# Mining the Situation: Spatiotemporal Traffic Prediction with Big Data 

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#### Abstract

With the vast availability of traffic sensors from which traffic information can be derived, a lot of research effort has been devoted to developing traffic prediction techniques, which in turn improve route navigation, traffic regulation, urban area planning and etc. One key challenge in traffic prediction is how much to rely on prediction models that are constructed using historical data in real-time traffic situations, which may differ from that of the historical data and change over time. In this paper, we propose a novel online framework that could learn from the current traffic situation (or context) in real-time and predict the future traffic by matching the current situation to the most effective prediction model trained using historical data. As real-time traffic arrives, the traffic context space is adaptively partitioned in order to efficiently estimate the effectiveness of each base predictor in different situations. We obtain and prove both short-term and long-term performance guarantees (bounds) for our online algorithm. The proposed algorithm also works effectively in scenarios where the true labels (i.e. realized traffic) are missing or become available with delay. Using the proposed framework, the context dimension that is the most relevant to traffic prediction can also be revealed, which can further reduce the implementation complexity as well as inform traffic policy making. Our experiments with real-world data in reallife conditions show that the proposed approach significantly outperforms existing solutions.


## I. Introduction

Traffic congestion causes tremendous loss in terms of both time and energy wasted. According to a recent report from the Texas Transportation Institute [1], in 2007, all 439 metropolitan areas experienced 4.2 billion vehicle-hours of delay, which is equivalent to 2.8 billion gallons in wasted fuel and $\$ 87.2$ billion in lost productivity, or about $0.7 \%$ of the nation's GDP. Traffic congestion is caused when the traffic demand approaches or exceeds the available capacity of the traffic system. In United States, Federal Highway Administration [2] [3] has observed that the number of miles of vehicle travel increased by 76 percent from 1980 to 1999, while the total miles of highway increased merely by 1.5 percent, which hardly accommodates growth in travel. It is now generally conceded that it is impossible to build our way out of congestion, mainly because increased capacity results in induced demand. These factors motivate an information-based approach to address these problems.
Fortunately, due to thorough sensor instrumentations of road networks in major cities as well as the vast availability of

[^0]auxiliary commodity sensors from which traffic information can be derived (e.g., CCTV cameras, GPS devices), a large volume of real-time and historical traffic data at very high spatial and temporal resolutions has become available. Several companies, such as Inrix, now sell both types and at our research center we have had access to both datasets from Los Angeles County for the past three years. As shown by many studies [4] [5] [6] [7] [8], these traffic datasets can be used to predict traffic congestion, which in turn enables drivers to avoid congested areas (e.g., through intelligent navigation systems), policy makers to decide about changes to traffic regulations (e.g., replace a carpool lane with a toll lane), urban planners to design better pathways (e.g., adding an extra lane) and civil engineers to plan better for construction zones (e.g., how a short-term construction would impact traffic).
One major challenge in predicting traffic is how much to rely on the prediction model constructed using historical data in the real-time traffic situation, which may differ from that of the historical data due to the fact that traffic situations are numerous and changing over time. Previous studies showed that depending on the traffic situation one prediction model may be more useful than the other. For example, in [7] it is shown that a hybrid forecasting model that selects in real-time depending on the current situation between Auto-Regressive Integrated Moving Average (ARIMA) model and Historical Average Model (HAM) model yields significant better prediction accuracy. It is shown that the ARIMA prediction model is more effective in predicting the speed in normal conditions but at the edges of the rush-hour time (i.e., the beginning and the end of rush hour), the HAM model is more useful. This becomes even more challenging when considering different causes for congestion, e.g., recurring (e.g., daily rush hours), occasional (e.g., weather conditions), unpredictable (e.g., accidents), and temporarily - for short term (e.g., a basketball game) or long term (e.g., road construction) congestions. However there is no holistic approach on when and in which situations to switch from one prediction model to the other for a more effective prediction. The exhaustive method that trains for each traffic situation a prediction model is obviously impractical since it would induce extremely high complexity due to the numerous possible traffic situations.
Our main thesis in this paper is that we try to learn from the current traffic situation in real-time and predict the future traffic by matching the current situation to the most effective prediction model that we constructed using historical data. First, a finite (possibly small) number of traffic predictors are constructed for the same number of representative traffic conditions using historical data. Using a small set of base
predictors reduces the training and maintenance costs. Given this set of base predictors, we learn to select the most effective predictor that best suits the current traffic situation in realtime. For instance, suppose we have two traffic predictors trained on historical datasets in different weather conditions, sunny and rainy. We will learn online which predictor to use for prediction in a cloudy weather which does not have a predictor trained for it. The basic idea to learn and select the most effective predictor is based on estimating the reward of a predictor in different situations. The reward estimate is calculated based on how accurate each predictor has been in predicting, say, speed value, given the actual speed values we have observed in the recent past via the real-time data. However, significant challenges still remain as we will explain shortly.

Many features can be used to identify a traffic "situation", which henceforth are called context. Example features include: location, time, weather condition, number of lanes, area type (e.g., business district, residential), etc. Therefore, the context space is a multidimensional space with $D$ dimensions, where $D$ is the number of features. Since $D$ can be very large and the value space can be continuous, learning the most effective predictor in each individual context using reward estimates for this individual context (i.e. a $D$-dimensional point in the context space) can be extremely slow. For example, there are numerous possible weather conditions (characterized by temperature, humidity, wind speed etc.) but each specific weather condition only appears occasionally in real-time. Thus, we may initially group weather conditions into rough categories such as sunny, rainy, cloudy etc. and then refine each category to improve prediction. However, how to adaptively group contexts and partition the context space poses a significant challenge for fast learning of the best predictor for different traffic contexts. Moreover a rigourous performance characterization of such a method is missing. These are the problems that we are going to solve in this paper.
Our approach has three important byproducts. First, since the reward is continuously being updated/aggregated, we are utilizing what we learn in real-time to adapt to both shortterm and long-term changes. For example, suppose that on a location that is a 3-lane freeway, one of the lanes is closed for a week due to construction. Our approach that is originally using the predictor that is the most effective to a 3-lane subspace would start observing that the predictor does not work well for the 2-lane case and thus switching to the new predictor. Later once the construction is complete, the reward mechanism would guide our approach to go back to the previous 3-lane predictor. Second, our approach is agnostic to the congestion cause. For example, our reward mechanism may guide us to select a predictor that is trained for a rush-hour subspace, even though the current time is not a rush-hour time, but perhaps because of an incident (e.g., an unknown object in the middle of the freeway) that resulted in a similar traffic condition as that of a rush hour at that location. Therefore, we can still predict the traffic condition successfully in the presence of that event. Finally, since location and time are two features of our context space, our approach is inherently spatiotemporal and takes into consideration the sensor readings that are spatially
and temporally closest to each other.
To evaluate our approach, we obtain and prove both shortterm and long-term performance guarantees (bounds) for our online algorithm. This provides not only the assurance that our algorithm will converge over time to the optimal predictor for each possible traffic situation (i.e., there is no loss in terms of the average reward) but also provides a bound for the speed of convergence of our algorithm to the optimal predictor (i.e., our algorithm is fast to converge to the optimal performance). In addition, we conducted a number of experiments to verify our approach with real-world data in real-life conditions. The results show our approach significantly outperforms existing approaches that do not adapt to the varying traffic situations.
The remainder of the paper is organized as follows. Section II reviews the related work and highlights the distinctions of our approach. Section III formulates the traffic prediction problem and defines the performance metric. Section IV describes our context-aware adaptive traffic prediction algorithm. Section V discusses several ways to optimize our algorithm. Section VI reports our experimental results with real world traffic datasets. Section VII concludes the paper.

## II. Related work

In this related work section, we first compare our scheme against other existing traffic prediction works (i.e. applicationrelated work) and afterwards we compare our work against various classes of online learning techniques (i.e. algorithm and theory related work).

## A. Traffic prediction

Several traffic prediction techniques have been studied in the past. The majority of these techniques focus on predicting traffic in typical conditions (e.g., morning rush hours) [4] [7] [9] [10], and more recently in the presence of accidents, e.g., [5] [4]. Both qualitative [11] and quantitative [12] approaches have been used to measure the impact of an accident on road networks and various machine learning techniques have been applied to predict the typical traffic conditions and the impact of accidents, including Naive Bayesian [13], Decision Tree Learning [6], and Nearest Neighbor [14]. The main differences between our work and the existing studies on traffic prediction are: 1) All existing techniques for traffic prediction are aimed at predicting traffic in specific traffic situations, e.g. either typical conditions or when accidents occur. Instead, our scheme is applicable to all traffic situations and learns to match the current traffic situation to the best traffic prediction model, by exploiting spatiotemporal and other context similarity information. 2) All existing techniques used for traffic prediction deploy models learned offline (i.e. they rely on a priori training sessions) or that they are retrained after long periods and thus, they cannot adapt (learn from) dynamically changing traffic situations. Instead, our scheme is able to dynamically adapt to the changing traffic situations on the fly and improve the traffic prediction overtime as additional traffic data is received. 3) Most existing works are based on empirical studies and do not offer rigorous performance guarantees for traffic prediction. Instead, our scheme is able
to provide both short-term and long-term performance bounds in dynamically changing traffic situations.

## B. Ensemble learning

Our framework builds a hybrid traffic predictor on top of a set of base predictors and thus, it appertains to the class of ensemble learning techniques. Traditional ensemble schemes [15] [16] for data analysis are mostly focused on analyzing offline datasets; examples of these techniques include bagging [15] and boosting [16]. In the past decade much work has been done to develop online versions of such ensemble techniques. For example, an online version of Adaboost is described in [17]. Another strand of literature on online ensemble learning is represented by the weight update schemes [18] [19] [20] [21] [22] that maintain a collection of given predictors, predict using a weighted majority rule, and update online the weights associated to the learners. Most of these schemes develop multiplicative update rules [18] [19] [20]. For example, weighted majority in [18] decreases the weights of predictors in the pool that disagree with the true label whenever the ensemble makes a mistake. Additive weight update is adopted in [21] where the weights of learners that predict correctly are increased by a certain amount. In [22], weights of the learners are updated based on stochastic gradient descent. Most prior works on ensemble learning provide algorithms which are only asymptotically converging to an optimal or locally-optimal solution without providing any rates of convergence. On the contrary, we not only prove convergence results, but we are also able to explicitly characterize the performance loss incurred at each time step with respect to the omniscient oracle (i.e. complete knowledge) benchmark which knows the accuracies of all classifiers for the data. Specifically, we prove regret bounds that hold uniformly over time. Moreover, existing works do not exploit the contextual information associated with the data. In contrast, we focus on how contextual specialization of predictors can be discovered over time to create a strong predictor from many weak predictors in different contexts.

## C. Contextual multi-armed bandits

The proposed algorithm is analyzed using techniques developed for the contextual multi-armed bandits (MAB) problems. Previously, MAB methods were applied to solve problems in clinical trials [23] [24], multi-user communication networks [25], web advertising [26], recommender systems [27] [28] and stream mining systems [29] [30]. A key advantage of MAB methods as compared to other online learning methods is that they can provide a bound on the convergence speed as well as a bound on the loss due to learning compared to an oracle solution which requires knowledge of the stochastic model of the system, which is named regret. To the authors' best knowledge, this paper is the first attempt to develop MAB methods to solve spatiotemporal traffic prediction problems for intelligent transportation systems. In existing works on MAB, the learner can only observe the reward of the selected action (which in our case is the base predictor). In contrast, in this paper, the learner can observe the rewards of all predictors since the prediction action does not have an explicit impact
on reward realization. This makes the learning much faster than existing works. Moreover, this paper also studies which context dimension (or set of context dimensions) is more relevant to the traffic prediction problem, thereby reducing the implementation complexity and providing guidelines for traffic policy making by identifying what are the causes for various traffic situations.

## III. Problem Formulation

## A. Problem setting

Figure 1 illustrates the system model under consideration. We consider a set of locations $\mathcal{L}$ where traffic sensors are deployed. These locations can be either on the highways or arterial streets. We consider an infinite horizon discrete time system $t=1,2, \ldots$ where in each slot $t$ a traffic prediction request from one of the location $l_{o} \in \mathcal{L}$ arrives to the system in sequence. Given the current traffic speed $\boldsymbol{x}^{t}$ at this location, the goal is to predict the traffic speed $\hat{y}^{t}$ in some predetermined future time, e.g. in the next 15 minutes or in the next 2 hours. Note that the notation $t$ is only used to order the requests according to their relative arrival time. Each request can come from any location in $\mathcal{L}$ at any time in a day, thereby posing a spatiotemporal prediction problem.

Each request is associated with a set of traffic context information which is provided by the road sensors. The context information can include but is not limited to:

- The location context, e.g. the longitude and latitude of the requested location $l_{o}$, the location type (highway, arterial way), the area type (business district, residential).
- The time context, e.g. whether on weekday or weekend, at daytime or night, in the rush hour or not, etc.
- The incident context, e.g. whether there is a traffic incident occurred nearby and how far away from $l_{o}$, the type of the incident, the number of affected lanes etc.
- Other contexts such as weather (temperature, humidity, wind speed etc.), temporary events etc.
We use the notation $\theta^{t} \in \Theta$ to denote the context information associated with the $t$-th request where $\Theta$ is a $D$-dimensional space and $D$ is the number of types of context used. Without loss of generality, we normalize the context space $\Theta$ to be $[0,1]^{D}$. For example, time in a day can be normalized with respect to 24 hours.

The system maintains a set of $K$ base predictors $f \in \mathcal{F}$ that can take input of the current speed $\boldsymbol{x}^{t}$, sent by the road sensors, and output the predicted speed $f\left(\boldsymbol{x}^{t}\right)$ in the predetermined future at location $l_{o}$. These base predictors are trained and constructed using historical data for $K$ representative traffic situations before the system operates. However, their performance is unknown for the other traffic situations since they are changing over time. We aim to build a hybrid predictor that selects the most effective predictor for the real-time traffic situations by exploiting the traffic context information. Thus, for each request, the system selects the prediction result of one of the base predictors as the final traffic prediction result, denoted by $y^{t}$. The prediction result can be consumed by thirdparty applications such as navigation.


Fig. 1. System Diagram.

Eventually, the real traffic at the predetermined future for the $t$-th request, denoted by $\hat{y}^{t}$, is revealed. We also call $\hat{y}^{t}$ the ground-truth label for the $t$-th request. For now we assume that the label is revealed for each request at the end of each prediction. In reality, the label can arrive with delay or even be missing. We will consider these scenarios in Section V. By comparing the system predicted traffic $y^{t}$ and the true traffic $\hat{y}^{t}$, a reward $r^{t}$ is obtained according to a general reward function $r^{t}=R\left(y^{t}, \hat{y}^{t}\right)$. For example, a simple reward function indicates the accuracy of the prediction, i.e. $R\left(y^{t}, \hat{y}^{t}\right)=I\left(y^{t}=\hat{y}^{t}\right)$ where $I(\cdot)$ is the indicator function. The system obtains a reward of 1 only if the prediction is correct and 0 otherwise. Other reward functions that depend on how close the prediction is to the true label can also be adopted.

As mentioned, each based predictor is a function of the current traffic $\boldsymbol{x}^{t}$ which outputs the future traffic prediction $y^{t}$. Since for a given $\boldsymbol{x}^{t}$ the true future traffic $\hat{y}^{t}$ is a random variable, the reward by selecting a predictor $f$, i.e. $R\left(f\left(\boldsymbol{x}^{t}\right), \hat{y}^{t}\right)$, is also a random variable at each $t$. The effectiveness of a base predictor is measured by its expected reward, which depends on the underlying unknown joint distribution of $\boldsymbol{x}^{t}$ and $\hat{y}^{t}$. The effectiveness of a base predictor in a traffic context $\theta$ is thus its expected reward conditional on $\theta$ and is determined by the underlying unknown joint distribution of $\boldsymbol{x}^{t}$ and $\hat{y}^{t}$ conditional on the event $\theta$. Let $\pi_{f}(\theta)=E\{R(f(x), \hat{y}) \mid \theta\}$ be the expected reward of a predictor $f$ in context $\theta$. However, the base predictors are constructed using historical data can thus, their expected reward is unknown a priori for real-time situations which may vary over time. Therefore, the system will continuously revise its selection of base predictors as it learns better and better the base predictors' expected rewards in the current context.
B. Spatiotemporal prediction and multi-predictor diversity gain

By taking into consideration of the traffic context information when making traffic prediction, we are exploiting the multi-predictor diversity to improve the prediction performance. To get a sense of where the multi-predictor diversity gain comes from, consider the simple example in Figure 2, which shows the expected rewards of various base predictors. Since the traffic prediction is a spatiotemporal problem, we use both time and location of the traffic as the context information. Given a location at 5 miles from the reference location, we have three predictors constructed for three representative traffic situations - morning around 6 am , afternoon around 2 pm and evening around 7 pm . These predictors work effectively in their corresponding situations but may not work well in other time contexts due to the different traffic conditions in different time of the day. If we use the same predictor for the entire day, then the average prediction performance can be very bad. Instead, if we use the predictors for traffic situations that are similar to the representative situation, then much better prediction performance can be obtained. However, the challenge is when to use which predictor for prediction since the effectiveness of the base predictors is unknown for every traffic context. For example, the three base predictors ( 0 mile, 5 miles, 10 miles) given time 12 pm have complex expected reward curves which need to be learned over time to determine which predictor is the best at different locations.

## C. Performance metric for our algorithm

The goal of our system is to learn the optimal hybrid predictor which selects the most effective base predictor for each traffic situation. Since we do not have the complete knowledge of the performance of all base predictors for all


Fig. 2. Spatiotemporal prediction and multi-predictor diversity gain.
contexts in the online environment, we will develop online learning algorithms that learn to select the best predictors for different traffic contexts over time. The benchmark when evaluating the performance of our learning algorithm is the optimal hybrid predictor that is constructed by an oracle that has the complete information of the expected rewards of all base predictors in all situations. For a traffic context $\theta$, the optimal base predictor selected in the oracle benchmark is

$$
\begin{equation*}
f^{o p t}(\theta):=\arg \max _{f \in \mathcal{F}} \pi_{f}(\theta), \forall \theta \in \Theta \tag{1}
\end{equation*}
$$

Let $\sigma$ be a learning algorithm and $f^{\sigma(t)}$ be the predictor selected by $\sigma$ at time $t$, then the regret of learning by time $T$ is defined as the aggregate reward difference between our learning algorithm and the oracle solution up to $T$, i.e.

$$
\begin{equation*}
\operatorname{Reg}(T):=\sum_{t=1}^{T} \pi_{f^{*}\left(\theta^{t}\right)}\left(\theta^{t}\right)-E\left[\sum_{t=1}^{T} R\left(f^{\sigma(t)}\left(x^{t}\right), \hat{y}^{t}\right)\right] \tag{2}
\end{equation*}
$$

where the expectation is taken with respect to the randomness of the prediction, true traffic realization and predictors selected. The regret characterizes the loss incurred due to the unknown transportation system dynamics and gives the convergence rate of the total expected reward of the learning algorithm to the value of the optimal hybrid predictor in (1). The regret is non-decreasing in the total number of requests $T$ but we want it to increase as slow as possible. Any algorithm whose regret is sublinear in $T$, i.e. $\operatorname{Reg}(T)=O\left(T^{\gamma}\right)$ such that $\gamma<1$, will converge to the optimal solution in terms of the average reward, i.e. $\lim _{T \rightarrow \infty} \frac{\operatorname{Reg}(T)}{T}=0$. The regret of learning also gives a measure for the rate of learning. A smaller $\gamma$ will result in a faster convergence to the optimal average reward and thus, learning the optimal hybrid predictor is faster if $\gamma$ is smaller.

## IV. Context-Aware Adaptive Traffic Prediction

A natural way to learn a base predictor's performance in a non-representative traffic context is to record and update its sample mean reward as additional data (i.e. traffic requests) in the same context arrives. Using such a sample mean-based approach to construct a hybrid predictor is the basic idea of our learning algorithm; however, significant challenges still remain.

One the one hand, exploiting the context information can potentially boost the prediction performance as it provides ways to construct a strong hybrid predictor as suggested in Section III(B). Without the context information, we would only learn the average performance of each predictor over all contexts and thus, a single base predictor would always be selected even though on average it does not perform well. On the other hand, building the optimal hybrid predictor can be very difficult since the context space $\Theta$ can be very large and the value space can even be continuous. Thus, the sample mean reward approach would fail to work efficiently due to the small number of samples for each individual context $\theta$.

Our method to overcome this problem is to dynamically partition the entire context space into multiple smaller context subspaces and maintain and update the sample mean reward estimates for each subspace. This is due to the fact that the expected rewards of a predictor are likely to be similar for similar contexts. For instance, similar weather conditions would have similar impacts on the traffic on close locations. Next, we will propose an online prediction algorithm that adaptively partitions the context space according to the traffic prediction request arrivals on the fly and guarantees the sublinear learning regret.

## A. Algorithm description

In this subsection, we describe the proposed online adaptive traffic prediction algorithm (CA-Traffic). First we introduce several useful concepts for describing the proposed algorithm.

- Context subspace. A context subspace $C$ is a subspace of the entire context space $\Theta$, i.e. $C \subseteq \Theta$. In this paper, we will consider only context subspaces that are created by uniformly partitioning the context space on each dimension, which is enough to guarantee sublinear learning regrets. Thus, each context subspace is a $D$ dimensional hypercube with side length being $2^{-l}$ for some $l$. We call such a hypercube a level- $l$ subspace. For example, when the entire context space is $[0,1]$, namely the context dimension is $D=1$, the entire context space is a level-0 subspace, $[0,1 / 2)$ and $[1 / 2,1]$ are two level1 subspaces, $[0,1 / 4),[1 / 4,1 / 2),[1 / 2,3 / 4),[3 / 4,1]$ are four level-2 subspaces etc.
- Context space partition. A context space partition $\mathcal{P}$ is a set of non-overlapping context subspaces that cover the entire context space. For example, when $D=1,\{[0,1]\}$, $\{[0,1 / 2),[1 / 2,3 / 4),[3 / 4,1]\}$ are two context space partitions. Since our algorithm will adaptively partition the context space by adaptively removing subspaces from the partition and adding new subspaces into the partition, the context space partition is time-varying depending on the context arrival process of the traffic requests. Initially, the context space partition includes only the entire context space, i.e. $\mathcal{P}^{0}=\{\Theta\}$.
- Active context subspace. A context subspace $C$ is active if it is in the current context space partition $\mathcal{P}^{t}$, at time $t$. For each active context subspace $C \in \mathcal{P}^{t}$, the algorithm maintains the sample mean reward estimates $\bar{r}_{f}^{t}(C)$ for each for the predictor for the context arrivals to this subspace. For each active subspace $C \in \mathcal{P}^{t}$, the algorithm also maintains a counter $M_{C}^{t}$ that records the number of context arrivals to $C$.
The algorithm works as follows (See Algorithm 1). We will describe the algorithm in two parts. The first part (line 3-8) is the predictor selection and reward estimates update. When a traffic prediction request comes, the current and past traffic speed vector $\boldsymbol{x}^{t}$ along with the traffic context information $\theta^{t}$ are sent to the system. The algorithm first checks to which active subspace $C^{t} \in \mathcal{P}^{t}$ in the current partition $\mathcal{P}^{t}$ the context $\theta^{t}$ belongs (line 3). Next, the algorithm activates all predictors and obtains their predictions $f\left(\boldsymbol{x}^{t}\right), \forall f \in \mathcal{F}$ given the input $\boldsymbol{x}^{t}$ (line 4). However, it selects only one of the prediction as the final prediction $y^{t}$, according to the selection as follows (line 5)

$$
\begin{equation*}
y^{t}=\tilde{f}\left(\boldsymbol{x}^{t}\right) \quad \text { where } \quad \tilde{f}=\arg \max _{f} \bar{r}_{f}^{t}\left(C^{t}\right) \tag{3}
\end{equation*}
$$

In words, the selected base predictor has the highest reward estimate for the context subspace $C^{t}$ among all predictors. This is an intuitive selection based on the sample mean rewards. Next the counter $M_{C}^{t}$ steps by 1 since we have one more sample in $C$. When the true traffic pattern $\hat{y}^{t}$ is revealed (line 6), the sample mean reward estimates for all predictors are then updated (line 7-8).


Fig. 3. An illustration of the context space partitioning in a 2-dimensional space: the lower left subspace is further partitioned into 4 smaller subspaces because the partition condition is satisfied.

The second part of the algorithm, namely the adaptive context space partitioning, is the key of our algorithm (line 9-11). At the end of each slot $t$, the algorithm decides whether to further partition the current subspace $C^{t}$, depending on whether we have seen sufficiently many request arrivals in $C^{t}$. More specifically, if $M_{C}^{t} \geq A 2^{l p}$, then $C^{t}$ will be further partitioned (line 9), where $l$ is the subspace level of $C^{t}, A>0$ and $p>0$ are two design parameters. When partitioning is needed, $C^{t}$ is uniformly partitioned into $2^{D}$ smaller hypercubes (each hypercube is a level $l+1$ subspace with side-length half of that of $C^{t}$ ). Then $C^{t}$ is removed from the active context subspace set $\mathcal{P}$ and the new subspaces are added into $\mathcal{P}$ (line 11). In this way, $\mathcal{P}$ is still a partition whose subspaces are non-overlapping and cover the entire context space. Figure 3 provides an illustrative example of the context space partitioning for a 2-dimensional context space. The current context space partition $\mathcal{P}^{t}$ is shown in the left plot and the current subspace $C^{t}$ is the shaded bottom left square. When the partitioning condition is satisfied, $C^{t}$ is further split into four smaller squares. Intuitively, the context space partitioning process can help refine the learning in smaller subspaces. In the next subsection, we will show that by carefully choosing the design parameters $A$ and $p$, we can achieve sublinear learning regret in time, which implies that the optimal timeaverage prediction performance can be achieved.

## B. Learning regret analysis

In this subsection, we analyze the regret of the proposed traffic prediction algorithm. To enable this analysis, we make a technical assumption that each base predictor achieves similar expected rewards (accuracies) for similar contexts; this is formalized in terms of a Hölder condition.

Assumption. For each $f \in \mathcal{F}$, there exists $L>0, \alpha>0$ such that for all $\theta, \theta^{\prime} \in \Theta$, we have

$$
\begin{equation*}
\left|\pi_{f}(\theta)-\pi_{f}\left(\theta^{\prime}\right)\right| \leq L\left\|\theta-\theta^{\prime}\right\|^{\alpha} \tag{4}
\end{equation*}
$$

This is a natural and reasonable assumption in traffic prediction problems since similar contexts would lead to similar impact on the prediction outcomes. Note that $L$ is not required

```
Algorithm Context-aware Traffic Prediction (CA-Traffic)
    Initialize \(\mathcal{P}^{0}=\{\Theta\}, \bar{r}_{f}(\Theta)=0, \forall f \in \mathcal{F}, M_{\Theta}^{0}=0\).
    for each traffic prediction request (time slot \(t\) ) do
        Determine \(C^{t} \in \mathcal{P}^{t}\) such that \(\theta^{t} \in C^{t}\).
        Generate the predictions results for all predictors
    \(f\left(x^{t}\right), \forall f\).
        Select the final prediction \(y^{t}=f^{*}\left(x^{t}\right)\) according to
    (18).
        The true traffic pattern \(\hat{y}^{t}\) is revealed.
        Update the sample mean reward \(\bar{r}_{f}\left(C^{t}\right), \forall f\).
        \(M_{C}^{t}=M_{C}^{t}+1\).
        if \(M_{C}^{t} \geq A 2^{p l}\) then
            \(C^{t}\) is further partitioned.
        end if
    end for
```

to be known and that an unknown $\alpha$ can be estimated online using the sample mean estimates of accuracies for similar contexts, and our proposed algorithm can be modified to include the estimation of $\alpha$.

To facilitate the analysis, we artificially create two learning steps in the algorithm: for each traffic prediction request at time $t$, it belongs to either a virtual exploration step of a virtual exploitation step. We introduce a control function $\zeta(t)$ with the form $\zeta(t)=2^{2 \alpha l} \log (t)$ that controls which virtual step a time slot $t$ belongs to, where $l$ is the level of the current active subspace $C^{t}$. Specifically, it depends on the counter $M_{C}^{t}$ of the current active context subspace. If $M_{C}^{t} \leq \zeta(t)$, then it is a virtual exploration step; otherwise, it is a virtual exploitation step. Note that these steps are not part of the algorithm, they are solely used for the regret analysis.

Next, we introduce some notations here for the regret analysis. Let $\mathcal{E}_{f, C}^{t}$ be the set of rewards collected from choosing predictor $f$ by time $t$ for subspace $C$. For each subspace $C$ let $f_{C}^{*}$ be the predictor which is optimal for the center context in that subspace. Let $\bar{\pi}_{f, C}:=\sup _{\theta \in C} \pi_{f}(\theta)$ and $\underline{\pi}_{f, C}:=\inf _{\theta \in C} \pi_{f}(\theta)$. For a level- $l$ subspace $C$, we defined the set of suboptimal predictor as

$$
\begin{equation*}
\mathcal{S}_{C, l, B}:=\left\{f: \underline{\pi}_{f_{C}^{*}, C}-\bar{\pi}_{f, C}>B 2^{-\alpha l}\right\} \tag{5}
\end{equation*}
$$

where $l$ is the level of subspace $C, B>0$ is a constant, $L$ and $\alpha$ are the Hölder condition parameters. We also define $\beta_{a}:=\sum_{t=1}^{\infty} 1 / t^{a}$.

The prediction regret can be written as a sum of three terms:

$$
\begin{equation*}
\operatorname{Reg}(T)=\operatorname{Reg}_{e}(T)+\operatorname{Reg}_{s}(T)+\operatorname{Reg}_{n}(T) \tag{6}
\end{equation*}
$$

where $\operatorname{Reg}_{e}(T)$ is the regret due to virtual exploration steps by time $T, \operatorname{Reg}_{s}(T)$ is the regret due to sub-optimal predictor selection in the virtual exploitation steps by time $T$ and $\operatorname{Reg} g_{n}(T)$ is the regret due to near-optimal predictor selection in the virtual exploitation steps by time $T$. We will bound these three terms separately to obtain the complete regret.

The first lemma gives an upper bound on the highest level of the active subspace at any time $t$.
Lemma 1. Any active subspace $C \in \mathcal{P}^{t}$ has a level at most $\lceil\log (t) / p\rceil+1$.

The next lemmas bound the regrets for any subspace of level $l$.

Lemma 2. For any level-l subspace the regret due to virtual explorations by time $t$ is bounded above by $2^{2 \alpha l} \log (t)+1$.

Proof. The result follows from the fact that the number of exploration slots for contexts arriving to context subspace $C$ by time $T$ is upper bounded by $2^{2 \alpha l} \log T+1$.

Lemma 3. For any level-l subspace, given that $2 L\left(\sqrt{D} / 2^{l}\right)^{\alpha}+(2-B) 2^{-\alpha l} \leq 0$, the regret due to choosing a sub-optimal predictor in the exploitation slots is bounded above by $2 K \beta_{2}$.

Proof. We will bound the probability that the algorithm follows a suboptimal predictor in the virtual exploitation slots in a context subspace $C$ with level $l$. Thus, using this we will bound the expected number of times a sub-optimal predictor is selected by the algorithm in the virtual exploitation slots in $C$. Let $\lambda_{f, C}^{t}$ be the event that a sub-optimal predictor $f \in \mathcal{S}_{C}$ is selected at time $t$. Let $\gamma_{C}^{t}$ be the event that the request arrival to $C$ belongs to a exploitation slot. We have

$$
\begin{equation*}
R_{s, C}(T) \leq \sum_{t=1}^{T} \sum_{f \in \mathcal{S}_{C}} P\left(\lambda_{f, C}^{t}, \gamma_{C}^{t}\right) \tag{7}
\end{equation*}
$$

For some $H_{t}>0$, we have

$$
\begin{align*}
P\left(\lambda_{f, C}^{t}, \gamma_{C}^{t}\right) \leq & P\left(\bar{r}_{f}^{t}(C) \geq \bar{r}_{f^{*}}^{t}(C), \gamma_{C}^{t}\right) \\
\leq & P\left(\bar{r}_{f}^{t}(C) \geq \bar{\pi}_{f}^{t}(C)+H_{t}, \gamma_{C}^{t}\right) \\
& +P\left(\bar{r}_{f^{*}}^{t}(C) \leq \bar{\pi}_{f^{*}}^{t}(C)-H_{t}, \gamma_{C}^{t}\right) \\
& +P\left(\bar{r}_{f}^{t}(C) \geq \bar{r}_{f^{*}}^{t}(C),\right.  \tag{8}\\
& \bar{r}_{f}^{t}(C)<\bar{\pi}_{f}^{t}(C)+H_{t}, \\
& \left.\bar{r}_{f^{*}}^{t}(C)>\underline{\pi}_{f^{*}}^{t}(C)-H_{t}, \gamma_{C}^{t}\right)
\end{align*}
$$

and for a sub-optimal predictor $f \in \mathcal{S}_{C}$,

$$
\begin{align*}
& P\left(\bar{r}_{f}^{t}(C) \geq \bar{r}_{f^{*}}^{t}(C), \bar{r}_{f}^{t}(C)<\bar{\pi}_{f}^{t}(C)+H_{t}\right. \\
& \left.\bar{r}_{f}^{t}(C)>\bar{\pi}_{f^{*}}(C)-H_{t}, \gamma_{C}^{t}\right) \\
\leq \quad & P\left(\bar{r}_{f}^{t, b e s t}(C) \geq \bar{r}_{f^{*}}^{t, w o r s t}(C),\right. \\
& \bar{r}_{f}^{t, \text { best }}(C)<\bar{\pi}_{f}^{t}(C)+L\left(\sqrt{D} / 2^{-l}\right)^{\alpha}+H_{t} \\
& \left.\bar{r}_{f^{*}}^{t, w o r s t}(C)>\underline{\pi}_{f^{*}}^{t}(C)-L\left(\sqrt{D} / 2^{-l}\right)^{\alpha}-H_{t}, \gamma_{C}^{t}\right) \tag{9}
\end{align*}
$$

For $f \in \mathcal{S}_{C}$, when

$$
\begin{equation*}
2 L\left(\sqrt{D} / 2^{l}\right)^{\alpha}+2 H_{t}-B 2^{-\alpha l} \leq 0 \tag{10}
\end{equation*}
$$

we have (9) equal to zeros. Let $H_{t}=2^{-\alpha l}$. Assume that (10) holds. Using a Chernoff-Hoeffding bound, for any $f \in \mathcal{S}_{C}$, since on the event $\gamma_{C}^{t}$, the number of samples is greater than $2^{2 \alpha l} \log t$, we have

$$
\begin{equation*}
P\left(\bar{r}_{f}^{t}(C) \geq \bar{\pi}_{f}^{t}(C)+H_{t}, \gamma_{C}^{t}\right) \leq e^{-2\left(H_{t}\right)^{2} 2^{2 \alpha l} \log t}=\frac{1}{t^{2}} \tag{11}
\end{equation*}
$$

and similarly,

$$
\begin{equation*}
P\left(\bar{r}_{f^{*}}^{t}(C) \leq \frac{1}{t^{2}}\right. \tag{12}
\end{equation*}
$$

Hence, when (10) holds, summing over time and the suboptimal predictors, we get $R_{s, C}(T) \leq 2 K \beta_{2}$.

Lemma 4. For any level-l subspace, the regret due to selecting a near-optimal predictor in the virtual exploitation slots is bounded above by $A B 2^{l(p-\alpha)}$.

Proof. If a near optimal predictor is selected for a level $l$ subspace $C$, then the one-slot contribution to the regret is at most $B 2^{-\alpha l}$. We multiply this by $A 2^{p l}$, which is the maximum number of slots that $C$ can stay active.

Next, we combine the results from the previous lemmas to obtain our regret bound. Denote $W_{l}(T)$ be the number of level- $l$ subspaces that are activated by time $T$, we know that $W_{l}(T) \leq 2^{D l}$ for any $l$ and $T$. Moreover, from the result of Lemma 1, we know that $W_{l}(T)=0$ for $l>\lceil\log (t) / p\rceil+1$.

Theorem 1. The regret is upper bounded by

$$
\begin{align*}
& \operatorname{Reg}(T) \leq \\
& \sum_{l=1}^{\lceil\log (T) /(3 \alpha)\rceil+1} W_{l}(T)\left(2^{2 \alpha l}\left(2 L D^{\alpha / 2}+2+\log (T)\right)+2 K \beta_{2}\right) A
\end{align*}
$$

Proof. Consider a subspace $C$, the highest order of regret comes from $\operatorname{Reg}_{e, C}(T)$ and $\operatorname{Reg}_{n, C}(T)$. The former is on the order of $O\left(2^{2 \alpha l}\right)$ and the latter is on the order of $O\left(2^{l(p-\alpha)}\right)$. These two are balanced for $p=3 \alpha$. Although choosing $p$ smaller than $3 \alpha$ will not make the regret in $C$ larger, it will increase the number of subspaces activated by time $T$, causing an increase in the regret. Since we sum over all activated hypercubes, it is best to choose $p$ as large as possible. In order for the condition in Lemma 3 to hold, $B$ needs to satisfy $B \geq 2\left(L D^{\alpha / 2}+1\right)$. Choosing $B=2\left(L D^{\alpha / 2}+1\right)$ minimizes $\operatorname{Reg}_{n, C}(T)$.

The following corollary establishes the regret bound when the context arrivals are uniformly distributed over the context space. For example, if the context is the location, then the requests come uniformly from the area. This is the worst-case scenario because the algorithm has to learn over the entire context space.

Corollary 1. If the context arrival by time $T$ is uniformly distributed over the context space, we have

$$
\begin{align*}
\operatorname{Reg}(T) \leq & T^{\frac{D+2 \alpha}{D+3 \alpha}} 2^{(D+2 \alpha) l}\left(2 L D^{\alpha / 2}+2+\log (T)\right)  \tag{14}\\
& +T^{\frac{D}{D+3 \alpha}} 2^{D l} 2 K \beta_{2} \tag{15}
\end{align*}
$$

Proof. First we calculate the highest level when context arrivals are uniform. In the worst case, all level $l$ subspaces will stay active and then, they are deactivate till all level $l+1$ subspaces become active and so on. Let $l_{\max }$ be the maximum level subspace under this scenario. We have

$$
\begin{equation*}
\sum_{l=1}^{l_{\max }-1} 2^{D l} 2^{3 \alpha l}<T \tag{16}
\end{equation*}
$$

Thus, we must have $l_{\max }<1+\log T /(D+3 \alpha)$. Therefore,

$$
\begin{align*}
& \operatorname{Reg}(T) \\
& \leq \sum_{1+\log T /(D+3 \alpha)}^{1} 2^{D l}\left(2^{2 \alpha l}\left(2 L D^{\alpha / 2}+2+\log (T)\right)+2 K \beta_{2}\right) \\
& \leq T^{\frac{D+2 \alpha}{D+3 \alpha}} 2^{(D+2 \alpha) l}\left(2 L D^{\alpha / 2}+2+\log (T)\right) \\
& +T^{\frac{D}{D+3 \alpha}} 2^{D l} 2 K \beta_{2} \tag{17}
\end{align*}
$$

We have shown that the regret upper bound is sublinear in time, implying that the average traffic prediction rewards (e.g. accuracy) achieves the optimal reward as time goes to infinity. Moreover, it also provides performance bounds for any finite time $T$ rather than the asymptotic result. Ensuring a fast convergence rate is important for the algorithm to quickly adapt to the dynamically changing environment.

## V. EXtENSIONS

## Dimension reduction

In the previous section, the context space partitioning is performed on all context dimensions simultaneously. In particular, each context subspace $C$ has dimension $D$ and each time it is further partitioned, $2^{D}$ new subspaces are added into the context space partition $\mathcal{P}$. Thus, learning can be very slow when $D$ is large since many traffic requests are required to learn the best predictors for all these subspaces. One way to reduce the number of new subspaces created during the partitioning process is to maintain the context partition and subspaces and perform the partition for each dimension separately. In this way, each time a partitioning is needed for one dimension, only two new subspaces will be created for this dimension. Therefore, at most $2 D$ more subspaces will be created for each request arrival. Note, however, that most of the time a partitioning is not needed.

The modified algorithm works as follows. For each context dimension (e.g. time, type and distance), we maintain a similar context space and partition structure as in Section III (in other words the context space dimension is 1 but we have $D$ such spaces). Denote $\mathcal{P}_{d}^{t}$ as the context space partition for dimension $d$ and $C_{d}^{t}$ as the current context subspace for dimension $d$, at time $t$. Note now that since we consider only one dimension, $C_{d}^{t}$ is a one-dimensional subspace for each $d$. Each time a traffic instant $x^{t}$ with context $\theta^{t}$ arrives, we obtain the prediction results of all base predictors given $x^{t}$. The final prediction $y^{t}$ is selected according to a different rule than (18) as follows

$$
\begin{equation*}
y^{t}=\tilde{f}\left(x^{t}\right) \quad \text { where } \quad \tilde{f}=\arg \max _{f}\left\{\max _{d} \bar{r}_{f}^{t}\left(C_{d}^{t}\right)\right\} \tag{18}
\end{equation*}
$$

In words, the algorithm selects the predictor that has the highest reward estimate for all current subspace among all context dimensions. Figure 4 shows an illustrative example for the predictor selection when we only use the time and location as the contexts. In this example, the time context (10:05am) falls into the subspace at the most left quarter ( $7 \mathrm{am}-11 \mathrm{pm}$ ) and the location context ( 3.7 miles away from a reference location) falls into the right half subspace (2.5-5 miles).


Fig. 5. An illustrative example for context space partition with relevant context: partitioning only occurs on the location context since the partitioning condition is satisfied

According to the time context dimension, the predictor with the highest reward estimate is Predictor 1 while according to the location context dimension, the predictor with the highest reward estimate is Predictor 2. Overall, the best estimated predictor is Predictor 2, which is selected by the algorithm.

After the true traffic $\hat{y}^{t}$ is observed, the reward estimates for all predictors in all $D$ one-dimensional context subspace $C_{d}^{t}, \forall d$ are updated. The $D$ partitions $\mathcal{P}_{d}^{t}, \forall d$ are also updated in a similar way as before depending on whether there have been sufficiently many traffic requests with contexts in the current subspaces. Figure 5 illustrates the context space partition for each individual dimension. In this example, only the location context satisfies the partitioning condition and hence its right half subspace is further partitioned.

## B. Relevant context dimension

While using all context dimensions is supposed to provide the most refined information and thus leads to the best performance, it is equally important to investigate which dimension or set of dimensions is the most informative for a specific traffic situation. The benefits of revealing the most relevant context dimension (or set of dimensions) are manyfold, including reduced cost due to context information retrieval and transmission, reduced algorithmic and computation complexity and targeted active traffic control. In the extreme case, a context dimension (e.g. the time) is not informative at all if for all values of the context along this dimension, the best traffic predictor is the same. Hence, having this context dimension does not add benefits for the traffic prediction but only incurs additional cost.

For the expositional clarify, in the following we will focus only on the most relevant context. The extension to the $k$ most relevant context dimensions $(\forall k<D)$ is straightforward. Let $\pi_{f}\left(\theta_{d}\right)$ be the expected prediction reward of predictor $f \in \mathcal{F}$ when the context along the $d$-th dimension is $\theta_{d}$ and $f^{*}\left(\theta_{d}\right)=$ $\arg \max _{f} \pi_{f}\left(\theta_{d}\right)$ be the predictor with the highest expected reward given $\theta_{d}$. Then the expected reward if we only use $d$ th dimension context information is $R_{d}=E_{\theta_{d}}\left\{\pi_{f^{*}\left(\theta_{d}\right)}\left(\theta_{d}\right)\right\}$ where the expectation is taken over the distribution of the $d$ th dimension context. The most relevant context dimension is defined to be $d^{*}=\arg \max _{d} R_{d}$.

Our framework can be easily extended to determine the most relevant context dimension. For each dimension, we maintain the similar partition and subspace structure as in Section III
(with $D=1$ ). In addition, we maintain the time-average prediction reward $\bar{R}_{d}^{t}$ for each dimension $d$. The estimated most relevant dimension at time $t$ is thus $\left(d^{*}\right)^{t}=\arg \max _{d} \bar{R}_{d}^{t}$.

Theorem 2. The estimated most relevant dimension converges to the true most relevant dimension, i.e. $\lim _{t \rightarrow \infty}\left(d^{*}\right)^{t}=d^{*}$.

Proof. Since for each dimension $d$, the time-average regret tends to 0 as $t \rightarrow \infty$, the time-average reward also $\bar{R}_{d}^{t} \rightarrow R_{d}$ as $t \rightarrow \infty$. Therefore, the most relevant dimension can also be revealed when $t \rightarrow \infty$.

## C. Missing and delayed feedback

The proposed algorithm requires the knowledge of the true label $\hat{y}^{t}$ on the predicted traffic to update reward estimates of different predictors so that their true performance can be learned. In practice, the feedback about true traffic label $\hat{y}^{t}$ can be missing or delayed due to, for example, delayed traffic reports and temporary sensor down. In this subsection, we can make small modifications to the proposed algorithm to deal with such scenarios.

Consider the case when the feedback is missing with probability $p_{m}$. The algorithm is modified so that it updates the sample mean reward and performs context space partitioning only for requests in which the true label is revealed. Let $R e g^{m}(T)$ denote the regret of the modified algorithm with missing feedback, we have the following result.

Proposition 1. Suppose the feedback about the true label is missing with probability $p_{m}$, we have

$$
\begin{align*}
& \operatorname{Reg}^{m}(T) \\
& \leq \sum_{l=1}^{\lceil\log (T) /(3 \alpha)\rceil+1} W_{l}(T)\left(2 ^ { 2 \alpha l } \left(2 L D^{\alpha / 2}+2\right.\right.  \tag{19}\\
& \left.\left.+\frac{1}{1-p_{m}} \log (T)\right)+2 K \beta_{2}\right)
\end{align*}
$$

Proof. Missing labels cause more virtual exploration slots to learn the performance of base predictors accurately enough. In expectation, $\frac{1}{1-p_{m}}-1$ more virtual exploration slots are required in ratio. Hence, the regret due to virtual exploration increases to $\frac{1}{1-p_{m}}$ of before. The regret due to virtual exploitation slots is not affected since the the control function $\zeta(t)$ ensures the reward estimates are accurate enough. Using the original regret bound and taking into account the increased regret due to virtual exploration, we obtain the new regret bound.

Consider the case when the feedback is delayed. We assume that the true label of the request at $t$ is observed at most $L_{\text {max }}$ slots later. The algorithm is modified so that it keeps in its memory the last $L_{\max }$ labels and the reward estimates are updated whenever the corresponding true label is revealed. Let $\operatorname{Reg}^{d}(T)$ denote the regret of the modified algorithm with delayed feedback. We then have the following result

## Proposition 2. Suppose the feedback about the true label is

 delayed by at most $L_{\max }$ slots, then we have$$
\begin{equation*}
\operatorname{Reg}^{d}(T) \leq L_{\max }+\operatorname{Reg}(T) \tag{20}
\end{equation*}
$$

Proof. A new sample is added to sample mean accuracy whenever the true label of a precious prediction arrives. The


Fig. 4. An illustrative example for predictor selection with separately maintained context partition: a request with context (10:05am and 3.7 miles away from reference location) arrives; Predictor 1 is the best for the time context and Predictor 2 is the best for the location context; Predictor 2 is the finally selected predictor
worst case is when all labels are delayed by $L_{\max }$ time steps. This is equivalent to starting the algorithm with an $L_{\max }$ delay.

The above two propositions show that the missing and delayed labels reduce the learning speed. However, since the regret bounds are still sublinear in time $T$, the time average reward converges to the optimal reward as $T \rightarrow \infty$. This shows that our algorithm is robust to errors caused by uncertain traffic conditions.

## VI. Experiments

## A. Experimental setup

1) Dataset: Our experiment utilizes a very large real-world traffic dataset, which includes both real-time and historically archived data since 2010. The dataset consists of two parts: (i) Traffic sensor data from 9300 traffic loop-detectors located on the highways and arterial streets of Los Angeles County (covering 5400 miles cumulatively). Several main traffic parameters such as occupancy, volume and speed are collected in this dataset at the rate of 1 reading per sensor per minute; (ii) Traffic incidents data. This dataset contains the traffic incident information in the same area as in the traffic sensor dataset. On average, 400 incidents occur per day and the dataset includes detailed information of each incident, including the severity and location information of the incident as well as the incident type etc.
2) Evaluation Method: The proposed method is suitable for any spatiotemporal traffic prediction problem. In our experiments, the prediction requests come from a freeway segment of 3.4 miles on interstate freeway 405 (I-405) during daytime 8 am to 17 pm . Figure 6 shows the freeway segment used in the experiment. Locations will be referred using the distance from the reference location $A$. For each request from location $l_{o}$, the system aims to predict whether the traffic will be congested at $l_{o}$ in the next 15 minutes using the current traffic speed data. If the traffic speed drops below a threshold $\lambda$, then the location is labeled as congested, denoted by $\hat{y}=1$;


Fig. 6. Freeway segment used in the experiment.
otherwise, the location is labeled as not congested, denoted by $\hat{y}=-1$. We will show the results for different values of $\lambda$. We use the simple binary reward function for evaluation. That is, the system obtains a reward of 1 if the prediction is correct and 0 otherwise. Therefore, the reward represents the prediction accuracy. The context information that we use in the experiments include the time when the prediction request is made and the location where the request comes from. These contexts capture the spatiotemporal feature of the considered problem. Nevertheless, other context information mentioned in Section III(A) can also be adopted in our algorithm.

Using historical data, we construct 6 base predictors (Naive Bayes) for 6 representative situations with context information from the set $[8 \mathrm{am}, 12 \mathrm{pm}, 16 \mathrm{pm}] \times[0 \mathrm{mile}, 3.4 \mathrm{miles}]$. These are representative traffic situations since 8 am represents the morning rush hour, 12 pm represents non-rush hour, 16 pm represents the afternoon rush hour, " 0 mile" is at the freeway intersection and " 3.4 miles" is the farthest location away from the intersection in considered freeway segment.

|  | CA-Traffic | MU | AU | GDU |
| :---: | :---: | :---: | :---: | :---: |
| $\lambda=50 \mathrm{mph}$ | 0.94 | 0.83 | 0.83 | 0.82 |
| $\lambda=30 \mathrm{mph}$ | 0.91 | 0.82 | 0.80 | 0.78 |

TABLE I
OVERALL PREDICTION ACCURACY.
3) Benchmarks: Since our scheme appertains to the class of online ensemble learning techniques, we will compare our scheme against several such approaches. These benchmark solutions assign weights to base predictors but use different rules to update the weights. Denote the weight for base predictor $f$ by $w_{f}$. The final traffic prediction depends on the weighted combination of the predictions of the base predictors:

$$
y= \begin{cases}+1, & \text { if } \sum_{f \in \mathcal{F}} w_{f} y_{f} \geq 0  \tag{21}\\ -1, & \text { otherwise }\end{cases}
$$

Three approaches are used to update the weights:

- Multiplicative Update (MU) [18] [19]: If the prediction is correct for predictor $f$, i.e. $y_{f}=\hat{y}$, then $w_{f} \leftarrow \alpha w_{f}$ where $\alpha>1$ is a constant; otherwise, $w_{f} \leftarrow w_{f} / \alpha$.
- Additive Update (AU) [21]: If the prediction is correct for predictor $f$, i.e. $y_{f}=\hat{y}$, then $w_{f} \leftarrow w_{f}+1$; otherwise, $w_{f} \leftarrow w_{f}$.
- Gradient Descent Update (GDU) [22]: The weight of predictor $f$ is update as $w_{f} \leftarrow(1-\beta) w_{f}-2 \beta\left(w_{f} y_{f}-\hat{y}\right) \hat{y}$ where $\beta \in(0,1)$ is a constant.


## B. Prediction accuracies

In Table I, we report the prediction accuracies of our proposed algorithm (CA-Traffic) and the benchmark solutions for $\lambda=50 \mathrm{mph}$ and $\lambda=30 \mathrm{mph}$. Our algorithm outperforms the benchmark solutions by more than $10 \%$ in terms of prediction accuracy. Note that this is a huge improvement since a random guessing can already achieve $50 \%$ accuracy and thus, the improvement cannot be more than $50 \%$.

Table II and III further report the prediction accuracies in different traffic situations. In Table II, the location context is fixed at 0.8 miles from the reference location and the accuracies for various time contexts (i.e. $10 \mathrm{am}, 2 \mathrm{pm}$ and 5 pm ) are presented for our proposed algorithm and the benchmarks. In Table III, the time context is fixed at 10 am and the accuracies for various location contexts (i.e. 0.8 miles, 2.1 miles, 3.1 miles) are reported. In all traffic situations, the proposed algorithm significantly outperforms the benchmark solutions since it is able to match specific traffic situations to the best predictors.

## C. Convergence of learning

Since our algorithm is an online algorithm, it is also important to investigate its convergence rate. Figure 7 and 8 illustrate the prediction accuracies of our proposed algorithm over time, where the horizontal axis is the number of requests. As we can see, the proposed algorithm converges fast, requiring only a couple of hundreds of traffic prediction requests.

| $(\lambda=50 \mathrm{mph})$ | CA-Traffic | MU | AU | GDU |
| :---: | :---: | :---: | :---: | :---: |
| 10 am | 0.93 | 0.81 | 0.85 | 0.83 |
| 2 pm | 0.93 | 0.86 | 0.81 | 0.80 |
| 5 pm | 0.99 | 0.86 | 0.87 | 0.88 |
| $(\lambda=30 \mathrm{mph})$ | CA-Traffic | MU | AU | GDU |
| 10 am | 0.93 | 0.83 | 0.83 | 0.81 |
| 2 pm | 0.91 | 0.80 | 0.83 | 0.82 |
| 5 pm | 0.99 | 0.81 | 0.85 | 0.83 |

TABLE II
Traffic prediction accuracy at 0.8 miles.

| $(\lambda=50 \mathrm{mph})$ | CA-Traffic | MU | AU | GDU |
| :---: | :---: | :---: | :---: | :---: |
| 0.8 mile | 0.92 | 0.84 | 0.83 | 0.82 |
| 2.1 mile | 0.96 | 0.81 | 0.85 | 0.85 |
| 3.1 mile | 0.93 | 0.85 | 0.83 | 0.81 |
| $(\lambda=30 \mathrm{mph})$ | CA-Traffic | MU | AU | GDU |
| 0.8 mile | 0.92 | 0.81 | 0.83 | 0.82 |
| 2.1 mile | 0.93 | 0.83 | 0.82 | 0.81 |
| 3.1 mile | 0.94 | 0.81 | 0.82 | 0.82 |

TABLE III
TRAFFIC PREDICTION ACCURACY AT 10AM

## D. Missing context information

The context information associated with the requests may be missing occasionally due to, for example, missing reports and record mistakes. However, our modified algorithm (described in Section $V(A)$ ), denoted by CA-Traffic(R), can easily handle these scenarios. In this set of experiments, we show the performance of the modified algorithm for the extreme cases in which one type of context information is always missing. Table IV reports the accuracies of our algorithms (CA-Traffic and CA-Traffic(R)) as well as the benchmarks. Although CATraffic( R ) performs slightly worse than CA-Traffic when there is no missing context, it performs much better than CATraffic and the benchmark solutions when context can be


Fig. 7. Accuracy over time with different time contexts at 0.8 miles. $(\lambda=$ 50mph)


Fig. 8. Accuracy over time with different location contexts at 10am. $(\lambda=$ $50 m p h$


Fig. 9. Relative importance of contexts.
missing because it maintains the context partition separately for each context type and hence, it is robust to missing context information.

## E. Relevant context

In this set of experiments, we unravel the most relevant context that leads to the best prediction performance. To do so, we run the algorithm using only a single context (i.e. either time or location) and records the average reward. The most relevant context is the one leading to the highest average reward. Figure 9 shows the the relative importance (e.g. Reward(time) $/($ Reward(time) + Reward(location) $))$ of each context for different congestion threshold $\lambda=$ $20 \mathrm{mph}, 30 \mathrm{mph}, 50 \mathrm{mph}$. The result indicates that time is the more important context for the traffic prediction problem in our experiment.

## F. Missing and delayed labels

Finally, we investigate the impact of missing and delayed labels on the prediction accuracy, as shown in Figure 10 and


Fig. 10. Prediction accuracy with missing and delayed labels. $\lambda=50 \mathrm{mph}$.


Fig. 11. Prediction accuracy with missing and delayed labels. $\lambda=30 \mathrm{mph}$.
11. In the missing label case, the system observes the true traffic label with probability 0.8 . In the delayed label case, the true label of the traffic comes at most five prediction requests later. In both cases, the prediction accuracy is lower than that without missing or delayed labels. However, the proposed algorithm is still able to achieve very high accuracy which exceeds $90 \%$.

## VII. Conclusions

In this paper, we proposed a framework for online traffic prediction, which discovers online the contextual specialization of predictors to create a strong hybrid predictor from several weak predictors. The proposed framework matches the real-time traffic situation to the most effective predictor constructed using historical data, thereby self-adapting to the dynamically changing traffic situations. We systematically proved both short-term and long-term performance guarantees for our algorithm, which provide not only the assurance that our algorithm will converge over time to the optimal hybrid predictor for each possible traffic situation but also provide a bound for the speed of convergence to the optimal predictor.

| $(\lambda=50 \mathrm{mph})$ | CA-Traffic | CA-Traffic(R) | MU | AU | GDU |
| :---: | :---: | :---: | :---: | :---: | :---: |
| time\&distance | 0.94 | 0.89 | 0.83 | 0.83 | 0.82 |
| time | 0.78 | 0.92 | 0.76 | 0.76 | 0.78 |
| distance | 0.72 | 0.88 | 0.79 | 0.77 | 0.78 |
| $(\lambda=30 \mathrm{mph})$ | CA-Traffic | CA-Traffic(R) | MU | AU | GDU |
| time\&distance | 0.91 | 0.80 | 0.81 | 0.83 | 0.78 |
| time | 0.76 | 0.86 | 0.70 | 0.72 | 0.75 |
| distance | 0.75 | 0.89 | 0.72 | 0.71 | 0.74 |

TABLE IV
TRAFFIC PREDICTION ACCURACY WITH INCOMPLETE CONTEXT INFORMATION.

Our experiments on real-world dataset verified the efficacy of the proposed scheme and show that it significantly outperforms existing online learning approaches for traffic prediction. As a future work, we plan to extend the current framework to distributed scenarios where traffic data is gathered by distributed entities and thus, coordination among distributed entities are required to achieve a global traffic prediction goal [31] [32] [33].

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