Intervention in Power Control Games With Selfish Users

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Abstract—We study the power control problem in single-hop wireless ad hoc networks with selfish users. Without incentive schemes, selfish users tend to transmit at their maximum power levels, causing excessive interference to each other. In this paper, we study a class of incentive schemes based on intervention to induce selfish users to transmit at desired power levels. In a power control scenario, an intervention scheme can be implemented by introducing an intervention device that can monitor the power levels of users and then transmit power to cause interference to users if necessary. Focusing on first-order intervention rules based on individual transmit powers, we derive conditions on the intervention rates and the power budget to achieve a desired outcome as a (unique) Nash equilibrium with intervention and propose a dynamic adjustment process to guide users and the intervention device to the desired outcome. We also analyze the effect of using aggregate receive power instead of individual transmit powers. Our results show that intervention schemes can be designed to achieve any positive power profile while using interference from the intervention device only as a threat. Lastly, simulation results are presented to illustrate the performance improvement from using intervention schemes and the theoretical results.

Index Terms—Game theory, incentives, intervention, power control, wireless networks.

I. INTRODUCTION

POWER control is an essential resource allocation scheme to control signal-to-interference-and-noise ratios (SINRs) for efficient transmission in wireless networks. Extensive studies have been done on power control (see [1] and references therein for an overview of the literature in this topic). In many earlier works on power control, each user has a fixed minimum SINR requirement and then minimizes its transmit power subject to the SINR requirement [1, Ch. 2]–[3]. This approach is suitable for fixed-rate communications with voice applications. However, with the growing importance of data and multimedia applications, users are no longer satisfied with a fixed SINR requirement, but they seek to maximize their utility reflecting the quality of service (QoS). To this end, most recent works formulate the problem in the network utility maximization

framework. In this framework, a central controller can compute optimal transmit power levels when the utility functions are such that the network utility maximization problem is convex, and then assigns the optimal power levels to users. Assuming that users are *obedient* to the central controller, the problem can also be solved in a distributed manner [1, Ch. 4], [4], [5].

Besides the network utility maximization framework, many works use noncooperative games to model the distributed power control problem, in which each user maximizes its own utility, instead of maximizing the network utility. In a noncooperative game model with a single frequency channel, each user tends to transmit at its maximum power level to obtain high throughput, causing excessive interference to other users. This outcome may be far from the global optimality of social welfare [1], [4], [6], especially when interference among users is strong [7]. To improve the noncooperative outcome, various power control schemes have been proposed based on pricing [8]–[12], auctions [13], and mechanism design [14], [15]. These works aim to achieve a better outcome by modifying the objective functions of users using taxation and developing a distributed method based on the optimization of the modified objective functions. Users are assumed to be obedient in that they accept the objective functions and follow the rule prescribed by the designer, and prices are used as control signals to guide users to a desired outcome. However, *selfish* users may have their own innate objectives which are different from the assigned objectives and may ignore control signals and deviate from the prescribed rule if they are better off by doing so.

In summary, the methods in most existing works are not suitable for power control with selfish users. Selfish behavior of users can arise in many practical scenarios without central controllers, such as wireless ad hoc networks, where each user transmits information from its own transmitter to its own receiver, and multi-cell cellular networks, where the base station cannot control the interfering mobile stations in other cells. Hence, it is important to design an incentive scheme to induce selfish users to achieve a desirable outcome in power control scenarios. One method to provide incentives for selfish users is to impose taxation as real money payment. However, in order to achieve a desired outcome with a pricing scheme, the designer needs to know how payment affects the payoffs of users, which is often the private information of users. The designer may use a mechanism design approach as in [14], [15] to elicit private information, but it generally requires heavy communication overheads.[1] Another method to provide incentives is to use repeated games [16], [17]. However, effective incentive schemes

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[1]Another drawback of [14] and [15] is the assumption that each user's utility function is jointly concave in all the users' power levels, which seems to be unrealistic in power control scenarios.

based on repeated games require users to have long-run frequent interactions and to be sufficiently patient [18].

Recently, a new class of incentive schemes has been proposed based on the idea of intervention [19]–[21]. To implement an intervention scheme, we need an *intervention device* that can monitor the actions of users and intervene in their interaction if necessary. The *monitoring technology* of the intervention device determines what it can observe about the actions of users, while its *intervention capability* determines the extent to which it can intervene in their interaction. An *intervention rule* prescribes the action that the intervention device should take as a function of its observation. Among existing works on intervention schemes, [19] and [20] applied intervention schemes to contention games in the medium access control (MAC) layer, while [21] studied the impact of the monitoring technology and the intervention capability on the system performance in an abstract model. We also note that [22] proposed a packet-dropping mechanism for queueing games using an idea similar to intervention. In this paper, we focus on a power control scenario and study intervention schemes in this particular scenario.

In the power control scenario considered in this paper, the intervention device estimates the individual transmit power of each user or the aggregate receive power at its receiver and then transmits at a certain power level following the intervention rule prescribed by the designer. In order to achieve a target operating point, the designer can use an intervention rule such that the intervention device transmits minimum, possibly zero, power if users are transmitting at the desired power levels, while transmitting at a high power level to reduce the SINRs of users if a deviation is detected. In this way, an intervention scheme can punish the misbehavior of users and regulate the power transmission of selfish users. We first consider a monitoring technology with which the intervention device can estimate the individual transmit power of each user without errors. While focusing on a simple class of intervention rules called first-order intervention rules, we study the requirements for the parameters of first-order intervention rules to achieve a given target power profile as a (unique) Nash equilibrium (NE). We propose a dynamic adjustment process that the designer can use to guide users to the target power profile through intermediate targets. We then relax the monitoring requirement and consider a monitoring technology with which the intervention device can estimate only the aggregate receive power. We show that with aggregate observation, intervention rules can be designed to achieve a given target as an NE but rarely as a unique NE. Our results provide a systematic design principle based on which a designer can choose an intervention scheme (an intervention device and an intervention rule) to achieve a desired outcome. Our analysis suggests that, unlike pricing schemes, it is possible for the designer to design effective intervention schemes without having knowledge about how users value their SINRs, as long as their utility is monotonically increasing with their own SINRs. We also propose a method based on intervention for the designer to estimate the cross-channel gains, the noise powers, and maximum transmit power levels of users without any cooperative behavior of users such as sending pilot signals for channel estimation and reporting the estimates to the designer. After obtaining relevant information, the designer

TABLE I
SUMMARY OF MAJOR NOTATION

Symbol	Description
N	Number of (regular) users in the network
$\mathcal{N}$	Set of users, $\mathcal{N} \triangleq \{1, \dots, N\}$
P_i	Maximum transmit power level of user i; $i \in \mathcal{N}$
P_0	Power budget of the intervention device, or the intervention capability
$\mathcal{P}_i$	User i's action space $\mathcal{P}_i \triangleq [0, P_i]$; $i \in \mathcal{N} \cup \{0\}$
p_i	Transmit power level of user i, $p_i \in \mathcal{P}_i$; $i \in \mathcal{N} \cup \{0\}$
$\mathbf{p}$	Power profile of the users, $\mathbf{p} = (p_1, \dots, p_N) \in \mathcal{P}$, where $\mathcal{P} = \triangleq \prod_{i=1}^N \mathcal{P}_i$
$\mathbf{p}^\star$	Target power profile, $\mathbf{p}^\star = (p_1^\star, \dots, p_N^\star)$
$\mathbf{P}$	Profile of the maximum power levels, $\mathbf{P} = (P_1, \dots, P_N)$
h_{ij}	Channel gain from user j's transmitter to user i's receiver, where $i, j \in \mathcal{N} \cup \{0\}$
n_i	Noise power at user i's receiver
γ_i	SINR at user i's receiver
$\mathcal{Y}$	Set of possible signals obtained by the intervention device
ρ	Signal determination function, $\rho : \mathcal{P} \to \mathcal{Y}$
f	Intervention rule, $f : \mathcal{Y} \to \mathcal{P}_0$
$\mathcal{E}(f)$	Set of power profiles sustained by f
$\mathcal{F}_K(\mathbf{p}^\star)$	Set of Kth-order intervention rules with the target power profile $\mathbf{p}^\star$
$PB_K(\mathbf{p}^\star)$	Minimum power budget to sustain $\mathbf{p}^\star$ using Kth-order intervention rules
$PB_K^s(\mathbf{p}^\star)$	Minimum power budget to strongly sustain $\mathbf{p}^\star$ using Kth-order intervention rules
f_1^I	First-order intervention rule based on individual transmit powers
α_i	Intervention rate for user i in f_1^I; $i \in \mathcal{N}$
f_1^A	First-order intervention rule based on aggregate receive power
α_0	Aggregate intervention rate in f_1^A
$\tilde{\mathcal{N}}(\mathbf{p}^\star)$	Set of users whose target powers are less than the maximum power levels, $\tilde{\mathcal{N}}(\mathbf{p}^\star) = \{i \in \mathcal{N} : p_i^\star < P_i\}$
N'	Number of users whose target powers are less than the maximum power levels, $N' = |\tilde{\mathcal{N}}(\mathbf{p}^\star)|$
$\{(f^t, \mathbf{p}^t)\}_{t=1}^T$	Sequence of intervention rules and power profiles in the dynamic adjustment process
$\{\tilde{\mathbf{p}}^t\}_{t=1}^T$	Sequence of intermediate target power profiles in the dynamic adjustment process
$d(\mathbf{p}, \mathbf{p}')$	Relative distance between two power profiles $\mathbf{p}$ and $\mathbf{p}'$, $d(\mathbf{p}, \mathbf{p}') = \sum_{i=1}^N (p_i - p_i')/p_i$
$T^\star(\mathbf{p}^\star)$	Minimum convergence time (in steps) for the dynamic adjustment process to reach $\mathbf{p}^\star$

can configure an intervention rule to achieve a target operating point as NE.

The rest of the paper is organized as follows. In Section II, we describe the system model of power control with intervention. In Section III, we propose design criteria for intervention rules, performance characteristics to evaluate intervention rules, and classes of intervention rules. In Section IV, we study the design of first-order intervention rules to achieve a target power profile. In Section V, we discuss implementation issues related to intervention. Simulation results are presented in Section VI. Finally, Section VII concludes the paper. For the ease of reference, we summarize major notation used in this paper in Table I.

II. MODEL OF POWER CONTROL WITH INTERVENTION

We consider a single-hop wireless ad hoc network, where a fixed set of N users and an intervention device transmit in a single frequency channel. The set of the users is denoted by

$\mathcal{N} \triangleq \{1, 2, \ldots, N\}$. Each user has a transmitter and a receiver. Each user i chooses its transmit power p_i in the set $\mathcal{P}_i \triangleq [0, P_i]$, where $P_i > 0$ for all $i \in \mathcal{N}$. The power profile of all the users is denoted by $\mathbf{p} = (p_1, \ldots, p_N) \in \mathcal{P} \triangleq \prod_{i=1}^{N} \mathcal{P}_i$, and the power profile of all the users other than user i is denoted by $\mathbf{p}_{-i}$.

In the network, there is an intervention device, sometimes referred to as user 0, that consists of a transmitter and a receiver. The receiver of the intervention device can monitor the power profile of the users, while the transmitter can create interference to the users by transmitting power. Once the users choose their power profile, the intervention device obtains a signal $y \in \mathcal{Y}$, where $\mathcal{Y}$ is the set of all possible signals. We assume that the signal is realized deterministically given a power profile and use the signal determination function $\rho : \mathcal{P} \rightarrow \mathcal{Y}$ to denote the signal given the power profile $\mathbf{p}$.[2] After observing a signal, the intervention device chooses its own transmit power p_0 in the set $\mathcal{P}_0 \triangleq [0, P_0]$, where $P_0 > 0$. We call $(\mathcal{Y}, \rho)$ the monitoring technology of the intervention device, and call P_0 its intervention capability. The ability of an intervention device is characterized by its monitoring technology and intervention capability. We call the transmit powers of the intervention device and the users $(p_0, \mathbf{p}) \in \mathcal{P}_0 \times \mathcal{P}$ an outcome.

The QoS obtained by user i is determined by its SINR, denoted by γ_i. We use a block-fading channel model with sufficiently long fading blocks, as in [2]–[5], and [7]–[17]. In one block, for $i, j \in \mathcal{N} \cup \{0\}$, let h_{ij} be the channel gain from user j's transmitter to user i's receiver, and let n_i be the noise power at user i's receiver. If code-division multiple accessing (CDMA) is used, the channel gain is defined as the effective channel gain taking into account the spreading factor. In one block, the users and the intervention device transmit at constant power levels $\mathbf{p}$ and p_0, respectively.[3] Then the SINR of user $i \in \mathcal{N}$ is given by

$$\gamma_i(p_0, \mathbf{p}) = \frac{h_{ii} p_i}{h_{i0} p_0 + \sum_{j \neq i} h_{ij} p_j + n_i}.^{[4]} \tag{1}$$

We assume that each user $i \in \mathcal{N}$ has monotonic preferences on its own SINR in the sense that it weakly prefers γ_i to γ_i' if and only if $\gamma_i \geq \gamma_i'$. Our analysis does not require any other properties of preferences (for example, preferences do not need to be represented by a concave utility function).[5]

In our setting, the intervention device has a receiver to measure the aggregate receive power from all the users. Furthermore, if the receiver moves and takes measurement at different

locations, it can estimate the individual transmit power of each user as well. Thus, in this paper we will focus on two types of monitoring technology with which the intervention device can estimate individual transmit powers $\mathbf{p}$ or an aggregate receive power $\sum_{i=1}^{N} h_{0i} p_i$. In other words, we consider two signal determination functions, $\rho(\mathbf{p}) = \mathbf{p}$ and $\rho(\mathbf{p}) = \sum_{i=1}^{N} h_{0i} p_i$.

III. DESIGN OF INTERVENTION RULES

A. Design Criteria

Since the intervention device transmits its power after it obtains a signal, its strategy can be represented by a mapping $f : \mathcal{Y} \rightarrow \mathcal{P}_0$, which is called an intervention rule. The SINR of user i when the intervention device uses an intervention rule f and the users choose a power profile $\mathbf{p}$ is given by $\gamma_i(f(\rho(\mathbf{p})), \mathbf{p})$. With an abuse of notation, we will use $f(\mathbf{p})$ to mean $f(\rho(\mathbf{p}))$. Given an intervention rule f, the interaction among the users that choose their own power levels selfishly can be modeled as a noncooperative game, whose strategic form is given by

$$\Gamma_f = \langle \mathcal{N}, (\mathcal{P}_i)_{i \in \mathcal{N}}, (\gamma_i(f(\cdot), \cdot))_{i \in \mathcal{N}} \rangle. \tag{2}$$

We can predict the power profile chosen by the users given an intervention rule using the concept of Nash equilibrium.

Definition 1: A power profile $\mathbf{p}^* \in \mathcal{P}$ is a Nash equilibrium of the game Γ_f if

$$\gamma_i(f(\mathbf{p}^*), \mathbf{p}^*) \geq \gamma_i(f(p_i, \mathbf{p}_{-i}^*), p_i, \mathbf{p}_{-i}^*) \tag{3}$$

for all $p_i \in \mathcal{P}_i$ and all $i \in \mathcal{N}$.

When a power profile $\mathbf{p}^*$ is a NE of Γ_f, no user has the incentive to deviate from $\mathbf{p}^*$ unilaterally provided that the intervention device uses intervention rule f. Moreover, if $\mathbf{p}^*$ is a unique NE of Γ_f, intervention has robustness in that the designer does not need to worry about coordination failure (i.e., the possibility that the users get stuck in a "wrong" equilibrium).

Definition 2: An intervention rule f *(strongly) sustains* a power profile $\mathbf{p}^*$ if $\mathbf{p}^*$ is a (unique) NE of the game Γ_f.

We use $\mathcal{E}(f)$ to denote the set of all power profiles sustained by f. Suppose that the designer has a welfare function $U_0(\gamma_1, \ldots, \gamma_N)$, defined on the users' SINRs. Then the objective of the designer is to find a target power profile that maximizes the welfare function and an intervention rule that (strongly) sustains the target power profile. Formally, the design problem solved by the designer can be written as

$$\max_{\mathbf{p}} \max_f \quad U_0(\gamma_1(f(\mathbf{p}), \mathbf{p}), \ldots, \gamma_N(f(\mathbf{p}), \mathbf{p}))$$
$$\text{s.t.} \quad \mathbf{p} \in \mathcal{E}(f). \tag{4}$$

If uniqueness is desired, we can replace $\mathbf{p} \in \mathcal{E}(f)$ in (4) with $\{\mathbf{p}\} = \mathcal{E}(f)$. Note that a solution to the design problem (4), $\mathbf{p}^\star$ and $f^\star$, must satisfy $f^\star(\mathbf{p}^\star) = 0$, namely no intervention if the users choose the target power profile. Based on this observation, the design problem (4) can be solved in two steps. In the first step, we obtain a target power profile $\mathbf{p}^\star$ by solving

$$\max_{\mathbf{p}} \{U_0(\gamma_1(0, \mathbf{p}), \ldots, \gamma_N(0, \mathbf{p})) : \mathbf{p} \in \mathcal{P}\}. \tag{5}$$

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[2]More generally, we can assume that the signal is realized randomly given a power profile and use $\rho(\mathbf{p})$ to represent the probability distribution of signals given $\mathbf{p}$. We leave the analysis of this general case for future research.

[3]In practice, there is a time lag between when the users transmit and when the intervention device transmits because the intervention has to monitor the users' power profile in order to decide its transmit power level. In this paper, we assume that this time lag is short and negligible compared to the length of fading blocks, although in principle we can take into account the time lag in the users' utility functions. See, for example, [20] for a model that takes into account the time lag explicitly.

[4]Throughout the paper, we use $j \neq i$ with the summation operator to mean $j \in \mathcal{N} \setminus \{i\}$, not $j \in \mathcal{N} \cup \{0\} \setminus \{i\}$.

[5]The preferences of a user may be defined on dimensions other than its SINR. For example, a user may use the ratio of SINR to transmit power as the utility function, as in [9], because it may care about its energy consumption as well. Our approach can be applied to such a scenario if utility functions representing the users' preferences are known to the designer.

There always exists a solution to the optimization problem (5) as long as the welfare function U_0 is continuous in $\gamma_1, \ldots, \gamma_2$. Under some welfare functions, e.g., $U_0(\gamma_1, \ldots, \gamma_N) = \sum_{i=1}^{N} \log(\gamma_i)$ as in [4] and [5], the optimization problem (5) is convex, and thus easy to solve. If (5) is solved offline, the designer can choose other welfare functions even though the resulting optimization problem is nonconvex. In the second step, we look for an intervention rule f such that $f(\mathbf{p}^\star) = 0$ and $\mathbf{p}^\star \in \mathcal{E}(f)$ (or $\{\mathbf{p}^\star\} = \mathcal{E}(f)$), given the target power profile $\mathbf{p}^\star$ obtained in the first step.

The first step of solving the design problem (4), namely finding the target power profile $\mathbf{p}^\star$, requires knowledge about the parameters in the model, h_{ij}, n_i, and P_i for all $i, j \in \mathcal{N} \cup \{0\}$. In Section V-A, we propose a method for the designer to estimate the relevant system parameters needed to solve (5) based on intervention. Note, however, that solving the problem (5) does not require the designer to know the details of the users' preferences on their SINRs since knowing the expressions for SINRs suffices to evaluate the welfare function and to check the incentive constraints. To highlight the informational advantage of our approach, let us consider a pricing scheme, in which each user i is charged $\pi_i(\mathbf{p})$ when the users choose a power profile $\mathbf{p}$. Suppose that each user i has quasilinear preferences on its own SINR and payment which are represented by a utility function of the form $u_i(\gamma_i) - \pi_i$. Then in order to find a pricing scheme that sustains a target profile $\mathbf{p}^\star$, the designer needs to know u_i for all $i \in \mathcal{N}$. Since a pricing scheme uses an outside instrument to influence the decisions of the users, the designer needs to know how the users value SINRs relative to payments, which is subjective and thus hard to measure. On the contrary, an intervention scheme affects the users through their SINRs, and thus the designer does not need to know how the users value their SINRs.

In the subsequent discussion of this paper, we focus on the second step of solving the design problem (4), assuming that a target power profile $\mathbf{p}^\star$ has been found. That is, we aim to find an intervention rule f that (strongly) sustains $\mathbf{p}^\star$, namely $\mathbf{p}^\star \in \mathcal{E}(f)$ (or $\{\mathbf{p}^\star\} = \mathcal{E}(f)$), and that satisfies $f(\mathbf{p}^\star) = 0$. Since user i can guarantee a positive SINR by choosing a positive power, it is impossible to provide an incentive for user i to choose $p_i = 0$ using any intervention rule. Thus, we assume that $\mathbf{p}^\star \in \prod_i (0, P_i]$. We say that $(f, \mathbf{p}^\star)$ is an *(intervention) equilibrium* if $\mathbf{p}^\star \in \mathcal{E}(f)$ and $f(\mathbf{p}^\star) = 0$. At an equilibrium, no user has an incentive to deviate unilaterally while the designer fulfills his design criteria. Thus, an equilibrium can be considered as a stable configuration of an intervention rule and a power profile. An equilibrium can be achieved following the procedure described as follows.

1) The designer chooses a target power profile $\mathbf{p}^\star$ and an intervention rule f.
2) The users choose their power profile $\mathbf{p}$, knowing the intervention rule chosen by the designer.[6]
3) The intervention device obtains a signal $\rho(\mathbf{p})$ and chooses its power $p_0 = f(\mathbf{p})$.

To execute the above procedure, we may consider an adjustment process (e.g., one based on best-response updates) for the users and the intervention device to reach an equilibrium (see Section IV-B), as well as an estimation process for the intervention device to obtain a signal (see Section V-A). It is an implicit underlying assumption of our analysis that the time it takes to reach a final outcome (i.e., the duration of the procedure) is short relative to the time for which the final outcome lasts. This justifies that in our model, the users fully take into account interference from the intervention device that is realized at the final outcome when they make decisions about their powers. When a network parameter changes (e.g., some users leave or join the network, or move to different locations), a new target is chosen and the procedure is repeated to achieve a new equilibrium. Thus, our analysis holds as long as network parameters do not change frequently, whereas providing incentives using a repeated game strategy usually requires an infinite horizon and sufficiently patient players.

Another important underlying assumption in our analysis is that the designer can commit to the intervention rule it chooses. Since U_0 is increasing in each γ_i and each γ_i is decreasing in p_0, the designer prefers not to intervene at all, i.e., it prefers to choose $p_0 = 0$. However, if p_0 is held fixed at 0, the users will choose $\mathbf{P}$. The role of the intervention device is to provide a punishment mechanism for the users to choose a desired power profile other than $\mathbf{P}$; the device should choose a high power level if a deviation is detected. Without the designer's commitment to the intervention rule, the designer would choose $p_0 = 0$ (e.g., by disabling the intervention device) regardless of the power profile chosen by the users. Predicting this behavior of the designer, the users would choose $\mathbf{P}$, resulting in the same outcome as with no intervention. Therefore, in order for the proposed intervention schemes to provide incentives successfully, it is critical that the designer executes punishment as promised to make punishment credible. In practice, credibility can be achieved by programming the intervention rule in the intervention device and making the program difficult to manipulate.

Finally, the benchmark is the welfare when there is no intervention device in the network, i.e., p_0 is held fixed at 0. In this case, γ_i is always strictly increasing in p_i, and thus $\mathbf{P} \triangleq (P_1, \ldots, P_N)$ is the unique NE of the game $\langle \mathcal{N}, (\mathcal{P}_i)_{i \in \mathcal{N}}, (\gamma_i(0, \cdot))_{i \in \mathcal{N}} \rangle$.[7] Thus, the case of $\mathbf{p}^\star = \mathbf{P}$ is trivial, because there is no need to use an incentive scheme in order to achieve the power profile $\mathbf{P}$. Hence, our main interest lies in the case of $\mathbf{p}^\star \neq \mathbf{P}$, although our analysis does not exclude the case of $\mathbf{p}^\star = \mathbf{P}$. The case of $\mathbf{p}^\star \neq \mathbf{P}$ arises in many situations, for example, when interference among users is strong and welfare is measured by the sum of utilities or the minimum of utilities [1], [4], [6], [7].

B. Performance Characteristics

Given a target power profile $\mathbf{p}^\star$, there are potentially many intervention rules f that satisfy the design criteria $\mathbf{p}^\star \in \mathcal{E}(f)$

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[6]The intervention rule can be broadcast to the users, or learned by the users from experimentation.

[7]This is true for any constant intervention rule, where p_0 is chosen independently of the observation of the intervention device. This shows the inability of traditional Stackelberg games, where the leader (the intervention device) takes an action before the followers (the users) do, to provide incentives in our setting.

and $f(\mathbf{p}^\star) = 0$. Thus, below we propose several performance characteristics with which we can evaluate different intervention rules satisfying the design criteria.

1) Monitoring requirement: The minimum amount of information about the power profile that is required for the intervention device to execute a given intervention rule (assuming perfect estimation).
2) Intervention capability requirement: The minimum intervention capability needed for the intervention device to execute a given intervention rule, i.e., $\sup_{\mathbf{p} \in \mathcal{P}} f(\mathbf{p})$. (Even though there is no intervention at an equilibrium, the intervention device should have an intervention capability $P_0 \geq \sup_{\mathbf{p} \in \mathcal{P}} f(\mathbf{p})$ in order to make the intervention rule f credible to the users.)
3) Strong sustainment: Whether a given intervention rule strongly sustains the target power profile $\mathbf{p}^\star$.
4) Complexity: The complexity of a given intervention rule in terms of design, broadcast/learning, and computation.

C. Classes of Intervention Rules

Without loss of generality, we can express an intervention rule f satisfying $f(\mathbf{p}^\star) = 0$ as $f(\mathbf{p}) = [g(\mathbf{p})]_0^{P_0}$, where $[x]_a^b = \min\{\max\{x, a\}, b\}$, for some function $g : \mathcal{P} \to \mathbb{R}$ such that $g(\mathbf{p}^\star) = 0$. Also, since the designer desires to achieve $\mathbf{p}^\star$, it is natural to consider functions g that increase as the users deviate from $\mathbf{p}^\star$. Hence, we consider the following classes of intervention rules:

$$\mathcal{F}_K(\mathbf{p}^\star) = \left\{ f : f(\mathbf{p}) = \left[\sum_{i=1}^{N} \sum_{k=1}^{K} \alpha_{i,k} |p_i - p_i^\star|^k \right]_0^{P_0} \right.$$
$$\left. \text{for some } \alpha_{i,k} \geq 0 \text{ and } P_0 > 0 \right\} \quad (6)$$

for $K = 1, 2, \ldots$. We call an intervention rule $f \in \mathcal{F}_K(\mathbf{p}^\star)$ a *Kth-order intervention rule with target power profile* $\mathbf{p}^\star$. As K becomes larger, the set $\mathcal{F}_K(\mathbf{p}^\star)$ contains more intervention rules, but at the same time complexity increases. Simple intervention rules are desirable for the designer, the users, and the intervention device. As K is smaller, the designer has less design parameters and less burden of broadcasting the intervention rule; the users can more easily learn the intervention rule and find their best responses during an adjustment process; and the intervention device can more quickly compute the value of the intervention rule at the chosen power profile. Thus, our analysis mainly focuses on first-order intervention rules, the simplest among the above classes.

Let $\tilde{\mathcal{F}}_K(\mathbf{p}^\star)(\tilde{\mathcal{F}}_K^s(\mathbf{p}^\star))$ be the set of all Kth-order intervention rules that (strongly) sustains $\mathbf{p}^\star$, i.e., $\tilde{\mathcal{F}}_K(\mathbf{p}^\star) = \{f \in \mathcal{F}_K(\mathbf{p}^\star) : \mathbf{p}^\star \in \mathcal{E}(f)\}$ and $\tilde{\mathcal{F}}_K^s(\mathbf{p}^\star) = \{f \in \mathcal{F}_K(\mathbf{p}^\star) : \{\mathbf{p}^\star\} = \mathcal{E}(f)\}$. We define the *minimum power budget*[8] for a Kth-order intervention rule to (strongly) sustain $\mathbf{p}^\star$ by

$$PB_K(\mathbf{p}^\star) = \inf_{f \in \tilde{\mathcal{F}}_K(\mathbf{p}^\star)} \sup_{\mathbf{p} \in \mathcal{P}} f(\mathbf{p}) \quad (7)$$

[8]As to be shown later, for strong sustainment, we need P_0 to exceed a certain value. Thus, we actually mean "infimum" power budget by minimum power budget.

and

$$PB_K^s(\mathbf{p}^\star) = \inf_{f \in \tilde{\mathcal{F}}_K^s(\mathbf{p}^\star)} \sup_{\mathbf{p} \in \mathcal{P}} f(\mathbf{p}). \quad (8)$$

Thus, with an intervention capability $P_0 > PB_K(\mathbf{p}^\star)(P_0 > PB_K^s(\mathbf{p}^\star))$, there exists a Kth-order intervention rule that (strongly) sustains $\mathbf{p}^\star$. We set $PB_K(\mathbf{p}^\star) = +\infty(PB_K^s(\mathbf{p}^\star) = +\infty)$ if there is no Kth-order intervention rule that (strongly) sustains $\mathbf{p}^\star$ (i.e., $\tilde{\mathcal{F}}_K(\mathbf{p}^\star)(\tilde{\mathcal{F}}_K^s(\mathbf{p}^\star))$ is empty). Since $\mathcal{F}_K(\mathbf{p}^\star) \subset \mathcal{F}_{K'}(\mathbf{p}^\star)$ for all K, K' such that $K \leq K'$, both $PB_K(\mathbf{p}^\star)$ and $PB_K^s(\mathbf{p}^\star)$ are weakly decreasing in K for all $\mathbf{p}^\star$. This suggests a tradeoff between complexity and the minimum power budget. Also, since $\tilde{\mathcal{F}}_K^s(\mathbf{p}^\star) \subset \tilde{\mathcal{F}}_K(\mathbf{p}^\star)$, we have $PB_K(\mathbf{p}^\star) \leq PB_K^s(\mathbf{p}^\star)$ for all K and $\mathbf{p}^\star$. The difference $PB_K^s(\mathbf{p}^\star) - PB_K(\mathbf{p}^\star)$ can be interpreted as the price of strong sustainment in terms of the minimum power budget.

IV. ANALYSIS OF FIRST-ORDER INTERVENTION RULES

A. Design of Intervention Rules

We consider first-order intervention rules of the form

$$f_1^I(\mathbf{p}) = \left[\sum_{i=1}^{N} \alpha_i |p_i - p_i^\star| \right]_0^{P_0}. \quad (9)$$

Under a first-order intervention rule, the intervention device increases its transmit power linearly with the deviation of each user from the target power, $|p_i - p_i^\star|$, in the range of its intervention capability. We call α_i the *intervention rate* for user i, which measures how sensitive intervention reacts to a deviation of user i. Let $\tilde{\mathcal{N}}(\mathbf{p}^\star) = \{i \in \mathcal{N} : p_i^\star < P_i\}$. Without loss of generality, we label the users in such a way that $i \in \tilde{\mathcal{N}}(\mathbf{p}^\star)$ if and only if $i \leq N'$, where $N' = |\tilde{\mathcal{N}}(\mathbf{p}^\star)|$. Since the users have natural incentives to choose their maximum powers in the absence of intervention, we need to provide incentives only for the users in $\tilde{\mathcal{N}}(\mathbf{p}^\star)$. The following theorem shows that when the intervention capability is sufficiently large, the designer can always find intervention rates to have a given target power profile $\mathbf{p}^\star$ sustained by a first-order intervention rule.

Theorem 1: For any $\mathbf{p}^\star \in \prod_i (0, P_i]$, $\mathbf{p}^\star \in \mathcal{E}(f_1^I)$ if and only if

$$\alpha_i \geq \frac{\sum_{j \neq i} h_{ij} p_j^\star + n_i}{p_i^\star h_{i0}} \quad (10)$$

and

$$P_0 \geq \frac{(P_i - p_i^\star)\left(\sum_{j \neq i} h_{ij} p_j^\star + n_i\right)}{p_i^\star h_{i0}} \quad (11)$$

for all $i \in \tilde{\mathcal{N}}(\mathbf{p}^\star)$.

Proof: See [23, Appendix B]. ∎

We can interpret Theorem 1 as follows. As h_{i0} is larger, intervention causes more interference to user i with the same transmit power, and thus the intervention rate α_i can be chosen smaller to yield the same interference. When $\sum_{j \neq i} h_{ij} p_j^\star + n_i$ is large, interference to user i from other users and its noise power are already strong, and thus the intervention rate α_i

should be large in order for intervention to be effective. Hence, $h_{i0}/(\sum_{j \neq i} h_{ij} p_j^\star + n_i)$ can be interpreted as the effectiveness of intervention to user i. Without intervention, users have natural incentives to increase their transmit powers. Thus, as the target power for user i, $p_i^\star$, is smaller, the incentive for user i to deviate is stronger, and thus a larger intervention rate α_i is needed to prevent deviation. Note that $(P_i - p_i^\star)$ is the maximum possible deviation by user i (in the direction where it has a natural incentive to deviate). The minimum intervention capability in the right-hand side of (11) is increasing with the maximum possible deviation and the strength of the incentive to deviate while decreasing with the effectiveness of intervention.

A first-order intervention rule f_1^I satisfying the conditions in Theorem 1 may have a NE other than the target power profile $\mathbf{p}^\star$. For example, if $P_0 \leq \sum_{j \neq i} \alpha_j (P_j - p_j^\star)$ for all $i \in \tilde{\mathcal{N}}(\mathbf{p}^\star)$, $\mathbf{P}$ is also sustained by f_1^I. The presence of this extra NE is undesirable since it brings a possibility that the users still choose $\mathbf{P}$ while the intervention device causes interference to the users by transmitting its maximum power P_0. Obviously, this outcome $(P_0, \mathbf{P})$ is worse for every user than the outcome at the unique NE without intervention $(0, \mathbf{P})$. In order to eliminate this possibility, the designer may want to choose an intervention rule that strongly sustains the target power profile. The following theorem provides a sufficient condition for a first-order intervention rule to strongly sustain a given target power profile.

Theorem 2: For any $\mathbf{p}^\star \in \prod_i (0, P_i]$, $\{\mathbf{p}^\star\} = \mathcal{E}(f_1^I)$ if

$$\alpha_i > \frac{1}{p_i^\star} \sum_{j>i} \alpha_j (P_j - p_j^\star) + \frac{\sum_{j<i} h_{ij} p_j^\star + \sum_{j>i} h_{ij} P_j + n_i}{p_i^\star h_{i0}} \quad (12)$$

and

$$P_0 > \frac{P_i}{p_i^\star} \sum_{j>i} \alpha_j (P_j - p_j^\star)$$
$$+ \frac{(P_i - p_i^\star) \left(\sum_{j<i} h_{ij} p_j^\star + \sum_{j>i} h_{ij} P_j + n_i \right)}{p_i^\star h_{i0}} \quad (13)$$

for all $i \in \tilde{\mathcal{N}}(\mathbf{p}^\star)$.[9]

Proof: See [23, Appendix C]. ∎

By comparing Theorems 1 and 2, we can see that the requirements for the intervention rates and the intervention capability is higher when we impose strong sustainment. For any given power profile, the intervention rates can be chosen sequentially to satisfy the condition (12) starting from user N' down to user 1. We can set $\alpha_i = 0$ for all $i \notin \tilde{\mathcal{N}}(\mathbf{p}^\star)$. Unlike Theorem 1, the choice of the intervention rates affects the minimum required intervention capability. For strong sustainment, the intervention capability is required to be larger as the designer chooses larger intervention rates. A main reason for the existence of an extra NE is that the region of power profiles on which the maximum intervention power is applied is so wide that the users cannot escape the region by unilateral deviation. Thus, a larger intervention capability is needed to reduce the region and guarantee the uniqueness of NE.

[9] We define $\sum_{j \in J} x_j = 0$ if J is empty. Similarly, we define $\prod_{j \in J} x_j = 1$ if J is empty.

From Theorem 1, we obtain

$$PB_1(\mathbf{p}^\star) = \max_i \frac{(P_i - p_i^\star) \left(\sum_{j \neq i} h_{ij} p_j^\star + n_i \right)}{p_i^\star h_{i0}}. \quad (14)$$

Since Theorem 2 gives a sufficient condition for strong sustainment, we obtain an upper bound on $PB_1^s(\mathbf{p}^\star)$,

$$\overline{PB}_1^s(\mathbf{p}^\star) = \sum_{i=1}^N \left[\left(\prod_{j=1}^{i-1} \frac{P_j}{p_j^\star} \right) \right.$$
$$\left. \cdot \frac{(P_i - p_i^\star) \left(\sum_{j<i} h_{ij} p_j^\star + \sum_{j>i} h_{ij} P_j + n_i \right)}{p_i^\star h_{i0}} \right]. \quad (15)$$

Note that $PB_1(\mathbf{p}^\star) \leq \overline{PB}_1^s(\mathbf{p}^\star)$ with equality if and only if $N' \leq 1$. Combining these results, we can bound $PB_1^s(\mathbf{p}^\star)$ by

$$PB_1(\mathbf{p}^\star) \leq PB_1^s(\mathbf{p}^\star) \leq \overline{PB}_1^s(\mathbf{p}^\star). \quad (16)$$

By Theorems 1 and 2, we know that first-order intervention rules can sustain the set $\prod_i (0, P_i]$. Note that among all the efficient power profiles, those with $p_i = 0$ for some i have probability measure zero. Hence, we obtain almost the entire set of feasible payoffs by using first-order intervention rules. As argued in Section III, it is impossible to provide an incentive for user i to choose $p_i = 0$ by intervention rules of any orders. Thus, we actually obtain the largest set of sustainable payoffs by using first-order intervention rules. This discussion suggests that the potential gain from using higher-order intervention rules is not from what they can sustain but how they sustain a target power profile.

Remark 1: An extreme intervention rule f_e^I [21], defined by

$$f_e^I(\mathbf{p}) = \begin{cases} 0, & \text{if } \mathbf{p} = \mathbf{p}^\star, \\ P_0, & \text{if } \mathbf{p} \neq \mathbf{p}^\star \end{cases} \quad (17)$$

can be considered as the limiting case of first-order intervention rules as each α_i goes to infinity in that the area of the region $\{\mathbf{p} : f_e^I(\mathbf{p}) \neq f_1^I(\mathbf{p})\}$ approaches zero in the limit. With this class of intervention rules, we have $\mathcal{E}(f_e^I) = \{\mathbf{p}^\star, \mathbf{P}\}$ if $P_0 \geq PB_1(\mathbf{p}^\star)$ and $\mathcal{E}(f_e^I) = \{\mathbf{P}\}$ otherwise. Therefore, it is impossible to construct an extreme intervention rule that strongly sustains a target power profile, except in the uninteresting case $\mathbf{p}^\star = \mathbf{P}$. This motivates us to study intervention rules other than extreme intervention rules.

B. Dynamic Adjustment Processes

Previously, we have derived the conditions under which a target power profile is (strongly) sustained. Now we propose a dynamic adjustment process, in order to guide the intervention device and the users to achieve the target power profile as NE. The adjustment occurs at discrete steps, labeled as $t = 1, 2, \ldots$. We allow the use of different intervention rules in different steps. Thus, the beginning of each step is triggered by the intervention device's announcement of the intervention rule to be used in that step. Users are synchronized by the announcement of

the intervention rules. The adjustment process is based on the myopic best-response updates of the users and is described in Algorithm 1.

Algorithm 1 A dynamic adjustment process

1: **Initialization**: $t = 0$

2:   The users choose an initial power profile $\mathbf{p}^0$.

3:   The designer announces the use of first-order intervention rules $f^t \in \mathcal{F}_1$ with power budget P_0.

4: **while** $\mathbf{p}^t \neq \mathbf{p}^\star$ or $f^t(\mathbf{p}^t) \neq 0$ **do**

5:   $t \leftarrow t + 1$

6:   Given $\mathbf{p}^{t-1}$, the designer chooses and broadcasts the intervention rates α_i^t and the target power profile $\tilde{\mathbf{p}}^t$ for the current time slot t.

7:   Given $f^t(\mathbf{p}) = [\sum_{i=1}^{N} \alpha_i^t |p_i - \tilde{p}_i^t|]_0^{P_0}$, each user i chooses a best response to $\mathbf{p}_{-i}^{t-1}$:

8:   $p_i^t \in BR_i(f^t, \mathbf{p}_{-i}^{t-1}) \triangleq \arg\max_{p_i \in \mathcal{P}_i} \gamma_i(f^t, p_i, \mathbf{p}_{-i}^{t-1})$.

9: **end while**

During the adjustment process, the designer may use different intervention rules, as well as intermediate target profiles different from the final target $\mathbf{p}^\star$. That is, we have $f^t \in \mathcal{F}_1(\tilde{\mathbf{p}}^t)$, where $\tilde{\mathbf{p}}^t$ is the intermediate target power profile at step t. In the adjustment process, the designer chooses a sequence of intervention rules. Suppose that the designer uses an update rule $\psi : \mathcal{P} \to \mathcal{F}_1$ to determine an intervention rule based on the power profile in the previous step. Then given an initial power profile $\mathbf{p}^0$, an update rule yields a sequence of intervention rules and power profiles $\{(f^t, \mathbf{p}^t)\}_{t=1}^{\infty}$.[10] We can evaluate an update rule by the following two criteria.

1) *Convergence*: Does the induced sequence reach an equilibrium $(f, \mathbf{p}^\star)$? If so, how many steps are needed?

2) *Minimum power budget*: How much power budget is needed to execute $\{f^t\}_{t=1}^{\infty}$, i.e., $\sup_t \sup_{\mathbf{p} \in \mathcal{P}} f^t(\mathbf{p})$?

The following theorem shows that when the target power profile $\mathbf{p}^\star$ is close to the maximum power profile $\mathbf{P}$ and the intervention capability P_0 is large, we can obtain fast convergence as well as strong sustainment.

Theorem 3: For any $\mathbf{p}^\star \in \prod_i (0, P_i]$ such that $\sum_{i=1}^{N} (P_i - p_i^\star)/P_i < 1$, there exists $(\alpha_1, \ldots, \alpha_N) \in \mathbb{R}_+^N$ such that

$$\alpha_i > \frac{1}{p_i^\star} \sum_{j \neq i} \alpha_j \left(P_j - p_j^\star\right) + \frac{\sum_{j \neq i} h_{ij} P_j + n_i}{p_i^\star h_{i0}} \quad (18)$$

for all $i \in \tilde{\mathcal{N}}(\mathbf{p}^\star)$. Suppose that $f_1^s \in \mathcal{F}_1(\mathbf{p}^\star)$ satisfies (18) and

$$P_0 > \frac{P_i}{p_i^\star} \sum_{j \neq i} \alpha_j \left(P_j - p_j^\star\right) + \frac{(P_i - p_i^\star)\left(\sum_{j \neq i} h_{ij} P_j + n_i\right)}{p_i^\star h_{i0}} \quad (19)$$

for all $i \in \tilde{\mathcal{N}}(\mathbf{p}^\star)$. Then $\{\mathbf{p}^\star\} = \mathcal{E}(f_1^s)$. Moreover, starting from an arbitrary initial power profile $\mathbf{p}^0 \neq \mathbf{p}^\star$, the adjustment process with $f^t = f_1^s$ for all $t = 1, 2, \ldots$ reaches $(f_1^s, \mathbf{p}^\star)$ in at most two steps (and in one step if $\mathbf{p}^0 \in \prod_i [p_i^\star, P_i]$).

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[10]Since the best response correspondence is nonsingleton only in knife-edge cases, we focus on update rules that yield a deterministic sequence.

Proof: See [23, Appendix D]. ∎

The minimum power budget required to execute an intervention rule described in Theorem 3 is given by

$$\widetilde{PB}_1^s(\mathbf{p}^\star) = \frac{1}{1 - \sum_{i=1}^{N} \frac{P_i - p_i^\star}{P_i}} \sum_{i=1}^{N} \frac{(P_i - p_i^\star)\left(\sum_{j \neq i} h_{ij} P_j + n_i\right)}{P_i h_{i0}}.$$

Since the requirement for α_i in (18) is more stringent than that in (12), we have $\widetilde{PB}_1^s(\mathbf{p}^\star) \geq \overline{PB}_1^s(\mathbf{p}^\star)$ with equality if and only if $N' \leq 1$. The difference $\widetilde{PB}_1^s(\mathbf{p}^\star) - \overline{PB}_1^s(\mathbf{p}^\star)$ can be considered as the price of fast convergence to $\mathbf{p}^\star$ in terms of the minimum power budget. In addition to requiring a larger power budget, Theorem 3 imposes a restriction on the range of target profiles. That is, the target should be close enough to $\mathbf{P}$ for the result of Theorem 3 to hold. However, the target may not satisfy the restriction $\sum_{i=1}^{N} (P_i - p_i^\star)/P_i < 1$. In this case, the designer needs to use intermediate target power profiles that are successively close to one another in order to guide the users to the final target. The use of intermediate target power profiles is also necessary when the intervention device does not have a large enough power budget to strongly sustain a target power profile. In this case, the designer can use a sequence of intervention functions to drive the users to reach the target power profile as the unique outcome. This process requires smaller power budget than that required by strong sustainment, but may take longer time for the system to reach the target power profile.

Define the relative distance from $\mathbf{p}$ to $\mathbf{p}'$

$$d(\mathbf{p}, \mathbf{p}') = \sum_{i=1}^{N} \frac{p_i - p_i'}{p_i}. \quad (20)$$

Using the proofs of Theorems 2 and 3, we can show that, given $\mathbf{p}^{t-1}$, the designer can achieve the intermediate target at step t, i.e., $\mathbf{p}^t = \tilde{\mathbf{p}}^t$ only if $\tilde{\mathbf{p}}^t$ satisfies $d(\mathbf{p}^{t-1}, \tilde{\mathbf{p}}^t) < 1$. This imposes a bound on the relative distance between two successive intermediate targets. Below we provide two different methods for the designer to generate intermediate targets. The first method, which is summarized in Algorithm 2 and analyzed in Theorem 4, produces a sequence of intermediate targets reaching the final target whose successive elements have a relative distance of $\delta \in (0, 1)$ while requiring the minimum power budget in each step. This method can be used in a scenario where the power constraint of the intervention device does not bind; the designer can fix δ sufficiently close to 1, and the method will allow the system to reach the final target in the minimum number of steps. The second method, which is summarized in Algorithm 3 and analyzed in Theorem 5, yields a sequence of intermediate targets with the largest relative distance in each step while satisfying the power constraint. Thus, this method will allow the manager with a limited power budget to reach the final target as fast as possible.

Now suppose that a fixed relative distance between two successive targets is given. Algorithm 2 provides a method for the designer to generate intermediate targets in the most power-budget efficient way.

Algorithm 2 An algorithm that generates a sequence of intermediate target power profiles with a fixed relative distance

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Require: Fix $\delta \in (0,1)$ close to 1

1: **Initialization** $(t = 1)$: Set $\tilde{\mathbf{p}}^1 = \mathbf{P}$ {This step can be skipped if $\mathbf{p}^0 = \mathbf{P}$} and $\mathcal{M} = \mathcal{N}$

2: Set $\mu_i = p_i^\star / \tilde{p}_i^1$ for all $i \in \mathcal{N}$

3: **while** $\sum_{i=1}^N (\tilde{p}_i^t - p_i^\star)/\tilde{p}_i^t \geq 1$ **do**

4:   $t \leftarrow t + 1$

5:   **while** $\sum_{i=1}^N \mu_i < N - \delta$ **do**

6:     Choose $i^* \in \arg \max_{i \in \mathcal{M}} ((\sum_{j \neq i} h_{ij}\tilde{p}_j^{t-1} + n_i)/h_{i0})$

7:     Set $\mu_{i^*} = \min\{1, N - \delta - \sum_{j \neq i^*} \mu_j\}$, and $\mathcal{M} \leftarrow \mathcal{M} \setminus \{i^*\}$

8:   **end while**

9:   Set $\tilde{p}_i^t = p_i^\star / \mu_i$ for all $i \in \mathcal{N}$

10: **end while**

11: Set $\tilde{\mathbf{p}}^t = \mathbf{p}^\star$

The following theorem shows that the designer can lead the users to the final target by using intermediate targets generated by Algorithm 2 provided that the intervention capability is sufficiently large.

Theorem 4: For any $\mathbf{p}^\star \in \prod_i (0, P_i]$, if $\delta \geq 1 - \min_i (p_i^\star / P_i)$, then Algorithm 2 terminates at a finite step T with $T \leq N' + 1$. Let $(\tilde{\mathbf{p}}^t)_{t=1}^T$ be the sequence of power profiles generated by Algorithm 2. Then there exists a sequence of intervention rules $(f^t)_{t=1}^T$ with $f^t \in \mathcal{F}_1(\tilde{\mathbf{p}}^2)$ such that the adjustment process with $(f^t)_{t=1}^T$ yields $\mathbf{p}^t = \tilde{\mathbf{p}}^t$ for all $t = 1, \ldots, T$ starting from any $\mathbf{p}^0 \in \mathcal{P}$.

Proof: See [23, Appendix F]. ∎

Now we consider a scenario where the intervention capability P_0 should be taken into consideration while generating intermediate targets.[11] In this scenario, in order to induce the users to follow intermediate targets during the adjustment process, the intermediate targets $(\tilde{\mathbf{p}}^t)_{t=2}^T$ should satisfy not only

$$\sum_{i=1}^N \frac{\tilde{p}_i^{t-1} - \tilde{p}_i^t}{\tilde{p}_i^{t-1}} < 1 \tag{21}$$

but also

$$P_0^t \triangleq \max_{i \in \mathcal{N}(\tilde{\mathbf{p}}^t)} \left\{ \frac{P_i}{\tilde{p}_i^{t-1}} \frac{\sum_{j=1}^N \frac{(\tilde{p}_j^{t-1} - \tilde{p}_j^t)(\sum_{k \neq j} h_{jk}\tilde{p}_k^{t-1} + n_j)}{\tilde{p}_j^{t-1} h_{j0}}}{1 - \sum_{j=1}^N \frac{\tilde{p}_j^{t-1} - \tilde{p}_j^t}{\tilde{p}_j^{t-1}}} + \frac{(P_i - \tilde{p}_i^{t-1})(\sum_{j \neq i} h_{ij}\tilde{p}_j^{t-1} + n_i)}{\tilde{p}_i^{t-1} h_{i0}} \right\} < P_0 \tag{22}$$

for all $t = 2, \ldots, T$. In order to reach the final target in the minimum number of steps, we need to maximize the relative distance between successive targets while satisfying the constraints

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[11] Note that to sustain $\mathbf{p}^\star$ as the NE, we require P_0 to satisfy the conditions in Theorem 1.

(21) and (22). Thus, the problem to obtain $\tilde{\mathbf{p}}^t$ given $\tilde{\mathbf{p}}^{t-1}$ can be written as

$$\max_{\hat{\mathbf{p}}^t} \quad \sum_{i=1}^N \frac{\tilde{p}_i^{t-1} - \tilde{p}_i^t}{\tilde{p}_i^{t-1}} \tag{23}$$

$$s.t. \quad \max_{i \in \mathcal{N}(\tilde{\mathbf{p}}^t)} \left\{ \frac{P_i}{\tilde{p}_i^{t-1}} \frac{\sum_{j=1}^N \frac{\tilde{p}_j^{t-1} - \tilde{p}_j^t}{\tilde{p}_j^{t-1}} b_j^{t-1}}{1 - \sum_{j=1}^N \frac{\tilde{p}_j^{t-1} - \tilde{p}_j^t}{\tilde{p}_j^{t-1}}} \right.$$

$$\left. + \frac{P_i - \tilde{p}_i^{t-1}}{\tilde{p}_i^{t-1}} b_i^{t-1} \right\} \leq P_0 - \varepsilon_1 \tag{24}$$

$$\sum_{i=1}^N \frac{\tilde{p}_i^{t-1} - \tilde{p}_i^t}{\tilde{p}_i^{t-1}} \leq 1 - \varepsilon_2 \tag{25}$$

$$\mathbf{p}^\star \leq \tilde{\mathbf{p}}^t \leq \tilde{\mathbf{p}}^{t-1} \tag{26}$$

for small $\varepsilon_1, \varepsilon_2 > 0$. Algorithm 3 formalizes the method to generate a sequence of intermediate target power profiles, which has maximal relative distances (MRD) between successive target power profiles given an intervention capability P_0. We call the sequence generated by Algorithm 3 the MRD sequence. Note that the major complexity in solving the above problem is line 10 in Algorithm 3. This search on $\tilde{p}_{i^*}^t$ can be done efficiently by bisection method, because P_0^t is decreasing with $\tilde{p}_{i^*}^t$.

Algorithm 3 An algorithm that generates a sequence of intermediate target power profiles with maximal relative distances given an intervention capability

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Require: Small $\varepsilon_1 \in (0,1)$ and $\varepsilon_2 \in (0,1)$;
$P_0 - \varepsilon_1 > \max_i \{((P_i - p_i^\star)/p_i^\star) \cdot ((\sum_{j \neq i} h_{ij}P_j + n_i)/h_{i0})\}$

1: **Initialization** $(t = 1)$: Set $\tilde{\mathbf{p}}^1 = \mathbf{P}$ {This step can be skipped if $\mathbf{p}^0 = \mathbf{P}$}

2: **while** $\tilde{\mathbf{p}}^t \neq \mathbf{p}^\star$ **do**

3:   $t \leftarrow t + 1$, $\mathcal{M} = \{i : \tilde{p}_i^{t-1} > p_i^\star\}$, $\tilde{\mathbf{p}}^t = \tilde{\mathbf{p}}^{t-1}$

4:   **repeat**

5:     $i^* = \min_{i \in \mathcal{M}} b_i^{t-1}$, $\tilde{p}_{i^*}^t = \max\{p_{i^*}^\star, (N - 1 + \varepsilon_2 - \sum_{i \neq i^*} \tilde{p}_i^t / \tilde{p}_i^{t-1}) \cdot \tilde{p}_{i^*}^{t-1}\}$

6:     calculate P_0^t by (22)

7:     **if** $P_0^t < P_0 - \varepsilon_1$ **then**

8:       $\mathcal{M} \leftarrow \mathcal{M} \setminus \{i^*\}$

9:     **else if** $P_0^t > P_0 - \varepsilon_1$ **then**

10:      use bisection methods to find $\tilde{p}_{i^*}^t$ in the interval $[\max\{p_{i^*}^\star, (N - 1 + \varepsilon_2 - \sum_{i \neq i^*} (\tilde{p}_i^t / \tilde{p}_i^{t-1})) \cdot \tilde{p}_{i^*}^{t-1}\}, \tilde{p}_{i^*}^{t-1}]$ such that $P_0^t = P_0 - \varepsilon_1$

11:    **end if**

12: **until** $P_0^t = P_0 - \varepsilon_1$ **or** $\mathcal{M} = \emptyset$

13: **end while**

14: Set $\tilde{\mathbf{p}}^t = \mathbf{p}^\star$

Given the power budget P_0, we are interested in the minimum convergence time for the dynamic adjustment process to reach the target power profile $\mathbf{p}^\star$, defined by $T^\star(\mathbf{p}^\star) = \inf\{T : \tilde{\mathbf{p}}^T = \mathbf{p}^\star\}$, where the infimum is taken over the set of sequences satisfying (21) and (22) starting from $\mathbf{p}^1 = \mathbf{P}$. In order to obtain

an upper bound for $T^{\star}(\mathbf{p}^{\star})$, we use an upper bound for the convergence time of a geometric sequence of T intermediate target power profiles in the following form:

$$\tilde{p}_i^t = (\eta_i)^{t-1} P_i, \quad \forall i, t = 1, \dots, T \tag{27}$$

where $\eta_i = (p_i^{\star}/P_i)^{1/(T-1)}, i = 1, \dots, N$.

Theorem 5: If $\mathbf{p}^0 \neq \mathbf{p}^{\star}$, $\sum_{i=1}^{N} (P_i - p_i^{\star})/P_i \geq 1$, and

$$P_0 > \left(\max_i \frac{P_i}{p_i^{\star}} - 1 \right) \max_i \frac{\sum_{j \neq i} h_{ij} p_j^{\star} + n_i}{h_{i0}} \tag{28}$$

then $T^{\star}(\mathbf{p}^{\star}) > 2$ and $T^{\star}(\mathbf{p}^{\star})$ satisfies

$$\sum_{i=1}^{N} \left(\frac{p_i^{\star}}{P_i} \right)^{\frac{1}{T^{\star}(\mathbf{p}^{\star})-2}} < N - 1 + \frac{1}{C} \tag{29}$$

where

$$C = \frac{P_0}{\max_i \frac{\sum_{j \neq i} h_{ij} p_j^{\star} + n_i}{h_{i0}} \cdot \max_i \frac{P_i}{p_i^{\star}}} + \frac{1}{\max_i \frac{P_i}{p_i^{\star}}}. \tag{30}$$

Proof: See [23, Appendix G]. ∎

The inequality (29) provides an upper bound for $T^{\star}(\mathbf{p}^{\star})$, since the left-hand side of (29) increases in $T^{\star}(\mathbf{p}^{\star})$ and approaches N as $T^{\star}(\mathbf{p}^{\star}) \to \infty$ while the right-hand side is smaller than N given (28). From Theorem 5, we can see that the convergence time is small if the power budget P_0 is large, the target power profile is close to the maximum power (i.e., $\max_i(P_i/p_i^{\star})$ is small), or SINR is relatively small compared to the channel gain from the intervention device (i.e., $\max_i((\sum_{j \neq i} h_{ij} p_j^{\star} + n_i)/h_{i0})$ is small).

C. Relaxation of Monitoring Requirement

The results in this section so far relies on the ability of the intervention device to estimate individual transmit powers. However, estimating individual transmit powers requires larger monitoring overhead for the intervention device than estimating aggregate receive power. In order to study intervention rules that can be executed with the monitoring of aggregate receive power, we consider a class of intervention rules that can be expressed as

$$f_1^A(\mathbf{p}) = \left[\alpha_0 \left| \left(\sum_{i=1}^{N} h_{0i} p_i \right) - p_A^{\star} \right| \right]_0^{P_0} \tag{31}$$

for some $\alpha_0 \geq 0$, $P_0 > 0$, and $p_A^{\star}$. We call an intervention rule in this class a first-order intervention rule based on aggregate receive power or, in short, an intervention rule based on aggregate power. We call α_0 the aggregate intervention rate, and call $p_A^{\star}$ the target aggregate power, which is set as the aggregate receive power at the target power profile, i.e., $p_A^{\star} = \sum_{i=1}^{N} h_{0i} p_i^{\star}$. We first give a necessary and sufficient condition for an intervention rule based on aggregate power to sustain a target power profile.

Theorem 6: For any $\mathbf{p}^{\star} \in \prod_i (0, P_i]$, $\mathbf{p}^{\star} \in \mathcal{E}(f_1^A)$ if and only if

$$\alpha_0 \geq \max_{i \in \tilde{\mathcal{N}}(\mathbf{p}^{\star})} \frac{\sum_{j \neq i} h_{ij} p_j^{\star} + n_i}{h_{0i} p_i^{\star} h_{i0}} \tag{32}$$

and

$$P_0 \geq \max_{i \in \tilde{\mathcal{N}}(\mathbf{p}^{\star})} \frac{(P_i - p_i^{\star}) \left(\sum_{j \neq i} h_{ij} p_j^{\star} + n_i \right)}{p_i^{\star} h_{i0}}. \tag{33}$$

Proof: See [23, Appendix H]. ∎

The minimum intervention capability required to sustain a target profile is not affected by using aggregate receive power instead of individual transmit powers. However, the aggregate intervention rate should be chosen high enough to prevent a deviation of any user, whereas with the monitoring of individual transmit powers the intervention rates can be chosen individually for each user. This suggests that strong sustainment is more difficult with intervention rules based on aggregate power. For example, $\mathbf{P}$ is also sustained by f_1^A if $P_0 \leq \alpha_0 \sum_{j \neq i} (h_{0j} P_j - p_j^{\star})$ for all $i \in \tilde{\mathcal{N}}(\mathbf{p}^{\star})$, which is weaker than the corresponding condition in the case of intervention rules based on individual powers, $P_0 \leq \sum_{j \neq i} \alpha_j (P_j - p_j^{\star})$ for all $i \in \tilde{\mathcal{N}}(\mathbf{p}^{\star})$. With the monitoring of individual powers, a deviation of each user can be detected and punished. This leads to the property that the best response of user i is almost always either $p_i^{\star}$ or P_i under first-order intervention rules based on individual powers. This implies that a power profile sustained by a first-order intervention rule based on individual powers almost always belongs to the set $\prod_i \{p_i^{\star}, P_i\}$. In contrast, with the monitoring of aggregate power, only an aggregate deviation can be detected. This yields a possibility that an intervention rule based on aggregate power sustains a power profile that is different from the target but yields the same aggregate power. This possibility makes the problem of coordination failure more worrisome because if the users are given only the target aggregate power $p_A^{\star}$ they may not know which power profile to select among those that yield the aggregate power $p_A^{\star}$.[12] The problem arising from the increased degree of non-uniqueness can be considered as the cost of reduced monitoring overhead. To state the result formally, let $\alpha_0^i = (\sum_{j \neq i} h_{ij} p_j^{\star} + n_i)/(h_{0i} p_i^{\star} h_{i0})$ and $P_0^i = (P_i - p_i^{\star})(\sum_{j \neq i} h_{ij} p_j^{\star} + n_i)/(p_i^{\star} h_{i0})$ for all $i \in \tilde{\mathcal{N}}(\mathbf{p}^{\star})$. Also, let $\bar{\alpha}_0 = \max_{i \in \tilde{\mathcal{N}}(\mathbf{p}^{\star})} \alpha_0^i$ and $\bar{P}_0 = \max_{i \in \tilde{\mathcal{N}}(\mathbf{p}^{\star})} P_0^i$.

Theorem 7: Suppose that, for $\mathbf{p}^{\star} \in \prod_i (0, P_i]$, there exist $i, j \in \tilde{\mathcal{N}}(\mathbf{p}^{\star})$ such that 1) $\bar{\alpha}_0 = \alpha_0^i > \alpha_0^j$ or $\alpha_0^i, \alpha_0^j < \bar{\alpha}_0$, and 2) $\bar{P}_0 = P_0^i > P_0^j$ or $P_0^i, P_0^j < \bar{P}_0$. Then for any f_1^A such that $\mathbf{p}^{\star} \in \mathcal{E}(f_1^A)$ and for any $\epsilon > 0$, there exists $\tilde{\mathbf{p}} \neq \mathbf{p}^{\star}$ such that $\tilde{\mathbf{p}} \in \mathcal{E}(f_1^A)$, $\sum_{i=1}^{N} h_{0i} p_i^{\star} = \sum_{i=1}^{N} h_{0i} \tilde{p}_i$, and $|\tilde{\mathbf{p}} - \mathbf{p}^{\star}| < \epsilon$.

Proof: See [23, Appendix I]. ∎

Theorem 7 provides a sufficient condition under which the strong sustainment of a given target power profile is impossible with intervention rules based on aggregate power. We argue that the sufficient condition is mild. First, note that, for almost all $\mathbf{p}^{\star} \in \prod_i (0, P_i]$, α_0^i's and P_0^i's can be ordered strictly. With strict ordering of α_0^i's and P_0^i's, we can always find a pair of users $i, j \in \tilde{\mathcal{N}}(\mathbf{p}^{\star})$ satisfying the condition in Theorem 7 if there are at least three users in $\tilde{\mathcal{N}}(\mathbf{p}^{\star})$. That is, strong sustainment is generically impossible with intervention rules based on aggregate power when $|\tilde{\mathcal{N}}(\mathbf{p}^{\star})| \geq 3$.

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[12]A way to overcome this problem is to broadcast the target power profile $\mathbf{p}^{\star}$ to the users in order to make $\mathbf{p}^{\star}$ as a focal point [24].

V. IMPLEMENTATION ISSUES

In this section, we discuss some implementation issues. First, we provide algorithms for the intervention device to estimate the system parameters and individual transmit powers without user cooperation. Then we compare the communication overhead of the intervention scheme with that of other incentive schemes.

A. Estimation of System Parameters and Individual Transmit Powers

As we have seen from previous results (e.g., Theorems 1–3), in order to determine the intervention rates and the power budget requirement, the designer needs to know the normalized cross channel gains $\{h_{ij}/h_{i0}\}_{j=1,j\neq i}^{N}, \forall i \in \mathcal{N}$, the normalized noise powers $n_i/h_{i0}, \forall i \in \mathcal{N}$, the maximum power levels $P_i, \forall i \in \mathcal{N}$, and the target power profile $\mathbf{p}^\star$. We propose a method to estimate the normalized cross channel gains, normalized noise powers, and the maximum power levels without user cooperation. Based on the above parameters, the designer can determine the target power profile $\mathbf{p}^\star$ by solving (4). In addition, we propose a method to estimate the individual transmit powers without user cooperation.

1) Normalized Cross Channel Gains and Noise Powers: The designer estimates the normalized cross channel gains and the normalized noise powers by adjusting the intervention rules and observing the reaction of the users. First, the designer broadcast the intervention capability P_0 and a temporary target power profile $\tilde{\mathbf{p}} < \mathbf{P}$. Then it makes N rounds of measurements at N different locations. We assume that during the measurements, the users always choose the power levels that maximize their SINRs given the intervention rule. Thus, we exclude the strategic behavior of the users to influence the measurements in their favor. We also assume that the intervention device can move its receiver to N different locations, or it has N receivers located at different locations.

Algorithm 4 The nth Round of measurement performed by the intervention device

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Require: Error tolerance $\varepsilon > 0$

1: **Initialization**: Broadcast $\alpha_j = 0, \forall j \in \mathcal{N}$

2: **for** index $= 0$ to $N - 1$ **do**

3:　Set $i = (n + \text{index}) \bmod N$, $\underline{\alpha}_i = 0$, $\bar{\alpha}_i = 0$, α_i any positive value (preferably large)

4:　Measure the aggregate receive power $\bar{R}_i$ and set the current aggregate receive power $R_i = \bar{R}_i$

5:　**while** $\bar{\alpha}_i = 0$ **or** $\alpha_i - \underline{\alpha}_i > \varepsilon$ **do**

6:　　**if** $\bar{\alpha}_i = 0$ **and** $R_i = \bar{R}_i$ **then**

7:　　　$\alpha_i \leftarrow 2 \cdot \alpha_i$

8:　　**else if** $R_i = \bar{R}_i$ **then**

9:　　　$\underline{\alpha}_i \leftarrow \alpha_i, \alpha_i \leftarrow (\underline{\alpha}_i + \bar{\alpha}_i)/2$

10:　　**else**

11:　　　$\bar{\alpha}_i \leftarrow \alpha_i, \alpha_i \leftarrow (\underline{\alpha}_i + \bar{\alpha}_i)/2$

12:　　**end if**

13:　　Broadcast the index i and the new α_i, and measure the current aggregate receive power R_i

14:　　**if** $R_i < \bar{R}_i$ **then**

15:　　　Set $\underline{R}_i = R_i$

16:　　**end if**

17:　**end while**

18: **end for**

In round n, the designer adjusts the intervention rates one by one, starting from α_n to α_N, then from α_1 to α_{n-1}. All measurements are made at the receiver at location n. When adjusting α_i, the aim is to find α_i^*, the minimum intervention rate at which user i's best response is $\tilde{p}_i$. We can calculate α_i^* as

$$\alpha_i^* = \begin{cases} \dfrac{\sum_{j=1}^{i-1} h_{ij}\tilde{p}_j + \sum_{j=i+1}^{n-1} h_{ij}P_j + \sum_{j=n}^{N} h_{ij}\tilde{p}_j + n_i}{h_{i0}}, & i < n \\[2ex] \dfrac{\sum_{j=1}^{n-1} h_{ij}P_j + \sum_{j=n}^{i-1} h_{ij}\tilde{p}_j + \sum_{j=i+1}^{N} h_{ij}P_j + n_i}{h_{i0}}, & i \geq n \end{cases}$$
(34)

The designer tunes α_i to find α_i^* according to the change of the aggregate receive power. When $\alpha_i > \alpha_i^*$, user i's best response is P_i, and the aggregate receive power at location n is

$$\bar{R}_i^n = \begin{cases} \sum_{j=1}^{i-1} h_{0j}^n \tilde{p}_j + \sum_{j=i}^{n-1} h_{0j}^n P_j \\ \quad + \sum_{j=n}^{N} h_{0j}^n \tilde{p}_j + n_0, & i < n \\ \sum_{j=1}^{n-1} h_{0j}^n P_j + \sum_{j=n}^{i-1} h_{0j}^n \tilde{p}_j \\ \quad + \sum_{j=i}^{N} h_{0j}^n P_j + n_0, & i \geq n \end{cases}$$

where h_{0j}^n is the channel gain from user j's transmitter to the intervention device's receiver at location n. When $\alpha_i > \alpha_i^*$, user i's best response is the target power profile $\tilde{p}_i$, and the aggregate receive power decreases to

$$\underline{R}_i^n = \begin{cases} \sum_{j=1}^{i} h_{0j}^n \tilde{p}_j + \sum_{j=i+1}^{n-1} h_{0j}^n P_j \\ \quad + \sum_{j=n}^{N} h_{0j}^n \tilde{p}_j + n_0, & i < n \\ \sum_{j=1}^{n-1} h_{0j}^n P_j + \sum_{j=n}^{i} h_{0j}^n \tilde{p}_j \\ \quad + \sum_{j=i+1}^{N} h_{0j}^n P_j + n_0, & i \geq n \end{cases}$$

During the measurement, the designer maintains an upper bound $\bar{\alpha}_i$, at which the aggregate receive power is $\underline{R}_i$, and a lower bound $\underline{\alpha}_i$, at which the aggregate receive power is $\bar{R}_i$. By bisection methods, an estimate of α_i^*, denoted by m_{ni}, is obtained within the error tolerance ε. The nth-round measurement is summarized in Algorithm 4.

Round n returns a set of measurements $\{\bar{R}_i^n, \underline{R}_i^n\}_{i=1}^{N}$, from which we obtain maximum power levels $\{P_i\}_{i=1}^{N}$. Note that $\bar{R}_i^n - \underline{R}_i^n = h_{0i}^n \cdot (P_i - \tilde{p}_i)$ for all $i, n \in \mathcal{N}$. Thus, we have

$$h_{0i}^n = h_{0i}^1 \cdot \frac{\bar{R}_i^n - \underline{R}_i^n}{\bar{R}_i^1 - \underline{R}_i^1}, \quad \forall i, n \in \mathcal{N}. \tag{35}$$

Since $\underline{R}_{n-1}^n = \sum_{j=1}^{N} h_{0j}^n \tilde{p}_j (\underline{R}_0^1 = \underline{R}_N^1$ when $n = 1)$, using the above relationship between h_{0i}^n and h_{0i}^1, we have the following N linear equations

$$\sum_{j=1}^{N} h_{0j}^1 \cdot \left(\frac{\bar{R}_j^n - \underline{R}_j^n}{\bar{R}_j^1 - \underline{R}_j^1} \tilde{p}_j \right) = \underline{R}_{n-1}^n, \quad n = 1, \dots, N \tag{36}$$

which we can solve for $\{h_{0j}^1\}_{j=1}^{N}$. Given $\{h_{0j}^1\}_{j=1}^{N}$, we can calculate $\{P_i\}_{i=1}^{N}$ by using $\bar{R}_i^1 - \underline{R}_i^1 = h_{0i}^1 \cdot (P_i - \tilde{p}_i), \forall i \in \mathcal{N}$.

Round n also returns another set of measurements $\mathbf{m}_n = [m_{n1}, \dots, m_{ni}, \dots, m_{nN}]$, where we assume $m_{ni} = \alpha_i^*$. Given the measurements $\{\mathbf{m}_n\}_{n=1}^{N}$, we can obtain the normalized cross channel gains and the normalized

TABLE II
COMPARISON OF COMMUNICATION OVERHEAD OF DIFFERENT FRAMEWORKS

Framework	Users	The Designer	Overall
Intervention	0	$2N^2 \log_2(1/\varepsilon) + 2N \cdot T^\star(\mathbf{p}^\star)$	$2N^2 \log_2(1/\varepsilon) + 2N \cdot T^\star(\mathbf{p}^\star)$
Network utility maximization [4][5]	N ([4]) or 0 ([5]) in each step	0 ([4]) or N ([5]) in each step	N in each step
Game theoretic control based on taxation [8]–[13]	N in each step	N in each step	$2N$ in each step
Mechanism design [14][15]	N^2 in each step	N in each step	$N^2 + N$ in each step

noise powers. Specifically, from $\mathbf{m}_n$ and $\mathbf{m}_{n+1}, \forall n < N$, we have

$$m_{n+1,i} - m_{ni} = \frac{h_{in}P_n - h_{in}\tilde{p}_n}{h_{i0}}, \quad \forall i \neq n \qquad (37)$$

from which we can get $h_{in}/h_{i0} = (m_{n+1,i} - m_{ni})/(P_n - \tilde{p}_n), \forall i \neq n$.

To sum up, we can get $\{h_{in}/h_{i0}\}_{i \neq n}$ according to $\mathbf{m}_n$ and $\mathbf{m}_{n+1}$ for all $n < N$, and get $\{h_{iN}/h_{i0}\}_{i \neq N}$ according to $\mathbf{m}_N$ and $\mathbf{m}_1$. Now that we know all the normalized channel gains, we can get the normalized noise powers n_i/h_{i0} from (34).

2) Individual Transmit Powers: The byproduct of the above estimation is the channel gains from the users to the intervention device at N different locations: $\{h_{0i}^n\}_{i=1}^N, n = 1, \ldots, N$. At any time, the intervention device can measure the aggregate received power at all the N locations

$$\sum_{j=1}^N h_{0j}^n p_j + n_0^n \qquad (38)$$

where n_0^n is the noise power known to the intervention device. Since the designer knows the values of N different linear combinations of the N individual transmit powers, it can solve the group of N linear equations to obtain the individual transmit powers. The complexity of this operation is of the order N^3.

B. Comparison of Communication Overhead

Now we compare the communication overhead of the intervention scheme with that of other frameworks, including network utility maximization [4], [5], game theoretic control based on taxation [8]–[13], and mechanism design [14], [15]. Before the comparison, we would like to emphasize that the intervention scheme works for selfish users, who have no incentive to provide any information to anyone else. As shown in Table II, intervention requires no information flow from the users to the designer. On the contrary, the other works assume that the users are obedient to exchange information with the designer or among each other according to some prescribed rules.

The communication overhead is measured by the total amount of information flow before the system reaches the desired operating point. Specifically, the amount of information flow is measured by the number of real numbers broadcast by the users and the intervention device, ignoring quantization and coding. The communication overhead can be further categorized into the communication overhead on the users and that on the designer. We summarize the comparison in Table II. In each framework, we select representative algorithms to calculate the communication overhead. Hence, the numbers in the table are not precise for all the algorithms, but are correct in terms of the order.

We can see from Table II that in intervention, users have zero communication overhead. In other words, the intervention device does not rely on the users to provide information. While in other frameworks, users may be required to broadcast some information. Hence, intervention is more suitable when users are selfish and unwilling to provide information truthfully. Another advantage of intervention is that its communication overhead can be bounded analytically. While in other frameworks, the convergence speed to the desired operating point is not guaranteed. The detailed analysis is as follows.

1) Intervention: The communication overhead of intervention is comprised of two parts. The first part is the overhead of estimating normalized cross channel gains and noise powers. The temporary target power profile $\tilde{\mathbf{p}}$ and the intervention capability P_0 is broadcast at the beginning, which we omit here. There are N rounds of measurement. In each round, the intervention device needs to broadcast the initial intervention rates (line 1 in Algorithm 4), and the indices and the values of the intervention rates it is adjusting (line 13 in Algorithm 4). Since it uses bisection methods to adjust the intervention rate, the number of adjustments is $\log_2(1/\varepsilon)$ for each intervention rate, where ε is the error tolerance in estimating intervention rates in Algorithm 4. In sum, the communication overhead in estimating system parameters is $N(2N \log_2(1/\varepsilon) + 1) \approx 2N^2 \log_2(1/\varepsilon)$ for all intervention rates.

The second part is the communication overhead during the convergence to NE. In general, this overhead depends on the target power profile and the intervention capability. If the target power profile is close to the maximum power level $(\sum_{i=1}^N (P_i - p_i^\star)/P_i < 1))$ and the intervention capability is large (Theorem 3), the convergence is in one step. If the target power profile is not close enough or the intervention capability is limited, we need to use the dynamic adjustment process. The convergence is in $N' + 1$ steps if the intervention capability is large (Theorem 4), and is in $T^\star(\mathbf{p}^\star)$ steps if the intervention capability is limited (Theorem 5). In each step, the intermediate target power profile and the intervention rates are broadcast. In summary, the worst-case communication overhead during the convergence to NE is $2N \cdot T^\star(\mathbf{p}^\star)$.

2) Other Frameworks: The communication overhead of other frameworks is the total information flow during the convergence to the desired operating point. Unlike intervention, the number of steps during the convergence is not bounded. Hence, the communication overhead could be arbitrarily large.

If users cooperate with the designer to maximize the assigned utility function, as in [4], [5] and [8]–[13], the information flow in one step can be less than that in intervention. In network utility maximization, either each user or the base station will
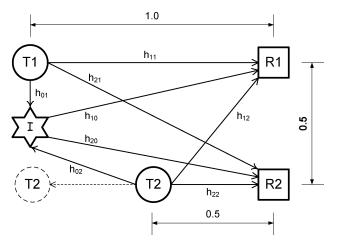
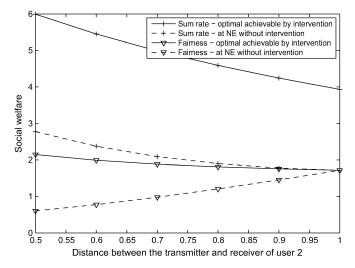Fig. 1.   Example single-hop wireless ad-hoc network with two users.

Fig. 2.   Optimal social welfare achievable by intervention and the social welfare at NE without intervention, when user 2's transmitter moves away from its receiver.

broadcast at least N signals for all the users. In [4], each user broadcast its own interference price. In [5], the base station broadcast "load" and "spillage." In game theoretic control based on taxation [8]–[13], the base station broadcast N prices, usually different for different users. To obtain the optimal pricing, users report back some information, such as their payoffs or interferences, to adjust prices.

In mechanism design [14], [15], similar to intervention, the designer does not know the utility function of each user. In this case, the information flow in one step is of the same order as that in intervention. Specifically, each user broadcast its own version of optimal power allocation vector in each step, and the designer broadcast the reference power allocation vector. Hence, the amount of information flow is $N^2 + N$ in each step.

VI. SIMULATION RESULTS

We consider a two-user network shown in Fig. 1. User 2's transmitter is near to user 1's receiver, causing significant interference to user 1. The distance from user 1's transmitter to its receiver is normalized to 1. Originally, the distance from user 2's transmitter to its receiver is 0.5. The vertical distance between the two users' transmitters and that between the two users' receivers are both 0.5. Without specific notice, we assume that the positions of the transmitters and receivers of both users remain the same. In the simulation of Fig. 2, we let user 2's transmitter move away from its receiver as shown by the dashed left arrow, resulting in less interference to user 1. The channel gain h is reciprocal to the distance d with the path loss exponent a, namely $h = d^{-a}$. We assume an indoor environment where $a = 3$ [25]. The noise powers at the receivers of both users are 0.2. The power budgets of both users are 10.

A. Performance Improvement by Intervention

Now we examine the performance improvement by using intervention mechanisms. We let user 2's transmitter moves away from its receiver. In Fig. 2, we show the performance achieved by intervention and that at the NE without intervention, under two criteria for social welfare. The sum rate is define by

$$\log(1 + \gamma_1) + \log(1 + \gamma_2) \qquad (39)$$

and the fairness is defined by the "max-min" fairness [1, pp. 392], [5], [26]

$$\min\{\log(1 + \gamma_1), \quad \log(1 + \gamma_2)\}. \qquad (40)$$

As we can see from Fig. 2, the sum rate achievable by intervention doubles that at the NE without intervention in all the cases. The fairness achievable by intervention is much larger than that at the NE without intervention in most cases. When the distance from user 2's transmitter to its receiver is 1.0, the network is symmetric. Only at this point is the NE without intervention optimal in terms of fairness.

B. Minimum Power Budget

Now we show the power budget requirement for different intervention rules. In Fig. 3, we show the contour of the minimum power budget for different intervention rules under different target power profiles. Fig. 3(a) shows minimum power budget to sustain a target power profile using first-order intervention based on individual transmit powers obtained by Theorem 1 and that using first-order intervention based on aggregate receive power obtained by Theorem 6. Since the power budget requirements are the same for these two intervention rules, we show them in the same figure. Fig. 3(b) shows the minimum power budget to strongly sustain a target power profile obtained by simulation. As we expect, the power budget requirement for strong sustainment is higher. Fig. 3(c) shows the upper bound on the minimum power budget to strongly sustain a target power profile obtained by Theorem 2. We can see that the result in Theorem 2 serves as a good upper bound. Finally, Fig. 3(d) shows the upper bound on the minimum power budget for strong sustainment and fast convergence obtained by Theorem 3. In this case, the system reaches NE in at most two time slots. To achieve this fast convergence, the intervention device needs a much higher power budget. In addition, not all the target power profiles can be sustained. The target power profiles that cannot be sustained lie in the shadow area in the figure.
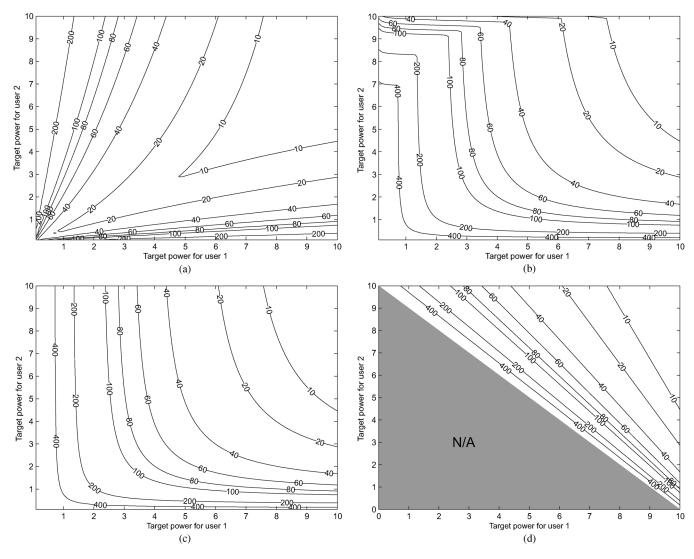
Fig. 3. Contour of the minimum power budget of first-order intervention under different target power profiles. (a) Minimum power budget to sustain a target power profile obtained by Theorem 1 and Theorem 6. (b) Minimum power budget to strongly sustain a target power profile obtained by simulation. (c) Upper bound on the minimum power budget to strongly sustain a target power profile obtained by Theorem 2. (d) Upper bound on the minimum power budget for strong sustainment and fast convergence obtained by Theorem 3.

C. Power Budget and Convergence Time Tradeoff in Dynamic Adjustment Process

Now we study the tradeoff between the power budget and the convergence time in the dynamic adjustment process. Here the convergence time is measured as the number of steps in the adjustment process. To better illustrate the tradeoff, we use a five-user network in the simulation for Fig. 4 and Fig. 5. The channel gains and noise powers used in the simulation are one realization of the random variables. Since different realizations result in similar tradeoff, we only show the results for one realization. The target power profile is $p_1^\star = P_1$ and $p_i^\star = 0.1P_i$ for $i > 1$. Since the relative distance between the maximum power profile and the target power profile is $\sum_{i=1}^{5}((P_i - p_i^\star)/P_i) = 3.6 > 1$, we cannot reach the target power profile from the maximum power profile directly using Theorem 3. Instead, we need a sequence of intermediate target power profile before we reach the final target power profile.

First, suppose that there is no power budget requirement. Without power budget limit, our goal is to reach the final target power profile in as few time slots as possible. Since it is not easy to construct a sequence of intermediate target power profiles given a desired convergence time, we construct the sequence according to the desired relative distance between adjacent intermediate target power profiles, which is an indicator for the convergence time. In Fig. 4, we show the convergence time and the power budget requirement of the MRD sequence generated by Algorithm 2 and the geometric sequence under different relative distances. We can see from Fig. 4(a) that a larger relative distance results in a faster convergence for both sequences. Thus, we can use the relative distance, a metric amenable for the construction of the sequence, to control the convergence speed of the adjustment process. In particular, when the relative distance is $\delta = 0.9 = 1 - \min_i\{p_i^\star/P_i\}$, the MRD sequence converges in $N' + 1 = 5$ steps as predicted by Theorem 4. In Fig. 4(b), we can see that for both sequences, the power budget requirement is decreasing with the relative distances in most
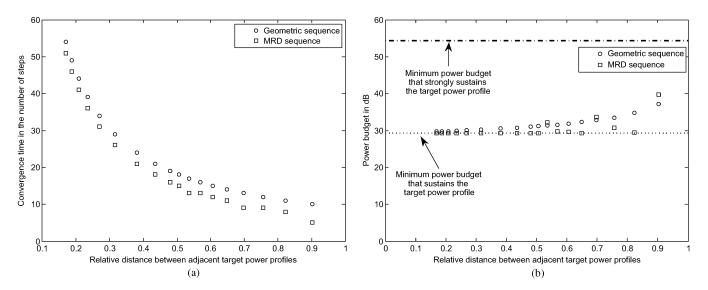
Fig. 4. Given the relative distance between adjacent target power profiles, the convergence time and the power budget requirement of different sequences of intermediate target power profiles in a five-user network. The relative distance between the maximum power profile and the target power profile is $\sum_{i=1}^{5}((P_i - p_i^\star)/P_i) = 3.6 > 1$. (a) Convergence time. (b) Power budget requirement.
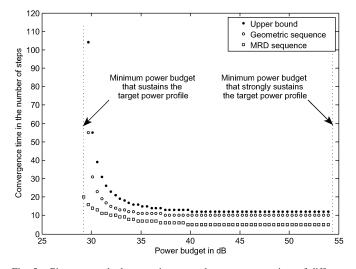

Fig. 5. Given power budget requirements, the convergence time of different sequences of intermediate target power profiles in a five-user network. The relative distance between the maximum power profile and the target power profile is $\sum_{i=1}^{5}((P_i - p_i^\star)/P_i) = 3.6 > 1$.

cases. The power budget is lower for the MRD sequence. For both sequences, it requires much less power by the dynamic adjustment process than by the strong sustainment condition in Theorem 2.

Second, suppose that there is a power budget requirement. Given different power budget requirements, we show the convergence time of MRD and geometric sequences and the upper bound on the convergence time in Fig. 5. We can see from the figure that under most power budget requirements, the convergence time of MRD sequence is roughly half of that of the naive geometric sequence. When the power budget is small (near to the minimum power budget that sustains the target power profile), the fast convergence of MRD sequence is even more significant compared to that of the geometric sequence. When the power budget approaches the minimum requirement that strongly sustains the target power profile, the convergence time of MRD sequence is 5, which is half of that of the geometric sequence.

VII. CONCLUSION

In this paper, we proposed incentive schemes based on intervention for power control in single-hop wireless ad hoc networks with selfish users. We formulated a game-theoretic model of power control with an intervention device and proposed design criteria that desirable intervention rules should satisfy. Focusing on a simple class of intervention rules called first-order intervention rules, we provided requirements for intervention rules to sustain a target power profile when the intervention device estimates individual transmit powers or aggregate receive power. We also proposed dynamic adjustment processes to guide users to a target power profile through intermediate targets. We discussed implementation issues and presented simulation results. To the best of our knowledge, this work is the first to investigate intervention schemes in a power control scenario. For future research, we can apply intervention schemes to different power control scenarios, for example, a scenario where users can allocate their power budgets across multiple channels and a scenario where users care about their energy consumption as well as their data rates.

REFERENCES

[1] M. Chiang, P. Hande, T. Lan, and C. W. Tan, "Power control in wireless cellular networks," *Foundat. Trends Network.*, vol. 2, no. 4, pp. 381–533, Apr. 2008.

[2] R. D. Yates, "A framework for uplink power control in cellular radio systems," *IEEE J. Sel. Areas Commun.*, vol. 13, no. 7, pp. 1341–1347, Sep. 1995.

[3] E. Altman and Z. Altman, "S-modular games and power control in wireless networks," *IEEE Trans. Autom. Control*, vol. 48, no. 5, pp. 839–842, May 2003.

[4] J. Huang, R. A. Berry, and M. L. Honig, "Distributed interference compensation for wireless networks," *IEEE J. Sel. Areas Commun.*, vol. 24, no. 5, pp. 1074–1084, May 2006.

[5] P. Hande, S. Rangan, M. Chiang, and X. Wu, "Distributed uplink power control for optimal SIR assignment in cellular data networks," *IEEE/ACM Trans. Netw.*, vol. 16, no. 6, pp. 1420–1433, Dec. 2008.

[6] E. Altman, T. Boulogne, R. El-Azouzi, T. Jimenez, and L. Wynter, "A survey on networking games in telecommunications," *Comput. Operat. Res.*, vol. 33, no. 2, pp. 286–311, Feb. 2006.

[7] E. G. Larsson, E. A. Jorswieck, J. Lindblom, and R. Mochaourab, "Game theory and the flat-fading Gaussian interference channel," *IEEE Signal Process. Mag.*, vol. 26, no. 5, pp. 18–27, Sep. 2009.

[8] T. Alpcan, T. Basar, R. Srikant, and E. Altman, "CDMA uplink power control as a noncooperative game," *Wireless Netw.*, vol. 8, pp. 659–670, 2002.

[9] C. U. Saraydar, N. B. Mandayam, and D. J. Goodman, "Efficient power control via pricing in wireless data networks," *IEEE Trans. Commun.*, vol. 50, no. 2, pp. 291–303, Feb. 2002.

[10] M. Xiao, N. B. Shroff, and E. K. P. Chong, "A utility-based power control scheme in wireless cellular systems," *IEEE/ACM Trans. Netw.*, vol. 11, no. 2, pp. 210–221, Apr. 2003.

[11] G. Scutari, S. Barbarossa, and D. P. Palomar, "Potential games: A framework for vector power control problems with coupled constraints," in *Proc. ICASSP '06*, 2006, pp. 241–244.

[12] U. O. Candogan, I. Menache, A. Ozdaglar, and P. A. Parrilo, "Near-optimal power control in wireless networks: A potential game approach," in *Proc. IEEE INFOCOM '10*, 2010, pp. 1–9.

[13] J. Huang, R. A. Berry, and M. L. Honig, "Auction-based spectrum sharing," *Mobile Netw. Applicat.*, vol. 11, pp. 405–418, 2006.

[14] S. Sharma and D. Teneketzis, "An externalities-based decentralized optimal power allocation algorithm for wireless networks," *IEEE/ACM Trans. Netw.*, vol. 17, no. 6, pp. 1819–1831, Dec. 2009.

[15] S. Sharma and D. Teneketzis, "A game-theoretic approach to decentralized optimal power allocation for cellular networks," *Telecomm. Syst.*, pp. 1–16, 2010.

[16] Y. Wu, B. Wang, K. J. R. Liu, and T. C. Clancy, "Repeated open spectrum sharing game with cheat-proof strategies," *IEEE Trans. Wireless Commun.*, vol. 8, no. 4, pp. 1922–1933, Apr. 2009.

[17] C. Long, Q. Zhang, B. Li, H. Yang, and X. Guan, "Non-cooperative power control for wireless ad hoc networks with repeated games," *IEEE J. Sel. Areas Commun.*, vol. 25, no. 6, pp. 1101–1112, Aug. 2007.

[18] G. Mailath and L. Samuelson, *Repeated Games and Reputations: Long-Run Relationships*.   Oxford, U.K.: Oxford Univ. Press, 2006.

[19] J. Park and M. van der Schaar, "Stackelberg contention games in multiuser networks," *EURASIP J. Adv. Signal Process.*, vol. 2009, pp. 1–15, 2009.

[20] J. Park and M. van der Schaar, "Designing incentive schemes based on intervention: The case of imperfect monitoring," in *Proc. GameNets '11* [Online]. Available: http://medianetlab.ee.ucla.edu/papers/gamenets2011_jaeok.pdf

[21] J. Park and M. van der Schaar, "The theory of intervention games for resource sharing in wireless communications," *IEEE J. Sel. Areas Commun* [Online]. Available: http://arxiv.org/abs/1101.3052, accepted for publication

[22] Y. Gai, H. Liu, and B. Krishnamachari, "A packet dropping-based incentive mechanism for M/M/1 queues with selfish users," in *Proc. IEEE INFOCOM '11*, 2011, pp. 2687–2695.

[23] Y. Xiao, J. Park, and M. van der Schaar, "Intervention in power control games with selfish users," [Online]. Available: http://arxiv.org/abs/1104.1227v1 Tech. Rep.

[24] D. Fudenberg and J. Tirole, *Game Theory*.   Cambridge, MA: MIT Press, 1991.

[25] T. S. Rappaport, *Wireless Communications: Principles and Practice*. Upper Saddle River, NJ: Prentice-Hall, 2002.

[26] M. Schubert and H. Boche, "Solution of the multiuser downlink beamforming problem with individual SINR constraints," *IEEE Trans. Veh. Technol.*, vol. 53, no. 1, pp. 18–28, Jan. 2004.

[27] R. A. Horn and C. R. Johnson, *Matrix Analysis*.   Cambridge, U.K.: Cambridge Univ. Press, 1985.

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