Pricing and Incentives in Peer-to-Peer Networks

Jaeok Park
Electrical Engineering Department
University of California, Los Angeles
Los Angeles, CA 90095, USA
Email: jaeok@ee.ucla.edu

Mihaela van der Schaar
Electrical Engineering Department
University of California, Los Angeles
Los Angeles, CA 90095, USA
Email: mihaela@ee.ucla.edu

***Abstract*—Peer-to-peer (P2P) networks offer a cost effective and easily deployable framework for sharing user-generated content. However, intrinsic incentive problems reside in P2P networks as the transfer of content incurs costs both to uploaders and to downloaders while the benefit accrues only to downloaders. We investigate the issues of incentives in content production and sharing over P2P networks using a game theoretic model. Peers do not share produced content at all at non-cooperative equilibria whereas Pareto efficiency requires peers to fully share produced content. There is also a divergence in the total amount of produced content between non-cooperative equilibria and Pareto efficiency. By imposing full sharing, we decompose the inefficiency of non-cooperative equilibria into two parts, inefficiency due to no sharing and inefficiency due to underproduction. As a method to remedy the incentive problems in P2P networks, two classes of pricing schemes, MP pricing schemes and linear pricing schemes, are proposed. We show that the proposed pricing schemes can achieve Pareto efficiency as non-cooperative equilibria. We also examine a linear pricing scheme that maximizes the revenue of the network manager.**

I. Introduction

In today's Internet-based social communities, peer-to-peer (P2P) networks offer a cost effective and easily deployable framework for sharing user-generated content [1]. While P2P networks have many advantages such as scalability, resilience, and effectiveness in coping with dynamics and heterogeneity [2], they have intrinsic incentive problems in that the transfer of content incurs costs to uploaders as well as to downloaders but benefits only downloaders. Since the social cost of transfer (the sum of upload and download costs) exceeds the private cost of transfer (download costs), peers as downloaders have incentives to overutilize P2P networks as in the tragedy of the commons problem. On the other hand, since upload incurs costs to uploaders without benefiting them at all, peers as uploaders have incentives to underutilize P2P networks. The incentive problem that peers desire to benefit from P2P networks while not contributing to them is referred to as the free-rider (or freeloader) problem in the literature. The free-rider problem is observed in real P2P networks. For example, [3] reports that in the Gnutella P2P network, nearly 70% of peers share no files and nearly 50% of all uploads are made by the top 1% of sharing peers.

To mitigate the free-rider problem in P2P networks, the literature has considered two classes of incentive schemes: soft and hard schemes [4]. Under a soft scheme, a rating system is used to rate peers based on their past upload/download behavior, and peers determine their upload decisions depending on the ratings of peers that request download. A hard scheme uses monetary payment to compensate peers for their upload and charge peers for their download. The form of payment can be either virtual currency such as tokens and points or real currency. Previous work studying these two classes of schemes can be found in [4].

In the language of game theory, soft schemes can be framed as repeated games in which peers make upload/download decisions depending on their histories. It is a well-known idea in game theory that repeated interaction can support cooperation among self-interested individuals (see, for example, [5]). However, peers with occasional use of a P2P network have a weak incentive for cooperation as their expected return from cooperation is low. Hence, soft schemes may fail to induce sharing in a large P2P network with low usage peers, where the motive for cooperation using reward and punishment through repeated interaction is relatively weak. There are theoretical results in the literature on repeated games that cooperation can be sustained even in a community where a large number of individuals are randomly matched [6], [7]. The results in [6] and [7] are based on the idea of "contagious" punishments under which a single defection triggers a series of defections for some period. In principle, a rating system in a P2P network can be considered as a variation of "labels" in [6]. For example, [8] applies the idea of [6] to sustain cooperation in P2P routing. However, cooperation supported by contagious punishments is fragile in the presence of a malicious peer, which always seeks to free-ride.

This paper uses hard schemes based on one-shot game formulation. We first present an abstract model of content production and sharing over a P2P network. A distinctive feature of our model is that we consider peers' incentives to produce content as well as to share content. Most previous work assumes that peers are endowed with exogenous amounts of content and focuses on incentives for sharing (see, for example, [4], [9]). However, the amount of content a peer produces can be affected by the amount of content available in the P2P network. Our model captures the strategic link between producing and downloading content, by endogenizing the amount of content that peers produce. Using a game theoretic analysis, we find two discrepancies between non-cooperative equilibria and Pareto efficiency. First, there is no sharing at non-cooperative equilibria while Pareto efficiency requires full sharing. Second, each peer obtains a smaller amount of content at non-cooperative equilibria than at Pareto efficiency. By imposing full sharing on the basic model, we introduce inefficiency measures to quantify the effects of these two sources of inefficiency separately. Finally, we propose two classes of pricing schemes, MP pricing schemes and linear pricing schemes, which can be used to achieve Pareto efficiency as non-cooperative equilibria. The proposed pricing schemes can be implemented without requiring peers to have knowledge about the past behavior of other peers, and thus they can be applied to P2P networks in which peers do not interact frequently.

The remainder of this paper is organized as follows. In Section II, we describe our basic model of content production and sharing. In Section III, we analyze the model using two solution concepts, non-cooperative equilibrium and Pareto efficiency. In Section IV, we propose two classes of pricing schemes to achieve Pareto efficiency among self-interested peers. We conclude in Section V.

II. Basic Model

We consider a completely connected P2P network over which N peers can share content they produce, where $N \geq 2$. Examples of such user-generated content include photos, videos, news, and customer reviews. Using a game theoretic framework, we construct an abstract model of content production and sharing (CPS), which we call the CPS game. The CPS game is a sequential game consisting of the following three stages.

1) *Stage One (Production):* Each peer determines its level of production.[1] $x_i \in \mathbb{R}_+$ represents the amount of content produced by peer i and is known only to peer i.
2) *Stage Two (Sharing):* Each peer specifies its level of sharing. $y_i \in [0, x_i]$ represents the amount of content that peer i makes available to other peers. $(y_1, \dots, y_N)$ is known to all peers at the end of stage two.
3) *Stage Three (Transfer):* Each peer determines the amounts of content that it downloads from other peers. Peer i serves all the requests it receives from any other peer up to y_i. $z_{ij} \in [0, y_j]$ represents the amount of content that peer i downloads from peer $j \neq i$, or equivalently peer j uploads to peer i.

Let $\mathcal{N} \triangleq \{1, \dots, N\}$ be the set of peers in the P2P network. For notations, we define $\mathbf{x} \triangleq (x_1, \dots, x_N)$, $\mathbf{y} \triangleq (y_1, \dots, y_N)$, and $\mathbf{Z} \triangleq [z_{ij}]_{i,j \in \mathcal{N}}$, an N-by-N matrix whose (i,j)-entry is given by z_{ij}, where we set $z_{ii} = 0$ for all $i \in \mathcal{N}$. The download profile of peer i is given by the ith row of $\mathbf{Z}$, denoted by $\mathbf{z}_i \triangleq (z_{i1}, \dots, z_{iN})$. Similarly, the upload profile of peer i is given by the ith column of $\mathbf{Z}$, denoted by $\mathbf{z}^i \triangleq (z_{1i}, \dots, z_{Ni})$. Given the elements of $\mathbf{Z}$, we can compute the download volume of peer i by $d_i(\mathbf{z}_i) \triangleq \sum_{j=1}^N z_{ij}$ and its upload volume by $u_i(\mathbf{z}^i) \triangleq \sum_{j=1}^N z_{ji}$. For notational convenience, we suppress the dependence of d_i and u_i on $\mathbf{Z}$ and write d_i and u_i instead of $d_i(\mathbf{z}_i)$ and $u_i(\mathbf{z}^i)$, respectively. Also, we define $w(\mathbf{Z})$ to be the total transfer volume of the P2P network given $\mathbf{Z}$, i.e., $w(\mathbf{Z}) \triangleq \sum_{i=1}^N \sum_{j=1}^N z_{ij} = \sum_{i=1}^N d_i = \sum_{i=1}^N u_i$, which can be considered as a measure of the utilization of the P2P network.

The total amount of content that peer i obtains in the CPS game, which we call the consumption of peer i, is given by the sum of the amounts it produces and downloads, $x_i + d_i$. The utility of peer i is given by the benefit of consumption minus the costs of production, download, and upload:

$$v_i(\mathbf{x}, \mathbf{y}, \mathbf{Z}) = f(x_i + d_i) - \kappa x_i - \delta d_i - \sigma u_i.$$

We analyze the case of homogeneous peers in that f, κ, δ, and σ are the same for all peers. The benefit of consumption is measured by a concave function $f : \mathbb{R}_+ \to \mathbb{R}_+$ as in [10] and [11]. We assume that f is twice continuously differentiable and satisfies $f(0) = 0$, $f' > 0$ and $f'' < 0$ on $\mathbb{R}_{++}$. We also assume that $f'(0)$ is finite,[2] $f'(0) > \kappa$, and $\lim_{x \to \infty} f'(x) = 0$ so that for every $\alpha \in (0, f'(0)]$ there exists a unique $\hat{x}_\alpha \geq 0$ that satisfies $f'(\hat{x}_\alpha) = \alpha$. We use linear cost functions as widely adopted in the literature (for example, see [12] and [13]). The cost of producing the amount of content x_i is given by κx_i, where $\kappa > 0$ is the marginal cost of production. We assume that peers have sufficient bandwidth to serve all requests they face. However, download and upload create costs in terms of bandwidth overhead, and transferring the amount of content z_{ij} from peer i to peer j induces a cost of δz_{ij} to peer i (the downloader) and σz_{ij} to peer j (the uploader), where $\delta > 0$ and $\sigma > 0$ are the marginal costs of download and upload, respectively. The P2P network has a positive social value only if obtaining a unit of content through the P2P network costs less to peers than producing it privately. Hence, we assume that $\kappa > \delta + \sigma$ to ensure that the P2P network is socially valuable.

III. Equilibrium and Pareto Efficiency

We analyze the CPS game as a non-cooperative game and as a one-shot game. Each peer maximizes its own utility given the strategies of other peers, without considering past and future plays that may exist. In particular, peers cannot use personal reciprocation such as "choking" in [14]. Since the CPS game induces non-singleton information sets in stages two and three, we use a non-cooperative solution concept of sequential equilibrium (SE) in order to preserve the spirit of subgame perfection. SE requires that peers behave optimally in each stage given consistent beliefs [15].

An SE strategy profile of the CPS game can be found applying a backward induction argument. An information set of peer i in stage three is determined by the information it has at the beginning of stage three, $(x_i, \mathbf{y})$. Let $\mathbf{z}_i^*(x_i, \mathbf{y})$ be an optimal choice of peer i at the information set $(x_i, \mathbf{y})$ in stage three. Then for all $(\mathbf{x}, \mathbf{y})$, we have

$$v_i(\mathbf{x}, \mathbf{y}, \mathbf{z}_i^*(x_i, \mathbf{y}), \mathbf{z}_{-i}^*(\mathbf{x}_{-i}, \mathbf{y})) \geq v_i(\mathbf{x}, \mathbf{y}, \mathbf{z}_i, \mathbf{z}_{-i}^*(\mathbf{x}_{-i}, \mathbf{y}))$$

[1] In this paper, the term *production* is used in a broad sense to mean any method of obtaining content other than download in the P2P network.

[2] We use $f'(0)$ to represent the right derivative of f at 0.

for all feasible $\mathbf{z}_i$, i.e., $z_{ij} \in [0, y_j]$ for $j \neq i$ and $z_{ii} = 0$, where $\mathbf{z}^*_{-i}(\mathbf{x}_{-i}, \mathbf{y}) \triangleq (\mathbf{z}^*_j(x_j, \mathbf{y}))_{j \neq i}$. Note that $\mathbf{z}^*_i(x_i, \mathbf{y})$ is independent of the belief of peer i on $\mathbf{x}_{-i}$ due to the structure of the utility function.

In stage two, an information set of peer i is specified by its stage-one decision x_i. When making stage-two decisions, peers correctly predict that $\mathbf{Z}^*(\mathbf{x}, \mathbf{y}) \triangleq (\mathbf{z}^*_i(x_i, \mathbf{y}))_{i=1}^N$ will be played in stage three if $\mathbf{y}$ is chosen in stage two. An optimal decision of peer i at the information set x_i is denoted by $y^*_i(x_i)$ and satisfies

$$v_i(\mathbf{x}, y^*_i(x_i), \mathbf{y}^*_{-i}(\mathbf{x}_{-i}), \mathbf{Z}^*(\mathbf{x}, \mathbf{y}^*(\mathbf{x}))) \\ \geq v_i(\mathbf{x}, y_i, \mathbf{y}^*_{-i}(\mathbf{x}_{-i}), \mathbf{Z}^*(\mathbf{x}, y_i, \mathbf{y}^*_{-i}(\mathbf{x}_{-i})))$$

for all feasible $y_i \in [0, x_i]$, where $\mathbf{y}^*(\mathbf{x}) \triangleq (y^*_i(x_i))_{i=1}^N$ and $\mathbf{y}^*_{-i}(\mathbf{x}_{-i}) \triangleq (y^*_j(x_j))_{j \neq i}$. The belief of peer i on $\mathbf{x}_{-i}$ influences its stage-two decision through its prediction on $\mathbf{z}^*_{-i}(\mathbf{x}_{-i}, \mathbf{y})$. We assume that peers hold correct beliefs as required by SE.

In stage one, peers make production decisions simultaneously, predicting that the choice $\mathbf{x}$ will induce decisions in stages two and three as $\mathbf{y}^*(\mathbf{x})$ and $\mathbf{Z}^*(\mathbf{x}, \mathbf{y}^*(\mathbf{x}))$, respectively. Then the utility of peer i can be represented as determined entirely by $\mathbf{x}$:

$$\tilde{v}_i(\mathbf{x}) \triangleq v_i(\mathbf{x}, \mathbf{y}^*(\mathbf{x}), \mathbf{Z}^*(\mathbf{x}, \mathbf{y}^*(\mathbf{x}))).$$

Let $\mathbf{x}^*$ be a Nash equilibrium (NE) of the reduced game with payoff functions $(\tilde{v}_i)_{i=1}^N$. That is,

$$\tilde{v}_i(\mathbf{x}^*) \geq \tilde{v}_i(x_i, \mathbf{x}^*_{-i})$$

for all $x_i \geq 0$, for all $i \in \mathcal{N}$. Then an SE strategy profile is given by $(\mathbf{x}^*, \mathbf{y}^*(\mathbf{x}), \mathbf{Z}^*(\mathbf{x}, \mathbf{y})) = (x^*_i, y^*_i(x_i), \mathbf{z}^*_i(x_i, \mathbf{y}))_{i=1}^N$ while the SE outcome is $(\mathbf{x}^*, \mathbf{y}^*(\mathbf{x}^*), \mathbf{Z}^*(\mathbf{x}, \mathbf{y}^*(\mathbf{x}^*)))$.

Proposition 1: The unique SE outcome of the CPS game is $x_i = \hat{x}_\kappa$, $y_i = 0$, $\mathbf{z}_i = (0, \ldots, 0)$ for all $i \in \mathcal{N}$. Thus, $w(\mathbf{Z}) = 0$ at SE.

All the proofs of propositions in this paper are given in an appendix. Proposition 1 shows that even a small cost of upload makes the socially valuable P2P network never utilized because peers are not compensated for their upload. As a result of no sharing in the P2P network, peers choose the autarkic optimal level of production, $\hat{x}_\kappa$, which maximizes $f(x) - \kappa x$. The utility of a peer at SE is thus given by $f^*(\kappa) \triangleq f(\hat{x}_\kappa) - \kappa \hat{x}_\kappa$. The collapse of P2P networks is also shown in [9], which uses a different formulation. Specifically, [9] assumes that each peer has a given number of files and that upload and download choices are discrete.

We now characterize Pareto efficient (PE) allocations, which maximize social welfare defined by the sum of the utilities of peers, i.e., $\Pi(\mathbf{x}, \mathbf{y}, \mathbf{Z}) \triangleq \sum_{i=1}^N v_i(\mathbf{x}, \mathbf{y}, \mathbf{Z})$.

Proposition 2: Let $\beta \triangleq \frac{1}{N}\kappa + \frac{N-1}{N}(\delta + \sigma)$. At Pareto efficiency (PE), we have $\sum_{i=1}^N x_i = \hat{x}_\beta$ and $x_i = y_i = z_{ji}$ for all $j \neq i$, for all $i \in \mathcal{N}$. Thus, $w(\mathbf{Z}) = (N-1)\hat{x}_\beta$ at PE.

Proposition 2 shows that at PE peers produce $\hat{x}_\beta$ in aggregate and share produced content fully so that each peer consumes the total amount produced. Note that β can be interpreted as the cost of obtaining one unit of content under full sharing. The utility of a peer at PE is given by

$$f(\hat{x}_\beta) - \delta \hat{x}_\beta - [\kappa + (N-1)\sigma - \delta] x_i, \tag{1}$$

and social welfare at PE is $\Pi^{PE} = N f^*(\beta)$, where $f^*(\beta) \triangleq f(\hat{x}_\beta) - \beta \hat{x}_\beta$. Social welfare at SE is given by $\Pi^{SE} = N f^*(\kappa)$. Since $\beta < \kappa$, the consumption of a peer and social welfare are smaller at SE than at PE, i.e., $\hat{x}_\kappa < \hat{x}_\beta$ and $\Pi^{SE} < \Pi^{PE}$, which proves the inefficiency of SE.

The utility at the autarkic optimum, $f^*(\kappa)$, can be considered as the reservation utility of a peer. Hence, if a peer receives a utility lower than $f^*(\kappa)$, it would opt out of the P2P network. An allocation $(\mathbf{x}, \mathbf{y}, \mathbf{Z})$ satisfies the participation (or individual rationality) constraint for peer i if $v_i(\mathbf{x}, \mathbf{y}, \mathbf{Z}) \geq f^*(\kappa)$. An allocation $(\mathbf{x}, \mathbf{y}, \mathbf{Z})$ is participation-efficient if it is PE and satisfies the participation constraint for every peer. Since the utility of a peer at PE is decreasing in its share of production as shown in (1), imposing the participation constraint on PE allocations yields an upper bound on the level of production.

Corollary 1: An allocation $(\mathbf{x}, \mathbf{y}, \mathbf{Z})$ is participation-efficient if $\sum_{i=1}^N x_i = \hat{x}_\beta$, $x_i = y_i = z_{ji}$ for all $j \neq i$, and

$$x_i \leq \frac{f(\hat{x}_\beta) - \delta \hat{x}_\beta - f^*(\kappa)}{\kappa + (N-1)\sigma - \delta},$$

for all $i \in \mathcal{N}$.

Note that there is a continuum of participation-efficient allocations because the symmetric PE production profile, $x_i = \hat{x}_\beta / N$ for all i, satisfies the participation constraint with strict inequality and the utility of a peer changes continuously in $\mathbf{x}$.

A network manager can implement a PE allocation if he is able to compute the value of $\hat{x}_\beta$ with the knowledge of the utility functions of peers and able to enforce the sharing *levels* of peers. On the contrary, consider a scenario in which the network manager can make peers share all produced content but cannot specify the desired levels of sharing. In this scenario, the stage-two choices of peers are fixed as $y_i = x_i$ for all $i \in \mathcal{N}$. We call the modified game the CPS game with enforced full sharing.

Proposition 3: Let $\gamma \triangleq \kappa + (N-1)\sigma$. Define $\tilde{x}_\gamma$ by $\tilde{x}_\gamma = \hat{x}_\gamma$ if $\gamma \leq f'(0)$ and $\tilde{x}_\gamma = 0$ otherwise (i.e., $\tilde{x}_\gamma = \arg\max_{x \geq 0}\{f(x) - \gamma x\}$). At SE of the CPS game with enforced full sharing, we have $\sum_{i=1}^N x_i = \tilde{x}_\gamma$ and $x_i = y_i = z_{ji}$ for all $j \neq i$, for all $i \in \mathcal{N}$. Thus, $w(\mathbf{Z}) = (N-1)\tilde{x}_\gamma$ at SE with enforced full sharing.

In the proof of Proposition 3, it is shown that peers download all shared content at SE, and thus with enforced full sharing the effective marginal cost of production becomes $\gamma = \kappa + (N-1)\sigma$, which includes the marginal cost of upload to $(N-1)$ peers. As a result, peers produce $\tilde{x}_\gamma$ in aggregate, which is smaller than the autarkic optimal production level, $\hat{x}_\kappa$, and social welfare at SE with enforced full sharing is given by

$$\Pi^{FS} = N[f(\tilde{x}_\gamma) - \beta \tilde{x}_\gamma].$$

Now we compare social welfare achieved in Propositions 1 to 3. We define the *price of anarchy* (PoA)[3] to be the ratio of social welfare at the worst SE to that at PE, i.e.,

$$PoA \triangleq \frac{\Pi^{SE}}{\Pi^{PE}} = \frac{f^*(\kappa)}{f^*(\beta)}. \quad (2)$$

The *price of no sharing* (PoNS) compares social welfare at SE without and with enforced full sharing, i.e.,

$$PoNS \triangleq \frac{\Pi^{SE}}{\Pi^{FS}} = \frac{f^*(\kappa)}{f(\tilde{x}_\gamma) - \beta\tilde{x}_\gamma} \ (= \infty \text{ if } \tilde{x}_\gamma = 0).$$

The *price of underproduction* (PoU) compares social welfare at SE with enforced full sharing and at PE, i.e.,

$$PoU \triangleq \frac{\Pi^{FS}}{\Pi^{PE}} = \frac{f(\tilde{x}_\gamma) - \beta\tilde{x}_\gamma}{f^*(\beta)}. \quad (3)$$

The PoA is a widely used measure of the inefficiency of non-cooperative equilibria. The PoNS can be considered as a measure of an efficiency loss due to the absence of enforced full sharing. The PoU measures inefficiency due to under-production caused by the selfish behavior of peers assuming that full sharing is enforced. When $\tilde{x}_\gamma > 0$, the PoA can be decomposed as the product of the PoNS and the PoU, i.e., $PoA = PoNS \times PoU$. The following proposition examines the range of values that each measure of inefficiency can take when we vary the utility specification of the model, f, κ, δ, and σ.

Proposition 4: $PoA \in (0,1)$, $PoNS \in (0,\infty]$, and $PoU \in [0,1)$. These bounds are tight.

Since $\beta < \kappa < \gamma$, it is easy to see from (2) and (3) that $PoA, PoU < 1$. Proposition 4 shows that the relative size of Π^{SE} and Π^{FS} is ambiguous, which implies that the enforcement of full sharing may make peers worse off. This is because enforced full sharing has two offsetting effects on social welfare. On one hand, full sharing has a positive effect on welfare by reducing the cost of obtaining one unit of content to β, compared to κ in the case of no sharing. On the other hand, full sharing has a negative effect by increasing the effective cost of production from κ to γ. Therefore, the overall welfare implication of enforced full sharing is determined by the stronger of the two effects.

IV. Pricing Schemes

As mentioned in the previous section, if the network manager can compute the PE level of production and enforce sharing levels, he can implement a PE allocation by specifying a desired sharing level to each peer, taking into account participation constraints if necessary. In this section, we consider a scenario in which the network manager cannot enforce sharing levels but can enforce payments from peers. We continue to assume that the network manager knows the utility functions of peers.[4] In this scenario, the network manager can use a pricing scheme to achieve a PE allocation among self-interested peers. We consider two classes of pricing schemes: marginal product (MP) pricing schemes, where payments are based on the stage-two choices, and linear pricing schemes, where payments are based on the stage-three choices.

A. Marginal Product Pricing Schemes

We first propose MP pricing schemes, which rely on the main idea of the renowned VCG mechanism [16]–[18] and are related to potential games [19]. To make a comparison with a mechanism design problem, we interpret the production level of peer i, x_i, as its "type" and the sharing level of peer i, y_i, as its "announcement" of its type. The main difference between our model and the standard mechanism design theory is that in our model peers choose their own types in a strategic decision making process whereas in mechanism design types are either given or determined by the nature in a random process.

In an MP pricing scheme, payments are determined by the announcements of peers, $\mathbf{y}$. The payment to a peer is its contribution to other peers assuming that every peer announced its true type, i.e., $x_i = y_i$ for all $i \in \mathcal{N}$, and that every peer will download all shared content, i.e., $z_{ij} = y_j$ for all $j \neq i$, for all $i \in \mathcal{N}$. Formally, the payment to peer i can be expressed as

$$t_i^V(\mathbf{y}) = \sum_{j \neq i} v_j(\mathbf{y}, \mathbf{y}, \tilde{\mathbf{Z}}(\mathbf{y})) - r_i(\mathbf{y}_{-i}), \quad (4)$$

where $\tilde{\mathbf{Z}}(\mathbf{y})$ is a matrix whose (i,j)-entry is given by $\tilde{z}_{ij} = y_j$ for $j \neq i$ and $\tilde{z}_{ij} = 0$ for $j = i$, for all $i \in \mathcal{N}$, and $r_i(\mathbf{y}_{-i})$ is the reservation utility of all peers but peer i with types $\mathbf{x}_{-i} = \mathbf{y}_{-i}$. Note that $r_i(\mathbf{y}_{-i})$ is independent of the announcement of peer i, y_i. An MP pricing scheme is specified by reservation utility functions, $(r_1, \ldots, r_N)$.

The payoff function of peer i in the CPS game with an MP pricing scheme is given by

$$\begin{aligned}\pi_i(\mathbf{x}, \mathbf{y}, \mathbf{Z}) &= v_i((x_i, \mathbf{y}_{-i}), \mathbf{y}, \mathbf{Z}) + t_i^V(\mathbf{y}) \\ &= v_i((x_i, \mathbf{y}_{-i}), \mathbf{y}, \mathbf{Z}) + \sum_{j \neq i} v_j(\mathbf{y}, \mathbf{y}, \tilde{\mathbf{Z}}(\mathbf{y})) - r_i(\mathbf{y}_{-i}). \quad (5)\end{aligned}$$

Suppose that the assumptions to compute the payments in (4), i.e., $\mathbf{x} = \mathbf{y}$ and $\mathbf{Z} = \tilde{\mathbf{Z}}(\mathbf{y})$, are correct. Then (5) can be considered as the MP of peer i when $r_i(\mathbf{y}_{-i})$ represents social welfare that is achieved with all peers but peer i with types $\mathbf{y}_{-i}$. MP is also referred to as wonderful life utility [20]. The following proposition shows that any MP pricing scheme makes the set of SE allocations equal to the set of PE allocations.

Proposition 5: At SE of the CPS game with any MP pricing scheme, we have $\sum_{i=1}^N x_i = \hat{x}_\beta$ and $x_i = y_i = z_{ji}$ for all $j \neq i$, for all $i \in \mathcal{N}$.

In the proof of Proposition 5, it is shown that taking into account optimal decisions in stages two and three, the stage-

[3] Since the SE outcome of the CPS game is unique, the price of anarchy and the price of stability coincide for the CPS game.

[4] If the network manager does not know the utility functions of peers, he can achieve a PE outcome using a mechanism design approach, which requires reports from peers to the network manager.

one payoff function of peer i can be written as

$$\hat{\pi}_i(\mathbf{x}) = Nf\left(\sum_{i=1}^{N} x_i\right) - [\kappa + (N-1)(\delta+\sigma)]\sum_{i=1}^{N} x_i - r_i(\mathbf{x}_{-i}).$$

Thus, the reduced stage-one game has an exact potential of $\hat{\Pi}(\mathbf{x}) \triangleq \Pi(\mathbf{x}, \mathbf{x}, \tilde{\mathbf{Z}}(\mathbf{x}))$. In essence, an MP pricing scheme transforms the CPS game into a potential game where the potential function measures social welfare, and thus the NE of the reduced stage-one game, which corresponds to the SE of the CPS game with the MP pricing scheme, maximizes social welfare.

Now we focus on a class of MP pricing schemes that have constant reservation utility, i.e., $r_i(\mathbf{y}_{-i}) = C$ for some constant C for all $i \in \mathcal{N}$. The payment to peer i at an SE outcome $(\mathbf{x}^*, \mathbf{x}^*, \tilde{\mathbf{Z}}(\mathbf{x}^*))$ is given by

$$\begin{aligned} t_i^V(\mathbf{x}^*) &= (N-1)f(\hat{x}_\beta) - [\kappa + (N-1)\sigma + (N-2)\delta]\hat{x}_\beta \\ &\quad + [\kappa + (N-1)\sigma - \delta]x_i^* - C, \end{aligned} \tag{6}$$

and its equilibrium payoff is

$$\hat{\pi}_i(\mathbf{x}^*) = Nf^*(\beta) - C.$$

Comparing (6) with (1), we find that MP pricing schemes with constant reservation utility just compensate a peer for its effective cost of production at a PE allocation so that the net payoff does not depend on the individual production level. Since

$$\sum_{i=1}^{N} t_i^V(\mathbf{x}^*) = (N-1)Nf^*(\beta) - NC,$$

imposing budget balance (i.e., $\sum_{i=1}^{N} t_i^V(\mathbf{x}^*) = 0$) yields the following result.

Corollary 2: At SE of the CPS game with the MP pricing scheme with constant reservation utility $(N-1)f^*(\beta)$, we have $\hat{\pi}_i(\mathbf{x}^*) = f^*(\beta)$ and $\sum_{i=1}^{N} t_i^V(\mathbf{x}^*) = 0$.

In other words, the MP pricing scheme with constant reservation utility $(N-1)f^*(\beta)$ satisfies efficiency, budget balance, and incentive compatibility (in the sense of SE). Note that this result contrasts with a standard result in mechanism design, which states that the three properties cannot be easily satisfied at the same time (see, for example, Proposition 23.C.6. of [21]). In our model, the network manager can choose a constant reservation utility level to have budget balance satisfied at PE because he knows the utility functions of peers. On the contrary, when the utility functions of peers are unknown to the network manager, setting constant reservation utility in this manner usually results in r_i dependent on the announcement of peer i, which will distort its incentive for truth-telling.

B. Linear Pricing Schemes

We now propose another class of pricing schemes, called linear pricing schemes. Under a linear pricing scheme, the payment to a peer is linearly increasing in its upload volume and linearly decreasing in its download volume. Formally, the payment to peer i can be expressed as

$$t_i^L(\mathbf{Z}) = -pd_i(\mathbf{z}_i) + qu_i(\mathbf{z}^i),$$

where $p > 0$ is the price of download and $q > 0$ is the price of upload. Thus, a linear pricing scheme is fully specified by two prices, (p, q). The payoff to peer i in the CPS game with a linear pricing scheme (p, q) is given by

$$\begin{aligned} \pi_i(\mathbf{x}, \mathbf{y}, \mathbf{Z}) &= v_i(\mathbf{x}, \mathbf{y}, \mathbf{Z}) + t_i^L(\mathbf{Z}) \\ &= f(x_i + d_i) - \kappa x_i - (p+\delta)d_i + (q-\sigma)u_i. \end{aligned}$$

In effect, a linear pricing scheme (p, q) increases the cost of download from δ to $p + \delta$ and decreases the cost of upload from σ to $\sigma - q$. If the reward for upload exceeds the cost of upload, i.e., $q > \sigma$, then peers receive a net benefit from upload, which provides them with an incentive for sharing. The net revenue of the network manager can be computed by

$$-\sum_{i=1}^{N} t_i^L(\mathbf{Z}) = p\sum_{i=1}^{N} d_i - q\sum_{i=1}^{N} u_i = (p-q)w(\mathbf{Z})$$

We consider two scenarios depending on the objective of the network manager. In the monopolistic scenario, the network manager sets prices to maximize his net revenue. The optimization problem of a monopolistic network manager can be written as

$$\begin{aligned} &\max_{p,q>0} (p-q)w(\mathbf{Z}) \\ &\text{subject to } (\mathbf{x}, \mathbf{y}, \mathbf{Z}) \in SE(p, q), \end{aligned}$$

where $SE(p, q)$ is the set of SE allocations in the CPS game with linear pricing scheme (p, q). In the perfectly competitive scenario, the network manager sets prices to maximize the social welfare of peers while making non-negative net revenue. That is, a perfectly competitive network manager solves

$$\begin{aligned} &\max_{p,q>0} \sum_{i=1}^{N} \pi_i(\mathbf{x}, \mathbf{y}, \mathbf{Z}) \\ &\text{subject to } (\mathbf{x}, \mathbf{y}, \mathbf{Z}) \in SE(p, q) \text{ and } p \geq q. \end{aligned}$$

We can interpret these two scenarios as resulting from the different goals of the network manager. The network manager is motivated by self-interest in the monopolistic scenario whereas he is benevolent in the perfectly competitive scenario. It is also possible that a self-interested network manager aims to maximize social welfare if he can extract all consumer surplus from peers using a lump-sum fee. The degree of competition that the network manager faces can also determine which scenario is more relevant. If peers do not have an alternative P2P network, then the monopolistic scenario arises. On the other hand, if many P2P networks compete for peers, then the resulting outcome can be described by the perfectly competitive scenario.

We first present a preliminary result as a lemma and then analyze the monopolistic scenario.

Lemma 1: Suppose that $p + \delta \leq \kappa - (N-1)(q-\sigma)$ and $q > \sigma$. Then at SE of the CPS game with linear pricing scheme

(p, q), we have $\sum_{i=1}^N x_i = \hat{x}_{[\kappa-(N-1)(q-\sigma)]}$ and $x_i = y_i = z_{ji}$ for all $j \neq i$, for all $i \in \mathcal{N}$.

Proposition 6: Define a function $h : (0, f'(0)] \to [0, \infty)$ by $h = (f')^{-1}$. Suppose that h is twice continuously differentiable and satisfies $h(\kappa) + (\kappa - \beta)h'(\kappa) < 0$ and $2h'(\eta) + (\eta - \beta)h''(\eta) < 0$ for all $\eta \in (0, f'(0))$. Then there exists a unique $\eta^* \in (\beta, \kappa)$ that maximizes $(\eta - \beta)h(\eta)$ on $(0, f'(0)]$, and the optimal linear pricing scheme for a monopolistic network manager is given by

$$p^m = \eta^* - \delta \text{ and } q^m = \sigma + \frac{1}{N-1}(\kappa - \eta^*).$$

At SE of the CPS game with linear pricing scheme (p^m, q^m), we have $\sum_{i=1}^N x_i = \hat{x}_{\eta^*}$ and $x_i = y_i = z_{ji}$ for all $j \neq i$, for all $i \in \mathcal{N}$. The payoff to peer i at SE is given by

$$\begin{aligned}
\pi_i^m &= f(\hat{x}_{\eta^*}) - \kappa x_i - (p^m + \delta)(\hat{x}_{\eta^*} - x_i) \\
&\quad + (q^m - \sigma)(N-1)x_i \\
&= f(\hat{x}_{\eta^*}) - (p^m + \delta)\hat{x}_{\eta^*} \\
&\quad - [\kappa - (N-1)(q^m - \sigma) - (p^m + \delta)]x_i \\
&= f(\hat{x}_{\eta^*}) - \eta^* \hat{x}_{\eta^*} = f^*(\eta^*)
\end{aligned}$$

Note that π_i^m is independent of x_i since any linear pricing scheme (p, q) that satisfies $p + \delta = \kappa - (N-1)(q - \sigma)$ makes peers indifferent between being a supplier of content (i.e., to produce and upload content) and being a buyer of content (i.e., to download content). Since $\beta < \eta^* < \kappa$, we have $f^*(\kappa) < f^*(\eta^*) < f^*(\beta)$. Hence, the monopolistic linear pricing scheme (p^m, q^m) improves social welfare compared to SE of the CPS game, but does not achieve PE. This is because the monopolistic network manager wants to create a positive margin, $p - q$, which limits the utilization of the P2P network. The maximum net revenue of the monopolistic network manager is given by $N(\eta^* - \beta)\hat{x}_{\eta^*}$.

Next we analyze the perfectly competitive scenario.

Proposition 7: The optimal linear pricing scheme for a perfectly competitive network manager is given by

$$p^{pc} = q^{pc} = [\kappa + (N-1)\sigma - \delta]/N.$$

Note that $p^{pc} + \delta = \kappa - (N-1)(q^{pc} - \sigma) = \beta$. Hence, the perfectly competitive linear pricing scheme (p^{pc}, q^{pc}) achieves allocations with $\sum_{i=1}^N x_i = \hat{x}_\beta$ and $x_i = y_i = z_{ji}$ for all $j \neq i$, for all $i \in \mathcal{N}$ as SE outcomes. The payoff to each peer is given by $f^*(\beta)$, regardless of its production level, while the net revenue of the perfectly competitive network manager is 0. Since $p^{pc} = q^{pc}$, the perfectly competitive linear pricing scheme transfers payments from downloaders to uploaders in a way to achieve PE.

As an illustrative example, consider a P2P network with 50 peers, i.e., $N = 50$, whose utility functions are specified by $f(x) = \log(1 + x)$, $\kappa = 0.3$, $\delta = 0.01$, and $\sigma = 0.04$. At SE of the CPS game, each peer produces and consumes $\hat{x}_\kappa = 2.3333$ and obtains a utility of $f^*(\kappa) = 0.5040$. At PE, each peer consumes $\hat{x}_\beta = 17.1818$ and obtains an average utility of $f^*(\beta) = 1.9554$. This outcome can be achieved with the perfectly competitive linear pricing scheme $p^{pc} = q^{pc} = 0.0450$. The monopolistic linear pricing scheme is given by $p^m = 0.2245$ and $q^m = 0.0413$, where the margin $p^m - q^m = 0.1832$ can be considered as a brokerage fee paid to the network manager. With the monopolistic linear pricing scheme, each peer consumes $\hat{x}_{\eta^*} = 3.2640$ and obtains a utility of $f^*(\eta^*) = 0.6847$, while the network manager earns a net revenue of $N(\eta^* - \beta)\hat{x}_{\eta^*} = 29.2979$.

V. Conclusion

In this paper, we have constructed and analyzed an abstract model of content production and sharing over P2P networks. Using a game theoretic analysis, we have shown that peers do not share content they produce at non-cooperative equilibria whereas Pareto efficiency requires that peers share all produced content. There is also a divergence in the total amount of produced content between non-cooperative equilibria and Pareto efficiency. By imposing full sharing, we have decomposed the inefficiency of non-cooperative equilibria into two parts, inefficiency due to no sharing and inefficiency due to underproduction. We have proposed two classes of pricing schemes, MP pricing schemes and linear pricing schemes, which can achieve Pareto efficiency as non-cooperative equilibria. We have also obtained a linear pricing scheme that maximizes the revenue of the network manager. We have analyzed our model assuming one-shot interaction among homogeneous peers. Our model can be extended to take dynamic decision-making into consideration, where peers can share content they downloaded in the past. Also, investigating the case of peers with heterogeneous benefit and cost functions will offer an interesting future research topic.

Appendix
Proofs of Propositions and Lemmas

A. Proof of Proposition 1

Proof: The stage-three optimization problem for peer i at the information set $(x_i, \mathbf{y})$ can be written as

$$\max_{d_i} f(x_i + d_i) - \delta d_i$$
$$\text{subject to } 0 \leq d_i \leq \sum_{j \neq i} y_j.$$

The optimality condition for d_i is given by

- If $x_i + \sum_{j \neq i} y_j \leq \hat{x}_\delta$, then $d_i = \sum_{j \neq i} y_j$, i.e., $z_{ij} = y_j$ for all $j \neq i$.
- If $x_i + \sum_{j \neq i} y_j > \hat{x}_\delta$, then $d_i = \max\{\hat{x}_\delta - x_i, 0\}$.

We denote optimal d_i given $(x_i, \mathbf{y})$ by $d_i^*(x_i, \mathbf{y})$ and an optimal download profile of peer i by $\mathbf{z}_i^*(x_i, \mathbf{y})$. Note that $\mathbf{z}_i^*(x_i, \mathbf{y})$ is not unique if $x_i < \hat{x}_\delta$ and $x_i + \sum_{j \neq i} y_j > \hat{x}_\delta$, and we can choose an optimal download profile arbitrarily in that case. Based on $(\mathbf{z}_j^*(x_j, \mathbf{y}))_{j \neq i}$, we can derive the upload volume of peer i, written as $u_i^*(\mathbf{x}_{-i}, \mathbf{y})$.

The stage-two optimality condition for y_i at the information set x_i with a correct belief on $\mathbf{x}_{-i}$ is given by

- If $u_i^*(\mathbf{x}_{-i}, \mathbf{y}) = 0$, then $y_i \in [0, x_i]$.
- If $u_i^*(\mathbf{x}_{-i}, \mathbf{y}) > 0$, then $y_i = 0$.

Setting $y_i > 0$ can only result in incurring a cost of upload when there is a request from another peer in stage three while upload does not benefit the uploader at all. Hence, not sharing at all, i.e., choosing $y_i(x_i) = 0$ for all x_i, is a weakly dominant strategy in stage two.

We show that an SE outcome cannot have $y_i > 0$ for any i. Suppose that peer i chooses $y_i > 0$ in stage two at SE. Then it must be the case that peer i is expecting no upload in stage three, i.e., $u_i^*(\mathbf{x}_{-i}, \mathbf{y}) = 0$. By the optimality condition in stage three, this is possible only if $x_j + \sum_{k \neq j} y_k > \hat{x}_\delta$ for all $j \neq i$. Since $\hat{x}_\delta > 0$, there must exist a peer $j \neq i$ such that $x_j > 0$. Then peer j can decrease x_j in stage one, which will lead to an increase in download from peer i, z_{ji}, by the same amount in stage three. This allows peer j to consume the same amount of content with less cost, which violates optimality in stage one. Therefore, $y_i^*(x_i) = 0$ for all x_i, for all i at SE. Then the stage-one optimization problem for peer i expecting $\mathbf{y} = 0$ in stage two and thus no upload and download in stage three can be written as

$$\max_{x_i \geq 0} f(x_i) - \kappa x_i,$$

which is solved at $x_i = \hat{x}_\kappa$. ■

B. Proof of Proposition 2

Proof: Note that β is a convex combination of κ and $(\delta + \sigma)$. Since $\kappa > \delta + \sigma$, we have $\kappa > \beta > \delta + \sigma$ for any $N \geq 2$, and thus there exists a unique $\hat{x}_\beta$ that satisfies $f'(\hat{x}_\beta) = \beta$.

The social welfare function can be written as

$$\begin{aligned}\Pi(\mathbf{x}, \mathbf{y}, \mathbf{Z}) &= \sum_{i=1}^N [f(x_i + d_i) - \kappa x_i - \delta d_i - \sigma u_i] \\ &= \sum_{i=1}^N f(x_i + d_i) - \kappa \sum_{i=1}^N x_i - (\delta + \sigma) w(\mathbf{Z}).\end{aligned}$$

Thus, a PE allocation $(\mathbf{x}, \mathbf{y}, \mathbf{Z})$ solves

$$\max_{\mathbf{x}, \mathbf{y}, \mathbf{Z}} \sum_{i=1}^N [f(x_i + d_i) - \kappa x_i - (\delta + \sigma) d_i]$$

$$\text{subject to } x_i \geq 0, 0 \leq y_i \leq x_i, 0 \leq d_i \leq \sum_{j \neq i} y_j, \text{for all } i.$$

First, consider the optimal choice of d_i given $(\mathbf{x}, \mathbf{y})$. The optimality condition for d_i is given by

- If $x_i + \sum_{j \neq i} y_j \leq \hat{x}_{(\delta+\sigma)}$, then $d_i = \sum_{j \neq i} y_j$, i.e., $z_{ij} = y_j$ for all $j \neq i$.
- If $x_i + \sum_{j \neq i} y_j > \hat{x}_{(\delta+\sigma)}$, then $d_i = \max\{\hat{x}_{(\delta+\sigma)} - x_i, 0\}$.

Suppose that $\sum_{i=1}^N x_i \leq \hat{x}_{(\delta+\sigma)}$. Then given $\mathbf{x}$, it is optimal to set $y_i = x_i$ and $z_{ij} = y_j$ for all $j \neq i$, for all i. We want to show that $\sum_{i=1}^N x_i > \hat{x}_{(\delta+\sigma)}$ is not possible at PE. Suppose on the contrary that $\sum_{i=1}^N x_i > \hat{x}_{(\delta+\sigma)}$ at PE. Then we have $d_i < \sum_{j \neq i} x_j$ for all i, and thus the objective function can be improved by choosing peer i such that $x_i > 0$, reducing x_i, and increasing d_i by the same amount, where the amount adjusted is chosen so that $\mathbf{d}_{-i} \triangleq (d_1, \ldots, d_{i-1}, d_{i+1}, \ldots, d_N)$ is left unaffected.

Knowing that $y_i = x_i$ and $d_i = \sum_{j \neq i} y_j$ for all i at PE, we can reduce the problem to

$$\max_{\mathbf{x} \geq 0} N f\left(\sum_{i=1}^N x_i\right) - [\kappa + (N-1)(\delta + \sigma)] \sum_{i=1}^N x_i. \quad (7)$$

By the first-order optimality condition, any $\mathbf{x} \geq 0$ such that $\sum_{i=1}^N x_i = \hat{x}_\beta$ solves (7), and the result follows. ■

C. Proof of Proposition 3

Proof: Imposing $\mathbf{y} = \mathbf{x}$ in stage two, the stage-three optimality condition can be written as

- If $\sum_{i=1}^N x_i \leq \hat{x}_\delta$, then $d_i = \sum_{j \neq i} x_j$, i.e., $z_{ij} = x_j$ for all $j \neq i$.
- If $\sum_{i=1}^N x_i > \hat{x}_\delta$, then $d_i = \max\{\hat{x}_\delta - x_i, 0\}$.

We denote optimal d_i given the stage-one variable $\mathbf{x}$ by $d_i^*(\mathbf{x})$. The upload volume of peer i can be derived from the download decisions of other peers as in the proof of Proposition 1 and is written as $u_i^*(\mathbf{x})$.

The stage-one optimization problem for peer i can be written as

$$\max_{x_i \geq 0} f(x_i + d_i^*(\mathbf{x})) - \kappa x_i - \delta d_i^*(\mathbf{x}) - \sigma u_i^*(\mathbf{x})$$

given $\mathbf{x}_{-i}$. Suppose that $\sum_{i=1}^N x_i > \hat{x}_\delta$ at SE. Then there exists peer i such that $x_i > 0$. If $x_i \geq \hat{x}_\delta$, then peer i can increase its utility by reducing x_i. If $x_i < \hat{x}_\delta$, then peer i can improve its utility by reducing production x_i and increasing downloading d_i by the same amount. Hence, $\sum_{i=1}^N x_i \leq \hat{x}_\delta$ at SE, and thus $d_i^*(\mathbf{x}) = \sum_{j \neq i} x_j$ and $u_i^*(\mathbf{x}) = (N-1)x_i$. Then the stage-one problem can be reduced to

$$\max_{x_i \geq 0} f\left(\sum_{i=1}^N x_i\right) - [\kappa + (N-1)\sigma] x_i - \delta \sum_{j \neq i} x_j$$

given $\mathbf{x}_{-i}$. If $f'(0) < \kappa + (N-1)\sigma$, then no peer has an incentive to produce and thus $x_i = 0$ for all i at SE. If $f'(0) \geq \kappa + (N-1)\sigma$, then the optimality condition for peer i is given by

- If $\sum_{j \neq i} x_j \leq \hat{x}_\gamma$, then $x_i = \hat{x}_\gamma - \sum_{j \neq i} x_j$.
- If $\sum_{j \neq i} x_j > \hat{x}_\gamma$, then $x_i = 0$.

Hence, we obtain $\sum_{i=1}^N x_i = \tilde{x}_\gamma$ at SE. ■

D. Proof of Proposition 4

Proof: First, since $\beta < \kappa$, we have $\Pi^{PE} > \Pi^{SE} > 0$ and thus $PoA \in (0, 1)$. PoA can be made arbitrarily close to 1 by making β close to κ, or making $(\delta + \sigma)$ close to κ. It can also be made arbitrarily close to 0 by having κ close to $f'(0)$, thereby making $\hat{x}_\kappa$ and $\hat{x}_\beta$ close to 0 and a positive number, respectively.

We have $\Pi^{FS} \geq 0$, with equality when $\tilde{x}_\gamma = 0$, in which case $PoNS = \infty$. $PoNS$ can be made arbitrarily close to 0 by having σ close to 0 and f almost linear on $[0, \hat{x}_\kappa]$ with $\hat{x}_\kappa \approx \tilde{x}_\gamma > 0$. Then $f^*(\kappa) \approx 0$ while $f(\tilde{x}_\gamma) - \beta \tilde{x}_\gamma > 0$.

Since $\hat{x}_\beta$ maximizes $f(x)-\beta x$ and $\gamma > \beta$, we have $\Pi^{PE} > \Pi^{FS}$ and thus $PoU < 1$. PoU can be made arbitrarily close to 1 by having $(f')^{-1}$ almost constant on $[\beta, \gamma]$ so that $\hat{x}_\beta \approx \tilde{x}_\gamma$. Also, $PoU = 0$ when $\tilde{x}_\gamma = 0$. ∎

E. Proof of Proposition 5

Proof: Since an MP pricing scheme does not affect the stage-three optimization problem, the optimality condition for the stage-three decisions is the same as in the proof of Proposition 1. We use $d_i^*(x_i, \mathbf{y}_{-i})$ and $u_i^*(\mathbf{x}_{-i}, \mathbf{y})$ to represent the download and upload volumes of peer i, respectively, derived from the optimal choices in stage three.

The payoff of peer i in stage-two given the stage-one choices $\mathbf{x}$ can be written as

$$\begin{aligned}&\tilde{\pi}_i(\mathbf{y}|\mathbf{x})\\ &= f(x_i + d_i^*(x_i, \mathbf{y}_{-i})) - \kappa x_i - \delta d_i^*(x_i, \mathbf{y}_{-i}) - \sigma u_i^*(\mathbf{x}_{-i}, \mathbf{y})\\ &+ \sum_{j \neq i} \left[f\left(\sum_{k=1}^N y_k\right) - \kappa y_j - \delta \sum_{k \neq j} y_k - \sigma(N-1)y_j \right]\\ &- r_i(\mathbf{y}_{-i}).\end{aligned}$$

We want to show that the constraints in stage two ($y_i \leq x_i$) and in stage three ($d_i \leq \sum_{j \neq i} y_j$) are binding for each peer i. Suppose that at least one of the constraints is slack. Then the stage-two constraint must be slack since otherwise we obtain a contradiction. Peer i can improve its payoff by reducing x_i in stage one without changing its choices in stages two and three, contradicting SE. Hence, we obtain $y_i = x_i$ and $z_{ij} = y_j$ for all $j \neq i$, for all i, and the stage-one payoff function of peer i can be written as

$$\begin{aligned}&\hat{\pi}_i(\mathbf{x})\\ &= Nf\left(\sum_{i=1}^N x_i\right) - [\kappa + (N-1)(\delta + \sigma)] \sum_{i=1}^N x_i - r_i(\mathbf{x}_{-i}),\end{aligned}$$

and an optimal choice in stage one is characterized by $\sum_{i=1}^N x_i = \hat{x}_\beta$. ∎

F. Proof of Lemma 1

Proof: With a linear pricing scheme (p, q), the stage-three optimization problem for peer i can be written as

$$\begin{aligned}&\max_{d_i} f(x_i + d_i) - \kappa x_i - (p+\delta)d_i + (q-\sigma)u_i\\ &\text{subject to } 0 \leq d_i \leq \sum_{j \neq i} y_j,\end{aligned}$$

given $(x_i, \mathbf{y})$ and $\mathbf{Z}_{-i}$. Note that the condition $p + \delta \leq \kappa - (N-1)(q-\sigma)$ guarantees a positive demand for download at SE as it makes peers prefer download to production as a way of obtaining content. The optimality condition for d_i is given by

- If $x_i + \sum_{j \neq i} y_j \leq \hat{x}_{(p+\delta)}$, then $d_i = \sum_{j \neq i} y_j$, i.e., $z_{ij} = y_j$ for all $j \neq i$.
- If $x_i + \sum_{j \neq i} y_j > \hat{x}_{(p+\delta)}$, then $d_i = \max\{\hat{x}_{(p+\delta)} - x_i, 0\}$.

We use $d_i^*(x_i, \mathbf{y}_{-i})$ to denote the optimal download volume of peer i. We look for an equilibrium in which each peer i downloads from other peers proportionally to their sharing levels, i.e.,

$$z_{ij}^*(x_i, \mathbf{y}_{-i}) = \frac{y_j}{\sum_{k \neq i} y_k} d_i^*(x_i, \mathbf{y}_{-i}). \quad (8)$$

Then the upload volume of peer i is given by $u_i^*(\mathbf{x}_{-i}, \mathbf{y}) = \sum_{j \neq i} z_{ji}^*(x_j, \mathbf{y}_{-j})$.

In stage two, peer i solves

$$\begin{aligned}&\max_{y_i} f(x_i + d_i^*(x_i, \mathbf{y}_{-i})) - \kappa x_i\\ &\qquad - (p+\delta)d_i^*(x_i, \mathbf{y}_{-i}) + (q-\sigma)u_i^*(\mathbf{x}_{-i}, \mathbf{y})\\ &\text{subject to } 0 \leq y_i \leq x_i,\end{aligned}$$

given x_i, $\mathbf{y}_{-i}$, and a belief on $\mathbf{x}_{-i}$. Since $q - \sigma > 0$, it is optimal for peer i to choose $y_i = x_i$ if u_i^* is increasing in y_i. Given (8), the only case where we may have $y_i < x_i$ optimal is when $u_i^*(\mathbf{x}_{-i}, \mathbf{y}) = 0$. This implies that $x_j \geq \hat{x}_{(p+\delta)}$ for all $j \neq i$. Suppose that $x_i > \hat{x}_{(p+\delta)}$. Then $d_i = u_i = 0$ and thus $\pi_i = f(x_i) - \kappa x_i$, but π_i can be increased by reducing x_i as $p + \delta < \kappa$, violating optimality in stage one. Now suppose that $x_i \leq \hat{x}_{(p+\delta)}$. If $x_i + \sum_{j \neq i} y_j \leq \hat{x}_{(p+\delta)}$, then $z_{ij} = y_j$ and $\pi_j = f(x_j) - \kappa x_j + (q-\sigma)y_j$ for all $j \neq i$. There must exist peer $j \neq i$ such that $y_j < x_j$, and peer j can increase π_j by increasing y_j. If $x_i + \sum_{j \neq i} y_j > \hat{x}_{(p+\delta)}$, then $\pi_i = f(\hat{x}_{(p+\delta)}) - \kappa x_i - (p+\delta)(\hat{x}_{(p+\delta)} - x_i)$. Since π_i is decreasing in x_i, π_i can be improved if $x_i > 0$, violating optimality in stage one. However, if $x_i = 0$, then $y_i = 0$ is the only feasible choice in stage two. Therefore, it is optimal for peer i to choose $y_i = x_i$ in stage two.

Predicting optimal decisions in stages two and three, peer i solves in stage one

$$\begin{aligned}&\max_{x_i \geq 0} f(x_i + d_i^*(x_i, \mathbf{x}_{-i})) - \kappa x_i\\ &\qquad - (p+\delta)d_i^*(x_i, \mathbf{x}_{-i}) + (q-\sigma)u_i^*(\mathbf{x}_{-i}, \mathbf{x})\end{aligned}$$

given $\mathbf{x}_{-i}$. Suppose that $\sum_{i=1}^N x_i > \hat{x}_{(p+\delta)}$. If $x_i > \hat{x}_{(p+\delta)}$, then $\pi_i = f(x_i) - \kappa x_i + (q-\sigma)u_i^*(\mathbf{x}_{-i}, \mathbf{x})$. Then

$$\frac{\partial \pi_i}{\partial x_i} = f'(x_i) - \kappa + (q-\sigma)\frac{\partial u_i^*}{\partial x_i}.$$

Note that $\partial u_i^* / \partial x_i < N - 1$ when $\sum_{i=1}^N x_i > \hat{x}_{(p+\delta)}$, and $x_i > \hat{x}_{(p+\delta)}$ implies $f'(x_i) < p + \delta$. Hence,

$$\frac{\partial \pi_i}{\partial x_i} < p + \delta - \kappa + (N-1)(q-\sigma) \leq 0,$$

and π_i can be increased by decreasing x_i, yielding a contradiction. Thus, we have $x_i \leq \hat{x}_{(p+\delta)}$ for all i. Then $\pi_i = f(\hat{x}_{(p+\delta)}) - \kappa x_i - (p+\delta)(\hat{x}_{(p+\delta)} - x_i) + (q-\sigma)u_i^*(\mathbf{x}_{-i}, \mathbf{x})$, and

$$\begin{aligned}\frac{\partial \pi_i}{\partial x_i} &= -\kappa + (p+\delta) + (q-\sigma)\frac{\partial u_i^*}{\partial x_i}\\ &< -\kappa + (p+\delta) + (N-1)(q-\sigma) \leq 0.\end{aligned}$$

Since $\sum_{i=1}^N x_i > \hat{x}_{(p^*+\delta)}$, there must exist peer i with $x_i > 0$, and peer i can improve π_i by decreasing x_i, yielding a contradiction again.

Therefore, at SE of the CPS game with the linear pricing scheme (p,q), we should have $\sum_{i=1}^{N} x_i \leq \hat{x}_{(p+\delta)}$. Then $d_i^*(x_i, \mathbf{x}_{-i}) = \sum_{j\neq i} x_j$ and $u_i^*(\mathbf{x}_{-i}, \mathbf{x}) = (N-1)x_i$, and the stage-one problem for peer i becomes

$$\max_{x_i \geq 0} f\left(\sum_{i=1}^{N} x_i\right) - \kappa x_i - (p+\delta)\sum_{j\neq i} x_j + (q-\sigma)(N-1)x_i$$

given $\mathbf{x}_{-i}$. Using a first-order optimality condition, we can show that SE satisfies $\sum_{i=1}^{N} x_i = \hat{x}_{[\kappa-(N-1)(q-\sigma)]}$. ■

G. Proof of Proposition 6

Proof: In order to induce sharing, the network manager needs to set $q > \sigma$. Also, we assume that the network manager chooses (p,q) that satisfies $p+\delta \leq \kappa-(N-1)(q-\sigma)$.[5] Since $w(\mathbf{Z}) = (N-1)\hat{x}_{[\kappa-(N-1)(q-\sigma)]}$ at SE by Lemma 1, the objective function of the monopolistic network manager is given by $(N-1)(p-q)\hat{x}_{[\kappa-(N-1)(q-\sigma)]}$. Hence, given q, it is optimal to set p such that $p+\delta = \kappa-(N-1)(q-\sigma)$. Let η denote the common value of $p+\delta$ and $\kappa-(N-1)(q-\sigma)$. Then we have $p = \eta-\delta$,

$$q = \sigma + \frac{1}{N-1}(\kappa-\eta), \text{ and } p-q = \frac{N}{N-1}(\eta-\beta).$$

Also, $q > \sigma$ is equivalent to $\eta < \kappa$. The problem of the monopolistic network manager can be rewritten as

$$\max_{0<\eta<\kappa} N(\eta-\beta)h(\eta). \tag{9}$$

The first-order necessary condition for η^* to be a solution of (9) is given by

$$h(\eta^*) + (\eta^*-\beta)h'(\eta^*) = 0,$$

while the second-order sufficient condition is given by

$$2h'(\eta) + (\eta-\beta)h''(\eta) < 0 \text{ for all } \eta \in (0, f'(0)).$$

Note that $h(\beta) = \hat{x}_\beta > 0$, and thus $\eta^* > \beta$. Also, the condition $h(\kappa) + (\kappa-\beta)h'(\kappa) < 0$ implies that $\eta^* < \kappa$. Thus, we obtain the optimal linear pricing scheme for the monopolistic network manager (p^m, q^m) as in Proposition 6. ■

H. Proof of Proposition 7

Proof: Consider a linear pricing scheme (p,q) such that $p+\delta \leq \kappa-(N-1)(q-\sigma)$ and $q > \sigma$. Then using Lemma 1, the social welfare of peers at SE of the CPS game with the linear pricing scheme (p,q) is given by

$$\sum_{i=1}^{N} \pi_i = Nf(\hat{x}_{[\kappa-(N-1)(q-\sigma)]}) - [\kappa+(N-1)(p-q+\delta+\sigma)]\hat{x}_{[\kappa-(N-1)(q-\sigma)]}. \tag{10}$$

[5]If the condition $p+\delta \leq \kappa-(N-1)(q-\sigma)$ is not satisfied, SE depends on how $u_i^*(\mathbf{x}_{-i}, \mathbf{y})$ responds to y_i. If each peer i perceives that z_{ji}^* does not react to y_i when $d_j^* > \sum_{k\neq j} y_k$ for all $j \neq i$, then Proposition 6 holds without requiring $p+\delta \leq \kappa-(N-1)(q-\sigma)$.

Since $\sum_{i=1}^{N} \pi_i$ is decreasing in p, it is optimal for the perfectly competitive network manager to set $p = q$. Then (10) is reduced to

$$\begin{aligned}\sum_{i=1}^{N} \pi_i &= Nf(\hat{x}_{[\kappa-(N-1)(q-\sigma)]}) \\ &\quad - [\kappa+(N-1)(\delta+\sigma)]\hat{x}_{[\kappa-(N-1)(q-\sigma)]} \\ &= N[f(\hat{x}_{[\kappa-(N-1)(q-\sigma)]}) - \beta f(\hat{x}_{[\kappa-(N-1)(q-\sigma)]})],\end{aligned}$$

which is maximized when $\kappa-(N-1)(q-\sigma) = \beta$. Rearranging this yields q^{pc} as in Proposition 7. ■

References

[1] H. Park and M. van der Schaar, "A framework for foresighted resource reciprocation in P2P networks," *IEEE Trans. Multimedia*, vol. 11, no. 1, pp. 101–116, Jan. 2009.

[2] J. Liu, S. G. Rao, B. Li, and H. Zhang, "Opportunities and challenges of peer-to-peer internet video broadcast," in *Proc. IEEE, Special Issue on Recent Advances in Distributed Multimedia Communications*, 2007.

[3] E. Adar and B. A. Huberman, "Free riding on Gnutella," *First Monday*, vol. 5, no. 10, 2000.

[4] O. Loginova, H. Lu, and X. H. Wang, "Incentive schemes in peer-to-peer networks," *B.E. J. Theoretical Econ.*, vol. 9, no. 1, Article 2, 2009.

[5] R. Axelrod, *The Evolution of Cooperation*. New York: Basic Books, 1984.

[6] M. Kandori, "Social norms and community enforcement," *Review Econ. Stud.*, vol. 59, no. 1, pp. 63–80, Jan. 1992.

[7] G. Ellison, "Cooperation in the prisoner's dilemma with anonymous random matching ," *Review Econ. Stud.*, vol. 61, no. 3, pp. 567–588, Jul. 1994.

[8] A. Blanc, Y.-K. Liu, and A. Vahdat, "Designing incentives for peer-to-peer routing," in *Proc. INFOCOM 2005*, pp. 374–385.

[9] P. Golle, K. Leyton-Brown, and I. Mironov, "Incentives for sharing in peer-to-peer networks," in *Proc. 3rd ACM Conf. Electronic Commerce*, Tampa, FL, Oct. 2001, pp. 264–267.

[10] A. Galeotti and S. Goyal, "The law of the few," *Amer. Econ. Review*, to be published.

[11] Y. Li, Z. Li, M. Chiang, and A. R. Calderbank, "Video transmission scheduling for peer-to-peer live streaming systems," in *Proc. 2008 Int. Conf. Multimedia and Expo*, Hannover, Germany, pp. 653–656.

[12] C. Buragohain, D. Agrawal, and S. Suri, "A game theoretic framework for incentives in P2P systems," in *Proc. 3rd Int. Conf. Peer-to-Peer Computing*, Linkoping, Sweden, 2003, pp. 48–56.

[13] M. van der Schaar, D. S. Turaga, and R. Sood, "Stochastic Optimization for Content Sharing in P2P Systems," *IEEE Trans. Multimedia*, vol. 10, no. 1, pp. 132–144, Jan. 2008.

[14] B. Cohen, "Incentives build robustness in BitTorrent," in *Proc. P2P Econ. Workshop*, Berkeley, CA, 2003.

[15] D. Fudenberg and J. Tirole, *Game Theory*. Cambridge, MA: MIT Press, 1991.

[16] W. Vickrey, "Counterspeculation, auctions, and competitive sealed-tenders," *J. Finance*, vol. 16, no. 1, pp. 8–37, Mar. 1961.

[17] E. H. Clarke, "Multipart pricing of public goods," *Public Choice*, vol. 11, no. 1, pp. 17–33, Sep. 1971.

[18] T. Groves, "Incentives in teams," *Econometrica*, vol. 41, no. 4, pp. 617–631, Jul. 1973.

[19] D. Monderer and L. S. Shapley, "Potential games," *Games and Econ. Behavior*, vol. 14, no. 1, pp. 124–143, May 1996.

[20] D. Wolpert and K. Tumor, "An overview of collective intelligence," in *Handbook of Agent Technology*, J. M. Bradshaw, Ed. AAAI Press/MIT Press, 1999.

[21] A. Mas-Colell, M. D. Whinston, and J. R. Green, *Microeconomic Theory*, Oxford, U.K.: Oxford Univ. Press, 1995.