Peer-to-Peer Procotol Designs based on Social Norms

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Abstract—*Free-riding represents a key challenge to the proliferation of today's peer-to-peer (P2P) networks, since there is no deterrent in the system that prevents peers which are not contributing their resources to utilize the resources produced by other peers. Overcoming the free-riding problem is particularly difficult because of the unique challenges that P2P networks put forward: asymmetric interests of peers, large population of peers interacting infrequently, peers dynamically joining and leaving the network, network errors, and low-cost identity whitewashing. Incentive protocols play a crucial role in overcoming the free-riding problem. In this paper, we build an analytical framework for the design and analysis of a new family of incentive protocols that is based on indirect reciprocity and utilizes the idea of social norms, which has been shown to be successful in shaping group behaviors in societies with large population. Under this framework, we formally analyze how the unique features of P2P networks as well as the peer's foresightedness impact the robustness of protocols. We are able to prove that if peers are foresighted to consider the impact of their current actions on their future utilities, our protocols can sustain subgame perfect equilibria in the network from which no peer has incentives to deviate since it cannot gain a higher utility by unilaterally changing its sharing strategy. Hence, we prove that it becomes in the self-interest of peers to contribute their resources to the P2P system rather than free-ride. Subsequently, we formalize the problem of finding the optimal protocols that maximize the network performance among the achievable subgame perfect equilibria. The structure of the optimal protocols is studied and an efficient searching algorithm to obtain a near-optimal protocol is proposed. We prove that the family of protocols based on social norms can induce self-interested peers to achieve a desirable level of cooperation in the network which is close to Pareto efficiency.*

Keywords- Peer-to-Peer incentive protocols, Social Norm, Reputation Scheme, Subgame Perfect Equilibrium.

I. INTRODUCTION

Peer-to-Peer (P2P) networks have introduced a new distributed paradigm in resource sharing. By pooling together the resources of many autonomous devices, P2P networks are able to provide a scalable and low-cost platform for distributed computing, storage, routing, and file sharing [7][14][27][31]. While P2P networks have many advantages such as scalability, resilience, and effectiveness in coping with network dynamics and peer heterogeneity [22], they are vulnerable to intrinsic incentive problems since the upload service incurs costs to both the uploader and the downloader, but benefits only the downloader. As contributing their content does not directly benefit uploaders, peers tend to avoid uploading while trying to download content from other peers, a behavior commonly known as free-riding. Empirical studies with P2P file sharing systems like Gnutella [1], Napster [27], and KaZaA [14] confirm the concern that free-riding is prevailing and resource owners are not altruistic [1][21][31][32]. For example, 70% of all peers in Gnutella do not share files, and 50% of all requests are satisfied by the top 1% sharing hosts [1].

Such studies demonstrate that designing a correct and robust incentive protocol to encourage cooperation and mitigate the free-rider problem is crucial to maintain the performance of P2P networks. To achieve this goal, a large body of research was dedicated to this area [3][6][11][33]. Many of these existing mechanisms rely on game-theoretical approaches and can be classified into two categories: pricing and reciprocity [23].

Pricing mechanisms rely on implementing a currency-based system that is resistant to forgery and double-spending [2][10][35]. Peers are incentivized by the reward with virtual currency for uploading and the charge for downloading. However, they are regarded as impractical because they require an accounting infrastructure to track the transactions of peers [3], which further necessitates a public key infrastructure, a web of trust, or threshold cryptography techniques [19]. Furthermore, these systems usually make use of auction for price setting, which might slow their convergence.

The principle of reciprocity states that peers need to contribute their resources such as data and bandwidth to accomplish their tasks in order to receive in return such resources from others. Differential service schemes are deployed in reciprocity-based protocols to determine how peers should make their upload decisions: a peer with a higher rating receives more resources than a peer with a lower rating [4][28][29]. Since a peer with a high rating is treated preferentially by other peers, incentives are provided for peers to cooperate in order to build up high ratings. Depending on how a peer's rating is generated, reciprocity-based protocols can be classified into two types: direct reciprocity and indirect reciprocity.

In direct reciprocity, each peer rates a specific peer individually [3][5]. Hence the interaction between two peers is only influenced by past interactions between them. Though easy to implement, direct reciprocity requires frequent interactions between two peers in order to establish accurate mutual ratings, which is restricted by the typical high churn rate in P2P networks. For example, the investigation in [1] shows that over 70% P2P traffic take place in networks with more than 1000 peers, which implies that a peer normally interacts with a stranger (i.e. with whom it was randomly matched) about whom it has no prior history and has no expectation to meet again in the future. Hence, protocols based on direct reciprocity such as tit-for-tat perform well only for networks dominated by long-lived relationships, where peers have ample opportunities to mutually reciprocate.

Due to the random matching feature in large P2P networks, indirect reciprocity becomes a more appropriate mechanism in designing incentive protocols. Most protocols based on indirect reciprocity use reputation mechanisms [8][15][30]. A peer is globally rated with a reputation calculated by its past behaviors in the network. In order to make a decision, a peer does not need to know the entire action history but the reputation of its opponent. However, the majority of existing works on P2P reputation mechanisms are concerned with system design issues and focus on effective information gathering techniques which only differ in how the global reputation is calculated and propagated, e.g. efficient information aggregation [15], secure peer identification [30], etc. An analytical framework that is able to study how peers can be incentivized to cooperate in P2P networks and what is the resulting impact on the network performance when various reputation mechanisms are deployed, is still missing.

In this paper, we aim to provide a rigorous framework for studying reputation mechanisms in P2P file-sharing applications such as Gnutella and BitTorrent, and propose effective indirect reciprocity based incentive protocols which can sustain cooperation among peers and achieve optimal performance in the network. To formalize the idea of indirect reciprocity, we apply the concept of *social norm* introduced in [18] by Kandori, which consists of a reputation scheme and a social strategy. We model the interactions among peers as repeated random-matching games [18], and design an appropriate social norm to punish the peers' deviations from the selected social strategy. The peers determine their upload services to a specific peer based on this peers' reputation, as well as their own status in the P2P system (i.e. their own reputations). To sustain the desired level of cooperation among the network peers, the social norm is designed such that myopic deviations by a peer for hope of immediate gains result in worse performance in the long-term for this peer, as other peers will reduce their future uploads to this peer in response to its deviation.

While analyzing the effectiveness of social norms, we also consider the unique features and constraints of P2P networks when designing our framework:

- Asymmetry of interests. Existing reciprocity protocols [5][37] model in general the stage game played between peers as a prisoner's dilemma, where both parties in the game upload and download from their opponents. Our work extends this model by considering the asymmetry of interest for peers in the same stage game. In other words, there is a mismatch in the services provided and demanded by peers.
- *Network errors*. Existing reputation-based mechanisms [8][15][30] assume that peers' reputations are truthfully updated by the reputation scheme. This is an idealized assumption which is hard to realize in realistic networks due to various types of network errors, which can be incurred due to error-prone data transmission, the peer's inaccurate knowledge of the network dynamics or other peers' reputations etc. In contrast, our framework explicitly takes into consideration that the reputation update may be subject to network errors and considers how protocols can be efficiently designed given various levels of network errors.
- *Dynamic population and whitewashers*. We also consider dynamic P2P networks where peers can leave and join the network freely. Each time a peer joins the network, it is granted a new identity. This further brings in the whitewashing problem [8].

Our proposed framework for building P2P protocols based on social norms is able to address the following questions:

- How the network conditions and the designing parameters of protocols affect peers' incentives?
- Under what condition subgame perfect equilibria (in which no peer can gain a higher utility by unilaterally changing its sharing strategy at any stage of the repeated interaction) exist in the network?
- When multiple subgame perfect equilibria exist, how to select among them the equilibrium (or equilibria) that is able to maximize the network performance and appropriately design the protocol that is able to achieve these equilibria?

The remainder of the paper is organized as follows. In Section II, a rigorous analytical framework is proposed to analyze the P2P network. In Section III, conditions for the existence of subgame perfect equilibrium are analyzed. Section IV investigates the design of optimal norm-based protocol. Section V discusses the optimal whitewashing prevention mechanism. After showing the simulation results with an illustrative example in Section VI, we conclude the paper in Section VII.

II. System Model

A. Network assumption

Without loss of generality, we consider an unstructured P2P file-sharing network such as Bittorrent [5][20], Gnutella [1], and KaZaA [14] [1], in which files are uniformly distributed with the distribution unrelated to the network topology. We assume that each peer possesses a certain amount of files that are shared publicly which can be accessed by the whole network. Peers can exchange entire files or fraction of files [5][26]. Trackers maintain and update periodically the file map that records the file possession of each peer and is responsible for helping the downloader find the list of peers who have the requested file [5]. In general, we assume that peers in the network are self-interested trying to maximize their individual utilities, and therefore, they will only upload content if this has a positive impact on their future downloads.

We utilize the widely-used continuum model to characterize the peer population, implying that each individual peer is negligible in the network. In practice, the continuum model well approximates the real peer population if there is a sufficiently large population in the network [7][8]. The network is modelled as a discrete-time system. By dividing time into slots of equal length, the peer population dynamically changes across adjacent slots. Specifically, we assume that a percentage α of the current population leaves and the same amount of new peers enters the network at the end of each slot, where α is referred to as the turnover rate [8]. We assume that each peer generate service requests complying with the same stationary stochastic process. By selecting the length of a slot small enough, each peer expects to generate

[1] The results obtained in this paper can be applied in P2P applications other than file-sharing without any change.

one service request on average in each slot [25][37].

Once a peer generates a service request, it is randomly matched with a peer from the list provided by the tracker. The matching is uniformly random such that all peers in the list have an equal probability to be chosen [25][37]. We further assume that the size of the list is also large in the continuum population model, which further implies that 1) a peer also serves one download request on average per slot; 2) the probability that two peers select the same peer as the server in the same slot is close to 0 and thus, the probability that a peer receives multiple download requests in a slot is also 0.

B. Stage game

We model the stage game played by a pair of peers as an asymmetric gift-giving game [13] to characterize the asymmetry of interests among peers. To avoid confusion, the peer who requests for download service is called a *client* and the peer who is being requested is called a *server*. In the stage game, only the server is strategic: it determines how to select its actions of contribution or not contributing content in order to maximize its utility. The server's action will not only impact its own utility, but also that of its requesting peers, i.e. its clients. Depending on the volume d of the content uploaded from the server to the client, the client receives a benefit of $r(d)$ and the server consumes a cost of $\rho(d)$. To consider a finite action space of the server, we quantize the volume that a server uploads into discrete values [8][36]. Without the loss of generality, we use a binary-level space as the example in our paper, i.e. $d \in \{0,1\}$ [8][36] [2]. When $d = 1$, the client receives a volume of content that is above its minimum requirement and receives a benefit of r, while the server consumes a cost of ρ; when $d = 0$, both the client and the server receive a utility of 0. Correspondingly, the server selects its action from a binary set, as $a \in \mathcal{A} = \{C, D\}$, where C stands for "cooperation" and represents the case when the server uploads at least the minimum volume of content requested by the client with $d = 1$; whereas D stands for "defection", which represents the case when the server contributes less than the minimum requested content and $d = 0$. In this paper, we assume $r > \rho$ to ensure that the P2P network is socially valuable [3].

The social welfare of the network is quantified by the social utility U that is defined as the average utility received by all peers in the network. Since $r > \rho$, it is obvious that the social utility is optimized when all servers choose $a = C$ in their stage games. Nevertheless a self-interested server who has the incentive to free-riding will always choose $a = D$ if it wants to maximize its stage-game utility myopically, which gives rise to an undesirable outcome of zero utilities for both peers in the stage game. This phenomenon is widely known as the tragedy of the commons [12].

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[2] Our framework can also be applied to the case when d has multiple levels without changing the analysis.

[3] If $r < \rho$, each upload service brings a positive net loss on the total utility of both peers, which makes the file sharing in the network not benefitable.

C. Repeated game

In a P2P network, however, the peers interact repeatedly and hence, the peers will play the same stage game multiple times. To formalize this, we deploy a general repeated game formulation. To ensure that the emerging equilibria of the repeated game is efficient, unlike in the stage game, we deploy the idea of a social norm, which is a protocol that encourages the peers to cooperate with each other by threatening them with future punishments [9][24]. Since each protocol design is based on a specific social norm, we use the two terms of "protocol" and "social norm" interchangeably in the rest of the paper.

In the repeated game, each peer is tagged with a reputation θ representing its social status. θ is a natural number from the finite set $\Theta = \{0, 1, 2, \cdots, L\}$. For notational convenience, a peer holding reputation θ is referred to as a $\theta -$peer. A high reputation relates to a better social status and we also call an $L -$peer as innocent and a $\theta -$peer as guilty with $\theta < L$.

As only the server is strategic in the stage game, we update a peer's reputation in accordance with its behavior when being a server. The reputation should truthfully reflect a peer's social status, and it is maintained and updated in our proposed framework by a trustworthy third-party device - the tracker. At the end of each slot, the tracker compares the server's action reported by the client to the social norm. If the action is in accordance to that specified by the social norm, the server will be rewarded with an increased reputation; otherwise, if the action is against the social norm, the server will be punished by a decreasing reputation. In practical networks, the report is usually a binary value and subject to a small error probability ε of being reverted. Here ε is referred to as the *reputation update error*.

A social norm κ is composed of a social strategy σ and a reputation scheme τ. σ is a reputation-based behavioral strategy. It specifies the correct action a server should play depending on the reputation of the client as well as its own reputation. "Correctness" here indicates that the action is appreciated by the social norm and the server will be rewarded by an increased reputation when playing it. Mathematically, σ can be represented by a mapping $\sigma : \Theta \times \Theta \to \mathcal{A}$, where the first Θ represents the server's reputation, the second Θ represents the client's reputation, and $\mathcal{A}$ represents the server's action. It should be noted that the peer's sharing strategy implemented in the existing reputation mechanisms [15][30] are special cases of the social strategy proposed here, as they determine the server's action simply based upon the client's reputation. τ serves as the reward and punishment system in the social norm and it specifies mathematically how a peer's reputation is updated according to its behavior. Specifically, τ maps the server-client pair's reputations and the server's action to the server's new reputation: $\tau : \Theta \times \Theta \times \mathcal{A} \to \Theta$[4].

Essentially, if cooperative behavior is encouraged in σ and is distinguished by sufficiently large

[4] Here what τ uses in reputation update is actually the client's assessment instead of the server's real action, which is subject to the reputation update error.

reward and punishment in τ, it can be enforced among peers. The working procedure of τ in this paper can be briefly specified as follows:

(1) If a peer always plays with σ, τ classifies it as an innocent peer and assigns to it a reputation L, i.e.

$$\tau\left(L,\tilde{\theta},\sigma\left(L,\tilde{\theta}\right)\right)=L,\ \forall\tilde{\theta}\in\Theta \tag{1}$$

(2) Once a peer deviates from σ by play actions against it, τ will then give it a reputation 0 and start an $L-$slot punishment on it for breaking the social norm, i.e.

$$\tau\left(\theta,\tilde{\theta},a\right)=0,\ \forall\theta,\tilde{\theta}\in\Theta\ \ and\ \ a\neq\sigma\left(\theta,\tilde{\theta}\right) \tag{2}$$

(3) During the punishment phase, the guilty peer has to follow the social strategy in its services to improve its reputation, we assume that by the peer's reputation is increased with one point by following the social strategy once, i.e.

$$\tau\left(\theta,\tilde{\theta},\sigma\left(\theta,\tilde{\theta}\right)\right)=\theta+1,\ \forall 0\leq\theta<L\ \ and\ \ \forall\tilde{\theta}\in\Theta\,. \tag{3}$$

Therefore, L is also referred to as the punishment length. τ's rules are in accordance to the social reality that a peer with a better social status faces larger threat of punishment by breaking the social norm and thus has less incentive to do so. In summary, τ can be written as follows

$$\tau\left(\theta,\tilde{\theta},a\right)=\begin{cases}L & if\ \ a=\sigma\left(\theta,\tilde{\theta}\right)\ \ and\ \ \theta=L\\ \theta+1 & if\ \ a=\sigma\left(\theta,\tilde{\theta}\right)\ \ and\ \ \theta<L\\ 0 & if\ \ a\neq\sigma\left(\theta,\tilde{\theta}\right)\end{cases}. \tag{4}$$

We also assign a new peer who just enters the network at the beginning of a slot a constant initial reputation $K\in\Theta$. The selection of K is discussed in Section V in the aim of preventing whitewashing. We assume $K=L$ in all the other sections without the loss of generality. A schematic representation of a social norm is provided in Figure 1

D. Utility function

A peer's long-term utility depends on the probabilities of meeting peers with certain reputations in the future. The fraction of the total population with a reputation θ is denoted as $\eta(\theta)$. Since we assume that the list of servers provided by the tracker is large, we use the approximation that the reputation distribution of the list is the same as that of the total population. Since in one slot, each peer receives one service request on average, its reputation is also updated once correspondingly. As a result, the peer reputation distribution $\{\eta(\theta)\}$ in the network evolves over time. When all peers following the social strategy, the evolution of $\{\eta(\theta)\}$ across slots can be characterized by the following iterative equations, where t is the index of time slots.

$$
\begin{aligned}
&\eta^{t+1}(L) = (1-\alpha)(1-\varepsilon)\eta^{t}(L) + (1-\alpha)(1-\varepsilon)\eta^{t}(L-1) \\
&\eta^{t+1}(\theta) = (1-\alpha)(1-\varepsilon)\eta^{t}(\theta-1),\ 1 \le \theta \le L-1 \text{ and } \theta \ne K \\
&\eta^{t+1}(K) = (1-\alpha)(1-\varepsilon)\eta^{t}(K-1) + \alpha \\
&\eta^{t+1}(0) = (1-\alpha)\varepsilon
\end{aligned}
\quad . \tag{5}
$$

With the dynamical update of $\{\eta(\theta)\}$, the stage-game utility as well as the long-term utility of a peer also changes over time. To simplify our analysis, we focus on the stationary reputation distribution in this paper, which is defined as follows.

Definition 1 {Stationary distribution}. The stationary reputation distribution $\{\eta_\tau(\theta)\}$ of a network remains unchanged while being updated by Eq. (5) and can be computed using the following set of equations

$$
\begin{aligned}
&\eta_\tau(L) = (1-\alpha)(1-\varepsilon)\eta_\tau(L) + (1-\alpha)(1-\varepsilon)\eta_\tau(L-1) \\
&\eta_\tau(\theta) = (1-\alpha)(1-\varepsilon)\eta_\tau(\theta-1),\ 1 \le \theta \le L-1 \text{ and } \theta \ne K \\
&\eta_\tau(K) = (1-\alpha)(1-\varepsilon)\eta_\tau(K-1) + \alpha \\
&\eta_\tau(0) = (1-\alpha)\varepsilon
\end{aligned}
\quad . \tag{6}
$$

In the following lemma, we prove that the stationary reputation distribution $\{\eta_\tau(\theta)\}$ is unique in the network for a specific social norm and show that $\{\eta(\theta)\}$ always converges to $\{\eta_\tau(\theta)\}$ when all peers follow the social strategy.

Lemma 1. When all peers follow the social strategy σ, the reputation distribution of the network converges to a unique stationary point $\{\eta_\tau(\theta)\}$, which are independent with σ.

Proof: It is easy to verify that the coefficient matrix of the equation set in Eq. (6) is full-ranked and its spectrum radius is smaller than 1. Therefore, $\{\eta^t(\theta)\}$ in Eq. (5) converges to a unique stationary point which is the solution of Eq. (6). ■

Therefore, the expected stage-game utility of a $\theta-$peer in a network whose reputation distribution is stationary can be specified as

$$
v_\kappa(\theta) = \sum_{\tilde{\theta}\in\Theta} \eta_\tau(\tilde{\theta}) r_\sigma(\tilde{\theta},\theta) + \sum_{\tilde{\theta}\in\Theta} \eta_\tau(\tilde{\theta}) \rho_\sigma(\theta,\tilde{\theta}), \tag{7}
$$

where $r_\sigma(\tilde{\theta},\theta)$ is the stage-game benefit the $\theta-$peer can receive when the server has a reputation $\tilde{\theta}$ and follows σ; and $\rho_\sigma(\theta,\tilde{\theta})$ is the stage-game cost the $\theta-$peer will consume if it follows σ and the client's reputation is $\tilde{\theta}$.

To evaluate a peer's long-term utility, we use the infinite-horizon discounted sum criterion, and a peer's expected overall utility in the repeated game starting from any slot t_0, when following the social strategy, can be expressed as

$$v_\kappa^\infty\left(\theta^{t_0}\right) = \mathbb{E}\left(\sum_{t=t_0}^{\infty}\delta^t v_\kappa\left(\theta^t\right)\right) = v_\kappa\left(\theta^{t_0}\right) + \delta\sum_{\theta'} p_\kappa\left(\theta' \mid \theta\right) v_\kappa^\infty\left(\theta'\right). \tag{8}$$

where $\delta = \beta(1-\alpha)$ is the weight that a peer gives to its utility that can be received in the future. It is a decreasing linear function of the turnover rate α. As α increases, a peer has higher probability to leave the network in the next slot, and thus gives less weight δ for the future. $\beta \in [0,1)$ is a peer-defined discount factor. In this paper, we assume that all peers adopt the same β. $p_\kappa\left(\theta' \mid \theta\right)$ is the transition probability of a peer's reputation across slots. When a peer follows the social strategy, it is always the case that it will be rewarded with probability $1-\varepsilon$ and be punished with probability ε regarding to the reputation update error, and thus

$$p_\kappa\left(\theta' \mid \theta\right) = \begin{cases} (1-\varepsilon), \ \theta = \theta' = L \\ (1-\varepsilon), \ \theta < L, \ \theta' = \theta + 1 \\ \varepsilon, \ \theta' = 0 \\ 0, \ otherwise \end{cases}. \tag{9}$$

The social utility of the network is defined as the expected stage-game utility averaged among all peers when the network is stationary as

$$U_\kappa = \sum_\theta \eta_\tau(\theta) v_\kappa(\theta). \tag{10}$$

As discussed in Section II.B, the social strategy should encourage cooperation among innocent peers. Consequently, we consider the family of social strategies which features the following properties:

(1) A peer does not receive better service than any other peer with a higher reputation, i.e. $\sigma\left(\theta,\tilde{\theta}\right) = C$ only if $\sigma\left(\theta,\tilde{\theta}'\right) = C$ for all $\tilde{\theta}' > \tilde{\theta}$. This introduces a service threshold reputation $m_\sigma(\theta)$ for every peer, such that

$$\sigma\left(\theta,\tilde{\theta}\right) = \begin{cases} C, \ if \ \tilde{\theta} > m_\sigma(\theta) \\ D, \ if \ \tilde{\theta} \leq \ m_\sigma(\theta) \end{cases}. \tag{11}$$

(2) $m_\sigma(\theta)$ is non-decreasing on θ such that a peer with a lower reputation has to provide more service to others,

$$m_\sigma(\theta_1) \geq m_\sigma(\theta_2), \ if \ \theta_1 > \theta_2 \tag{12}$$

(3) Innocent peers always help other innocent peers, and thus $m_\sigma(L) \leq L-1$.

In the following lemma, we shows any social strategy σ which satisfies the above constraints grants a peer of higher reputation with better expected overall utility so as to providing peers correct incentives to follow it, i.e. $v_\kappa^\infty(\theta) > v_\kappa^\infty(\theta')$ if $\theta > \theta'$.

Lemma 2. A social strategy σ satisfying the constraints in Eq. (11) – Eq. (12) ensures the resulting expected overall utility monotonically increases with a peer's reputation, i.e. $v_\kappa^\infty(\theta) > v_\kappa^\infty(\theta')$ if $\theta > \theta'$.

Proof: See Appendix A. ■

If in a social norm with a punishment length L, the innocent peers are devised to help some of the guilty peers in the social strategy, i.e. $m_\sigma(L) = L - l,\ l > 1$, we prove that such social norm is identical to another social norm in which the innocent peers only help the innocent peers.

Lemma 3. A social norm with the punishment length L and $m_\sigma(L) = L - l,\ l > 1$ is identical to the social norm with the punishment length $L - l + 1$ and $m_\sigma(L - l + 1) = L - l$.

Proof: See Appendix B. ■

Lemma 3 proves that we can always set $m_\sigma(L) = L - 1$ when analyzing a social norm without reducing the set of social norms we consider.

III. Social Norm Equilibrium

During the implementation of a social norm in a P2P network, two criteria need to be considered. The first is whether the social norm can be sustained in the network, where the self-interested peers will follow the prescribed protocol without having the incentive of disobeying it. The second is how the punishment in a social norm impacts the efficiency of the network and how to design the most efficient protocol to maximize the social utility. These two issues are discussed in the next two sections, with the incentive issue being addressed first, which is called the *social norm equilibrium*.

A. Definitions

Definition 3 {Social norm equilibrium}. The social norm $\kappa = (\sigma, \tau)$ is a social norm equilibrium if

$$r_\sigma\left(\theta,\tilde{\theta}\right) + \delta\sum_{\theta'} p_\kappa\left(\theta' \mid \theta\right) v_\kappa^\infty\left(\theta'\right) \geq r_{\sigma'}\left(\theta,\tilde{\theta}\right) + \delta\sum_{\theta'} p\left(\theta' \mid \theta,\tilde{\theta},\sigma'\left(\theta,\tilde{\theta}\right)\right) v_{\kappa,\sigma'}^\infty\left(\theta'\right),\ \forall\left(\theta,\ \tilde{\theta}\right) and\ \forall\sigma' . \quad (13)$$

where $p\left(\theta' \mid \theta,\tilde{\theta},\sigma'\left(\theta,\tilde{\theta}\right)\right)$ is the atransition probability of a peer's reputation from θ to θ' when it plays $\sigma'\left(\theta,\tilde{\theta}\right)$ to a client with a reputation $\tilde{\theta}$; and $v_{\kappa,\sigma'}^\infty(\theta)$ is the expected overall utility when the peer plays σ' in the network implementing the social norm κ.

By comparing the sum of a peer's instant utility in the current slot and the expected future utility, the peer will not have the incentive to deviate if it will receive a lower sum of the overall utility by doing so. As we can see, the social norm equilibrium prohibits a peer to gain in any situation of the repeated game, and hence, it is at the same time a *subgame perfect equilibrium* [24].

The definition of social norm equilibrium requires us to verify σ 's optimality over all possible behavioral strategies under κ, whose computation complexity is high. By employing the criterion of unimprovability in Markov decision theory [34], we establish the one-shot deviation principle for social norm equilibrium to simplify the computation, which at the same time serves as a sufficient and necessary condition for the existence of social norm equilibrium.

Lemma 4 (One-shot Deviation Principle). κ is a social norm equilibrium if and only if for any pair $\left(\theta,\tilde{\theta}\right)$, there is no profitable one-shot deviation, i.e.

$$r_{\sigma'}\left(\theta,\tilde{\theta}\right)-r_{\sigma}\left(\theta,\tilde{\theta}\right)\le\delta\left[\sum_{\theta'}p_{\kappa}\left(\theta'\mid\theta\right)v_{\sigma}^{\infty}\left(\theta'\right)-\sum_{\theta'}p\left(\theta'\mid\theta,\tilde{\theta},\sigma'\left(\theta,\tilde{\theta}\right)\right)v_{\sigma}^{\infty}\left(\theta'\right)\right] \tag{14}$$

Proof: If $\kappa=(\sigma,\tau)$ is an equilibrium, then clearly there are no profitable one-shot deviations.

Conversely, if κ is not an equilibrium, we need to show that there will be at least one profitable one-shot deviation. Since $r_{\sigma'}\left(\theta,\tilde{\theta}\right)$ is bounded for an arbitrary σ', this is always true due to the unimprovability of Markov decision theory [16][17].

Lemma 4 shows that if a peer cannot gain by unilaterally deviating from σ only in the current slot and following σ afterwards, it also cannot gain by switching to any other strategy σ', and vice versa.

B. The existence of social norm equilibrium

Note that the family of protocols that we are interested in can be fully defined using two elements: the punishment length L and the set of service thresholds $\{m_{\sigma}(\theta)\}$. In this section, we use the one-shot deviation principle to analyze how to select L and $\{m_{\sigma}(\theta)\}$ depending on the network condition ε and α, as well as the peer's discount factor β, in order to design a social norm equilibrium.

When an innocent peer receives the service request from another innocent peer, it should always provide the service as specified by σ. By following it, the peer expects to receive an overall utility of $V_L(C)=-\rho+\delta\left[(1-\varepsilon)v_{\kappa}^{\infty}(L)+\varepsilon v_{\kappa}^{\infty}(0)\right]$ as its reputation will remain at L if updated correctly. On the other hand, if the peer drops the request by playing $a=D$, it saves the instant serving cost of ρ in the current slot, and its reputation falls to 0 starting from the next slot with a high probability $1-\varepsilon$ [5]. The overall utility the peer can expect is thus $V_L(D)=\delta\left[(1-\varepsilon)v_{\kappa}^{\infty}(0)+\varepsilon v_{\kappa}^{\infty}(L)\right]$. As the one-shot deviation principle specifies, the peer has no incentive to drop the service request deliberately if $V_L(C)\ge V_L(D)$, which can be transformed in to the following inequality

[5] It does not affect the analysis when the reputation does not fall to 0 as in a general reputation mechanism, though peers will have a different form of incentive.

$$\rho \le \delta(1-2\varepsilon)\left[v_\kappa^\infty(L) - v_\kappa^\infty(0)\right]; \tag{15}$$

For a guilty peer, the situation is similar. When a θ – peer meets a $\tilde{\theta}$ – peer which it is required to serve by σ, i.e. $\tilde{\theta} > m_\sigma(\theta)$, the condition that needs to be met in order to prevent it from deviating deliberately is

$$\rho \le \delta(1-2\varepsilon)\left[v_\kappa^\infty(\theta+1) - v_\kappa^\infty(0)\right]. \tag{16}$$

When a peer meets another peer whose reputation is below its service threshold, $\tilde{\theta} \le m_\sigma(\theta)$, the peer will not deliberately deviate from σ since this will lead to both worse instant utility as well as worse expected future utility. Hence, there is no incentive constraint that needs to be considered in this case.

The first result we derive by analyzing Eq. (15) and Eq. (16) is that a protocol can be sustained as a social norm equilibrium if and only if the discount factor a peer adopts is large enough. In other words, when a peer gives enough weight to its future utility, it will have no incentive to deviate from the protocol.

Proposition 1. A protocol $\kappa = (\sigma, \tau)$ can be sustained as a social norm equilibrium if and only if the discount factor is larger than a threshold β_κ.

Proof: See Appendix C. ■

β_κ measures the incentive a protocol κ brings to peers. When β_κ is smaller, the peers need less foresightedness in order to obey κ, and the incentive brought by κ is thus larger. The value of β_κ is also influenced by the turnover rate α and the reputation update error ε. Thus we can also determine constraints on them for the existence of the social norm equilibrium. These constraints are instructive for determining whether it is possible to design protocols that can be obeyed by all peers depending on the network conditions.

Corollary 1. When $\alpha > \alpha_n = 1 - \frac{\rho}{r+\rho}$ or $\varepsilon > \varepsilon_n = \frac{1}{2}\left[1 - \frac{\rho}{r+\rho}\right]$, there is no protocol which can be sustained as the social norm equilibrium.

Proof: See Appendix D. ■

Corollary 1 provides a necessary condition for the existence of social norm equilibrium in the network. When the turnover rate α is large, the weight a peer gives to its future utility is small even if it adopts the largest discount factor $\beta = 1$. This makes a protocol fail to provide sufficient threat through the future punishment so as to offset a peer's instant gain from deviation, no matter how long the punishment length it deploys. When the reputation update error ε is large, both the probabilities that a peer being falsely punished while following the protocol and rewarded while breaking the protocol are high. Thus the peers will lose the incentive to obey the protocol.

Corollary 2. When $\alpha < \alpha_s = 1 - \rho / r$, the protocol that can be sustained as the social norm equilibrium always exists when $\beta \to 1$ and $\varepsilon \to 0$.

Proof: See Appendix D. ■

Corollary 3. When $\varepsilon < \varepsilon_s = \frac{1}{2}(1 - \rho / r)$, the protocol that can be sustained as the social norm equilibrium always exists when $\beta \to 1$ and $\alpha \to 0$.

Proof: See Appendix D. ■

Corollary 2 and 3 provide sufficient conditions for the existence of social norm equilibrium. Their proofs can be found in Appendix F. In contrast to Corollary 1, when α and ε are small, the existence of social norm equilibrium can be ensured as long as the peers are sufficiently foresighted with β arbitrarily close to 1.

Figure 3 plots α_n, α_s, ε_n, and ε_s as functions of the ratio between the benefit r and the cost ρ per service. For a specific pair of r and ρ, if the values of α and ε are above the red lines with cross markers, no social norm equilibrium exists. As a result, there will always be some peers who deliberately deviate from the protocol no matter how the punishment scheme is designed. On the other hand, if the values of α and ε are below the blue lines with triangle markers, social norm equilibria always exist, and hence we can always find at least one protocol which can be followed by all peers. It should also be noted that when $\frac{r}{\rho}$ increases, i.e. the service cost becomes negligible compared with the service benefit, the gaps between the blue and red lines diminish. In this case, we can use either line as an approximation of the sufficient and necessary condition for the existence of the social norm equilibrium.

The above analytical results can help us construct a method of designing a protocol as the social norm equilibrium. The procedure is specified as follows (here we use α_s and ε_s as the approximations of the sufficient and necessary condition):

- *Step 1*: Determine α and ε in the network. If $\alpha > \alpha_s$ or $\varepsilon > \varepsilon_s$, the social norm equilibrium does not exist and thus, no suitable protocol can be constructed to achieve cooperation.
- *Step 2*: If $\alpha < \alpha_s$ and $\varepsilon < \varepsilon_s$, the method searches the desirable protocol from the minimum punishment length $L = 1$.
- *Step 3*: Specify the service threshold $\{m_\sigma(\theta)\}$ and determine the discount factor's threshold β_κ [6]. If β_κ is smaller than the discount factor β that is adopted in the network, the protocol κ formed by L

[6] The method of determining β_κ can be Appendix D.

and $\{m_\sigma(\theta)\}$ is a social norm equilibrium, the method ends with κ. Otherwise impose more severe punishment by increasing the punishment length $L := L+1$ and restart step 3.

IV. The Optimal Social Norm Equilibrium

We end up the above section by providing a method of searching for the social norm equilibrium. However, multiple protocols may exist in the network that can be sustained as social norm equilibria. To select among these protocols, we would like to find the one that optimizes the network's performance, which is defined as the social utility in Eq. (10). The protocol designer's problem can be formalized as follows, with the resulting protocol being the *optimal social norm equilibrium*:

$$\begin{array}{ll} \underset{(L,\{m_\sigma(\theta)\})}{maximize} & U_\kappa = \sum_\theta \eta_\tau(\theta) v_\kappa(\theta) \\ subject\ to: & \beta(1-\alpha)(1-2\varepsilon)\left(v_\kappa^\infty(\theta) - v_\kappa^\infty(0)\right) \ge \rho,\ \forall\theta \end{array}. \tag{17}$$

Since $r > \rho$, it is obvious that the social utility is maximized when full cooperation exists in the network, i.e. all peers provide services to others once is requested. This gives the most efficient outcome that the network can achieve, denoted as $\bar{U} = r - \rho$. Nevertheless, since the punishment implemented in a protocol always leads to some efficiency loss, we can show that full cooperation and hence $\bar{U}$ can be achieved if and only if there is no error in the network, i.e. $\varepsilon = 0$.

Lemma 5. The social utility of the network can reach $\bar{U} = r - \rho$ if and only if $\varepsilon = 0$.

Proof: See Appendix E. ■

When $\varepsilon > 0$, an efficiency loss always exists since there are always some peers that will be falsely punished even if all peers follow the protocol and no one deviates deliberately. Since innocent peers always punish guilty peers in a protocol, it is straightforward that a protocol with a punishment length L can achieve its optimal social utility if its service thresholds are specified as: $m_\sigma(\theta) = -1, 0 \le \theta \le L-1$ and $m_\sigma(L) = L-1$, in which the guilty peers always mutually help each other. The corresponding optimal social utility is

$$U(L) = \sum_{\theta=0}^{L-1} \eta_\tau(\theta)\left[\sum_{\theta=0}^{L-1} \eta_\tau(\theta) r - \rho\right] + \eta_\tau(L)\left[r - \eta_\tau(L)\rho\right]. \tag{18}$$

By varying the punishment length L, we can compute the difference between $U(L)$ and $U(L-1)$ as

$$\begin{aligned} &\Delta U(L) = U(L) - U(L-1) \\ &= -\eta_\tau(L-1)\left\{(r-\rho)\left(\eta_\tau(L) - \sum_{\theta=0}^{L-2} \eta_\tau(\theta)\right)\right\} \\ &= -(1-\alpha)(1-\varepsilon)^{L-2} \varepsilon \left\{(r-\rho)\left[2\alpha - 1 + (1-\alpha)(1-\varepsilon)^{L-1} + (1-\alpha)(1-\varepsilon)^L\right]\right\} \end{aligned}. \tag{19}$$

whose polarity is determined by the polarity of the term $\left[2\alpha-1+(1-\alpha)(1-\varepsilon)^{L-1}+(1-\alpha)(1-\varepsilon)^{L}\right]$, which monotonically decreases with L. Therefore, the optimal social utility that can be achieved by a protocol when $\varepsilon>0$ is reached when the punishment length is either $L=1$ or $L\to\infty$. In the first case, the protocol has a minimum punishment length; and in the second case, the protocol has infinite punishment such that all peers are guilty and mutually help each other, which is obviously infeasible in practical P2P networks.

Since a shorter punishment length requires a smaller set of reputations, which in turn saves storage space and communication overheads from/to the tracker, it is desirable if we can minimize the punishment length to achieve the optimal social utility. With $L=1$, there are only two reputation levels and the service thresholds can be specified as $m_\sigma(0)=-1$ and $m_\sigma(1)=0$. The resulting protocol is denoted as κ_1. In the following proposition, we provide sufficient conditions for κ_1 to achieve the optimal social utility when $\varepsilon>0$.

Proposition 2. If $\alpha>\frac{1}{2}$ or $\varepsilon\le\alpha$, κ_1 is the protocol which achieves the optimal social utility.

Proof: From Eq. (19), $L=1$ is the optimal punishment length when (1) $\alpha\ge\frac{1}{2}$ or (2) $\alpha<\frac{1}{2}$ and $U(1)>\lim_{L\to\infty}U(L)$. We have

$$U(1)=(1-\alpha)\varepsilon\left[(1-\alpha)\varepsilon r-\rho\right]+\left[\alpha+(1-\alpha)(1-\varepsilon)\right]\left[r-\left(\alpha+(1-\alpha)(1-\varepsilon)\right)\rho\right] \tag{20}$$

and

$$\lim_{L\to\infty}U(L)=\left(1-\alpha+\alpha^2\right)(r-\rho). \tag{21}$$

When $\varepsilon<\alpha$,

$$\begin{aligned}U(1)&>\left[\alpha+(1-\alpha)(1-\varepsilon)\right]\left[r-\left(\alpha+(1-\alpha)(1-\varepsilon)\right)\rho\right]\\&>\left[\alpha+(1-\alpha)(1-\varepsilon)\right][r-\rho]>\left[\alpha+(1-\alpha)(1-\alpha)\right][r-\rho]=\left(1-\alpha+\alpha^2\right)(r-\rho)\end{aligned}. \tag{22}$$

■

With the sufficient conditions provided by Proposition 2, κ_1 is the optimal social norm equilibrium that solves Eq. (17) if the incentive constraints are satisfied. Proposition 3 gives a sufficient condition that κ_1 is sustained as a social norm equilibrium.

Proposition 3. When $\varepsilon<\alpha<1-\left(\frac{\rho}{(1-2\varepsilon)\beta r}\right)^{\frac{1}{3}}$, κ_1 is the optimal social norm equilibrium, i.e. the optimal norm-based protocol that maximizes the social utility.

Proof: With κ_1 implemented, the reputation distribution of the population is

$$\begin{aligned}\eta(0) &= (1-\alpha)\varepsilon \\ \eta(1) &= 1-(1-\alpha)\varepsilon\end{aligned}. \tag{23}$$

Substituting this into Eq. (7), we get the expected stage-game utilities of κ_1 as

$$\begin{aligned}v_{\kappa_1}(0) &= (1-\alpha)\varepsilon r - \rho \\ v_{\kappa_1}(1) &= r - \left[1-(1-\alpha)\varepsilon\right]\rho\end{aligned}. \tag{24}$$

From Eq. (43), we have

$$\begin{aligned}v_{\kappa_1}^{\infty}(0) &= v_{\kappa_1}(0) + \delta\left[(1-\varepsilon)v_{\kappa_1}^{\infty}(1) + \varepsilon v_{\kappa_1}^{\infty}(0)\right] \\ v_{\kappa_1}^{\infty}(1) &= v_{\kappa_1}(1) + \delta\left[(1-\varepsilon)v_{\kappa_1}^{\infty}(1) + \varepsilon v_{\kappa_1}^{\infty}(0)\right]\end{aligned}, \tag{25}$$

and thus $v_{\kappa_1}^{\infty}(1) - v_{\kappa_1}^{\infty}(0) = v_{\kappa_1}(1) - v_{\kappa_1}(0)$.

Consequently, the following inequality needs to be satisfied in order to sustain κ_1 as an equilibrium

$$\begin{aligned}\rho &\le \beta(1-\alpha)(1-2\varepsilon)\left(v_{\kappa_1}(1) - v_{\kappa_1}(0)\right) \\ &= \beta(1-\alpha)(1-2\varepsilon)\left[\left(1-(1-\alpha)\varepsilon\right)r + (1-\alpha)\varepsilon\rho\right]\end{aligned} \tag{26}$$

Since we already have $\varepsilon < \alpha$, this inequality is satisfied when

$$(1-\alpha)\left(1-(1-\alpha)\varepsilon\right)r \ge (1-\alpha)^3 r \ge \frac{\rho}{\beta(1-2\varepsilon)}. \tag{27}$$

As $1-\varepsilon > 1-\alpha$, Inequality (26) is again satisfied when

$$(1-\alpha)^3(1-2\varepsilon) \ge \frac{\rho}{\beta r}, \tag{28}$$

which leads to our conclusion. ■

The reason that α is upper-bounded is due to the fact that a peer needs to put sufficiently large weight on its future utility to have the incentive to follow κ_1. On the contrary, α should also be larger than ε such that there are more innocent peers than guilty peers at the beginning of each slot. As a result, $L=1$, rather than $L \to \infty$, can maximize the social utility. As a byproduct of proving Proposition 3, we can show that when $\alpha < 1-\left(\frac{\rho}{r}\right)^{\frac{1}{3}}$, κ_1 can be sustained as the equilibrium social norm as long as the discount factor β is sufficiently large and the reputation-update error approaches to 0.

Corollary 4. When $\alpha < 1-\left(\frac{\rho}{r}\right)^{\frac{1}{3}}$, κ_1 can always be sustained as the equilibrium social norm if $\beta \to 1$ and $\varepsilon \to 0$. ■

When κ_1 cannot be sustained as a social norm equilibrium, we need to extend the punishment length in the protocol to provide peers stronger incentives. Despite its existence, there is no simple explicit expression of the optimal social norm equilibrium. We therefore propose a general iterative method to find a social norm equilibrium which has close performance to the optimal social norm equilibrium.

- *Step 1*. Set $U_{\kappa_{opt}} = 0$. Determing whether κ_1 is the optimal social norm equilibrium by checking the condition in Proposition 3 satisfies the incentive constraints in Eq. (17). If it is satisfied, the method stops. We have the optimal social norm equilibrium $\kappa_{opt} = \kappa_1$ and $U_{\kappa_{opt}} = U_{\kappa_1}$.

If the condition in Proposition 3 is not satisfied, the method then enters the iterative process.

- *Step 2*. Increase the punishment length by 1 as $L := L + 1$.
- *Step 3*. Find the set of feasible social strategies B_L which satisfies the incentive constraints in Eq. (17) with the punishment length L.
- *Step 4*. If B is empty, go to step 2. Otherwise, find the strategy $\sigma_L \in B$ which maximizes the social utility in B. Let κ_L be the social norm formed by L and σ_L. If the corresponding social utility $U_{\kappa_L} > U_{\kappa_{opt}}$, set $\kappa_{opt} := \kappa_L$ and $U_{\kappa_{opt}} = U_{\kappa_L}$. The method goes to step 2. Otherwise, the method stops, and κ_{opt} is the near-optimal social norm equilibrium selected.

A schematic representation of this method is shown in Figure 2. The performance gap between the output of our method and the optimal social norm equilibrium is plotted in Figure 4, with the social utilities normalized by $\bar{U}$. As it shows, when the network conditions are good (i.e. α and ε are small), our method can well approximate the optimal social norm equilibrium with a performance gap close to 0. Meanwhile, Figure 4 (b) shows that either the optimal social norm equilibrium or the near-optimal protocol can achieve the most efficient outcome $\bar{U}$, i.e. full cooperation in the network, when ε approaches 0. It should also be noted that when $\alpha > \alpha_s = 1 - \frac{\rho}{r}$ and $\varepsilon > \varepsilon_s = \frac{1}{2}\left(1 - \frac{\rho}{r}\right)$ as pointed in Figure 3, a social norm equilibrium does not exist, and thus the social utility faces significant degradation beyond those points.

V. THE SOCIAL COST OF WHITEWASHING

In Section III, we show that the free-riding behavior can be effectively discouraged by the punishment executed in a protocol since no peer will deliberately deviate from it. However, the effectiveness of the punishment scheme is undermined by the whitewashing effect [8], which refers to the action of a peer leaving and rejoining the network in order to acquire a new identity. In particular, a free-rider might choose to whitewash in order to get an initial reputation K the same as a new peer when it rejoins the network. Since higher reputations lead to larger expected overall utilities, a peer with a reputation $\theta < K$ might have the incentive to whitewash itself if the cost of acquiring a new identity is low. The lower the cost of acquiring a new identity is, the more likely a peer will engage in whitewashing. Under our framework, a protocol assigns new peers an initial reputation $K < L$ to decrease the overall utility that a new peer expects to receive. A guilty peer's incentive to whitewash its identity is thus eliminated. Since the extra punishment on new peers will degrade the social utility of the network and it is getting severe as K decreases, we would like to find the largest initial reputation K_{opt} which ensures that whitewashing is eliminated in the networks.

A peer's incentive of whitewashing depends on the cost of acquiring a new identity, denoted as ρ_n. We have the following preliminary results:

- When $\rho_n = 0$, a peer can whitewash itself freely. This is also referred to as the *free-identity* problem [8]. In this case, a whitewasher with a reputation θ can gain a benefit of $v_\kappa^\infty(K) - v_\kappa^\infty(\theta)$ while paying no cost. Consequently, K should be set to 0 in order to eliminate the incentive of whitewashing for any peer.
- When $\rho_n \to \infty$, it is impossible for a peer to replace its identity. It is straightforward that no peer will have the incentive to whitewash itself and the protocol does not have to impose any punishment on new peers, and thus $K = L$. This is also referred to as the *permanent-identity* problem [8].

In the remainder of this section, the general case of $\rho_n \in (0, \infty)$ is discussed. Because a whitewasher leaves the network at the end of a slot, it is not affected by the future punishment imposed by the protocol beyond this slot. It can thus expect a future utility of $v_\kappa^\infty(K)$. Alternatively, if a $\theta-$peer stays in the network, it can expect a future utility of $(1-\varepsilon)v_\kappa^\infty(\theta+1) + \varepsilon v_\kappa^\infty(0)$. Whitewashing is effectively prevented on a $\theta-$peer if its loss on the future utility due to whitewashing sufficiently offsets its instant gain. As a peer's gain in a stage game by breaking the protocol is upper-bounded by ρ, no peer will have the incentive to whitewash itself if the following inequalities holds for any θ

$$\beta(1-\alpha)\left[(1-\varepsilon)v_\kappa^\infty(\theta+1) + \varepsilon v_\kappa^\infty(0) - v_\kappa^\infty(K)\right] \geq \rho - \rho_n, \forall\theta. \tag{29}$$

From Eq. (29), we can extend the above preliminary results. Proposition 4 shows that new peers should start at a low reputation when ρ_n is small, and quantifies the upper bound of ρ_n when $K_{opt} = 0$. Proposition 5 shows that new peers can start with a high reputation when ρ_n is large enough, and quantifies the lower bound of ρ_n when $K_{opt} = L$.

Proposition 4. If $\rho_n < \rho$, K_{opt} should be set as 0 in order to prevent whitewashing.

Proof: When $\rho_n < \rho$, we have

$$\beta(1-\alpha)\left[(1-\varepsilon)v_\kappa^\infty(1) + \varepsilon v_\kappa^\infty(0) - v_\kappa^\infty(K)\right] \geq \rho - \rho_n > 0. \tag{30}$$

Since $(1-\varepsilon)v_\kappa^\infty(1) + \varepsilon v_\kappa^\infty(0) - v_\kappa^\infty(K) < 0$ if and only if $K = 0$, we should assign a initial reputation of 0 in this case. ■

Proposition 5. When $\rho_n > \rho + (r+\rho)/(1-\beta(1-\alpha))$, K_{opt} can be set as L without giving any incentive to whitewashing.

Proof: If $K = L$ and whitewashing is prevented, we should have

$$\beta(1-\alpha)\left[(1-\varepsilon)v_\kappa^\infty(1) + \varepsilon v_\kappa^\infty(0) - v_\kappa^\infty(L)\right] \geq \rho - \rho_n . \tag{31}$$

From the proof of Proposition 3, we know that both $v_\kappa^\infty(1)$ and $v_\kappa^\infty(0)$ are smaller than $\left[1-\eta_\tau(L)\right]r - \eta_\tau(L)\rho$, while $v_\kappa^\infty(L) = r - \eta_\tau(L)\rho$. Therefore, the right side of Eq. (31) is larger than $-\beta(1-\alpha)\left[\alpha + (1-\alpha)(1-\varepsilon)\right]r$ and our conclusion is derived. ■

When $\rho < \rho_n < \rho + (r+\rho)/(1-\beta(1-\alpha))$, K_{opt} is determined through the following optimization

$$\begin{aligned} & K_{opt} = \arg\max K \\ & s.t.\ 0 \leq K \leq L \\ & \quad \delta\left[(1-\varepsilon)v_\kappa^\infty(\theta) + \varepsilon v_\kappa^\infty(0) - v_\kappa^\infty(K)\right] \geq \rho - \rho_n, \forall\theta \end{aligned} . \tag{32}$$

An example on the selection of K_{opt} is discussed in the next section together with the illustrative example of a specific indirect reciprocity based protocol.

VI. ILLUSTRATIVE EXAMPLE

We illustrate how to use the above analytical framework in practical protocol design by considering a specific set of social norms, whose service thresholds are $m_\sigma(\theta) = L-1$ for all θ. We call the set of the corresponding protocols as "Constant-service Protocol" or CP for short. In the CP, an innocent peer is always being served by all other peers, while a guilty peer cannot receive any service. Any peer's deviation from the above strategy triggers a punishment on it that lasts for L slots. During the punishment phase, if the guilty peer deviates from the social strategy again, the punishment is restarted from the beginning. The CP's rule can be simply summarized as follows

- If the client is innocent, the server cooperates.
- If the client is guilty, the server defects.

Formally, the CP's social strategy σ_{CP} can be represented as

$$\sigma_{CP}\left(\theta,\tilde{\theta}\right) = \begin{cases} C, & if\ \tilde{\theta} = L \\ D, & otherwise \end{cases}, \tag{33}$$

We can also obtain the corresponding stage-game utility by substituting Eq. (33) into Eq. (7)

$$v_{CP}(\theta) = \begin{cases} -\eta_{CP}(L)\rho, & if\ \theta < L \\ r - \eta_{CP}(L)\rho, & if\ \theta = L \end{cases}, \tag{34}$$

where $\eta_{CP}(L)$ is the fraction of innocent peers in CP's stationary reputation distribution.

A. Equilibrium analysis

We consider how to construct the CP as the social norm equilibrium. The analysis on the CP as in Section III shows that among all the peers, an innocent peer always has the largest incentive to follow as the right side of Eq. (16) is maximized, with the incentive monotonically increases with L; whereas a $0-$peer always has the smallest incentive to follow as the right side of Eq. (16) is minimized, with the incentive monotonically decreases with L. Therefore, in order to be sustained as a social norm equilibrium, the CP's punishment length should be large enough to provide the sufficient incentive for innocent peers and small enough to provide the sufficient incentive for $0-$peers. The result is quantified in the next proposition.

Proposition 6. The CP can be sustained as a social norm equilibrium if and only if its punishment length

$$\frac{\ln\left(\frac{1}{1-\beta(1-\alpha)}-\frac{\rho}{\beta(1-\alpha)(1-2\varepsilon)r}\right)}{\ln\beta(1-\alpha)} \leq L \leq \frac{\ln\frac{\rho(1-\varepsilon)}{r(1-2\varepsilon)}}{\ln\beta(1-\varepsilon)(1-\alpha)}. \tag{35}$$

Proof: The differences on the CP's stage-game utility can be derived from Eq. (34) as

$$\triangle v_{CP}(\theta)=\begin{cases}0, & if\ \theta<L\\ r, & if\ \theta=L\end{cases}, \tag{36}$$

and thus

$$v_{CP}^{\infty}(1)-v_{CP}^{\infty}(0)=\beta^{L-1}(1-\alpha)^{L-1}(1-\varepsilon)^{L-1}r. \tag{37}$$

Applying the equilibrium social norm's sufficient and necessary condition from Eq. (56), we have the conclusion that CP is an equilibrium if and only if

$$\beta(1-\alpha)(1-2\varepsilon)\beta^{L-1}(1-\alpha)^{L-1}(1-\varepsilon)^{L-1}r\geq\rho. \tag{38}$$

Solving this inequality leads to the right half of Eq. (35).

To provide the correct incentive for the innocent peers from deviation, we should have $\delta(1-2\varepsilon)\left[v_{CP}^{\infty}(L)-v_{CP}^{\infty}(0)\right]\geq\rho$. Since

$$v_{CP}^{\infty}(L)-v_{CP}^{\infty}(0)=\left(\frac{1-\delta^{L}}{1-\delta}\right)r, \tag{39}$$

we have

$$\delta(1-2\varepsilon)\left(\frac{1-\delta^{L}}{1-\delta}\right)r\geq\rho, \tag{40}$$

which leads to the left side of Eq. (35). ■

Stemming from Proposition 6, it can be shown that when the P2P network is socially valuable, i.e. $r > \rho$, there always exists at least one equilibrium CP, as long as ε and α are small enough and peers have sufficient rationality.

Corollary 5. There always exists at least one CP that can be sustained as a social norm equilibrium when $\varepsilon \to 0$, $\alpha \to 0$, and $\beta \to 1$.

Proof: From Proposition 6 by setting $L = 1$. ■

Next, we are interested in finding the optimal punishment length of the CP that maximizes the social utility. The social utility of the CP is

$$\begin{aligned} U_{CP}(L) &= -\eta_{CP}(L)\rho(1-\eta_{CP}(L)) + (r - \eta_{CP}(L)\rho)\eta_{CP}(L) \\ &= \eta_{CP}(L)(r-\rho) \end{aligned} . \quad (41)$$

Since $\eta_{CP}(L)$ also monotonically decreases with L, we can conclude that the social utility of CP is always maximized when $L = 1$.

B. Numerical results

In this section, we present the simulation results to show the CP's performance. Unless stated otherwise, the setting of the P2P network is as below: the number of peers ($N = 1000$), the benefit per service ($r = 20$), the cost per service ($\rho = 1$), and the initial reputation of the new peer ($K = L$). The social utilities are all normalized by $\bar{U}$.

The Impact of Network conditions: We first look at how the network conditions impact the CP's performance. Figure 6 illustrates the social utility achieved with different α and ε. The punishment lengths chosen for illustration are $L = 1,3,5$, respectively. As we can see, the social utility increases as the turnover rate α rises when $\alpha < \alpha_n$. This is due to the fact that the new peers are regarded as innocent. With larger α, more guilty old peers will be replaced by innocent new peers at the end of each slot. Since innocent peers are always being served, the corresponding social utility rises. However, high α will hurt a peer's incentive to follow the CP as the peer gives less weight to its future utility. As we showed in Corollary 1, when $\alpha > \alpha_n = 1 - \frac{\rho}{r+\rho} = 0.95$, no social norm equilibrium exists and all peers will choose to deviate, with the corresponding social utility falls to 0.

The situation is similar with the reputation update error ε. As ε rises, the social utility decreases since high error in reputation update hurts the incentive of peers to follow the CP. When $\varepsilon > \varepsilon_n = 0.5\left[1 - \frac{\rho}{r+\rho}\right] = 0.48$, no peer has the incentive to follow the CP and the social utility falls to 0.

On the other direction, when ε approaches 0, the CP can always achieve the most efficient outcome $\bar{U}$, regardless of the punishment length chosen.

The Impact of Peer Condition: We then discuss how the peer condition impact the CP's performance, specifically we look at the discount factor β and the rate that a peer generates download request, denoted as λ. The results are shown in Figure 6.

With β increases, a peer gives higher weight on its overall utility instead of its instant utility. As Proposition 1 shows, the social norm will be more effective in incentivizing peers to cooperate in such cases. It should also be noted that the CP with longer punishment has a better performance when β is small, even though the punishment prohibits the sharing activity between peers. The reason for this phenomenon is that innocent peers in the CP have stronger incentives if they face a longer punishment. As a result, when β is around 0.3, innocent peers with $L = 3$ and 5 already have the incentive to follow the protocol, and will mutually cooperate with each other; while innocent peers with $L = 1$ are still disincentivized and will not cooperate in the case.

In the above analysis, we assume that a peer generates and receives one service request on average per slot, i.e. $\lambda = 1$. Here we analyze how variable λ impacts the social utility. We assume that the arrival of service requests is non-bursty, i.e. $\lambda < 1$. If multiple service requests arrive at a peer simultaneously in a slot, a peer would serve only one request and drop all the rest. In this case, the peer will still be rewarded by the social norm since it provides the service as specified by the protocol, whereas it should in fact be punished since some other requests are dropped. The correct execution of a protocol will be violated in this case. As Figure 6 shows, the social utility increases with λ, which is intuitive since more data traffic in the network will generally make the network more socially valuable. λ also impacts a peer's incentive to follow the social norm. Since the expected future utility a peer receives will be discounted by λ and the instant utility a peer receives always remains unchanged, a lower λ decreases a peer's expected future utility and thus decreases the peer's incentive to follow the social norm. This is the reason for the discontinuities in Figure 6.

The Impact of Non-reciprocative Peers: Our analysis in this paper assumes that all peers in the network are reciprocative, which are self-interested and strategic while selecting their actions to play. Nevertheless, in practical P2P networks, there are also other types of peers who are not strategic and play constant actions. For example, the benevolent external helper, e.g. seeds, have all the files and serve other peers unconditionally; and there are also the malicious peers, who always refuse to help any other peer. We adjust the fractions of such peers in the peer population to examine how the social utility received by the reciprocative peers will be influenced by the non-reciprocative population. The fractions of benevolent helpers and malicious peers in the peer population are denoted as α_c and α_d, respectively.

As Figure 7 illustrates, the social utility of the reciprocative peers falls rapidly against α_d and gradually approaches 0. In the case of α_c, the social utility in general rises as α_c increases. However, because a large α_c ensures that a reciprocative peer has a high chance to be served and thus lowers its risk of being punished, it has less incentive to follow the protocol. Therefore, the social utility drops immediately when α_c achieves some level, as those are the points where reciprocative peers lose their incentives to follow the CP.

The Impact of the Initial Reputation: We finish the discussion of the CP by analyzing its selection of the optimal initial reputation K_{opt} for new peers. Figure 8 presents the selection of K_{opt} and the resulting social utility as functions of the new identity cost ρ_n. Since low turnover rate α makes a peer more foresighted when evaluating its overall utility, the penalty on the new peers becomes more effective on preventing whitewashing. Hence the required penalty is less severe with low α than that with high α, with the corresponding K_{opt} being larger. Also high α implies a more dynamical peer population with a high fraction of new peers in each slot. Therefore, the penalty on new peers decreases the social utility more severely. When $\alpha = 0$, the network is static and hence its social utility will not be influenced by the selection of K_{opt} .

VII. CONCLUSIONS

In this paper, we used the idea of "social norm" to establish a rigorous mathematical framework to analyze the incentive protocols based on indirect reciprocity in P2P file-sharing networks. We showed that under certain conditions, the protocol can be sustained as a social norm equilibrium in the network, in which no peer has the incentive to deviate deliberately. These conditions include: (1) a peer gives sufficient weight to its future utility; (2) the network error is sufficiently small; and (3) the network's turnover rate is sufficiently small. We also quantified the efficiency loss in the social norm equilibrium as a trade-off to the incentive given to peers to follow the protocol. We proved that full cooperation can be achieved when the network error approaches 0, in which case there is no trade-off between the efficiency loss and the incentive. We also studied the problem of finding the optimal social norm equilibrium in the aim of maximizing the network's social utility. An efficient searching algorithm for a near-optimal protocol was proposed. In addition, we proposed a whitewashing prevention mechanism and proved that the whitewashing effect can always be eliminated in indirect reciprocity mechanisms. The efficiency loss due to such a prevention mechanism was also studied. Our simulation measured the robustness and performance of a set of specific indirect reciprocity based protocols against different aspects of the P2P network, such as unreciprocative helpers, malicious peers, variable service rate, etc.

APPENDIX A

PROOF OF LEMMA 2

Eq. (7) can be rewrite as

$$v_\kappa(\theta) = \sum_{\tilde{\theta} \in H(\theta)} \eta_\tau\left(\tilde{\theta}\right) r - \sum_{\tilde{\theta} \in S(\theta)} \eta_\tau\left(\tilde{\theta}\right)\rho . \tag{42}$$

Define $n_\sigma(\theta) = \arg\max_{\tilde{\theta}} \left\{\theta > m_\sigma\left(\tilde{\theta}\right)\right\}$, $H(\theta) = \left\{\tilde{\theta} \mid \tilde{\theta} \le n_\sigma(\theta)\right\}$ is the set of peers which provide services to the $\theta-$peer in the social strategy σ. For the consistency of representation, here we let $n_\sigma(\theta) = -1$ if $H(\theta) = \Phi$. $S(\theta) = \left\{\tilde{\theta} \mid \tilde{\theta} > m_\sigma(\theta)\right\}$, on the other hand, is the set of peers which a $\theta-$peer provides service to in the social strategy σ.

Due to $m_\sigma(\theta)$'s non-decreasing property given by Eq. (12), $S(\theta)$ preserves a non-increasing inclusion partial order over θ, i.e. $S(\theta_1) \subseteq S(\theta_2)$ if $\theta_1 > \theta_2$. With simple manipulation, it is easy to determine that $n_\sigma(\theta)$ is also a non-decreasing function over θ, which ensures the non-decreasing inclusion partial order of $H(\theta)$, i.e. $H(\theta_1) \supseteq H(\theta_2)$ if $\theta_1 > \theta_2$. Thus, it is straightforward that $v_\kappa(\theta)$ is a non-decreasing function on θ.

From Eq. (8) and (9), the set of overall utilities can be represented by the set of stage-game utilities as follows:

$$\begin{aligned} &v_\kappa^\infty(L) = v_\kappa(L) + \delta\left[(1-\varepsilon)v_\kappa^\infty(L) + \varepsilon v_\kappa^\infty(0)\right] \\ &v_\kappa^\infty(\theta) = v_\kappa(\theta) + \delta\left[(1-\varepsilon)v_\kappa^\infty(\theta+1) + \varepsilon v_\kappa^\infty(0)\right],\ 0 \le \theta < L-1 \end{aligned} . \tag{43}$$

This is a full-ranked set of equations, which has a unique solution $\left\{v_\kappa^\infty(\theta)\right\}$ and can be solved by the iterative update below, as convergence is guaranteed.

$$\begin{aligned} &v_\kappa^\infty(L)\mid_{t+1} = v_\kappa(L) + \delta\left[(1-\varepsilon)v_\kappa^\infty(L)\mid_t + \varepsilon v_\kappa^\infty(0)\mid_t\right] \\ &v_\kappa^\infty(\theta)\mid_{t+1} = v_\kappa(\theta) + \delta\left[(1-\varepsilon)v_\kappa^\infty(\theta+1)\mid_t + \varepsilon v_\kappa^\infty(0)\mid_t\right],\ 0 \le \theta < L-1 \end{aligned} \tag{44}$$

Therefore, if we pick up an arbitrary initial $\left\{v_\kappa^\infty(\theta)\mid_0\right\}$ which satisfies the non-decreasing partial order as $v_\kappa^\infty(\theta_1)\mid_0 > v_\kappa^\infty(\theta_2)\mid_0$ if $\theta_1 > \theta_2$. It is easily verified that this partial order is preserved after each update, i.e.

$$v_\kappa^\infty(\theta_1)\mid_t > v_\kappa^\infty(\theta_2)\mid_t \quad \Rightarrow \quad v_\kappa^\infty(\theta_1)\mid_{t+1} > v_\kappa^\infty(\theta_2)\mid_{t+1} v_\kappa^\infty(\theta_1)\mid_{t+1} > v_\kappa^\infty(\theta_2)\mid_{t+1}, \tag{45}$$

and hence also holds on the unique solution Eq. (44) converges to, which is the solution of Eq. (43). Therefore, the overall utility always monotonically increases with the reputation θ. ■

APPENDIX B

PROOF OF LEMMA 3

In order to show that two social norms are identical, we only have to show that a peer receives the same utility in every stage game with these two social norms.

For a social norm $\kappa = (\sigma, \tau)$ with a punishment length L and $m_\sigma(L) = L - l,\ l > 1$, we construct another social norm $\kappa' = (\sigma', \tau')$ which has the following properties: (1) the punishment length of κ' is $L - l + 1$; (2) $m_{\sigma'}(\theta) = m_\sigma(\theta),\ \theta < L - l + 1$ and $m_{\sigma'}(L - l + 1) = L - l$. Denote the stationary distributions of these two social norms as $\{\eta_\tau(\theta)\}$ and $\{\eta_{\tau'}(\theta)\}$, respectively. It is easy to verify that

$$\begin{aligned} &\eta_\tau(\theta) = \eta_{\tau'}(\theta),\ \theta < L - l + 1 \\ &\sum_{\theta = L-l+1}^{L} \eta_\tau(\theta) = \eta_{\tau'}(L - l + 1) \end{aligned} \tag{46}$$

Substituting Eq. (46) into Eq. (42), we have that

$$\begin{aligned} &v_\kappa(\theta) = v_{\kappa'}(\theta),\ \theta < L - l + 1 \\ &v_\kappa(\theta) = v_{\kappa'}(L - l + 1),\ L - l + 1 \le \theta \le L \end{aligned} \tag{47}$$

Therefore, it is obvious that peers with reputation higher than $L - l$ in κ receive the same utility in every stage game. Since there is no difference on the utilities they receive, they can be put into the same category as innocent peers in κ. In this way, κ also has peers with $L - l + 2$ reputation levels as κ', while the peer with every reputation level receives the same stage-game utility in both social norms. We can thus conclude that κ and κ' are identical, and hence a social norm with $m_\sigma(L - l + 1) = L - l$ can always be replaced by another social norm in which the innocent peers only mutually help each other. A peer with the same behavioral strategy receives the same utility after such replacement. ■

APPENDIX C

PROOF OF PROPOSITION 1

As Lemma 2 proves that the overall utility $v_\kappa^\infty(\theta)$ monotonically increases with θ, and hence the constraints in Eq. (15) and Eq. (16) are binding. Because the maximum gain a peer can achieve in stage-game utility from the deviation is ρ, we only have to examine the validity of the following inequality to verify a social norm κ's equilibrium property

$$\rho \le \beta(1-\alpha)(1-2\varepsilon)\left(v_\kappa^\infty(1) - v_\kappa^\infty(0)\right) \tag{48}$$

Expanding $v_\kappa^\infty(1) - v_\kappa^\infty(0)$

$$
\begin{aligned}
&v_\kappa^\infty(1)-v_\kappa^\infty(0)\\
&=v_\kappa(1)+\delta\left[(1-\varepsilon)v_\kappa^\infty(2)+\varepsilon v_\kappa^\infty(0)\right]-v_\kappa(0)-\delta\left[(1-\varepsilon)v_\kappa^\infty(1)+\varepsilon v_\kappa^\infty(0)\right].\\
&=v_\kappa(1)-v_\kappa(0)+\delta(1-\varepsilon)\left[v_\kappa^\infty(2)-v_\kappa^\infty(1)\right]
\end{aligned}
\tag{49}
$$

Similarly, we have

$$
\begin{aligned}
&v_\kappa^\infty(2)-v_\kappa^\infty(1)=v_\kappa(2)-v_\kappa(1)+\beta(1-\alpha)(1-\varepsilon)\left[v_\kappa^\infty(3)-v_\kappa^\infty(2)\right]\\
&\vdots\\
&v_\kappa^\infty(L-1)-v_\kappa^\infty(L-2)=v_\kappa(L-1)-v_\kappa(L-2)+\beta(1-\alpha)(1-\varepsilon)\left[v_\kappa^\infty(L)-v_\kappa^\infty(L-1)\right].\\
&v_\kappa^\infty(L)-v_\kappa^\infty(L-1)=v_\kappa(L)-v_\kappa(L-1)
\end{aligned}
\tag{50}
$$

Substituting Eq. (50) iteratively into Eq. (49), $v_\kappa^\infty(1)-v_\kappa^\infty(0)$ can be represented in terms of the stage-game utilities as

$$
\begin{aligned}
&v_\kappa^\infty(1)-v_\kappa^\infty(0)\\
&=\sum_{\theta=1}^{L}\beta^{\theta-1}(1-\alpha)^{\theta-1}(1-\varepsilon)^{\theta-1}\left[1-\beta(1-\alpha)(1-\varepsilon)\right]v_\kappa(\theta)+\beta^L(1-\alpha)^L(1-\varepsilon)^L v_\kappa(L)-v_\kappa(0).\\
&=\sum_{\theta=1}^{L}\beta^{\theta-1}(1-\alpha)^{\theta-1}(1-\varepsilon)^{\theta-1}\Delta v_\kappa(\theta)
\end{aligned}
\tag{51}
$$

As $\{v_\kappa(\theta)\}$ and L are fixed in a protocol κ, $v_\kappa^\infty(1)-v_\kappa^\infty(0)$ and hence the right side of Eq. (48) monotonically increases with β. Taking β_κ as the discount factor which equals both sides of Eq. (48), it is thus straightforward that κ can be sustained as a social norm equilibrium if and only the discount factor β adopted by peers is larger than β_κ. ■

APPENDIX D

PROOF OF COROLLARY 1 - 3

The stage-game utility $v_\kappa(\theta)$ can be represented by $m_\sigma(\theta)$ and $n_\sigma(\theta)$ as

$$
v_\kappa(\theta)=r\sum_{\tilde\theta\le n_\sigma(\theta)}\eta_\tau(\tilde\theta)-\rho\sum_{\tilde\theta>m_\sigma(\theta)}\eta_\tau(\tilde\theta).
\tag{52}
$$

Since all peers serve $L-$peers and $m_\sigma(L)=L-1$,

$$
\begin{aligned}
v_\kappa(L)&=r-\eta_\tau(L)\rho\\
&=r-\left[\alpha+(1-\alpha)(1-\varepsilon)^L\right]\rho
\end{aligned}.
\tag{53}
$$

Substitute Eq. (52) and Eq. (53) into Eq. (51), we note that an upper bound of $v_\kappa^\infty(1)-v_\kappa^\infty(0)$ can be achieved by setting $n_\sigma(0)=-1$ and $n_\sigma(\theta)=L, 0<\theta<L$, and at the same time $m_\sigma(0)=-1$ and $m_\sigma(\theta)=L-1, \forall\theta>0$, which can be represented as follows

$$
v_\kappa^\infty(1)-v_\kappa^\infty(0)<r+(1-\alpha)\rho .
\tag{54}
$$

Thereby, if $\beta(1-\alpha)(1-2\varepsilon)[r+(1-\alpha)\rho]<\rho$, then there is no social norm which can be sustained as an equilibrium. It is obvious that this inequality is satisfied when $(1-\alpha)[r+\rho]<\rho$ or $(1-2\varepsilon)[r+\rho]<\rho$. Therefore, there is no social norm equilibrium if $\alpha>\alpha_n=1-\frac{\rho}{r+\rho}$ or $\varepsilon>\varepsilon_n=\frac{1}{2}\left[1-\frac{\rho}{r+\rho}\right]$, and Corollary 1 is proved.

To prove the next two corollaries, we design a strategy σ by setting its $n_\sigma(0)=-1$, $n_\sigma(\theta)=L-1, 0<\theta<L$. Therefore, the service threshold $m_\sigma(\theta)=0, 0\le\theta<L$. The $\Delta v_\kappa(\theta)$ of σ is $(1-\eta_\tau(L))r$ when $\theta=1$, $\eta_\tau(L)r+(1-\eta_\tau(0)-\eta_\tau(L))\rho$ when $\theta=L$, and 0 otherwise. Setting $L=1$, a sufficient condition that σ can be enforced in a social norm equilibrium κ is thus

$$\beta(1-\alpha)(1-2\varepsilon)r\ge\rho. \tag{55}$$

Therefore, when $\alpha<\alpha_s=1-\rho/r$, Eq. (55) can always be satisfied when $\beta\to1$ and $\varepsilon\to0$, there is at least one social norm equilibrium with a punishment length $L=1$. Similarly, when $\varepsilon<\varepsilon_s=\frac{1}{2}(1-\rho/r)$, Eq. (55) can always be satisfied when $\beta\to1$ and $\alpha\to0$, we thus have the same conclusion. Therefore, Corollary 2 and 3 are proved. ■

APPENDIX E

PROOF OF LEMMA 5

When $\varepsilon=0$, we design the social strategy as $\{m_\sigma(\theta)=L-1,\forall\theta\}$, then we can compute the overall utilities as

$$\begin{aligned} v_\kappa^\infty(L) &= \frac{1}{1-\delta}r \\ v_\kappa^\infty(\theta) &= \frac{\delta^{L-\theta}}{1-\delta}r,\ 1\le\theta\le L-1 \\ v_\kappa^\infty(0) &= \frac{\delta^L}{1-\delta}r \end{aligned} \tag{56}$$

From Eq. (15), no $L-$peer has the incentive to deviate from the social norm deliberately when $\delta\left(\frac{1}{1-\delta}-\frac{\delta^L}{1-\delta}\right)r\ge\rho$. When $\delta\ge\frac{\rho}{r+\rho}$, this inequality can always be satisfied as long as L is large enough. Since there is no reputation update error, all peers will remain as innocent if following the social norm. In this case, full cooperation can be achieved with $\bar{U}$ being reached.

When $\varepsilon>0$, there is always a non-zero fraction of $0-$peers in the network, i.e. $\eta_\tau(0)=(1-\alpha)\varepsilon>0$. $\bar{U}$ can be reached in this case if and only if $\{m_\sigma(\theta)=-1,\forall\theta\}$, i.e. all peers in the network mutually help each other. In this case, the stage game utilities as well as the overall utilities for all peers are the same, denoted as v_σ and v_σ^∞. As the right side of Eq. (15) and Eq. (16) become 0, no social norm equilibrium can be sustained in this case with any positive service cost $\rho > 0$. Therefore, $\bar{U}$ can never be reached by a social norm equilibrium. ■

APPENDIX F

FIGURES

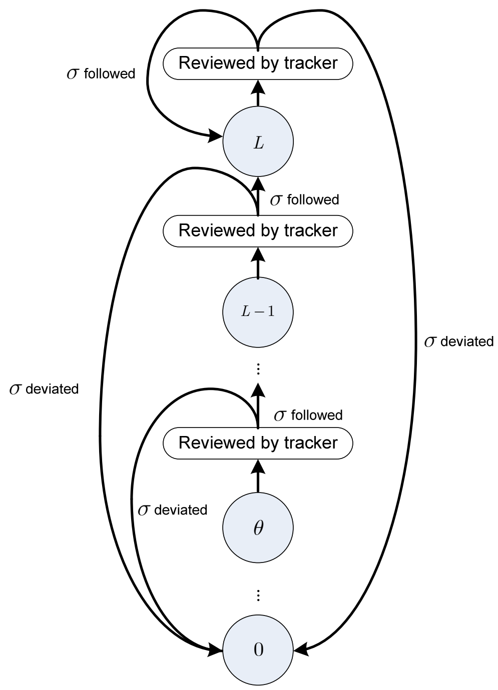

Figure 1. The schematic representation of a social norm

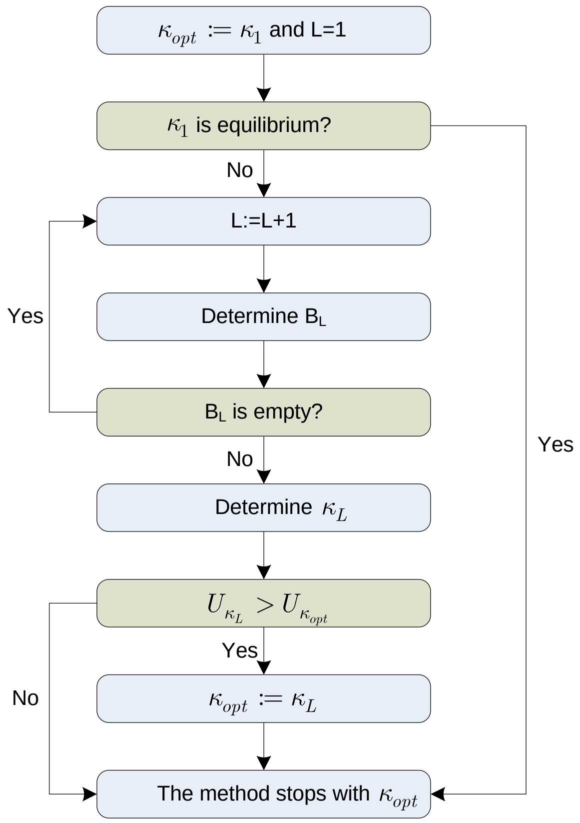

Figure 2. A schematic representation of the method to compute the optimal norm protocol

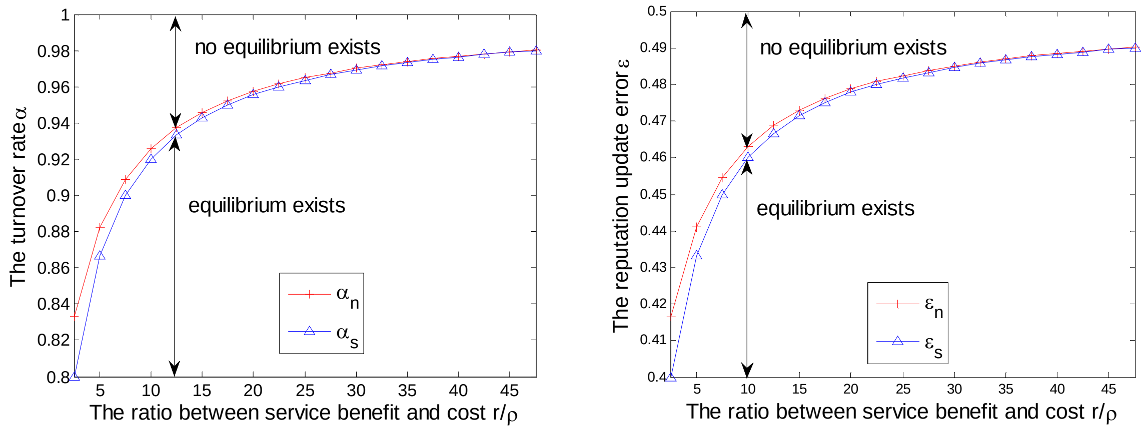

Figure 3. (a) The gap between α_n and α_s;

(b) The gap between ε_n and ε_s

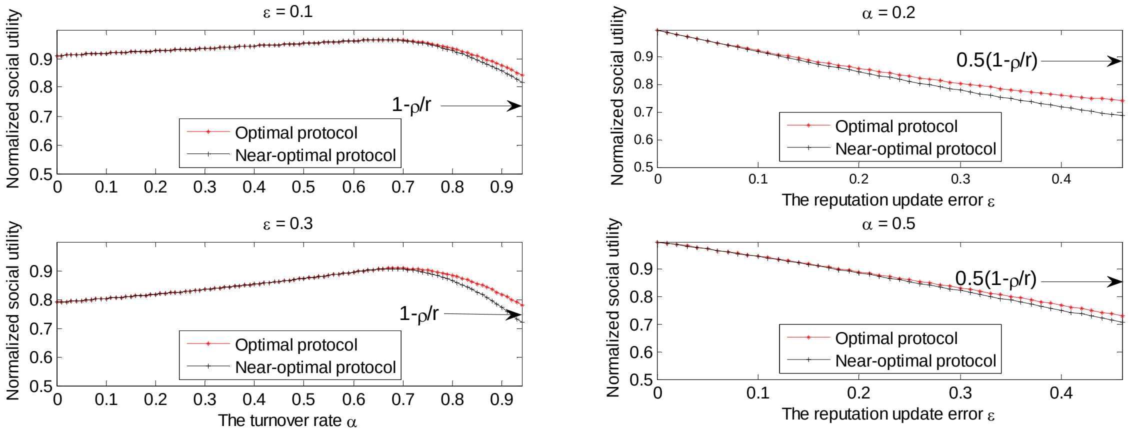

Figure 4. The performance gap between optimal protocol and close-optimal protocol (a) along with α ; (b) along with ε

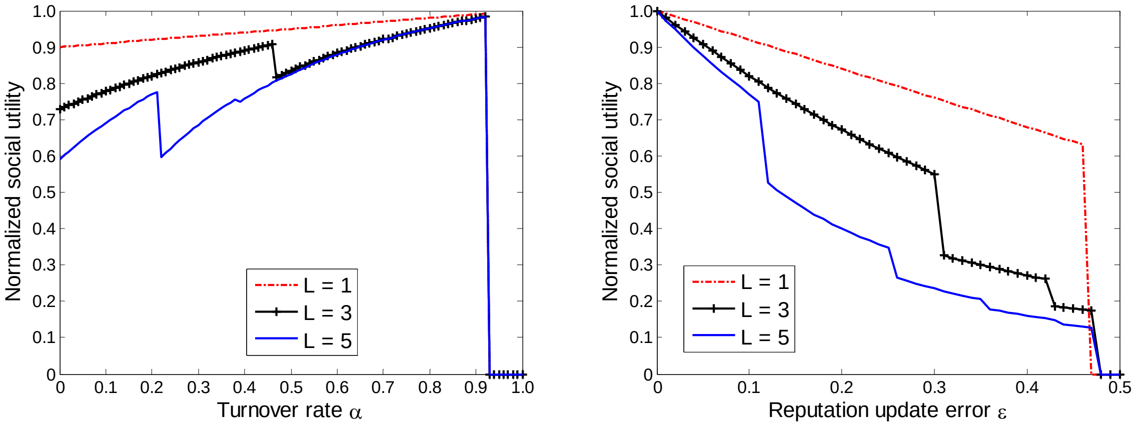

Figure 5. CP's performance along with α and ε ;

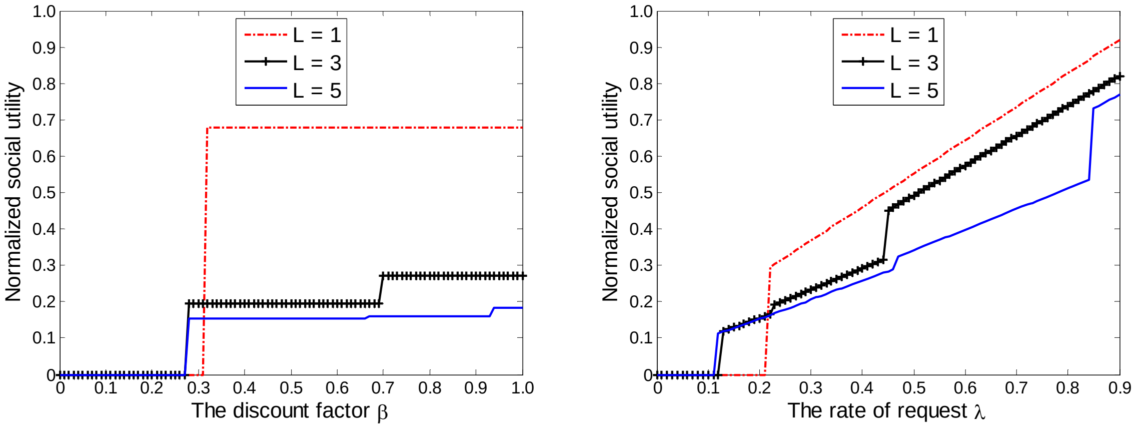

Figure 6. CP's performance along with β and λ

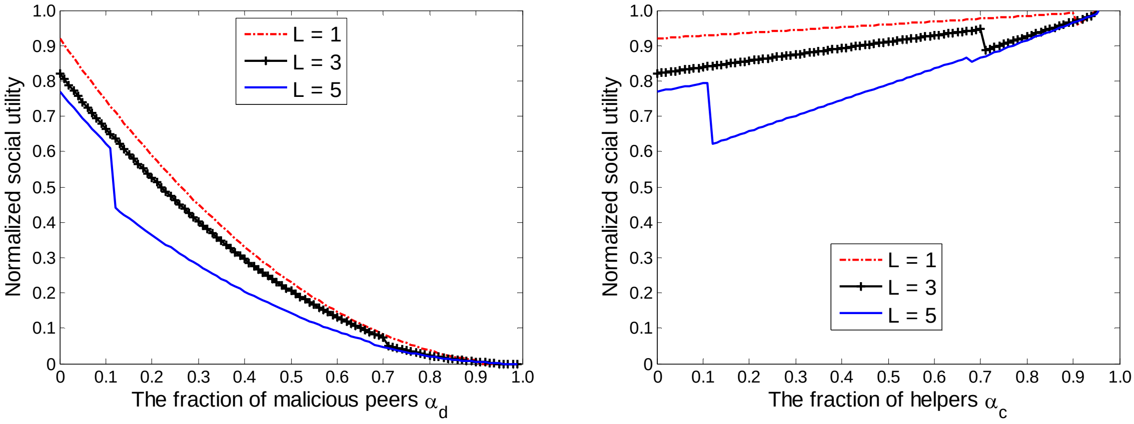

Figure 7. CP's performance along with α_d and α_c

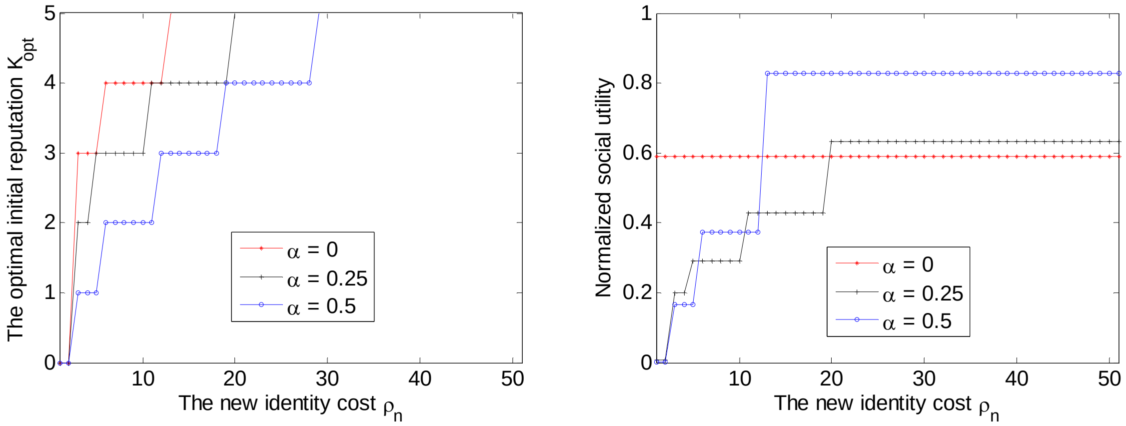

Figure 8. The optimal initial reputation and the resulting social utility along with the new identity cost

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