambda_k^n = 0$, i.e., there is a nonzero gap between the interference that users $2, \ldots, K$ experiences and their water-levels, it is straightforward to see that $\mathrm{sign}(\mathrm{NE}_n(\boldsymbol{P}_{\mathbf{1}})) = \mathrm{sign}(\mathrm{NE}_n(\boldsymbol{P}_{\mathbf{1}}^{n+})) = \mathrm{sign}(\mathrm{NE}_n(\boldsymbol{P}_{\mathbf{1}}^{n-}))$.

Without loss of generality, we temporarily assume that $P_k^n > 0$ for $k \in \{2, \ldots, K\}$. When users $2, \ldots, K$ reach the equilibrium, we have from the optimality conditions of water-filling solution:

$$(\boldsymbol{I} + \boldsymbol{G}) \cdot \mathrm{NE}_n(\boldsymbol{P}_{\mathbf{1}}) + \boldsymbol{g}^n P_1^n = \boldsymbol{\nu}$$

in which

$$\boldsymbol{G} = \begin{bmatrix} 0 & \alpha_{32}^n & \alpha_{42}^n & \cdots & \alpha_{K2}^n \\ \alpha_{23}^n & 0 & \alpha_{43}^n & \cdots & \alpha_{K3}^n \\ \alpha_{24}^n & \alpha_{34}^n & \ddots & \cdots & \vdots \\ \vdots & \vdots & \vdots & 0 & \alpha_{K,K-1}^n \\ \alpha_{2K}^n & \alpha_{3K}^n & \cdots & \alpha_{K-1,K}^n & 0 \end{bmatrix}$$

$$\boldsymbol{g}^n = \begin{bmatrix} \alpha_{12}^n \\ \alpha_{13}^n \\ \vdots \\ \alpha_{1K}^n \end{bmatrix}, \quad \boldsymbol{\nu} = \begin{bmatrix} \nu_2 \\ \nu_3 \\ \vdots \\ \nu_K \end{bmatrix}$$

$\nu_i (i = 2, \ldots, K)$ are the water-levels of all the water-filling users. We consider the channel realizations satisfying $\|\boldsymbol{G}\|_2 < 1$, which guarantees that the water-filling game has a unique NE [6, Theorem 8]. Therefore, $\boldsymbol{I} + \boldsymbol{G}$ is invertible. It then follows that

$$\mathrm{NE}_n(\boldsymbol{P}_{\mathbf{1}}) = (\boldsymbol{I} + \boldsymbol{G})^{-1}\boldsymbol{\nu} - (\boldsymbol{I} + \boldsymbol{G})^{-1}\boldsymbol{g}^n P_1^n. \tag{9}$$

We also have $\lim_{N \rightarrow \infty} (\partial \nu_i)/(\partial P_1^n) = 0$, because if the width of each frequency bin F_s/N is sufficiently small, the fluctuation of the water-level is negligible. In other words, if N is sufficiently large, we have $(\partial \nu_i)/(\partial P_1^n) \approx 0$. As a result, we have

$$\frac{\partial I_1^n}{\partial P_1^m} = \frac{\partial \boldsymbol{h}^n \cdot \mathrm{NE}_n(\boldsymbol{P}_{\mathbf{1}})}{\partial P_1^m} = \begin{cases} -\boldsymbol{h}^n (\boldsymbol{I} + \boldsymbol{G})^{-1} \boldsymbol{g}^n, & \text{if } m = n \\ 0, & \text{if } m \neq n \end{cases} \tag{10}$$

in which $\boldsymbol{h}^n = [\alpha_{21}^n \alpha_{31}^n \cdots \alpha_{K1}^n]$. Note that if $P_k^n = 0$ and $\lambda_k^n > 0$, all the derivations above still apply by removing the kth column and kth row from $\boldsymbol{G}, \mathrm{NE}_n(\boldsymbol{P}_{\mathbf{1}}), \boldsymbol{g}^n, \boldsymbol{\nu}$ correspondingly. Hence, we can conclude $(\partial I_1^n)/(\partial P_1^n)$ is a constant $c(\boldsymbol{\alpha}^{\boldsymbol{n}}, \mathrm{sign}(\mathrm{NE}_n(\boldsymbol{P}_{\mathbf{1}})))$ that depends on both $\boldsymbol{\alpha}^{\boldsymbol{n}}$ and the nonzero elements of $\mathrm{NE}_n(\boldsymbol{P}_{\mathbf{1}})$.

2) If there exists $k \in \{2, \ldots, K\}$ satisfying $P_k^n = 0$ and $\lambda_k^n = 0$, the stationary interference caused to user k is the same as its water-level ν_k. Therefore, a sufficiently small increment or decrement ε in user 1's allocated power P_1^n may cause $\mathrm{sign}(\mathrm{NE}_n(\boldsymbol{P}_{\mathbf{1}}^{n+}))$ and $\mathrm{sign}(\mathrm{NE}_n(\boldsymbol{P}_{\mathbf{1}}^{n-}))$ to be different, i.e., the stationary interference $\mathrm{NE}_n(\boldsymbol{P}_{\mathbf{1}})$ lies on the boundary between two polyhedra that have different piece-wise affine water-filling functions [6]. We need to treat the left-sided and the right-sided first derivatives respectively, and similar conclusions can be derived in the same way as in the first part. ■

Proposition 1 indicates that, the first derivative with respect to a foresighted user's allocated power in a certain channel is sufficient to capture how the stationary interference varies locally in that channel. We observe from (9) that $I_1^n = \boldsymbol{h}^n \cdot \mathrm{NE}_n(\boldsymbol{P}_{\mathbf{1}}) = \boldsymbol{h}^n(\boldsymbol{I} + \boldsymbol{G})^{-1}\boldsymbol{\nu} - \boldsymbol{h}^n(\boldsymbol{I} + \boldsymbol{G})^{-1}\boldsymbol{g}^n P_1^n$. Therefore, user 1 can define its belief function using the linear form [3] $\tilde{I}_1^n = \beta^n - \gamma^n P_1^n$, in which γ^n is the estimate of $-(\partial I_1^n)/(\partial P_1^n)$ and β^n is a constant representing the composite effect of user $2, \ldots, K$'s water-levels $\boldsymbol{\nu}$. This linear characterization of the stationary interference can greatly simplify the implicit functional expression $I_1^n(\boldsymbol{P}_{\mathbf{1}})$ given by the solution of the lower-level problem (7), while maintaining an accurate model of $I_1^n(\boldsymbol{P}_{\mathbf{1}})$ around the feasible operating point $\boldsymbol{P}_{\mathbf{1}}$.

B. Existence of Conjectural Equilibrium

Under the same known sufficient conditions discussed in [6], [7], [11] for guaranteeing the existence of NE and SE, the existence of CE can be proven by showing that the first two types of

[3] Note that as long as the channel realization is random, for a fixed N, the probability that the left-sided and right-sided derivatives in proposition 1 are not equal is zero. We will assume that the first derivative exists hereafter. If it does not exist, similar results can be derived by treating the left-sided and right-sided first derivatives separately.

TABLE I
COMPARISON AMONG NE, SE, AND CE IN WATER-FILLING GAMES

	User 1	User $2, \cdots, K$
Nash Equilibrium	$\{P_k^n\} = \arg\max_{\{P_k'^n\}\in\mathcal{A}_k} \sum_{n=1}^{N} \log_2\left(1+\frac{P_k'^n}{\sigma_k^n + I_k^n}\right)$	
Stackelberg Equilibrium	$\{P_1^n\} = \arg\max_{\{P_1'^n\}\in\mathcal{A}_1} \sum_{n=1}^{N} \log_2\left(1+\frac{P_1'^n}{\sigma_1^n + I_1^n(\boldsymbol{P}_1')}\right)$	$\{P_k^n\} = \arg\max_{\{P_k'^n\}\in\mathcal{A}_k} \sum_{n=1}^{N} \log_2\left(1+\frac{P_k'^n}{\sigma_k^n + I_k^n}\right)$
Conjectural Equilibrium	$\{P_k^n\} = \arg\max_{\{P_k'^n\}\in\mathcal{A}_k} \sum_{n=1}^{N} \log_2\left(1+\frac{P_k'^n}{\sigma_k^n + \tilde{I}_k^n}\right)$	
	$\tilde{I}_1^n = \beta^n - \gamma^n P_1^n, I_1^n = \sum_{i=2}^{K} \alpha_{i1}^n P_i^n, \tilde{I}_1^n = I_1^n$	$\tilde{I}_k^n = I_k^n = \sum_{i=1, i\neq k}^{K} \alpha_{ik}^n P_i^n$

equilibrium are special cases of CE. To this end, Table I compares the optimality conditions of the three types of equilibria in the water-filling game.

As shown in Table I, the information requirement for playing various equilibria differs. At NE, each user includes its stationary interference I_k^n as a constant in the optimization, and its action is the best response to I_k^n. To play SE, the foresighted user needs to know the functional expression of the stationary interference $I_1^n(\boldsymbol{P}_1')$ such that the bi-level program can be formed. Specifically, the required information includes both the system-wide channel state information $\boldsymbol{\alpha}^n$, the noise PSD σ_k^n, and the individual power constraint $\mathrm{P}_\mathrm{k}^{\max}$ for $\forall n \in \{1, \ldots, N\}, k \in \mathcal{K}$. In contrast, in the case of CE, the above information for playing SE is no longer required and the foresighted user behaves optimally with respect to its beliefs $\tilde{I}_1^n$ on how the stationary interference changes as a function of P_1^n.

Proposition 2 (NE and SE as CE): Both the Nash equilibrium and the Stackelberg equilibrium are special cases of conjectural equilibrium.

Proof: To solve the CE, the optimization solving CE in Table I is essentially

$$\max_{\{P_1^n\}} \sum_{n=1}^{N} \log_2\left(1+\frac{P_1^n}{\sigma_1^n + \beta^n - \gamma^n P_1^n}\right)$$
$$\text{s.t.} \quad P_1^n \geq 0, \beta^n - \gamma^n P_1^n \geq 0 \quad \text{and} \quad \sum_{n=1}^{N} P_1^n \leq \mathrm{P}_1^{\max}. \tag{11}$$

In order to show that both NE and SE are special cases of CE, we only need to verify that at NE and SE, user 1's action is optimal with respect to its belief and its belief agrees with its state. First, clearly, NE is a trivial CE with the parameters $\beta^n = \sum_{i=2}^{K} \alpha_{i1}^n P_i^n, \gamma^n = 0$ in user 1's belief functions. Next, denote $P_\mathrm{SE} = [P_\mathrm{SE}^1, \ldots, P_\mathrm{SE}^N]$ the optimal solution of the discretized version of problem (7). To prove SE is a CE, we need to find the corresponding β^n and γ^n and show that SE also solves problem (11). Consider the belief function in Table I with the parameters $\beta^n = (I_1^n - P_1^n \cdot (\partial I_1^n/\partial P_1^n))|_{\boldsymbol{P}_1=\boldsymbol{P}_\mathrm{SE}}$ and $\gamma^n = -(\partial I_1^n/\partial P_1^n)|_{\boldsymbol{P}_1=\boldsymbol{P}_\mathrm{SE}}$. As discussed before, such parameters preserve all the local information of the objective of (7) around $\boldsymbol{P}_\mathrm{SE}$ into problem (11). KKT conditions hold at $\boldsymbol{P}_\mathrm{SE}$ since it solves problem (7). A sufficient condition that ensures SE to be a CE is that (11) belongs to convex optimization, because KKT conditions are necessary and sufficient for convex programming to attain its optimum. Appendix A provides a sufficient condition SC 1 under which (11) is convex, thereby proving that SE is a special CE if these conditions are satisfied. ■

Proposition 2 indicates that the two isolating points, NE and SE, are both CE, if parameters $\boldsymbol{\beta} = \{\beta^n\}, \boldsymbol{\gamma} = \{\gamma^n\}$ are properly chosen. Therefore, CE can be viewed as an operational approach to attain the SE if the system-wide information required for solving SE is not available. It is because only the local information including stationary interference I_1^n and its first derivative $(\partial I_1^n/\partial P_1^n)$ is required to formulate (11), and this information can be obtained using measurements performed at the receiver.

In addition, we are interested in the existence of other CEs besides these two points. Denote the parameters of any CE, e.g., NE or SE, as $\boldsymbol{\beta}_* = \{\beta_*^n\}, \boldsymbol{\gamma}_* = \{\gamma_*^n\}$, and the optimal solution of (11) given parameters $\boldsymbol{\beta}, \boldsymbol{\gamma}$ as $\boldsymbol{P}_1(\boldsymbol{\beta}, \boldsymbol{\gamma})$. Let $F : \mathcal{R}^N \times \mathcal{R}^N \to \mathcal{R}^N$ be a mapping defined as $F(\boldsymbol{\beta}, \boldsymbol{\gamma}) = \{F^n(\boldsymbol{\beta}, \boldsymbol{\gamma})\}$ in which

$$F^n(\boldsymbol{\beta}, \boldsymbol{\gamma}) = \boldsymbol{h}^n \cdot \mathrm{NE}_n(\boldsymbol{P}_1(\boldsymbol{\beta}, \boldsymbol{\gamma})) - \beta^n - \gamma^n P_1^n(\boldsymbol{\beta}, \boldsymbol{\gamma}). \tag{12}$$

The following proposition gives a sufficient condition which ensures that infinite CEs exist.

Proposition 3 (Infinite Set of CE): Let $\mathcal{G}$ be a water-filling game that satisfies condition SC 1. Suppose that all the users form conjectures according to Table I. If there exist open neighborhoods $A \subset \mathcal{R}^N$ and $B \subset \mathcal{R}^N$ of $\boldsymbol{\beta}_*$ and $\boldsymbol{\gamma}_*$, respectively, such that $F(\cdot, \gamma) : A \to \mathcal{R}^N$ is locally one-to-one for any $\boldsymbol{\gamma} \in B$, then $\mathcal{G}$ admits an infinite set of conjectural equilibria.

Proof: See Appendix B.

In summary, Proposition 1, 2, and 3 characterize the existence and structure of conjectural equilibrium in water-filling games. As shown in Fig. 2, NE and SE can be both special cases of CE. Open sets of CE that contain NE and SE may exist in the $\boldsymbol{\beta}$-$\boldsymbol{\gamma}$ plane and different conjectural equilibria correspond to different values of $\boldsymbol{\beta}$ and $\boldsymbol{\gamma}$. SE attains the maximal data rate that

TABLE II
DYNAMIC UPDATES OF THE PLAY

	User 1	User $2,\cdots,K$
State $I_{k,t}$	$I_{k,t}^n = \sum_{i=1,i\neq k}^K \alpha_{i1}^n P_{i,t}^n$	
Belief function $\tilde{s}_k : \mathcal{A}_k \to \mathcal{S}_k$	$\beta_t^n, \gamma_t^n \leftarrow \mathbf{Update_1}\left(I_{1,t}^n, P_{1,t}^n\right)$ $\tilde{I}_{1,t}^n = \beta_t^n - \gamma_t^n P_{1,t}^n$	$\tilde{I}_{k,t}^n = I_{k,t}^n = \sum_{i=1,i\neq k}^K \alpha_{ik}^n P_{i,t}^n$
Action $a_{1,t},\cdots,a_{K,t}$	$\boldsymbol{P}_{1,t+1} \leftarrow \mathbf{Update_2}\left(\boldsymbol{P}_{1,t}, \tilde{\boldsymbol{I}}_{1,t}\right)$	$\boldsymbol{P}_{k,t} = \arg\max_{P_k' \in \mathcal{A}_k} \sum_{n=1}^N \log_2\left(1 + \frac{P_k'^n}{\sigma_k^n + \tilde{I}_{k,t}^n}\right)$

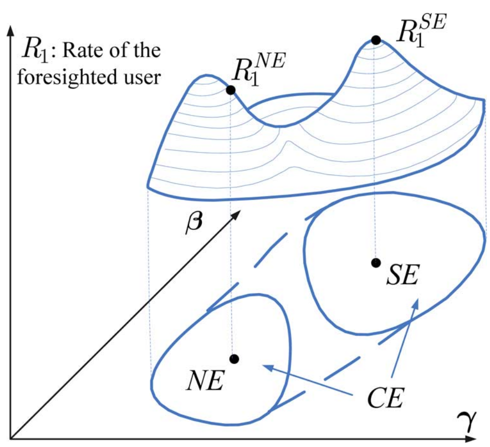

Fig. 2. Structure of conjectural equilibria in water-filling games.

a foresighted user can achieve. According to Proposition 3, if the foresighted user properly sets up its parameters $\boldsymbol{\beta}, \boldsymbol{\gamma}$, the solution of CE in (11) coincides with the solution of SE in (7). More importantly, as opposed to the SE in which the knowledge of the system-wide private information is required, CE assumes that the foresighted user knows only its stationary interference and the first derivatives with respect to the allocated power, which greatly simplifies the information acquisition. Therefore, in order to approach the performance upper bound given by SE, this paper adopts the approach of CE. Section IV will develop practical conjecture forming and updating algorithms to select out of the infinite CEs a desirable power allocation scheme that provides comparable achievable rates with SE.

IV. CONJECTURE-BASED RATE MAXIMIZATION

Since Proposition 3 shows that infinite CEs may exist and SE is the most desirable CE for a foresighted user, the parameters β^n, γ^n of belief functions should be wisely chosen in order to attain SE as a CE. Moreover, the one-shot game formulation and declarative conclusions in the previous sections provide no hint on how to approach the CE. In practice, it is also important to construct algorithmic mechanisms to attain the desirable CE. To arrive at a CE, a multiagent learning approach is proposed for the repeated game setting [20]. Let $\tilde{s}_{k,t}$ and $a_{k,t}$ denote user k's belief and action at time t. In the framework, at time t, the users update their beliefs $\tilde{s}_{k,t}$ and select their actions $a_{k,t}$ based on their past observations. If we define learning as the players' dynamic process of forming conjectures about the effects of their actions, CE captures the achieved outcome when consistency of conjectures within and across players emerges.

Similarly, this section proposes that users can update their beliefs in the repeated interaction setting and numerically examines their performance. Before going into the technical details, it should be pointed out that the pursuit of the practical solution's convergence to CE is not the principal goal of our investigation. Instead, computing power allocation strategies that require only local information and achieve comparable rates with SE (which requires global information) is the ultimate objective rather than the convergence. In other words, any power allocation strategy that lies outside the open CE set in Fig. 2 is favorable if it can improve the performance compared with NE.

Table II summarizes the dynamic updates of all users' states, belief functions, and optimal actions in the water-filling game. Specifically, at iteration t, users' states $I_{k,t}$ are determined by their opponents' power allocation. User 1 updates the parameters β_t^n, γ_t^n in its belief functions based on its state $I_{1,t}^n$ and allocated power $P_{1,t}^n$, and it also updates its power allocation $\boldsymbol{P}_{1,t+1}$ based on current operating points $\boldsymbol{P}_{1,t}$ and its belief $\tilde{\boldsymbol{I}}_{1,t}$. At the same time, myopic users $2,\ldots,K$ set their belief equal to their experienced interference and update their power allocation based on the water-filling strategy. Note that Table II implicitly assumes that user 1 will update after user $2,\ldots,K$'s IW algorithms converge such that user $2,\ldots,K$'s power allocations $\boldsymbol{P}_{k,t}$ at time t can be regarded as an equilibrium state. The outcome of this dynamic play is a CE if $\lim_{t\to\infty} \boldsymbol{P}_{k,t}$ exists and $\lim_{t\to\infty} \boldsymbol{I}_{k,t} = \lim_{t\to\infty} \tilde{\boldsymbol{I}}_{k,t}$. As discussed in the Proof of Proposition 2, it is equivalent to check the convergence of user 1's updates. We can see from Table II that user 1 needs to complete two updates at each iteration. The entire procedure in Table II that enables the foresighted user to build beliefs and improve its performance is named "*Conjecture-based Rate Maximization*." Appropriate rules for updating beliefs are discussed as follows.

$\mathbf{Update_1}$: β_t^n, γ_t^n: Note that, we have $I_1^n = \boldsymbol{h}^n(\boldsymbol{I}+\boldsymbol{G})^{-1}\boldsymbol{\nu} - \boldsymbol{h}^n(\boldsymbol{I}+\boldsymbol{G})^{-1}\boldsymbol{g}^n P_1^n$ from Proposition 1, user 1's belief function takes the form of $\tilde{I}_1^n = \beta^n - \gamma^n P_1^n$, and it satisfies $I_1^n = \tilde{I}_1^n$ at CE for any $n \in \{1,\ldots,N\}$. As discussed in the previous section, by setting the parameters $\beta^n = I_1^n - P_1^n \cdot (\partial I_1^n / \partial P_1^n)$ and $\gamma^n = -(\partial I_1^n/\partial P_1^n)$, we can preserve all the local information of the original SE problem (7) around current feasible operating point $\boldsymbol{P}_{1,t}$. Therefore, we

TABLE III
A DUAL ALGORITHM THAT SOLVES (11)

Algorithm 1 : A dual method that solves problem (11) using bisection update

input: $\{\sigma_1^n\}, \{\beta_t^n\}, \{\gamma_t^n\}, \mathbf{P_1^{max}}$

initialization : $\boldsymbol{\eta}_{\min}, \boldsymbol{\eta}_{\max}, \boldsymbol{\eta}_0 = (\boldsymbol{\eta}_{\min} + \boldsymbol{\eta}_{\max})/2, i = 0$

repeat

set $\boldsymbol{P_1} = \left[P_1^1 \cdots P_1^N\right]$ where $P_1^n = \arg \max_{P_1'^n \in \boldsymbol{dom}\, f_{\sigma_1^n, \beta^n, \gamma^n}} \log_2\left(1 + \frac{P_1'^n}{\sigma_1^n + \beta_t^n - \gamma_t^n P_1'^n}\right) - \boldsymbol{\eta}_i P_1'^n$.

if $\sum_n P_1^n < \mathbf{P_1^{max}}$, $\boldsymbol{\eta}_{\max} = \boldsymbol{\eta}_i$; else $\boldsymbol{\eta}_{\min} = \boldsymbol{\eta}_i$.

$\boldsymbol{\eta}_{i+1} \leftarrow (\boldsymbol{\eta}_{\min} + \boldsymbol{\eta}_{\max})/2$, $i = i + 1$.

until $\boldsymbol{\eta}_i$ converges

can update β_t^n and γ_t^n using $\beta_t^n = (I_1^n - P_1^n \cdot (\partial I_1^n / \partial P_1^n))|_{\boldsymbol{P_1} = \boldsymbol{P_{1,t}}}$ and $\gamma_t^n = -(\partial I_1^n / \partial P_1^n)|_{\boldsymbol{P_1} = \boldsymbol{P_{1,t}}}$. By Assumption 3, user 1 can approximate the parameters using

$$\frac{\partial I_1^n}{\partial P_1^n} \approx \frac{I_1^n\left(\{P_1^n + \varepsilon\} \cup \boldsymbol{P_1^{-n}}\right) - I_1^n\left(\{P_1^n - \varepsilon\} \cup \boldsymbol{P_1^{-n}}\right)}{2\varepsilon}$$

for small ε in which $\boldsymbol{P_1^{-n}} = \{P_1^1, \ldots, P_1^{n-1}, P_1^{n+1}, \ldots, P_1^N\}$.

After $\mathbf{Update_1}$ in each iteration, user 1 needs to solve (11). If Proposition 2's assumption is not satisfied, (11) belongs to the class of nonconvex optimization, which is generally hard to solve and standard optimization algorithms can only be used to determine local maxima [21]. However, in this application, we are able to show that, as long as the number of frequency bins N is sufficiently large, (11) satisfies the time-sharing condition [13], and its global optimum can be efficiently computed.

Definition 3 (Time-Sharing Condition [13]): Consider an optimization problem with the general form

$$\max \sum_{n=1}^{N} o_n(\boldsymbol{x}_n)$$
$$\text{s.t.} \sum_{n=1}^{N} \boldsymbol{c}_n(\boldsymbol{x}_n) \leq \boldsymbol{P} \tag{13}$$

where $o_n(\boldsymbol{x}_n)$ are objective functions that are not necessarily concave, $\boldsymbol{c}_n(\boldsymbol{x}_n)$ are constraint functions that are not necessarily convex. Power constraints are denoted by $\mathbf{P}$. Let $\boldsymbol{x}_n^*$ and $\boldsymbol{y}_n^*$ be optimal solutions to the optimization problem (13) with $\mathbf{P} = \mathbf{P_x}$ and $\mathbf{P} = \mathbf{P_y}$, respectively. An optimization problem of the form (13) is said to satisfy the time-sharing condition if for any $0 \leq v \leq 1$, there always exists a feasible solution $\mathbf{z}_n$, such that $\sum_{n=1}^N \boldsymbol{c}_n(\mathbf{z}_n) \leq v\mathbf{P_x} + (1-v)\mathbf{P_y}$, and $\sum_{n=1}^N o_n(\mathbf{z}_n) \leq v\sum_{n=1}^N o_n(\boldsymbol{x}_n^*) + (1-v)\sum_{n=1}^N o_n(\boldsymbol{y}_n^*)$.

Proposition 4 (Satisfaction of Time-Sharing Condition): As the total number of subcarriers N goes to infinity, (11) satisfies the time-sharing condition.

Proof: Specifically, for problem (11), $o_n(\mathrm{x}_n) = f_{\sigma_1^n, \beta^n, \gamma^n}(P_1^n), \mathbf{c}_n(\mathrm{x}_n) = P_1^n, \mathbf{P} = \mathbf{P_1^{max}}$. First, consider $\sigma_k(f)$ and $\alpha_{jk}(f)$ that are continuous functions of f.

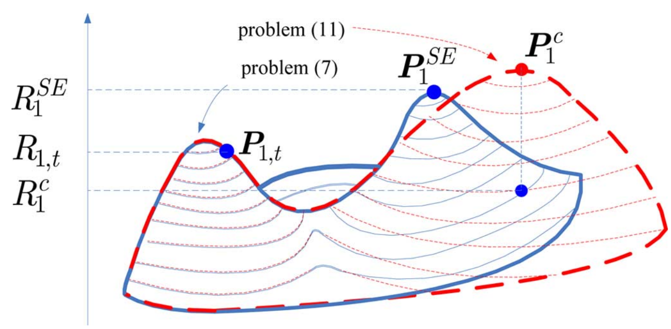

Fig. 3. Mismatch between (7) and (11).

By rule $\mathbf{Update_1}, \boldsymbol{\beta_t}(f)$ and $\boldsymbol{\gamma_t}(f)$ are piece-wise continuous, because Proposition 3 proves that $\boldsymbol{P_1}(\boldsymbol{\beta}, \boldsymbol{\gamma})$ and $F(\boldsymbol{\beta}, \boldsymbol{\gamma})$ are continuous in $(\boldsymbol{\beta}, \boldsymbol{\gamma})$. With the piece-wise continuity of $\boldsymbol{\beta}(f), \boldsymbol{\gamma}(f)$, it is easy to check that the time-sharing condition holds by following [13, Proof of Theorem 2]. ■

$\mathbf{Update_2}$*:* $\boldsymbol{P}_{1,t+1}$*:* It is shown in [13] that, if the optimization problem satisfies the time-sharing property, then it has a zero duality gap, which leads to efficient numerical algorithms that solve the nonconvex problem in the dual domain. Consider the dual objective function $d(\boldsymbol{\eta}) = \sum_{n=1}^N \{\max_{P_1^n} f_{\sigma_1^n, \beta^n, \gamma^n}(P_1^n) - \boldsymbol{\eta} P_1^n\} + \boldsymbol{\eta}\mathbf{P}_1^{\max}$. Since $d(\boldsymbol{\eta})$ is convex, a bisection or gradient-type search over the Lagrangian dual variable $\boldsymbol{\eta}$ is guaranteed to converge to the global optimum. Specifically, Algorithm 1 summarizes such a dual method that solves nonconvex problem (11) using bisection update. As long as the time-sharing condition is satisfied, Algorithm 1 converges to the global optimum. Hence, we can always solve problem (11) regardless of its convexity.

Table IV summarizes the procedure of algorithm "Conjecture-based Rate Maximization" (CRM). Next, we make several remarks about this algorithm. First, since we want to achieve better performance than NE, the initial operating point $\mathbf{P}_{1,0}$ is set to be the power allocation strategy P_1^{NE} that user 1 will choose if it adopts the IW algorithm. Second, in $\mathbf{Update_2}$, the global optimum $\boldsymbol{P_1^c}$ is not directly used to update $\boldsymbol{P}_{1,t+1}$. As shown in Fig. 3, this is because (11) is only a local approximation at $\boldsymbol{P}_{1,t}$ of the original SE problem (7) that we want to solve.

TABLE IV
CONJECTURE-BASED RATE MAXIMIZATION

Conjecture-based Rate Maximization

initialization : $t = 0, \boldsymbol{P}_{1,0} = \boldsymbol{P}_1^{NE}$

repeat

I. $\beta_t^n, \gamma_t^n \leftarrow \mathbf{Update_1}\left(I_{1,t}^n, P_{1,t}^n\right)$.

II. $\boldsymbol{P}_{1,t+1} \leftarrow \mathbf{Update_2}\left(\boldsymbol{P}_{1,t}, \tilde{\boldsymbol{I}}_{1,t}\right)$, which includes:

1) Consider problem $\max \sum_{n=1}^{N} f_{\sigma_1^n,\beta^n,\gamma^n}\left(P_1^n\right)$, *s.t.* $P_1^n \in \boldsymbol{dom}\ f_{\sigma_1^n,\beta^n,\gamma^n}$ *and* $\sum_{n=1}^{N} P_1^n \leq \mathbf{P_1^{max}}$.

2) Use Algorithm 1 to calculate the global optimum $\boldsymbol{P}_{\mathbf{1}}^c$ of the above problem.

3) Search in the interval of $v\boldsymbol{P}_{1,t} + (1-v)\boldsymbol{P}_{\mathbf{1}}^c\ (0 \leq v \leq 1)$ and find in the interval the power allocation $\boldsymbol{P}_{\mathbf{1}}^s$ that maximizes user 1's actual achievable rate R_1.

4) $\boldsymbol{P}_{\mathbf{1},t+1} \leftarrow \boldsymbol{P}_{\mathbf{1}}^s, t = t + 1$.

until no improvement can be made.

Using $\boldsymbol{P}_{\mathbf{1}}^c$ to update $\boldsymbol{P}_{1,t+1}$ may decrease the actual achievable rate R_1, if a mismatch between (7) and (11) exists for the solution $\boldsymbol{P}_{\mathbf{1}}^c$. Therefore, $\mathbf{Update_2}$ adopts line search and uses the transmit PSD that lies in this interval and maximizes the actual achievable rate to update $\boldsymbol{P}_{1,t+1}$. Therefore, it is guaranteed that the achievable rate will not decrease after each iteration. Third, as opposed to the two-user algorithm proposed in [11], CRM is designed for the general multiuser scenario regardless of the number of users. Last, CRM stops in limited iterations, but it is not guaranteed to converge to a CE. It is because the first step in $\mathbf{Update_2}$ may give $\boldsymbol{P}_{\mathbf{1},t} \neq \boldsymbol{P}_{\mathbf{1}}^c$ but the line search returns $\boldsymbol{P}_{\mathbf{1},t+1} = \boldsymbol{P}_{\mathbf{1},t}$. However, if $\boldsymbol{P}_{\mathbf{1},t} = \boldsymbol{P}_{\mathbf{1}}^c$, CRM converges to $\boldsymbol{P}_{\mathbf{1}}^c$ and the resulting outcome is a CE.

V. SIMULATION RESULTS

This section compares the performance of CRM with the IW algorithm and the two-user suboptimal algorithm (TSA) that searches SE assuming perfect knowledge of its opponent's private information [11]. We simulate a system with 50 subcarriers over the 15-MHz band. We consider frequency-selective channels using a four-ray Rayleigh model with the exponential power profile and 60 ns root mean square delay spread. The power of each ray is decreasing exponentially according to its delay.

We first simulate the two-user scenario with $P_1 = P_2 = 200$ and $\sigma_1(f) = \sigma_2(f) = 0.01$. The total power of all rays of $H_{11}(f)$ and $H_{22}(f)$ is normalized as one, and that of $H_{12}(f)$ and $H_{21}(f)$ is normalized as 0.5. Fig. 4 shows an example of user 1's power allocations when deploying different algorithms under the same conditions. In IW algorithm, user 1 water-fills the whole frequency band by regarding its competitor's interference as background noise. In contrast, user 1 will not water-fill if choosing CRM and TSA. It avoids the myopic behavior and improves its performance by explicitly considering the stationary interference caused by its opponent.

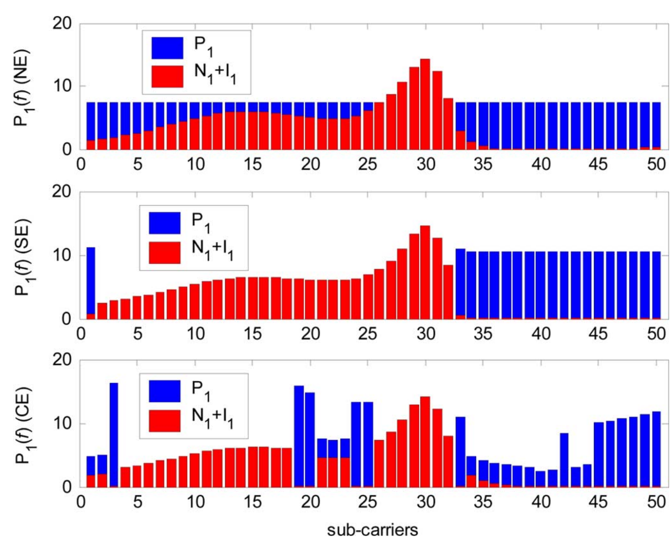

Fig. 4. User 1's power allocation using different algorithms.

To evaluate the performance, we tested 10^5 sets of frequency-selective fading channels that satisfy Assumption 4. Denote user i's achievable rate using CRM, IW, and TSA as R_i, R_i^{NE}, and $R_i'^{\mathrm{SE}}$ respectively. Fig. 5 shows the simulated cumulative probability of the ratio of R_i over R_i^{NE} and $R_i'^{\mathrm{SE}}$. The curve indicates that there is a probability of 59% that CRM returns the same power allocation strategy as IW. On the other hand, the average improvement for user 1 of CRM over IW is 24%, which achieves almost the same performance as TSA. As shown in Fig. 5, $R_1/R_1'^{\mathrm{SE}}$ is distributed symmetrically with respect to

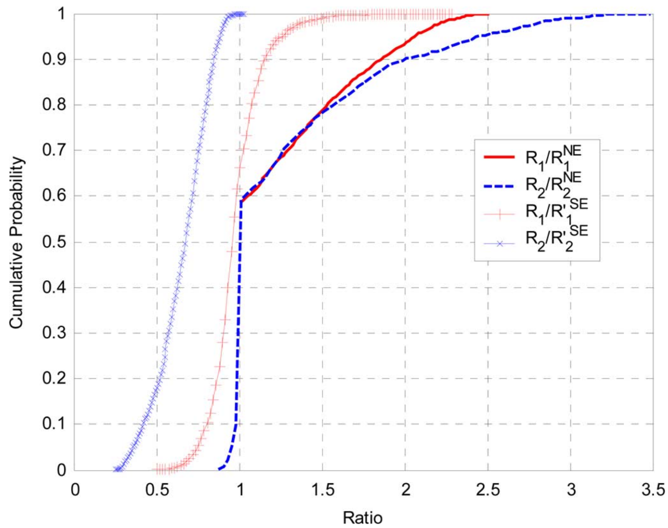

Fig. 5. Cdfs of R_i/R_i^{NE} and $R_i/R_i'^{\mathrm{SE}}(i=1,2)$.

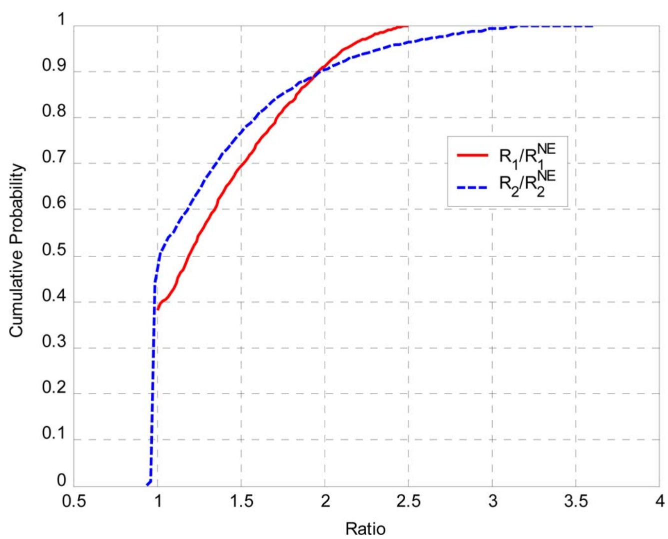

Fig. 6. Cdfs of $R_i/R_i^{\mathrm{NE}}(i=1,2)$ for modified CRM.

$R_1 = R_1'^{\mathrm{SE}}$. CRM improves on average user 2's data rate by 29% over IW, which is smaller than TSA. Similarly as in [11], in very few cases, CRM results in a rate R_2' smaller than R_2^{NE} in the IW algorithm.

The iteration time required by CRM is summarized in Table V. As mentioned above, CRM stops after just one iteration with a probability of 59% due to the problem mismatch shown in Fig. 3. In most scenarios, CRM terminates within 4 iterations and the average number of required iteration is only 1.75. To further improve the performance of CRM, we can modify the original CRM to handle the problem mismatch between (7) and (11). Notice that (11) is only a local approximation of (7) at $\boldsymbol{P}_{1,t}$. Additional constraints can be added in Algorithm 1, such that the optimum of (11) is searched only in a certain region around $\boldsymbol{P}_{1,t}$ rather than the whole domain of $f_{\sigma_1^n,\beta_t^n,\gamma_t^n}$. For example, $|P_1'^n - P_{1,t}^n|$ can be restricted within a certain threshold when performing Algorithm 1 for any $n \in \{1,\ldots,N\}$. We simulated the two-user scenarios with additional restriction of $|P_1'^n - P_{1,t}^n| \le 1$. Fig. 6 shows the simulated cumulative probability of R_i/R_i^{NE} for this modified

TABLE V
ITERATIONS REQUIRED BY DIFFERENT CRM ALGORITHMS

	Probability of required iterations				
	$t=1$	$t=2$	$t=3$	$t=4$	$t\ge 5$
CRM	0.59	0.29	0.06	0.04	0.02
Modified CRM	0.39	0.19	0.20	0.12	0.10

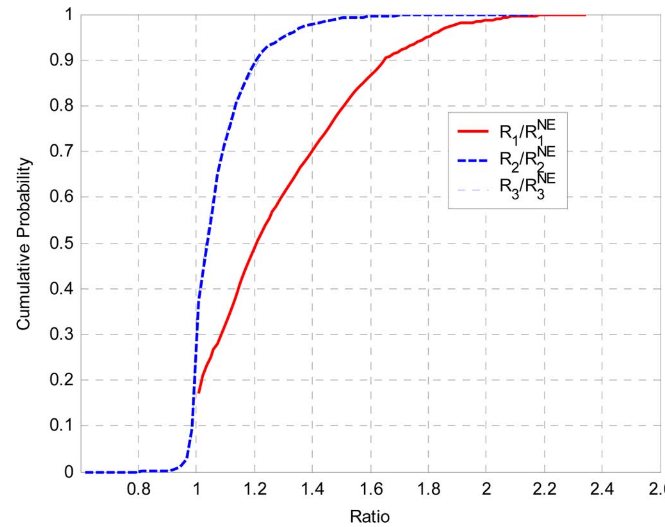

Fig. 7. Cdfs of $R_i/R_i^{\mathrm{NE}}(i=1,2,3)$ for modified CRM.

CRM. As opposed to CRM, the probability that the modified CRM returns the same power allocation strategy as IW is reduced to 39% and the average performance improvement is also increased for both users. Specifically, the average performance improvement for user 1 is 29% and that of user 2 is 31%. However, Table V shows that the improvement is achieved at the cost of more iterations.

We also tested performance of modified CRM in multiuser cases where TSA cannot be applied. We simulated the three-user scenarios with $\mathbf{P_k} = 200$ and $\sigma_k(f) = 0.01$. The total power of all rays of $H_{kk}(f)$ is normalized as one, and that of $H_{ij}(f)(i \ne j)$ is normalized as 0.33. Fig. 7 shows the simulated cumulative probability of R_i/R_i^{NE}. The average improvement for user 1 of modified CRM over IW is 29%, and that of the rest users is 8%. We can see that, it benefits on average most of the participants in the water-filling game if a foresighted user forms accurate conjectures and plays the conjecture equilibrium strategy.

VI. RELATED WORK AND DISCUSSION

Several recent works apply repeated or stochastic games, learning solutions, and Stackelberg formulations for power control in wireless networks [9], [24]–[31]. This section discusses the differences as well as the connections between our approach and this existing literature.

First, a typical repeated game approach for improving the inefficiency of the Nash equilibrium played in the one-shot power control game is to deploy a punishment mechanism [9], [30], [31]. The reason why the repeated game can gain better performance than the one-shot game is guaranteed by the Folk theorem [17]. In the proof of the Folk theorem, users usually deploy "trigger" strategies. A trigger strategy is a strategy which punishes an opponent for any deviation from a prescribed or agreed-upon behavior. For the Folk theorem to work, all the players must first agree to play a specific feasible outcome. Then, the players will deploy the trigger strategy under which any deviation from the prescribed/agreed-upon behavior is punished such that any gains made by the deviating player by differing from the prescribed behavior are at least cancelled out. However, as we see from the proof of the Folk theorem, most of the existing repeated game approaches require certain a priori negotiations before the game actually starts. For example, if they want to operate at the point that maximizes the network utility, they need to jointly determine their optimal actions before the game is played. The repeated nature of the game only provides the players with the opportunity to punish any misbehavior, such that the *a priori* selected equilibrium can be enforced. Our approach uses the repeated interaction among players in a different manner. Rather, we view it as an opportunity for the foresighted user to gain the knowledge required information to play the Stackelberg strategy. The key problem which we wanted to investigate is: if all the other players are not willing to cooperate (e.g., because they cannot explicitly exchange information) and they myopically play the best-response strategy, is it still possible to improve the performance as compared to the Nash equilibrium and how can this improvement be attained? This question is different than the traditional approach and it is important in heterogeneous communication environments in which devices built according to different standards can gather different information and exhibit various levels of "smartness" in determining their transmission strategies.

Second, stochastic game formulations and learning solutions have also been applied in wireless networks [24]–[29]. Most of these works either use well-known learning techniques (e.g., Q-learning, no-regret learning) in order to improve their achievable performance when playing the stochastic games, or apply different learning solutions to achieve various equilibria (Nash equilibrium, correlated equilibrium, etc.) to stabilize the communication network. The major difference of our work from the existing works is that we explore the problem structure of the wideband power control, investigate how to attain possible conjectural equilibria with different belief configurations, and design simple, yet efficient belief function and belief updating rule for the foresighted user to approach the Stackelberg equilibrium.

Last, there are several recent works that apply the traditional Stackelberg formulation in power control games [32]–[34]. In the traditional Stackelberg game, there exists a leader who declares its strategy before the other players and then enforces it [17]. The existing works usually use the Stackelberg formulation to model the interaction between entities with different priorities, e.g., primary users and secondary users in cognitive radio networks [32]. We interpret the Stackelberg leader in a different way. Since we investigate the steady state performance, the initial action order of the players does not matter. The leader and followers are differentiated based on their ability to forecast their competitors' response to their actions.

In addition, we also would like to mention that, the proposed formulation can be extended to the general communication scenario, in which multiple users are foresighted. However, the interaction in these cases becomes much more involved. Intuitively, since the degrees of freedom in the users' belief configuration increase as more users become foresighted learning users, the achievable performance region of the proposed framework will be a superset of the current region achieved by a single foresighted user. For these foresighted users, a reasonable outcome is to select an operating point in the set that achieves higher rates for all the foresighted users than the rates achieved by the IW algorithm:

$$\mathcal{R}^{n_f} = \left\{\left(R_1, \ldots, R_{n_f}\right) : R_i \geq R_i^{\mathrm{NE}}, \quad \text{for all } i = 1, \ldots, n_f\right\}$$

where n_f are the number of foresighted users and R_i^{NE} is user i's achievable rate if all the users play the Nash strategy. Similarly as [12], [13], since this selection involves solving a multiobjective optimization problem, coordination is generally required among these foresighted users to determine the desirable operation point. For example, they can exchange their local observations such that they can jointly optimize the multiobjective problem [17]. The game essentially evolve from a noncooperative game, played without explicit information exchange into a cooperative game, involving implicit or explicit information exchanges among foresighted users.

VII. Conclusion

This paper introduces the concept of conjectural equilibrium in noncooperative water-filling games and discusses how a foresighted user can model its experienced interference as a function of its own power allocation in order to improve its own data rate. The existence of conjectural equilibrium is proven and both game theoretic solutions, including Nash equilibrium and Stackelberg equilibrium, are shown to be special cases of this conjectural equilibrium. Practical algorithms based on conjectural equilibrium are developed to determine desirable power allocation strategies. Numerical results verify that a foresighted user forming proper conjectures can improves both its own achievable rate as well as the rates of other participants, even if it has no *a priori* knowledge of its competitors' private information. How to extend the framework to the scenarios in which multiple foresighted users coexist is a topic for future investigation. While this paper has focused on the water-filling game, the idea of forming conjectures based the available local information is also applicable to any communication system where making foresighted decisions is beneficial, e.g., distributed routing in wired network [35].

Appendix A

Define $f_{a_1,a_2,b}(x) = \ln(1 + (x)/(a_1 + a_2 - bx))$ where the term $a_1 \geq 0$ represents the noise PSD. The second derivative of $f_{a_1,a_2,b}(x)$ is

$$f''_{a_1,a_2,b}(x) = -\frac{(a_1 + a_2)[-(a_1 + a_2)(2b - 1) + 2b(b - 1)x]}{(a_1 + a_2 - bx)^2[a_1 + a_2 - (b - 1)x]^2}.$$

Clearly, if $b \neq 0$, $f''_{a_1,a_2,b}(x)$ is not always negative. We restrict the domain of $f_{a_1,a_2,b}$ to be

$$\mathbf{dom}\, f_{a_1,a_2,b} = \{x \geq 0\} \cap \{a_2 - bx \geq 0\}$$

because x is the transmitted power and $a_2 - bx$ represents the stationary interference, both of which are nonnegative. We de-

$$P_1^n(\boldsymbol{\beta},\boldsymbol{\gamma}) = \frac{1}{2\gamma^n(\gamma^n-1)}\left[(\sigma_1^n+\beta^n)(2\gamma^n-1) + \sqrt{(\sigma_1^n+\beta^n)^2(2\gamma^n-1)^2 - 4\gamma^n(\gamma^n-1)\left[(\sigma_1^n+\beta^n)^2 - \frac{\sigma_1^n+\beta^n}{\mu_1-\lambda_1^n}\right]}\right]. \tag{16}$$

rive a sufficient condition that guarantees $f_{a_1,a_2,b}(x)$ is concave in $\mathbf{dom}\ f_{a_1,a_2,b}$

$$a_2 > 0 \text{ and } b < 0.5\left(1-\frac{a_2}{a_1}\right). \tag{14}$$

This condition can be simply verified by using inequality analysis. Clearly, $a_2 > 0$ leads to $a_1 + a_2 > 0$ and $b < 0.5(1-(a_2)/(a_1)) < 1$. Therefore, $f''_{a_1,a_2,b}(x) < 0$ is equivalent to $-(a_1+a_2)(2b-1)+2b(b-1)x > 0$. We have $x \in \mathbf{dom}\, f_{a_1,a_2,b} \Rightarrow a_2 - bx \geq 0 \Rightarrow bx - a_2\cdot(a_1+a_2)/(a_2)\cdot(b-0.5)/(b-1) < 0 \Rightarrow -(a_1+a_2)(2b-1)+2b(b-1)x > 0$, because $(a_1+a_2)/(a_2)\cdot(b-0.5)/(b-1) > 1$ when $b < 0.5(1-(a_2)/(a_1))$. Hence, (14) leads to $f''_{a_1,a_2,b}(x) < 0$.

Based on sufficient condition (14), we can see, if $\beta^n > 0$ and $\gamma^n < 0.5(1-(\beta^n)/(\sigma_1^n))$ for any $n \in \{1,\ldots,N\}$, (11) belongs to convex programming. Therefore, if the following sufficient condition:

$$\mathsf{SC}\,1: \left(I_1^n - P_1^n\cdot\frac{\partial I_1^n}{\partial P_1^n}\right)\Bigg|_{\boldsymbol{P_1}=\boldsymbol{P}_{\mathrm{SE}}} > 0 \text{ and } -\frac{\partial I_1^n}{\partial P_1^n}\Bigg|_{\boldsymbol{P}_1=\boldsymbol{P}_{\mathrm{SE}}} < \frac{1}{2} - \frac{1}{2\sigma_1^n}\cdot\left(I_1^n - P_1^n\cdot\frac{\partial I_1^n}{\partial P_1^n}\right)\Bigg|_{\boldsymbol{P_1}=\boldsymbol{P}_{\mathrm{SE}}}$$

holds, SE satisfies the KKT optimality condition and solves the convex programming problem (11), i.e., SE is also a CE.

Appendix B

Proof of Proposition 3: If the water-filling game $\mathcal{G}$ satisfies condition $\mathsf{SC}\,1$, then (11) is convex. We can use the following "maximum theorem" [6], [22, p. 116] to show that $\boldsymbol{P}_1(\boldsymbol{\beta},\boldsymbol{\gamma})$ is continuous.

(*Maximum theorem*) Let $\phi(x,y)$ be a real-valued continuous function with domain $X\times Y$, where $X \subset \mathcal{R}^m$ and $Y \subset \mathcal{R}^n$ are closed and bounded sets. Suppose that $\phi(x,y)$ is strictly concave in x for each y. The functions $\Phi(y) = \arg\max\{\phi(x,y): x\in X\}$ is well defined for all $y \in Y$, and is continuous.

We can restrict the domain of parameters $\boldsymbol{\beta}$ and $\boldsymbol{\gamma}$ in closed and bounded set, e.g., $|\gamma^n| \leq M^+$, in which M^+ is a bound satisfying $M^+ \gg \max_n \boldsymbol{h}^n(\boldsymbol{I}+\boldsymbol{G})^{-1}\boldsymbol{g}^n$. Apply the maximum theorem with $\phi = R(\boldsymbol{P_1},\boldsymbol{\beta},\boldsymbol{\gamma}) = \sum_{n=1}^N f_{\sigma_1^n,\beta^n,\gamma^n}(P_1^n)$. The optimal solution $\boldsymbol{P_1}(\boldsymbol{\beta},\boldsymbol{\gamma})$ of (11) is the function Φ in the maximum theorem, and, hence, is a continuous function of $(\boldsymbol{\beta},\boldsymbol{\gamma})$. As a result, $F(\boldsymbol{\beta},\boldsymbol{\gamma})$ is also continuous in $(\boldsymbol{\beta},\boldsymbol{\gamma})$. Note that $\boldsymbol{P_1}(\boldsymbol{\beta},\boldsymbol{\gamma})$ and $F(\boldsymbol{\beta},\boldsymbol{\gamma})$ are not necessarily continuously differentiable.

By the definition of $F(\boldsymbol{\beta},\boldsymbol{\gamma})$ and conjectural equilibrium, we have that $F(\boldsymbol{\beta},\boldsymbol{\gamma}) = \mathbf{0}$ implies conjectural equilibrium. Note $F(\boldsymbol{\beta}_*,\boldsymbol{\gamma}_*) = \mathbf{0}$. If there exist open neighborhoods $A \subset \mathcal{R}^N$ and $B \subset \mathcal{R}^N$ of $\boldsymbol{\beta}_*$ and $\boldsymbol{\gamma}_*$, and for $\forall\boldsymbol{\gamma}\in B, F(\cdot,\boldsymbol{\gamma}): A \to \mathcal{R}^N$ is locally one-to-one, by the implicit function theorem [23], there exists open neighborhoods $A_0 \subset \mathcal{R}^N$ and $B_0 \subset \mathcal{R}^N$ of $\boldsymbol{\beta}_*$ and $\boldsymbol{\gamma}_*$ such that for each $\boldsymbol{\gamma}\in B_0$, there is a unique $\boldsymbol{\beta}(\boldsymbol{\gamma})$ satisfying $F(\boldsymbol{\beta}(\boldsymbol{\gamma}),\boldsymbol{\gamma}) = \mathbf{0}$. Therefore, $\mathcal{G}$ admits an infinite set of conjectural equilibria.

Alternatively, we can view $F(\boldsymbol{\beta},\boldsymbol{\gamma}) = \mathbf{0}$ as N equations with $2N$ unknowns, hence, the equilibrium is usually not a single point but a continuous surface. We can explore the structure of $\boldsymbol{P_1}(\boldsymbol{\beta},\boldsymbol{\gamma})$ to derive the expression of this surface. Particularly, under condition $\mathsf{SC}\,1$, the solution of convex problem (11) satisfies

$$\gamma^n(\gamma^n-1)(P_1^n)^2 - (\sigma_1^n+\beta^n)(2\gamma^n-1)P_1^n + (\sigma_1^n+\beta^n)^2 - \frac{\sigma_1^n+\beta^n}{\mu_1-\lambda_1^n} = 0 \tag{15}$$

where λ_1^n and μ_1 are the Lagrange multipliers as in (11). The optimal $\boldsymbol{P_1}(\boldsymbol{\beta},\boldsymbol{\gamma}) = \{P_1^n(\boldsymbol{\beta},\boldsymbol{\gamma})\}$ is given by (16), shown at the top of the page. Note that the other root of (15) is removed by checking its feasibility in $\mathbf{dom} f_{\sigma_1^n,\beta^n,\gamma^n}$. By substituting (16) into (11) and (12), we can explicitly express $F(\boldsymbol{\beta},\boldsymbol{\gamma})$ in terms of $\boldsymbol{\beta}$ and $\boldsymbol{\gamma}$, resulting in a very complex form of the surface. ■

References

[1] E. Altman and Z. Altman, "S-modular games and power control in wireless networks," *IEEE Trans. Autom. Control*, vol. 48, no. 5, pp. 839–842, May 2003.

[2] M. Chiang, P. Hande, T. Lan, and C. Wei Tan, "Power control in wireless cellular networks," *Foundations Trends in Netw.*, vol. 2, no. 4, pp. 381–533, http:/dx.doi.org/10.1561/1300000009.

[3] F. Meshkati, V. Poor, and S. Schwartz, "Energy-efficient resource allocation in wireless networks: An overview of game-theoretic approaches," *IEEE Signal Process. Mag.*, vol. 24, pp. 58–68, May 2007.

[4] W. Yu, G. Ginis, and J. Cioffi, "Distributed multiuser power control for digital subscriber lines," *IEEE J. Sel. Areas Commun.*, vol. 20, no. 5, pp. 1105–1115, Jun. 2002.

[5] Z. Luo and J. Pang, "Analysis of iterative waterfilling algorithm for multiuser power control in digital subscriber lines," *EURASIP J. Appl. Signal Process.*, vol. 2006, 2006, Article ID 24012.

[6] K. W. Shum, K.-K. Leung, and C. W. Sung, "Convergence of iterative waterfilling algorithm for gaussian interference channels," *IEEE J. Sel. Areas Commun.*, vol. 25, no. 6, pp. 1091–1100, Aug. 2007.

[7] G. Scutari, D. P. Palomar, and S. Barbarossa, "Competitive design of multiuser MIMO systems based on game theory: A unified view," *IEEE J. Sel. Areas Commun.*, vol. 26, no. 7, pp. 1089–1103, Sep. 2008.

[8] E. Altman, K. Avrachenkov, and A. Garnaev, "Closed form solutions for symmetric water filling games," in *Proc. IEEE INFOCOM 2008*, Apr. 2008, pp. 673–681.

[9] R. Etkin, A. Parekh, and D. Tse, "Spectrum sharing for unlicensed bands," *IEEE J. Sel. Areas Commun.*, vol. 25, no. 3, pp. 517–528, Apr. 2007.

[10] O. Popescu, D. Popescu, and C. Rose, "Simultaneous water filling in mutually interfering systems," *IEEE Trans. Wireless Commun.*, vol. 6, no. 3, pp. 1102–1113, Mar. 2007.

[11] Y. Su and M. van der Schaar, "A new look at multiuser power control games," in *Proc. IEEE ICC 2008*, May 2008, pp. 1072–1076, (long version available at http://arxiv.org/abs/0709.3880).

[12] R. Cendrillon, W. Yu, M. Moonen, J. Verlinden, and T. Bostoen, "Optimal multiuser spectrum balancing for digital subscriber lines," *IEEE Trans. Commun.*, vol. 54, no. 5, pp. 922–933, May 2006.

[13] W. Yu and R. Lui, "Dual methods for nonconvex spectrum optimization of multicarrier systems," *IEEE Trans. Commun.*, vol. 54, pp. 1310–1322, 2006.

[14] R. Cendrillon, J. Huang, M. Chiang, and M. Moonen, "Autonomous spectrum balancing for digital subscriber lines," *IEEE Trans. Signal Process.*, vol. 55, no. 8, pp. 4241–4257, Aug. 2007.
[15] J. Huang, R. Berry, and M. Honig, "Distributed interference compensation for wireless networks," *IEEE J. Sel. Areas Commun.*, vol. 24, no. 5, pp. 1074–1084, May 2006.
[16] E. Larsson and E. Jorswieck, "Competition versus cooperation on the MISO interference channel," *IEEE J. Sel. Areas Commun.*, vol. 26, no. 7, pp. 1059–1069, Sep. 2008.
[17] D. Fudenberg and J. Tirole, *Game Theory*. Cambridge, MA: MIT Press, 1991.
[18] E. Altman, T. Boulogne, R. El-Azouzi, T. Jimenez, and L. Wynter, "A survey on networking games," *Comput. Operat. Res.*, 2004.
[19] F. H. Hahn, "Exercises in conjectural equilibrium analysis," *Scandinavian J. Econom.*, vol. 79, pp. 210–226, 1977.
[20] M. P. Wellman and J. Hu, "Conjectural equilibrium in multiagent learning," *Machine Learn.*, vol. 33, pp. 179–200, 1998.
[21] S. Boyd and L. Vandenberghe, *Convex Optimization*. Cambridge, U.K.: Cambridge Univ. Press, 2004.
[22] C. Berge, *Topological Spaces*. New York: Dover, 1997.
[23] S. Kumagai, "An implicit function theorem: Comment," *J. Optimiz. Theory Appl.*, vol. 31, no. 2, pp. 285–288, Jun. 1980.
[24] S. Haykin, "Cognitive radio: Brain-empowered wireless communications," *IEEE J. Sel. Areas Commun.*, vol. 23, pp. 201–220, Feb. 2005.
[25] N. Nie and C. Comaniciu, "Adaptive channel allocation spectrum etiquette for cognitive radio networks," in *Proc. IEEE DySPAN*, Nov. 2005, pp. 269–278.
[26] C. Pandana and K. J. R. Liu, "Near optimal reinforcement learning framework for energy-aware wireless sensor communications," *IEEE J. Sel. Areas Commun.*, vol. 23, no. 4, pp. 788–797, Apr. 2005.
[27] C. Long, Q. Zhang, B. Li, H. Yang, and X. Guan, "noncooperative power control for wireless *ad hoc* networks with repeated games," *IEEE J. Sel. Areas Commun.*, vol. 25, pp. 1101–1112, Aug. 2007.
[28] D. Djonin and V. Krishnamurthy, "Q-learning algorithms for constrained markov decision processes with randomized monotone policies: Applications to MIMO transmission control," *IEEE Trans. Signal Process.*, vol. 55, no. 5, pp. 2170–2181, 2007.
[29] Y. Xing and R. Chandramouli, "Stochastic learning solution for distributed discrete power control game in wireless data networks," *IEEE/ACM Trans. Netw.*, pp. 932–944, Aug. 2008.
[30] Y. Wu, B. Wang, K. Liu, and T. Clancy, "Repeated open spectrum sharing game with cheat-proof strategies," *IEEE Trans. Wireless Commun.*, to be published.
[31] D. Niyato and E. Hossain, "Competitive pricing for spectrum sharing in cognitive radio networks: Dynamic game, inefficiency of Nash equilibrium, and collusion," *IEEE J. Sel. Areas Commun.*, vol. 26, no. 1, Jan. 2008.
[32] M. Bloem, T. Alpcan, and T. Baar, "A stackelberg game for power control and channel allocation in cognitive radio networks," in *Proc. Workshop on Game Theory in Commun. Netw. (GameComm)*, Oct. 2007.
[33] B. Wang, Z. Han, and K. J. R. Liu, "Stackelberg game for distributed resource allocation over multiuser cooperative communication networks," in *Proc. IEEE Globecom 2006*, Nov. 2006, pp. 1–5.
[34] A. Al Daoud, T. Alpcan, S. Agarwal, and M. Alanyali, "A stackelberg game for pricing uplink power in wide-band cognitive radio networks," in *Proc. IEEE Conf. Decision and Control*, Dec. 2008.
[35] Y. A. Korilis, A. A. Lazar, and A. Orda, "Architecting noncooperative networks," *IEEE J. Sel. Areas Commun.*, vol. 13, pp. 1241–1251, Sep. 1995.

Yi Su (S'08) received the B.E. and M.E. degrees from Tsinghua University, Beijing, China, in 2004 and 2006, respectively, both in electrical engineering.

He is currently working toward the Ph.D. degree in the Department of Electrical Engineering, University of California, Los Angeles.

Mihaela van der Schaar (SM'04) received the Ph.D. degree from Eindhoven University of Technology, The Netherlands, in 2001.

She is now an Associate Professor with the Electrical Engineering Department, University of California, Los Angeles.