# Energy-efficient Nonstationary Power Control in Cognitive Radio Networks

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Abstract—Spectrum sharing policies are essential for cognitive radio networks, where primary and secondary users aim to minimize their average energy consumptions subject to minimum throughput requirements. Most existing works proposed stationary spectrum sharing policies, in which users transmit simultaneously at *fixed* power levels, and need to transmit at high power levels due to multi-user interference. In this paper, we propose nonstationary spectrum sharing policies in which users transmit in a TDMA fashion (but not necessarily in a round-robin manner). Due to the absence of multi-user interference and the ability to let users adaptively switch between transmission and dormancy, our proposed policy greatly improves the spectrum and energy efficiency, and ensures no interference to primary users. Moreover, the proposed policy achieves high energy efficiency even when users have erroneous and binary feedback about their received interference and noise power levels. The proposed policy is also deviation-proof, namely the autonomous users find it in their self-interests to comply with the policy. The proposed policy can be implemented by each user running a low-complexity algorithm in a distributed fashion. Compared to existing policies, the proposed policies can achieve an energy saving of up to 80%.

#### I. INTRODUCTION

We study energy-efficient spectrum sharing policies in cognitive radio networks, in which primary users (PUs) and secondary users (SUs) aim to minimize their average energy consumption subject to their minimum throughput requirements. Spectrum sharing policies specify PUs' and SUs' transmission schedules and transmit power levels. Most existing works [1]– [8] restrict attention to *stationary* spectrum sharing policies, which require users to simultaneously transmit at *fixed* power levels. Stationary policies are *not* energy efficient, because due to multi-user interference, the users need to transmit at high power levels to fulfil their minimum throughput requirements.

Moreover, most existing works [1]–[12] assume that each user's receiver can perfectly estimate the local interference temperature (i.e. the interference and noise power level), and can accurately feed it back to its transmitter. However, in practice, users cannot perfectly estimate the interference temperature, and can only send limited (quantized) feedback.

In this paper, we study TDMA (time-division multiple access) spectrum sharing policies, a class of *nonstationary* policies in which the users transmit in a TDMA fashion. TDMA policies eliminate multi-user interference (including

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the interference from SUs to PUs), and allow users to adaptively switch on and off, depending on the average throughput they have achieved, for the purpose of energy saving. Note that in the optimal TDMA policies we propose, users usually do not transmit in the simple round-robin fashion, because of the heterogeneity in their minimum throughput requirements and channel conditions. The proposed policy enables users to achieve minimum throughput requirements with minimal energy consumptions, under erroneous and binary feedback. Moreover, the proposed policy is deviation-proof, namely a user cannot improve its energy efficiency over the proposed policy while still fulfilling its throughput requirement. The policy can be implemented by each user running a lowcomplexity algorithm in a distributed fashion.

We develop our design framework of nonstationary spectrum sharing policies based on the repeated game formalism. More specifically, we model the interaction among the users as a repeated game with imperfect monitoring. The repeated game formalism allows us to model and analyze nonstationary policies, and to potentially design the optimal policy. However, our results are not straightforward applications of existing results in repeated game theory, due to the following limitations of existing repeated game theory. First, the existing results in repeated games [13] are not constructive: they focus on what operating points can be achieved, but not how to achieve them. In contrast, given an operating point, we explicitly construct the policy to achieve it. Second, the existing results in repeated games [13] require a high-granularity feedback signal, namely the number of feedback signals should be proportional to the number of power levels a user can choose, while we prove that binary feedback signals are sufficient to achieve optimality in the spectrum sharing scenarios.

The rest of the paper is organized as follows. We review related works in Section II. Section III describes the system model for spectrum sharing. In Section IV, we formulate and solve the policy design problem. Simulation results are shown in Section V. Finally, Section VI concludes the paper.

#### II. RELATED WORKS

#### A. Stationary Spectrum Sharing Policies

In Table I, we compare the proposed existing stationary spectrum sharing policies based on two criteria: whether the policy is deviation-proof (against stationary or nonstationary policies), and what are the feedback requirements and the

We gratefully acknowledge support from NSF through Grant No. 1218136.

TABLE I.	Comparisons	AGAINST	STATIONARY	POLICIES.

	Feedback	Deviation-proof
[1]–[8]	Error-free, unquantized	Against stationary policies
[9]–[11]	Error-free, unquantized	Against stationary/nonstationary policies
Proposed	Erroneous, binary	Against stationary/nonstationary policies

TABLE II. COMPARISONS AGAINST NONSTATIONARY POLICIES.

	Energy-efficient	Power control	Feedback (Overhead)
[12]	No	Yes	Error-free, unquantized
[13]	No	Applicable	Erroneous, quantized
[14]	No	Yes	Erroneous, binary
Proposed	Yes	Yes	Erroneous, binary

corresponding overhead. The feedback here is the information on interference and noise power levels sent from a user's receiver to its transmitter.

Note that we put [9]–[11] in the category of stationary policies, although they design policies in a repeated game framework. This is because in the equilibrium where the system operates, the policies in [9]–[11] use fixed power levels. This is in contrast with [12], which uses time-varying power levels at equilibrium and is categorized as nonstationary policies in the next subsection.

#### B. Nonstationary Spectrum Sharing Policies

We summarize the major differences between the existing nonstationary policies and our proposed policy in Table II. The major limitation of the works based on repeated games with perfect monitoring [12] is the assumption of perfect monitoring, which requires error-free and unquantized feedback. The theory of repeated games with imperfect monitoring [13] allows erroneous and limited feedback, but requires that the amount of feedback increases with the number of power levels that the users can choose. In contrast, we only require binary feedback regardless of the number of power levels, which significantly reduces the feedback overhead.

Most related to this paper is our previous work [14], which designed optimal nonstationary polices to maximize the users' throughput subject to their transmit power constraints. However, due to this different objective, the design in [14] is significantly different. In [14], we aimed to maximize the users' total throughput without considering energy efficiency. Under this design objective, each user will transmit at the maximum power level in its slot. Hence, what we optimized is the transmission schedule of the users only. In this work, since we aim to minimize the energy consumption subject to the minimum throughput requirements, we need to optimize both the transmission schedule and the users' transmit power levels, which makes the design problem more challenging. Moreover, in [14], we consider a *single* PU and abstract it into an interference temperature constraint, while in this work, we consider *multiple* PUs and include their power control problem in the framework.

#### III. SYSTEM MODEL

### A. Model of Cognitive Radio Networks

Consider a cognitive radio network that consists of M PUs and N SUs transmitting in a single frequency channel. The set of PUs and that of SUs are denoted by  $\mathcal{M} \triangleq \{1, 2, \dots, M\}$  and  $\mathcal{N} \triangleq \{M + 1, M + 2, \dots, M + N\}$ , respectively. Each



Fig. 1. An example system model with two primary users (transmitter-receiver pairs 1 and 2) and a secondary user (transmitter-receiver pair 3). The solid line represents a link for intended data transmission, and the dotted line represents the interference from another user.

user<sup>1</sup> has a transmitter and a receiver. The channel gain from user *i*'s transmitter to user *j*'s receiver is  $g_{ij}$ . Each user *i* chooses its power level  $p_i$  from a compact set  $\mathcal{P}_i \subseteq \mathbb{R}_+$ . We assume that  $0 \in \mathcal{P}_i$ , namely user *i* can choose not to transmit. The set of joint power profiles is denoted by  $\mathcal{P} = \prod_{i=1}^{M+N} \mathcal{P}_i$ , and the joint power profile of all the users is denoted by  $\mathbf{p} = (p_1, \dots, p_{M+N}) \in \mathcal{P}$ . Let  $\mathbf{p}_{-i}$  be the power profile of all the users other than user *i*. Since the users cannot jointly decode their signals, each user *i* treats the interference from the other users as noise, and obtains the following throughput at the power profile  $\mathbf{p}$ :

$$r_i(\mathbf{p}) = \log_2 \left( 1 + \frac{p_i g_{ii}}{\sum_{j \in \mathcal{M} \cup \mathcal{N}, j \neq i} p_j g_{ji} + \sigma_i^2} \right), \qquad (1)$$

where  $\sigma_i^2$  is the noise power at user *i*'s receiver.

We define user *i*'s local interference temperature  $I_i(\mathbf{p}_{-i})$  as the interference and noise power level at its receiver, namely  $I_i(\mathbf{p}_{-i}) \triangleq \sum_{j \neq i} p_j g_{ji} + \sigma_i^2$ . Each user's receiver measures the interference temperature with errors and feedback the quantized measurement to its transmitter. We assume that each user *i* uses a *unbiased* estimator with an additive estimation error to obtain the estimate  $\hat{I}_i \triangleq I_i + \varepsilon_i$ , where  $\varepsilon_i$  is the estimation error with zero mean, whose probability distribution function  $f_{\varepsilon_i}$  is known to user *i*. We also assume that each user *i* uses the following simple *two-level* quantizer  $Q_i$ :

$$Q_i(\hat{I}_i(\mathbf{p}_{-i})) = \begin{cases} \bar{I}_i, & \text{if } \hat{I}_i(\mathbf{p}_{-i}) > \theta_i \\ \underline{I}_i, & \text{otherwise} \end{cases}, \forall \mathbf{p}_{-i} \in \mathcal{P} \setminus \mathcal{P}_i, \quad (2)$$

where  $\theta_i$  is user *i*'s quantization threshold, and  $I_i$  and  $\underline{I}_i$ are two reconstruction values. We assume that the quantizer preserves the mean value of  $\hat{I}_i(\mathbf{p}_{-i})$  when there is no multiuser interference. In other words, when  $\mathbf{p}_{-i} = \mathbf{0}$  (i.e. when  $I_i(\mathbf{p}_{-i}) = \sigma_i^2$ ), the quantizer should satisfy

$$\mathbb{E}_{\varepsilon_i}\{Q_i(\hat{I}_i(\mathbf{p}_{-i})|_{\mathbf{p}_{-i}=\mathbf{0}})\} = \mathbb{E}_{\varepsilon_i}\{\hat{I}_i(\mathbf{p}_{-i})|_{\mathbf{p}_{-i}=\mathbf{0}}\} = \sigma_i^2$$

This property can be easily satisfied by setting

$$I_{i} = \int_{x-\sigma_{i}^{2} \in \operatorname{supp}(f_{\varepsilon_{i}}), \ x \ge \theta_{i}} x \cdot f_{\varepsilon_{i}}(x-\sigma_{i}^{2}) dx I_{i} = \int_{x-\sigma_{i}^{2} \in \operatorname{supp}(f_{\varepsilon_{i}}), \ x < \theta_{i}} x \cdot f_{\varepsilon_{i}}(x-\sigma_{i}^{2}) dx ,$$
(3)

<sup>&</sup>lt;sup>1</sup>We refer to a primary user or a secondary user as a user in general, and will specify the type of users only when necessary.

where  $\operatorname{supp}(f_{e_i})$  is the support of the distribution function  $f_{e_i}$ . In practice, it is easy to implement an unbiased estimator and a simple two-level quantizer as in (2) and (3). As we will show later, such an estimator and a quantizer are sufficient to achieve the optimal performance.

The users can further reduce the feedback overhead as follows. Each user i's receiver informs its transmitter of the reconstruction values  $I_i$  and  $\underline{I}_i$  only once, at the beginning, after which the receiver sends a signal in the form of a simple probe, only when the estimated interference temperature  $I_i$ exceeds the quantization threshold  $\theta_i$ . The event of receiving or not receiving the probing signal, which is sent only when  $I_i > \theta_i$ , is enough to indicate user i's transmitter which one of the two reconstruction values it should choose. Since the probing signal indicates high interference temperature, we call it the distress signal as in [2][8]. With some abuse of definition, we denote user i's distress signal as  $y_i \in Y = \{0, 1\}$  with  $y_i = 1$  representing the event that user *i*'s distress signal is sent (i.e.  $I_i > \theta_i$ ). Write  $\rho_i(y_i | \mathbf{p})$  as the conditional probability distribution of user *i*'s distress signal  $y_i$  given power profile p, which is calculated as

$$\rho_i(y_i = 1 | \mathbf{p}) = \int_{x > \theta_i - I_i(\mathbf{p}_{-i})} f_{\varepsilon_i}(x) dx.$$
(4)

#### B. Spectrum Sharing Policies

The system is time slotted at t = 0, 1, 2, ... We assume as in [1]–[8] that the users are synchronized. At the beginning of time slot t, each user i chooses its transmit power  $p_i^t$ , and achieves the throughput  $r_i(\mathbf{p}^t)$ . At the end of time slot t, each user j who transmits  $(p_j^t > 0)$  sends its distress signal  $y_j^t = 1$ if the estimate  $\hat{I}_j$  exceeds the threshold  $\theta_j$ . We define  $y \in Y$ as the system distress signal, indicating whether there exists a user who has sent its distress signal, namely y = 1 if there exists j such that  $p_j > 0$  and  $y_j = 1$ , and y = 0 otherwise. The conditional distribution is denoted  $\rho(y|\mathbf{p})$ , which is calculated as  $\rho(y = 0|\mathbf{p}) = \prod_{j:p_j>0}\rho_j(y_j = 0|\mathbf{p})$ . Note that the system distress signal is not a physical signal sent in the system, but rather a logical signal summarizing the status of the system.

Each user *i* determines the transmit power level  $p_i^t$  based on the history of distress signals. The history of distress signals is  $h^t = \{y^0; \ldots; y^{t-1}\} \in Y^t$  for  $t \ge 1$ , and  $h^0 = \emptyset$  for t = 0. Then each user *i*'s strategy  $\pi_i$  is a mapping from the set of all the possible histories to its action set, namely  $\pi_i :$  $\cup_{t=0}^{\infty} Y^t \to \mathcal{P}_i$ . The *spectrum sharing policy*, denoted by  $\boldsymbol{\pi} = (\pi_1, \ldots, \pi_{M+N})$ , is the joint strategy profile of all the users. Hence, user *i*'s transmit power level at time slot *t* is determined by  $p_i^t = \pi_i(h^t)$ , and the users' joint power profile is determined by  $\mathbf{p}^t = \boldsymbol{\pi}(h^t)$ .

We classify all the spectrum sharing policies into two categories, stationary and nonstationary policies. A spectrum sharing policy  $\pi$  is *stationary* if and only if for all  $i \in \mathcal{N}$ , for all  $t \ge 0$ , and for all  $h^t \in Y^t$ , we have  $\pi_i(h^t) = p_i^{\text{stat}}$ , where  $p_i^{\text{stat}} \in \mathcal{P}_i$  is a constant. A spectrum sharing policy is *nonstationary* if it is not stationary. In this paper, we restrict our attention to a special class of nonstationary polices, namely TDMA policies (with fixed transmit power levels). A spectrum sharing policy  $\pi$  is a TDMA policy if at most one user transmits in each time slot, and each user *i* chooses the same power level  $p_i^{\text{TDMA}} \in \mathcal{P}_i$  when it transmits.



Fig. 2. The design framework to solve the policy design problem. The feasible operating points lie in different hyperplanes (red dash lines) that go through the vector of minimum throughput requirements (the blue square).

We characterize the spectrum and energy efficiency of a spectrum sharing policy by the users' *discounted* average throughput and *discounted* average energy consumption, respectively. Each user discounts its future throughput and energy consumption because of its delay-sensitive application (e.g. video streaming) [9]–[14]. A user running a more delaysensitive application discounts more (with a lower discount factor). Assuming as in [9]–[14] that all the users have the same discount factor  $\delta \in [0, 1)$ , user *i*'s average throughput is

$$R_{i}(\boldsymbol{\pi}) = (1-\delta) \left[ r_{i}(\mathbf{p}^{0}) + \sum_{t=1}^{\infty} \delta^{t} \sum_{y^{t-1} \in Y} \rho(y^{t-1} | \mathbf{p}^{t-1}) \cdot r_{i}(\mathbf{p}^{t}) \right],$$

where  $\mathbf{p}^0$  is determined by  $\mathbf{p}^0 = \boldsymbol{\pi}(\emptyset)$ , and when  $t \ge 1$ ,  $\mathbf{p}^t$  is determined by  $\mathbf{p}^t = \boldsymbol{\pi}(h^t) = \boldsymbol{\pi}(h^{t-1}; y^{t-1})$ . Similarly, user *i*'s average energy consumption is

$$P_{i}(\boldsymbol{\pi}) = (1 - \delta) \left[ p_{i}^{0} + \sum_{t=1}^{\infty} \delta^{t} \sum_{y^{t-1} \in Y} \rho(y^{t-1} | \mathbf{p}^{t-1}) \cdot p_{i}^{t} \right].$$

Each user *i* aims to minimize its average energy consumption  $P_i(\pi)$  while fulfilling a minimum throughput requirement  $R_i^{\min}$ . From one user's perspective, it has the incentive to deviate from a given spectrum sharing policy, if by doing so it can fulfill the minimum throughput requirement with a lower energy consumption. Hence, we can define deviation-proof policies as follows.

Definition 1 (Deviation-proof Policies): A spectrum sharing policy  $\pi$  is deviation-proof if for all  $i \in \mathcal{M} \cup \mathcal{N}$ , we have

$$\pi_i = \arg\min_{\pi'_i} P_i(\pi'_i, \pi_{-i}), \ s.t. \ R_i(\pi'_i, \pi_{-i}) \ge R_i^{\min},$$

where  $\pi_{-i}$  is the strategy profile of all the users except user *i*.

#### IV. THE DESIGN FRAMEWORK

We want to design a deviation-proof TDMA policy that fulfills all the users' minimum throughput requirements and minimizes the weighted sum of all the users' energy consumptions  $\sum_{i \in \mathcal{M} \cup \mathcal{N}} w_i \cdot P_i(\pi)$ , where  $w_i \ge 0$  and  $\sum_{i \in \mathcal{M} \cup \mathcal{N}} w_i =$ 1. Each user *i*'s weight  $w_i$  reflects its importance. We can differentiate PUs and SUs by setting higher weights for PUs. Given each user *i*'s minimum throughput requirement  $R_i^{\min}$ , we can formally define the policy design problem as

$$\begin{array}{ll} \min_{\boldsymbol{\pi}} & \sum_{i \in \mathcal{M} \cup \mathcal{N}} w_i \cdot P_i(\boldsymbol{\pi}) & (5) \\ s.t. & \boldsymbol{\pi} \text{ is a deviation} - \text{proof TDMA policy}, \\ & R_i(\boldsymbol{\pi}) \geq R_i^{\min}, \ \forall i \in \mathcal{M} \cup \mathcal{N}. \end{array}$$

In Fig. 2, we outline the proposed design framework to solve the policy design problem, which consists of three steps. We describe these three steps in details in the following.

#### A. Characterize the set of feasible operating points

The first step in solving the design problem (5) is characterize the feasible operating points that can be achieved by deviation-proof TDMA policies. The operating point of a TDMA policy is defined as  $\bar{\mathbf{r}} = (\bar{r}_1, \ldots, \bar{r}_{M+N})$ , a vector of each user *i*'s instantaneous throughput  $\bar{r}_i$  when it transmits. In a TDMA policy, each user *i*'s operating point is  $\bar{r}_i = \log_2 (1 + p_i^{\text{TDMA}} g_{ii} / \sigma_i^2)$ . Alternatively, given the operating point  $\bar{\mathbf{r}}$ , the users' power levels can be calculated as  $\mathbf{p}^{\text{TDMA}}(\bar{\mathbf{r}}) = (p_1^{\text{TDMA}}(\bar{r}_1), \ldots, p_{M+N}^{\text{TDMA}}(\bar{r}_{M+N}))$ .

We say an operating point  $\bar{\mathbf{r}}$  is feasible (for minimum throughput requirements  $\{R_i^{\min}\}$ ), if there exists a deviation-proof TDMA policy  $\pi$ , under which each user *i* achieves a throughput  $R_i(\pi) = R_i^{\min}$  with a transmit power level  $p_i^{\text{TDMA}}(\bar{r}_i)$  when it transmits.

Before stating our main result, we define  $\tilde{\mathbf{p}}^{i} = (p_{i}^{\text{TDMA}}(\bar{r}_{i}), \mathbf{p}_{-i} = \mathbf{0})$  as the joint power profile when user *i* transmits in a TDMA policy. We also define

$$b_{ij} = \sup_{p_j \in \mathcal{P}_j, p_j \neq \tilde{p}_j^i} \frac{\rho(y=1|\tilde{\mathbf{p}}^i) - \rho(y=1|p_j, \tilde{\mathbf{p}}^i_{-j})}{r_j(p_j, \tilde{\mathbf{p}}^i_{-j})/\bar{r}_j},$$
(6)

which can be interpreted as user j's benefit from deviation by interfering with user i's transmission. The numerator indicates how likely the deviation can be detected by the distress signal, reflected by the difference between the probabilities that a distress signal is triggered when user j does not and does deviate. The denominator indicates user j's gain in throughput if it deviates.

Now we state Theorem 1, which analytically characterizes the set of feasible operating points.

Theorem 1: An operating point  $\bar{\mathbf{r}}$  is feasible for the minimum throughput requirements  $\{R_i^{\min}\}_{i \in \mathcal{M} \cup \mathcal{N}}$ , if the following conditions are satisfied:

• Condition 1: the discount factor 
$$\delta$$
 satisfies  $\delta \geq \frac{\delta}{\Delta} \triangleq 1/\left(1 + \frac{1 - \sum_{i \in \mathcal{M} \cup \mathcal{N}} \underline{\mu}_i}{N - 1 + \sum_{i \in \mathcal{M} \cup \mathcal{N}} \sum_{j \neq i} (-\rho(y=1|\mathbf{\tilde{p}}^i)/b_{ij})}\right)$   
where  $\underline{\mu}_i \triangleq \max_{j \neq i} \frac{1 - \rho(y=1|\mathbf{\tilde{p}}^i)}{-b_{ij}}$ .

• Condition 2:  $\sum_{i \in \mathcal{M} \cup \mathcal{N}} R_i^{\min} / \bar{r}_i = 1$ , and  $\bar{r}_i \leq R_i^{\min} / \underline{\mu}_i, \forall i$ .

**Proof:** Due to space limit, we only outline the main idea of the proof (illustrated in Fig. 3). Please refer to [15, Appendix A] for the complete proof.

The proof heavily replies on the concept of self-generating sets [16]. Simply put, a self-generating set is a set in which every payoff is an equilibrium payoff [16]. Given the vector of



Fig. 3. The illustration of the proof of Theorem 1.

minimum throughput requirements (the blue square in Fig. 3), we first get M+N throughput vectors from the operating point  $\bar{\mathbf{r}}$  (e.g.  $(\bar{r}_1, 0)$  and  $(0, \bar{r}_2)$  in the two-user case, as illustrated by red dots in Fig. 3). The hyperplane determined by the M+Nthroughput vectors (the line connecting the red dots) should include the vector of minimum throughput requirements. Then we identify the largest self-generating set (the green line segment) in the hyperplane. If the self-generating set includes the vector of minimum throughput requirements, we say the operating point is feasible.

In the theorem, Condition 2 is the sufficient condition for the self-generating set to exist for a given operating point  $\overline{\mathbf{r}}$ . Since the boundary of the largest self-generating set is defined by  $\{\underline{\mu}_i\}_{i \in \mathcal{M} \cup \mathcal{N}}$ , Condition 2 ensures that the vector of minimum throughput requirements is in the self-generating set. In summary, Conditions 1 and 2 are the sufficient conditions for an operating point to be feasible.

Theorem 1 provides the sufficient conditions for the existence of feasible operating points. Condition 1 analytically specifies the requirement for discount factors. When Condition 1 is satisfied, Condition 2 determines the set of feasible operating points under given system parameters. We can choose any point satisfying Condition 2 as the feasible operating point.

#### B. Select the optimal operating point

Among all the feasible operating points, we select the optimal one  $\bar{\mathbf{r}}^*$  based on the following proposition.

*Proposition 1:* The optimal operating point  $\bar{\mathbf{r}}^*$  can be solved by the following *convex* optimization problem

$$\begin{split} \bar{\mathbf{r}}^{\star} &= \arg \min_{\bar{\mathbf{r}}} \quad \sum_{i \in \mathcal{M} \cup \mathcal{N}} w_i \cdot P_i(\bar{\mathbf{r}}) \\ s.t. \quad \sum_{i \in \mathcal{M} \cup \mathcal{N}} R_i^{\min} / \bar{r}_i = 1, \ \bar{r}_i \leq R_i^{\min} / \underline{\mu}_i, \end{split}$$
where  $\bar{P}_i(\bar{\mathbf{r}}) = \frac{R_i^{\min}}{\bar{r}_i} \cdot p_i^{\text{TDMA}}(\bar{r}_i).$ 
Proof: See [15, Appendix B].

## C. Construct the optimal deviation-proof policy

Given the optimal operating point  $\bar{\mathbf{r}}^*$ , each user *i* distributively runs the algorithm in Table III. The resulting policy satisfies Theorem 2.

Theorem 2: If each user *i* runs the algorithm in Table III, then each user *i* will achieve its minimum throughput requirement  $R_i^{\min}$  with energy consumption  $\bar{P}_i(\bar{\mathbf{r}}^{\star})$  that minimizes the weighted sum energy consumption. The policy implemented by the algorithm is deviation-proof: if a user does not follow



the algorithm, it will either fail to achieve the minimum throughput requirement, or achieve it with a higher energy consumption.

**Proof:** See [15, Appendix C]. 
$$\Box$$

As we can see from Table III, the computational complexity of implementing the optimal policy is very small. At each period t, each user only needs to compute M + Ndistances  $\{d_j(t)\}_{j \in \mathcal{M} + \mathcal{N}}$ , and M + N normalized throughput  $\{r'_j(t)\}_{j \in \mathcal{N}}$ , all of which can be calculated analytically. In addition, each SU only needs to store the M + N normalized throughput. The input to the algorithm can be obtained by each user in a decentralized manner. We refer interested readers to [15, Appendix D] for detailed description and discussions on implementation issues.

#### V. PERFORMANCE EVALUATION

We demonstrate the performance gain of our proposed policy over existing policies. We use the following system parameters. The noise powers at all the users' receivers are 0.05W. Direct channel gains are  $g_{ii} \sim C\mathcal{N}(0,1), \forall i$ , and the cross channel gains are  $g_{ij} \sim C\mathcal{N}(0,0.5), \forall i \neq j$ . The quantization threshold is 0.05 W for each user. The measurement error  $\varepsilon_i$  is Gaussian distributed with zeros mean and variance 0.1. The weight for each user  $w_i$  is the same. The discount factor is 0.95.

We compare the proposed policy against the optimal stationary policy in [1]–[8] and two (adapted) versions of the punish-forgive (PF) policies in [9]–[12]. Since the PF policies in [9]–[12] were originally proposed for network utility maximization problems (e.g. maximizing the sum throughput), we need to adapt them to solve the energy efficiency problem in (5). We describe the state-of-the-art policies that we compare against as follows.

- The optimal stationary policy [1]–[8]: each user transmits at a fixed power level that is just large enough to fulfill the throughput requirement under the interference from other users.
- The (adapted) stationary punish-forgive (SPF) policy [9]–[11]: the SPF policies are dynamic policies that have two phases. When the users have not received



Fig. 4. Illustration of different policies.

the distress signal, they transmit at optimal *stationary* power levels. When they receive a distress signal that indicates deviation, they switch to the punishment phase, in which all the users transmit at the Nash equilibrium power levels. In the energy efficiency formulation, the optimal stationary power levels are the Nash equilibrium power levels. Hence, the adapted SPF policy is essentially the same as the optimal stationary policy.

• The adapted nonstationary punish-forgive (NPF) policy: the punish-forgive policy in [12] is different from those in [9]–[11], in that *nonstationary* power levels are used when the users have not received the distress signal. In the simulation, we adapt the NPF policy in [12] such that the users transmit in the same way as in the proposed policy when they have not received the distress signal.

Since the SPF policy is the same as the optimal stationary policy, we simply refer to the NPF policy as the PF policy.

Fig. 4 illustrates the differences among stationary, PF, and the proposed policies in a simple case of two users, whose minimum throughput requirements are 1 bits/s/Hz and 2 bits/s/Hz, respectively. In stationary policies, users transmit simultaneously with fixed power levels (0.5 W and 0.9 W), which are higher than those (0.15 W and 0.75 W) in the proposed policy, because users need to overcome multi-user interference to achieve the minimum throughput requirements. In addition, users transmit all the time in stationary polices, which results in even higher average energy consumption.

The key difference between the proposed policy and the PF policy lies in time slot 5, after a distress signal is sent at t = 4. In the PF policy, users transmit together at the same high power levels as in the stationary policy at t = 5. In the proposed policy, user 2, the user who transmitted at t = 4, transmits again at t = 5. In summary, the punishment in the PF policy is the multi-user interference, which increases the energy consumptions of both users, while the punishment in the proposed policy is the delay in transmission, which keeps the energy consumptions low. This advantage of the proposed policy in terms of energy efficiency is also illustrated in Fig. 4.



Fig. 5. Comparisons of energy efficiency under different numbers of users.



Fig. 6. Comparisons of energy efficiency under different minimum throughput requirements.

In Fig. 5 and Fig. 6, we compare the energy efficiency of stationary, PF, and the proposed policies under different numbers of users and different minimum throughput requirements, respectively. The minimum throughput requirements are the same for all the users. The proposed policy significantly improves the spectrum and energy efficiency of existing policies in most scenarios. In particular, the proposed policy achieves an energy saving of up to 80%, when the number of users is large (when N > 7 in Fig. 5) and when the minimum throughput requirement is large (when  $R_i \approx 1.5$  bits/s/Hz in Fig. 6). Moreover, the proposed policy remains feasible even when the other policies are infeasible (i.e. when they fail to satisfy the minimum throughput requirements).

#### VI. CONCLUSION

We proposed deviation-proof TDMA spectrum sharing policies, which achieve high spectrum efficiency that is not achievable by existing policies, and are more energy efficient than existing policies under same minimum throughput requirements. It achieves high efficiency even when users have erroneous binary feedback of the interference temperature.

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