Strategic Networks: Information Dissemination and Link Formation Among Self-Interested Agents

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***Abstract*—This paper presents the first study of the endogenous formation of networks by strategic, self-interested agents who benefit from producing and disseminating information. This work departs from previous works on network formation (especially in the economics literature) which assume that agents benefit only by acquiring information produced by other agents. The strategic production and dissemination of information have striking consequences. We show first that the network structure that emerges (in equilibrium) typically displays a core-periphery structure, with the few agents at the core playing the role of connectors, creating and maintaining links to the agents at the periphery. We then determine conditions under which the networks that emerge are minimally connected and have short network diameters (properties that are important for efficiency). Finally, we show that the number of agents who produce information and the total amount of information produced in the network grow at the same rate as the agent population; this is in stark contrast to the "law of the few" that had been established in previous works which do not consider information dissemination.**

***Index Terms*—Information dissemination, network formation, self-interested agents.**

I. Introduction

THE TRADITIONAL analysis of communication networks assumes that the topology of the network is fixed exogenously or determined by the central designer and that the actions of users are obedient to the wishes of the designer. (It is typically assumed that the objective of the designer is to maximize social welfare [1] [2], but other objectives might be considered as well.) However, these assumptions do not apply at all to social networks such as Facebook [3] and Twitter [4], expert networks such as Amazon Mechanical Turk [5], vehicular networks [6], social mobile networks [7], peer-to-peer overlay routing systems [8], etc. To the contrary, the topologies of such networks are determined endogenously by the actions of self-interested and strategic users (which leads us to use the term "strategic networks"). Aspects of strategic networks that are of particular interests include the topology that emerges, the efficiency/inefficiency of behavior and especially the protocols that the designer might implement to promote social welfare even in the face of self-interested behavior by users.

A central aspect of the strategic networks we study here is that links in the network can be created and maintained by individual users. Creating and maintaining links are costly, so will only be carried out if they provide sufficient benefits for these individual users. In some earlier work, especially in the economics literature, the benefits of links are that they permit the acquisition of information (files in P2P networks, news on social networks, traffic/road conditions in vehicular networks, etc.); our point of departure in this paper is that we emphasize that links also permit the dissemination of information (advertising/marketing in social networks [9] [10] [18], routing traffic in overlay routing systems [11], etc.). The central message of this paper is that the emphasis on the dissemination of information leads to very different conclusions about the structure of networks and the behavior of users in those networks.

To be precise, we consider the behavior of a group of self-interested and strategic agents/users who may create and maintain links to other users, and produce and disseminate information. We formalize the strategic interactions among agents as a non-cooperative game, the Information Dissemination Game (IDG). Because we emphasize information dissemination, we assume that link formation is unilateral: the decision to create a link from one agent to another is made unilaterally by the first agent and the cost of creating that link is borne entirely by the agent who creates it; this is a reasonable description of behavior and cost when the benefit of creating a link is disseminating information (advertising) rather than gathering information (see e.g. [12] [14] [16]). We consider a setting in which agents and information are both heterogeneous: agents differ in terms of their locations, access to devices, information and link production capabilities and costs. Agents are self-interested: each intends to maximize its own benefit from information dissemination net of the cost of the links it forms. Our notion of solution in the Information Dissemination Game is a non-cooperative equilibrium.

We prove first that the typical network that emerges from the self-interested behavior of agents displays a core-periphery structure, with a smaller number of agents at the core (center) of the network and a larger number of agents at the periphery (edges) of the network. Agents in the core create many links and communicate with many other agents; agents in the periphery create few (or no) links and communicate mostly (or entirely) with agents in the core. We go on to show that the typical networks that emerge are minimally connected and have short network diameters, which are independent of the size of the network. When agents' strategic behavior incorporates both link formation and information production (with the objective of maximizing information dissemination), we show that the number of agents who produce information and the total amount of information produced grow with the size of the network; this is in sharp contrast with the "law of

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the few" which has been demonstrated in [14] with settings where the purpose of forming links is the acquisition of information.

Our analysis is important for a number of reasons. At the theoretical level: small diameters tend to make information dissemination efficient and minimal connectivity tends to minimize the total cost of constructing the network. At the empirical level: they are consistent with the findings of numerous empirical investigations. More generally, our analysis provides guidance and tools for network designers to create protocols providing incentives for agents to take actions that are consistent with self-interest and still promote social welfare.

The remainder of this paper is organized as follows. Section II describes our basic model of the IDG. Section III characterizes the non-cooperative equilibria that emerge in the basic model. Section IV analyzes the IDG with strategic information production. Section V discusses the related literature and Section VI concludes.

II. System Model

A. Settings

In this section, we propose a basic model to formulate the IDG, in order to capture the fundamental trade-offs between agents' benefit and cost from strategic information dissemination. Although simple, our formulation already provides qualitative insights on how the incentives of self-interested agents impact the network structure, and can be applied to numerous network applications (with slight modifications).

Let $N = \{1, 2, \ldots, n\}$ be the set of agents in the system with $n \geqslant 3$ and let i and j denote typical agents. Each agent i possesses some information in the amount $x_i \in \mathbb{R}^+$, which it finds in its own benefit to disseminate to other agents. We consider a non-cooperative game where each agent strategically determines whether to create links with other agents in order to disseminate its information. As in e.g. [14] [21], links are created by the unilateral actions of an agent who bears the entire cost [1]. Thus, the mutual consent of two agents is not required in order to create a link between them. The link formation strategy adopted by an agent i is denoted by a tuple $\mathbf{g}_i = (g_{ij})_{j\in\{1,\ldots,n\}/\{i\}} \in \{0,1\}^{n-1}$; $g_{ij} = 1$ if agent i forms a link with agent j and $g_{ij} = 0$ otherwise. The creation of a link incurs a cost to the creator and hence, the decision to form a link involves trading-off the benefit received from disseminating information using this link and the incurred cost. A strategy profile in the information dissemination game is defined as $\mathbf{g} \triangleq (\mathbf{g}_i)_{i=1}^n \in \mathbf{G}$, where $\mathbf{G}$ is a finite space.

The information flow across a link is assumed to be undirected. That is, given a link between any two agents, the information can be transmitted in both directions (i.e. from the creator to the recipient and vice versa) across this link. We thus define the *topology* of the network as $E_\mathbf{g} = \{(i,j) \in N \times N | i \neq j \ and \ \max\{g_{ij}, g_{ji}\} = 1\}$. In the rest of this paper, we will use the terms "topology" and "network" interchangeably. Given a topology $E_\mathbf{g}$, a path between two agents i and j is a sequence $path_{ij} = \{(i,j_1), (j_1,j_2), \ldots, (j_m,j)\}$ for some $m \geqslant 0$ such that $path_{ij} \subseteq E_\mathbf{g}$. Agent i can *reach* an agent j in a topology $E_\mathbf{g}$, denoted $i \to j$, if and only if there is at least one path from agent i to j in $E_\mathbf{g}$, otherwise i cannot reach agent j, denoted $i \nrightarrow j$. We assume that an agent i can disseminate its information to every agent j whom it can reach. Given this, the utility of an agent i in the IDG can be expressed as:

$$u_i(\mathbf{g}) = f\left(x_i |N_i(E_\mathbf{g})|\right) - \sum\nolimits_{j\in N_i(\mathbf{g})} k_{ij}. \qquad (1)$$

Here $N_i(E_\mathbf{g}) \triangleq \{j | i \to j\}$ is the set of agents whom agent i can reach, and $N_i(\mathbf{g}) \triangleq \{j | g_{ij} = 1\}$ is the set of agents with whom agent i forms links. $f\left(x_i |N_i(E_\mathbf{g})|\right)$ thus represents the total benefit that agent i receives from information dissemination, which depends on the amount of information it disseminates, i.e. x_i, as well as the total number of agents it can reach, i.e. $|N_i(E_\mathbf{g})|$. We assume that $f(\cdot)$ is twice continuously differentiable, increasing and concave with $f(0) = 0$. Hence, an agent's benefit increases, while the marginal benefit decreases, with x_i and $|N_i(E_\mathbf{g})|$. $\sum_{j\in N_i(\mathbf{g})} k_{ij}$ represents the total link formation cost of agent i, where $k_{ij} \in \mathbb{R}^+$ denotes the cost for agent i to form a link with agent j.

We assume that an agent cannot benefit from disseminating duplicated copies of its information to any other agent. That is, when there are multiple paths from agent i to agent j and multiple copies of agent i's information arrive at agent j, agent i receives a fixed benefit regardless of the number of copies that agent j receives. We assume that each agent benefits only from disseminating its own information, and forwarding the information that is received from other agents does not bring it any benefit.

B. Equilibrium and social welfare

We consider pure (not mixed) link formation strategies. Each agent maximizes its own utility given the strategies of others. A Nash equilibrium (NE) is defined as a strategy profile $\mathbf{g}^*$ such that the strategy of each agent i is a best response to the strategies of others:

$$u_i(\mathbf{g}_i^*, \mathbf{g}_{-i}^*) \geqslant u_i(\mathbf{g}_i, \mathbf{g}_{-i}^*), \forall \mathbf{g}_i \in \{0,1\}^{n-1}, \ \forall i \in N. \qquad (2)$$

Here $\mathbf{g}_{-i}$ represents the strategies of all agents other than agent i. The set of NE is defined as $\mathbf{G}^* = \{\mathbf{g}^* | \mathbf{g}^* \ satisfies \ (2)\}$. A strict NE is an NE such that the strategy of each agent i is a strict best response to the strategies of others (with the inequality in (2) being strict whenever $\mathbf{g}_i^* \neq \mathbf{g}_i$). It is shown in the online appendix [23] that a network will always converge to a strict NE in a dynamic link formation process. Therefore, a strict NE characterizes a steady state in the dynamic link formation process. Note that strict NE are NE and thus, the results below on NE also apply to strict NE.

The social welfare of the IDG is defined to be the sum of agents' individual utilities. For a strategy profile $\mathbf{g}$, the social welfare is given by $U(\mathbf{g}) \triangleq \sum_{i\in N} u_i(\mathbf{g})$. A strategy profile

[1]The precise formulations of link formation and information flow among participating agents in an IDG depend on details of the considered application. Due to the infeasibility of enumerating all possible models, we use a stylized model in this work as an example to formulate the IDG. Our current formulation has the great merit of being simple to work with and can be applied to most existing applications to date with slight modifications, e.g. the telephone networks [33] and the Voice over IP applications such as Skype [34].

$\mathbf{g}^{\#}$ is called socially optimal if it achieves the social optimum, denoted by $U^{\#}$, i.e.

$$U^{\#} \triangleq U(\mathbf{g}^{\#}) \geqslant U(\mathbf{g}),\ \forall \mathbf{g} \in \mathbf{G}. \quad (3)$$

III. Equilibrium and Efficiency Analysis of the IDG

This section studies the IDG described in Section II. First, we analyze the equilibrium link formation strategies of individual self-interested agents. Next, we explicitly compare the equilibrium social welfare of the IDG to the social optimum. The results provide important insights on the efficiency loss occurred due to the self-interested behavior of the agents in the IDG as compared to the case when the agents obediently follow the link formation actions dictated by some central designer.

A. Equilibrium analysis

Given a strategy profile $\mathbf{g}$, a *component* C is a set of agents such that $i \to j,\ \forall i, j \in C$ and $i \nrightarrow j',\ \forall i \in C$ and $\forall j' \notin C$. Hence, each component defines a connected sub-network in a network $E_{\mathbf{g}}$: any two agents in this component can mutually reach each other, whereas no agent in the component can reach any other agent outside the component. An agent who is not connected with any other agents in the network (i.e. an isolated agent) forms a component by itself, called a *singleton* component; a component that is not singleton is called a *non-singleton* component. A component C is called *minimal* if and only if there is only one path in $E_{\mathbf{g}}$ from any agent $i \in C$ to any other agent $j \in C$. The *shortest path* from agent i to j is the path that contains the minimum number of links. The *distance* $d_{\mathbf{g}}(i, j)$ between i and j is the number of links on a shortest path between them. By convention, $d_{\mathbf{g}}(i, j) \triangleq \infty$ when $i \nrightarrow j$. The *diameter* of a component C is defined as the largest distance between any two agents in it, which is denoted as $D_C \triangleq \max_{i,j \in C} d_{\mathbf{g}}(i, j)$. The diameter of a singleton component is defined to be 0. The diameter of the network is defined to be the largest diameter of all components it contains.

It should be noted that the strategy space for each agent in the IDG is compact and convex. Meanwhile, an agent's utility is quasi-concave over its link formation strategy. Hence, it has been shown in [22] that pure NE always exists in the IDG. We first derive some basic properties of the equilibria in the IDG. Although simple, these properties are important for characterizing the emerging equilibria later.

Proposition 1: Under an NE $\mathbf{g}^*$ of the IDG, each component is minimal.

Proof: Suppose that there is a component C such that there are two agents i and j who are connected by two paths $path_{ij}$, $path'_{ij} \subseteq E_{\mathbf{g}^*}$ with $path_{ij}/path'_{ij} \neq \phi$. Here $path_{ij}/path'_{ij}$ represents the relative complement of $path'_{ij}$ in $path_{ij}$ and contains all elements that belong to $path_{ij}$ but do not belong to $path'_{ij}$. Then there are always two agent i' and j' in C who satisfy: (1) $g^*_{i'j'} = 1$; and (2) there is a $path_{i'j'} \subseteq E_{\mathbf{g}^*}$ such that $path_{i'j'} \neq ((i', j'))$. Therefore, by setting $g_{i'j'} = 0$, agent i' always receives a strictly higher utility compared to what it can receive in $\mathbf{g}^*$, which contradicts the fact that $\mathbf{g}^*$ is an NE. Hence, this proposition follows. ■

Proposition 1 shows that in an equilibrium of the IDG, each connected sub-network (component) is minimal with no cycles in it. As we will show in Section III.B, the social optimum in the IDG is always achieved by networks consisting of minimal components and hence, the equilibria in the IDG can frequently achieve the social optimum (i.e. being efficient).

Proposition 1 characterizes individual components in the equilibrium network. However, it does not characterize the connectedness of the network, i.e. whether the network will be composed of a unique component where all agents are connecting with (and can disseminate information to) each other or several components that are isolated from each other. The following proposition provides a sufficient condition under which the network is connected at equilibrium.

Proposition 2: The network in each NE is always minimally connected if there is an agent i such that $f(x_i(|N| - 1)) - f(x_i(|N| - 2)) > \max_{j \in N}\{k_{ij}\}$.

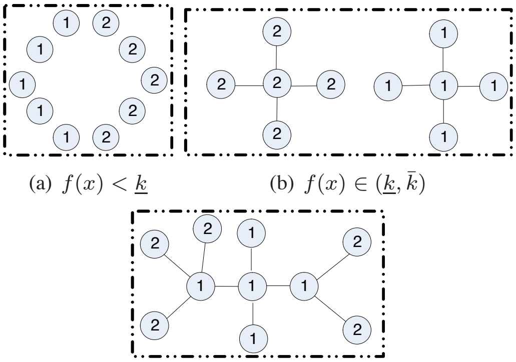
Proof: Suppose there is a NE $\mathbf{g}^*$ which contains more than one component. We consider two components C_1 and C_2. Suppose agent i is in C_1, then it can always increase its utility by forming a link to any other agent in C_2 since $f(x_i(|N| - 1)) - f(x_i(|N| - 2)) > \max\{k_{ij}\}_{i,j \in N}$, which contradicts the fact that $\mathbf{g}^*$ is an NE. This proposition thus follows. ■

Proposition 2 shows that the network will be connected at equilibrium when the benefit from information dissemination is sufficiently large (i.e. x_i is sufficiently large) with respect to the link formation cost. The properties of the network topology at equilibrium (i.e. the shape and diameter of the network) depends on the specific values of $\{x_i\}_{i \in N}$ and $\{k_{ij}\}_{i,j \in N}$. In the rest of this section, we analyze two exemplary networks with particular structures in order to obtain further insights on the equilibrium topology.

1) Networks with recipient-dependent costs: In the first example, we consider the network where the cost of forming a link is exclusively recipient specific. In particular, we have $k_{ij} = k_j, \forall i \in N/\{j\}$. This can capture the practical networks in which the link formation cost only depends on the type of the recipient and there are some agents to which it is easier to connect with than other agents (i.e. with smaller costs to form links with). For example, in networks where the link formation cost represents the subscription fee that the creator sends to the recipient, each agent charges the same price to any agent who wants to form a link with it. In the following theorem, we show that if the link formation cost in the network is not arbitrary but only takes values from a finite set $\{k^1, ..., k^L\}$, i.e. there are L different types of link formation costs and $k_i \in \{k^1, ..., k^L\}, \forall i \in N$ (e.g. the subscription fee is quantized to several discrete levels depending on the agents' types but not takes arbitrary values), then the diameter of the network at each strict equilibrium should be no more than $2L + 2$.

Theorem 1: Suppose that there are L different types of link formation costs, e.g. $\{k^1, ..., k^L\}$, such that $k_i \in \{k^1, ..., k^L\}, \forall i \in N$, then under any strict NE $\mathbf{g}^*$ of the IDG, the diameter of the network is at most $2L + 2$.

Proof: Consider a strict NE $\mathbf{g}^*$ and a non-singleton component in it, there is at least one agent i in the component such that $g^*_{ij} = 1$ for some j. Consider a path $path_{ij_d} = ((i, j), (j, j_1), (j_1, j_2), \ldots, (j_{d-1}, j_d))$. Since $g^*_{ij} = 1$, we



(a) $f(x) < \underline{k}$ (b) $f(x) \in (\underline{k}, \bar{k})$

(c) $f(x) > \bar{k}$

Fig. 1. The exemplary Nash equilibria in the network with groups

should have $\{k_j < k_{j_1}, k_j < k_{j_2}, ..., k_j < k_{j_d}\}$. Now there are two cases: $g^*_{j_1 j} = 1$ or $g^*_{j j_1} = 1$.

In the first case, suppose $g^*_{j_2 j_1} = 1$, then we have $k_{j_1} < k_j$, which leads to a contradiction to the fact that $k_j < k_{j_1}$. Hence, we have $g^*_{j_1 j_2} = 1$, which gives $k_{j_2} < k_{j_3}$. Using the same arguments, we have $g^*_{j_l j_{l+1}} = 1$ and $k_{j_{l+1}} < k_{j_{l+2}}$, for all $l \in \{1, ..., d-2\}$. Therefore, along the path $path_{jj_d} = ((j, j_1), (j_1, j_2), \ldots, (j_{d-1}, j_d))$, there are at least d different link formation costs. We thus have $d \leqslant L$ and the length of path $path_{ij_d}$ is smaller than $L + 1$.

Now consider the second case where $g^*_{jj_1} = 1$. Then we have $k_{j_1} < k_{j_2}$. Using arguments similar to that for the first case, we have that along $path_{jj_d} = ((j, j_1), \ldots, (j_{d-1}, j_d))$, there are at least d different link formation costs and hence the length of path $path_{ij_d}$ is still no more than $L + 1$.

Consider the longest path in this component, which is denoted as $((b_0, b_1), (b_1, b_2), \ldots, (b_{T-1}, b_T))$. Suppose $T > 2(L + 1)$ and consider the agents $b_{\lfloor T/2 \rfloor}$ and $b_{\lfloor T/2 \rfloor + 1}$. If $g^*_{b_{\lfloor T/2 \rfloor} b_{\lfloor T/2 \rfloor + 1}} = 1$, then the path $((b_{\lfloor T/2 \rfloor}, b_{\lfloor T/2 \rfloor + 1}), \ldots, (b_{T-1}, b_T))$ has a length longer than $(L + 1)$. If $g^*_{b_{\lfloor T/2 \rfloor + 1} b_{\lfloor T/2 \rfloor}} = 1$, then the path $((b_{\lfloor T/2 \rfloor + 1}, b_{\lfloor T/2 \rfloor}), \ldots, (b_1, b_0))$ has a length longer than $(L+1)$. Both scenarios contradict our argument above. Therefore, we can conclude that $T \leqslant 2(L + 1)$ always holds and this theorem follows. ■

The link formation cost thus plays an important role in shaping the equilibrium network in the IDG. As shown in Theorem 1, if there are only a finite number of different link formation costs in the network, then the size of each component (a connected sub-network) cannot be arbitrarily large but is upper-bounded by some constant value, which is independent of the population size but proportional to the number of different link formation costs. Based on Proposition 1 and Theorem 1, the "minimally connected" and "short diameter" properties of the equilibria in strategic networks are thus proven.

As a special case of Theorem 1, we prove in the following corollary that when the link formation cost is the same for all agents, each component in a strict NE forms a star topology, regardless of the values $\{x_i\}_{i \in N}$.

Corollary 1: If $k_{ij} = k, \forall i, j \in N$, then under a strict NE $\mathbf{g}^*$, each non-singleton component forms a star topology.

Proof: See Appendix. ■

Hence when the link formation cost is the same for all agents, each component at equilibrium preserves the "core-periphery" property with one single agent staying at the center of it and playing the role of the "connector" who connects (maintains links) with all other agents to support their information dissemination.

2) Networks with groups: We discuss a network where agents are divided into groups and agents within the same group have the same type. The cost of forming links within a group (i.e. between agents of the same type) is lower than the cost of forming links across groups (i.e. between agents of different types). Examples of strategic networks where such groups exist are users of close social relationships or close interests in a social network [24], devices or processing nodes located in the same area [25], etc.

Formally, we consider that all agents are divided into Z different groups $N_1, \ldots, N_Z$ with $|N_z| \geqslant 2$ for all $1 \leqslant z \leqslant Z$, such that $N = \cup\{N_z\}_{z=1}^{Z}$ and $N_z \cap N_{z'} = \phi$ for any $1 \leqslant z < z' \leqslant Z$. For two agents from the same group, the cost of forming a link between them is $\underline{k}$, while for two agents from different groups, the cost of forming a link between them is $\bar{k} > \underline{k}$. Here we assume that $x_i = x, \forall i \in N$ to make our analysis tractable. The following theorem characterizes the strict equilibria with the presence of groups and proves that each non-empty strict equilibrium preserves the "core-periphery" property.

Theorem 2: In the presence of groups, the Nash equilibria can be characterized as follows:

(i) When $f(x) < \underline{k}$, the unique strict NE $\mathbf{g}^*$ satisfies $g^*_{ij} = 0, \forall i, j$;

(ii) When $f(x) \in (\underline{k}, \bar{k})$, the unique strict NE consists of Z components, where each component contains only agents from the same group and the topology of each component is a star;

(iii) When $f(x) > \bar{k}$, in each strict NE $\mathbf{g}^*$, there is a group N_z and an agent $i \in N_z$ such that $g^*_{ij} = 1, \forall j \in N_z/\{i\}$. Also for each agent $j' \notin N_z$, there is an agent $j \in N_z$ such that $g^*_{jj'} = 1$.

Proof: See Appendix. ■

Several examples of the equilibrium topologies discussed in Theorem 2 are illustrated in Figure 1 in a network of $n = 10$ agents who are divided into 2 groups. The number on each node represents the group to which each agent belongs. Theorem 2 provides several important insights. First, in a strict equilibrium, agents from the same group always belong to the same component (i.e. are connected with each other). Second, each non-singleton component exhibits the "core-periphery" property. The agents that form the core are from the same group, while agents from other groups access the network via links maintained by the core. This analytical finding is reflected in numerous real world examples. For instance, in a large-scale overlay routing network [11], it is usually the case that a group of nodes who can inter-connect at a lower cost form the backbone of the network, while all other nodes connect to the network via this backbone. Third, in each component, there is always a central agent and all paths within this component initiate from this agent. Also, the distance from the central agent to any periphery agent is no more than 2.

Hence, the diameter of the network is no more than 4, which is also independent of the population size in the network.

B. Equilibrium efficiency of the IDG

In this section, we analyze the efficiency (social welfare) of the IDG. Because there are multiple equilibria, we use two metrics to measure the equilibrium efficiency: (i) the Price of Stability (PoS) is defined as the ratio between the social optimum and the highest social welfare that is achieved at equilibrium and measures the efficiency of the "best" equilibrium in the IDG, i.e. $PoS = U^{\#}/\max_{\mathbf{g}^* \in \mathbf{G}^*} U(\mathbf{g}^*)$; (ii) the Price of Anarchy (PoA) is defined as the ratio between the social optimum and the lowest social welfare that is achieved at equilibrium and measures the efficiency of the "worst" equilibrium in the IDG, i.e. $PoA = U^{\#}/\min_{\mathbf{g}^* \in \mathbf{G}^*} U(\mathbf{g}^*)$. In the rest of this section, we quantify the PoS and PoA in the IDG and show with multiple examples that the equilibria in the IDG frequently achieve the social optimum.

We first characterize the socially optimal strategy profiles. As with NE, it can be proven that the minimal property still holds in the network under any socially optimal strategy profile.

Proposition 3: In the IDG, each component under a socially optimal profile is always minimally connected.

Proof: This can be proven using the same idea as Proposition 1. ■

With the minimal property, we prove in the next theorem that when the link formation cost is recipient-dependent, i.e. $k_{ij} = k_j, \forall i \in N/\{j\}$, there is always an NE that can achieve the social optimum when the link formation cost is sufficiently small: the PoS of the IDG is always 1.

Theorem 3: If $k_{ij} = k_j, \forall i \in N/\{j\}$ and $\min_{i \in N} f(x_i) \geqslant \min_{i \in N} k_i$, the PoS of the IDG is always 1.

Proof: Let $k_{i_0} = \min_{i \in N} k_i$ and consider a periphery-sponsored star $\mathbf{g}$ with $g_{ji_0} = 1, \forall j \in N/\{i_0\}$ and $g_{jj'} = 0, \forall j, j' \in N/\{i_0\}$. It is obvious that $\mathbf{g}$ is both social optimal and an NE. Hence, this theorem is proven. ■

However, the PoA of the IDG is not necessarily 1 in this case, i.e. there are some NE that incur positive efficiency loss. This is quantified in the next proposition.

Proposition 4: If $k_{ij} = k_j, \forall i \in N/\{j\}$ and $\min_{i \in N} f(x_i) \geqslant \min_{i \in N} k_i$, the PoA of the IDG is upper-bounded by $\max_{i,j \in N} k_i/k_j$.

Proof: We consider an arbitrary NE $\mathbf{g}^*$. Let $k_{i_0} = \min_{i \in N} k_i$ and consider a component C_1 that contains i_0. Now consider another component C_2. If C_2 is a singleton component which contains a unique agent j, then j can always increase its utility by forming a link with i_0. If C_2 is a non-singleton component, then there is always an agent $j' \in C_2$ such that $g^*_{j'l} = 1, \exists l \in C_2$ with $\max\{g^*_{ll'}, g^*_{l'l}\} = 0, \forall l' \in N/\{l\}$. In this case, j' can also increase its utility by switching its link from l to i_0. Therefore, it can be concluded that $\mathbf{g}^*$ forms a connected network. From Proposition 3, we know that the network formed by $\mathbf{g}^*$ is also minimal which contains $|N|-1$ links and hence, $U^{\#}/U(\mathbf{g}^*) \leqslant \max_{i,j \in N} k_i/k_j$. Since this conclusion applies to any NE $\mathbf{g}^*$, Proposition 4 thus follows. ■

Proposition 4 shows that the upper bound of PoA depends on how the link formation cost varies among agents. In the special case where the link formation cost is the same for all agents, i.e. $k_{ij} = k, \forall i, j \in N$, each NE $\mathbf{g}^*$ achieves the social optimum: PoA is 1.

Corollary 2: If $k_{ij} = k, \forall i, j \in N$ and $\min_{i \in N} f(x_i) \geqslant k$, every NE $\mathbf{g}^*$ achieves the social optimum and the PoA of the IDG is 1.

Proof: This can be proven straightforwardly using Proposition 4. ■

IV. IDG WITH INFORMATION PRODUCTION

In the IDG discussed so far, we assumed that the information possessed by the each agent is exogenously determined and fixed during the game. Nevertheless in practical networks, it is usually the case that each agent i can proactively determine the amount of information that it wants to disseminate throughout the network, i.e. the value of x_i. In this section, we consider such IDG with strategic information production from individual agents.

In the IDG with information production, the strategy of an agent i can be represented as $(x_i, \mathbf{g}_i)$, and the agent jointly maximizes its decisions on the information production and link formation in order to maximize its overall utility from information dissemination. A strategy profile of the IDG with information production is written as $\mathbf{s} = (\mathbf{x}, \mathbf{g})$, where $\mathbf{x} = (x_i)_{i=1}^n$ denotes the information production decisions of all agents. Given a strategy profile, the utility of agent i is expressed as:

$$u_i(\mathbf{x}, \mathbf{g}) = f(x_i|N_i(E_{\mathbf{g}})|) - cx_i - \sum\nolimits_{j \in N_i(\mathbf{g})} k_{ij}. \tag{4}$$

Here cx_i represents the cost of producing an amount x_i of information, where c is the unit production cost.

A Nash equilibrium of the IDG with information production is a strategy profile $\mathbf{s}^* = (\mathbf{x}^*, \mathbf{g}^*)$ such that

$$u_i(\mathbf{s}_i^*, \mathbf{s}_{-i}^*) \geqslant u_i(\mathbf{s}_i, \mathbf{s}_{-i}^*), \forall \mathbf{s}_i \in \mathbb{R}^+ \times \{0,1\}^{n-1}, \forall i \in N. \tag{5}$$

When information production is a strategic choice, central questions are how many agents will produce information at equilibrium and how the total amount of information produced in the network changes with the population size. The seminal work in [20] analyzes the network formation game with information production where agents benefit from acquiring and consuming the information produced by other agents. It predicts the occurrence of the "law of the few" at equilibrium. That is, in each equilibrium there are only a small number of agents in the network who produce a positive amount of information (i.e. being information producers). As the population size grows to infinity, the fraction of information producers in the agent population goes to 0. Based on the "law of the few", [20] also predicts that the total amount of information that is produced (by all agents) in the network remains constant at equilibrium, which is independent of the population size.

The reason for the emergence of the "law of the few" in [20] is that each agent benefits solely from information consumption and hence its utility is not affected by how many agents it connects with and with whom it is connected so long as the total amount of its acquired information remains constant. Also, when there is a sufficient amount of information that has been acquired by an agent, it will stop producing information personally. Therefore, the information production at equilibrium is always dominated by a small fraction of information producers who produce all the information to be consumed by all agents.

In the rest of this section, we study the asymptotic information production behavior of agents in the IDG when the population size grows. It should be noted that in the IDG, the benefit of an individual agent is jointly determined by the amount of its own production, i.e. x_i, as well as the number of agents it connects with, i.e. $|N_i(E_{\mathbf{g}})|$, whereas the information produced by other agents has no influence on its information production decision. This makes the resulting asymptotic information production behavior at equilibrium exhibits significant differences to that in [20]. Importantly, we prove in the following theorem that the "law of the few" does not hold in the IDG. To illustrate this theorem, we define several auxiliary variables: S_N^* represents the set of equilibrium strategy profiles when the population is N, $I_N(\mathbf{s}) = \{i | i \in N \; and \; x_i^* > 0\}$ represents the set of information producers under the strategy profile $\mathbf{s}$, and $\bar{x}$ represents the solution of the equation $f'(x) = c$.

Theorem 4: In the IDG with information dissemination, when $\max\limits_{i,j \in N} k_{ij} < c\bar{x}$, (i) $\inf\limits_{\mathbf{s}^* \in S_N^*} \{I_N(\mathbf{s}^*)\}/|N| = 1$; (ii) the total amount of information produced in the network at equilibrium, i.e. $\sum\limits_{i \in N} x_i^*$, grows to infinity when the population size $|N| \to \infty$, i.e. $\lim\limits_{|N| \to \infty} \inf\limits_{\mathbf{s}^* \in S_N^*} \{\sum\limits_{i \in N} x_i^*\} \to \infty$.

Proof: To prove Statement (i), it is sufficient to see that each agent i will connect to at least one other agent in any NE and thus have $x_i^* > 0$ when $\max_{ij} k_{ij} < c\bar{x}$, which is independent of the population size $|N|$.

By taking the first order derivative of (4) over x_i, we have that $|N_i(E_{\mathbf{g}^*})| f'(|N_i(E_{\mathbf{g}^*})| x_i^*) = c$ and thus $|N_i(E_{\mathbf{g}^*})| x_i^* \geqslant \bar{x}$. Also, for any two agents i, j within the same component, we have $x_i^* = x_j^*$. Therefore for any component C, the total amount of information produced by agents within this component at equilibrium is $\sum\limits_{i \in C} x_i^*$, which satisfies $f'(\sum\limits_{i \in C} x_i^*) = c/|C|$ and $\sum\limits_{i \in C} x_i^* > \bar{x}$. Suppose that there is a sufficiently large constant W such that for any N we have $\inf\limits_{\mathbf{s}^* \in S_N^*} \{\sum\limits_{i \in N} x_i^*\} < W$. Select $\mathbf{s}_N^* = \arg \inf\limits_{s^* \in S_N^*} \{\sum\limits_{i \in N} x_i^*\}$. Due to the concavity of $f(\cdot)$, we have that $f'(\sum\limits_{i \in C} x_i^*) = c/|C| \geqslant f'(W)$ for any component C under $\mathbf{s}_N^*$. Hence, $|C| \leqslant c/f'(W)$, and we have $\sum\limits_{i \in N} x_i^* \geqslant |N| f'(W) \bar{x}/c$. This shows that there is always a sufficiently large $|N|$ such that $|N| f'(W) \bar{x}/c > W$ which contradicts the assumption that $\inf\limits_{\mathbf{s}^* \in S_N^*} \{\sum\limits_{i \in N} x_i^*\} < W$ for any N. Therefore, we have a contradiction and Statement (ii) follows. ■

Theorem 4 shows that when agents benefit from information dissemination instead of information consumption, both the number of information producers and the total amount of information produced in the network grow at least at a linear order of the population size at equilibrium. Therefore, the information production at equilibrium is no longer dominated by a small number of information producers and the "law of the few" predicted in [20] no longer holds.

V. Related Works

There is a broad literature studying the information dissemination in social networks [26]- [32], which focuses on explaining how the information (e.g. epidemics, job openings, etc.) is propagated in social networks and how the agents' actions (e.g. becoming "infected" or not, buying products or not, etc.) are influenced by the disseminated information. However, the analysis in these works is based on the assumption that the underlying topologies of the social networks are exogenously determined and none of them explicitly considers the strategic link formation of self-interested agents.

There are also numerous works in network science investigating the evolution of social and information networks whose topologies are formed endogenously by agents' self-interested actions [17]- [19]. These works focus on empirical measurements of existing social networks and they fail to provide theoretical foundations which can explain and emulate the relationship between agents' incentives to form links based on their own self-interest and the emerging network topologies.

Theoretical study of network formation in social and economic networks has been conducted by micro-economists as well as computer scientists (see e.g. [12]- [16] [20]), who analyze how the agents' self-interest in acquiring information from other agents leads to strategic link formation and particular network topologies. However, these works focus on the scenario in which agents benefit solely from consuming acquired information. In the rest of this section, we discuss the relationship and differences between our proposed information dissemination game and the existing models on network formation games with information acquisition.

1) Differences in agents' utilities and incentives: First, note that analysis of the IDG and that of the network formation game with information acquisition (which is referred to as the Information Acquisition Game (IAG) below) exhibit significant differences in both the agents' utilities and the problem formulation. To illustrate the differences, we write the utility function in the IDG below as well as an exemplary utility function in the IAG. To make the key differences even clearer, we assume that $k_{ij} = k, \forall i, j \in N$ throughout the analysis:

$$u_i^{IDG}(\mathbf{g}) = f(x_i |N_i(E_{\mathbf{g}})|) - k|N_i(\mathbf{g})|, \tag{6}$$

$$u_i^{IAG}(\mathbf{g}) = f(x_i + \sum\nolimits_{j \in N_i(E_{\mathbf{g}})} x_j) - k|N_i(\mathbf{g})|. \tag{7}$$

From these two utility functions, it can be observed that in the IAG, the benefit of an individual agent is determined by the total amount of information which it acquires, i.e. $x_i + \sum_{j \in N_i(E_{\mathbf{g}})} x_j$, and agents lose the incentive to form links as long as they are able to acquire sufficient information from the existing links, regardless of with how many agents they are connected and from which agents was the information acquired (i.e. the variety of agents). In contrast, in the IDG, the benefit of an individual agent is jointly determined by the amount of its own information, i.e. x_i, as well as the number of agents with whom it is connected, i.e. $|N_i(E_{\mathbf{g}})|$, while the information possessed by other agents has no influence on its link formation decision. The number and variety of agents that each agent is connected with thus form the most important factor that shapes its incentives.

It is important to note that in the IDG and the IAG, the amount of an agent's own information have opposite impacts on its incentive to form links with others: in the IAG, the more information an agent possesses, i.e. the larger x_i is, the smaller incentive it has to form links with other agents; whereas in the IDG, an agent with a larger x_i has a larger incentive to form links with others. As a result, the existing models used for IAG are not suitable to analyze the trade-off between the benefits and costs of information dissemination and link formation as well as the mutual impact between agents' strategic link formation decisions in an IDG. In fact, the existing IAG models can be applied to analyze the IDG only if agents are homogeneous with $x_i = x, \forall i$, so that the total amount of an agent's acquired information is proportional to the number of agents it connects with.

2) Differences in agents' equilibrium link formation behavior: A simple example show that these differences lead to highly different link formation behaviors thereby resulting in significant differences of the equilibrium topologies as opposed to those in the IAG. This point is further illustrated below using a simple example. For a fair comparison, we assume in the example that $f(y) = y^{\lambda}$ with $\lambda \in (0, 1)$. Suppose that there are n agents in the network. There is one agent i possessing an amount x_i of information with $x_i > \ln k/\lambda$ while all the other agents possess no information. Then in the IAG, it is easy to show that agent i forms no link at equilibrium, i.e. $g^*_{ij} = 0, \forall j \neq i$, and each agent $j \in N/\{i\}$ forms exactly one link with agent i, i.e. $g^*_{ji} = 1, \forall j \neq i$ and $g^*_{jj'} = 0, \forall j \neq i, j' \neq i$. Hence, the unique equilibrium in the IAG is a periphery-sponsored star. Nevertheless in the IDG, agent i forms at least one link with some other agent at equilibrium, i.e. $g^*_{ij} = 1, \exists j \neq i$, and each agent $j \in N/\{i\}$ forms no link with any other agent, i.e. $g^*_{jj'} = 0, \forall j \neq i, j' \in N$. Meanwhile, the larger x_i is, the more links that agent i forms in the IDG. We can show that when $x_i > \frac{1}{\lambda} \ln \frac{k}{(N-1)^{\lambda} - (N-2)^{\lambda}}$, agent i forms links with all other agents in the network, with the unique equilibrium being a center-sponsored star.

This example provides two important insights: (1) agents' link formation behaviors at equilibrium exhibit significant differences when they are playing the IAG or the IDG, even if they possess the same amount of information and incur the same link formation cost; (2) although the resulting equilibrium topologies in the IAG and IDG may exhibit some similarity with respect to their shapes (e.g. both IAG and IDG have the star topology as the unique equilibrium when $x_i > \frac{1}{\lambda} \ln \frac{k}{(N-1)^{\lambda} - (N-2)^{\lambda}}$), they may have completely different underlying structures which lead to different properties in practice, e.g. the center-sponsored star formed in the IDG has all its links supported by its center node and is more vulnerable to single-node failures than the periphery-sponsored star formed in the IAG.

3) Differences in emerging equilibrium topologies: The differences between the IDG and the IAG become even more distinct in the scenario where agents self-produce information. For instance, the "law of the few", which is shown as a robust feature at equilibrium in the IAG [20], no longer holds in the IDG as shown by Theorem 4.

VI. Conclusion

In this work, we investigated the problem of information dissemination and link formation in strategic networks. We rigorously determined how the agents' desire to disseminate their own information throughout the network impacts their interactions and the emerging connectivity/topology among them. Our analysis proved several important properties of the strategic networks (arising from the agents' strategic link formation) at equilibria, such as "core-periphery", "minimally connected", "short diameter". These properties are important because they characterize the efficiency and robustness of the resulting equilibrium networks. We also studied the strategic information production by individual agents and its impact on the equilibria of the information dissemination games. Importantly, we showed that when agents benefit from information dissemination, the information production at equilibrium is no longer dominated by a small number of information producers and hence, the "law of the few" derived for traditional network formation games where agents benefit from information consumption no longer holds.

Appendix A
Proof of Corollary 1

This can be proved using the same idea as Theorem 1. We first prove the following claim.

Claim 1. Given a strict NE $\mathbf{g}^*$ and when $k_{ij} = k, \forall i, j \in N$, if $g^*_{ij} = 1$ for some $i, j \in N$, then $\max\{g^*_{jj'}, g^*_{j'j}\} = 0$ for any $j' \neq i$ and $j' \neq j$.

Proof of Claim 1: Suppose, in contrast, $g^*_{ij} = 1$ and $\max\{g^*_{jj'}, g^*_{j'j}\} = 1$ for some $j' \neq i$ and $j' \neq j$. By deleting its link with j and forming a new link with j', i receives the same utility as what it receives in $\mathbf{g}^*$, which contradicts the fact that $\mathbf{g}^*$ is an (strict) equilibrium and hence this claim follows. ■

In the next step, we show that for each non-singleton component always has a star topology in a strict NE.

Without loss of generality, we select two agents $i, j \in C$ where C is a component in $E_{\mathbf{g}^*}$, such that $g^*_{ij} = 1$. According to Claim 1, we have that $\max\{g^*_{jj'}, g^*_{j'j}\} = 0$ for any $j' \in C$ and $j' \notin \{i, j\}$. According to Proposition 1, we should also have $g^*_{ji} = 0$, since agent j can strictly increase its utility otherwise by removing the link it forms to agent i.

Now suppose that $g^*_{j'i} = 1$ for some $j' \in C$ and $j' \notin \{i, j\}$. It is obvious that agent j' can switch its link from agent i to agent j without decreasing its utility, which gives a contradiction. Therefore, we can conclude that $g^*_{ij} = 1, \ \forall j \in C$ and $j \neq i$. Meanwhile, $g^*_{jj'} = 0, \ \forall j, j' \in C$ and $j, j' \neq i$. In other words, C has a star topology where agent i stays in the center and forms links with all other agents who stay in the periphery, while all the other agents do not form links mutually. This corollary thus follows. ■

APPENDIX B
PROOF OF THEOREM 2

(i) When $f(x) < \underline{k}$, suppose that there is an equilibrium $\mathbf{g}^*$ which contains a non-singleton component C. Let i and j be two agents in C such that $g^*_{ij} = 1$, it is obvious that agent i can strictly increase its utility by setting $g_{ij} = 0$. Hence, there is a contradiction and this statement follows.

(ii) When $f(x) \in (\underline{k}, \bar{k})$, consider a component C and one of its periphery agents i such that $\max\{g^*_{ij}, g^*_{ji}\} = 1, \exists j$ and $\max\{g^*_{ij'}, g^*_{j'i}\} = 0, \forall j' \neq j$.

Suppose $g^*_{ij} = 1$: If $k_{ij} = \bar{k}$ and $|C| > 2$, agent i can always switch its link to some other agent $j' \in C$ without decreasing its utility, If $k_{ij} = \bar{k}$ and $|C| = 2$, agent i can always increase its utility by switching its link to some other agent $j' \notin C$. Both cases contradict the fact that $\mathbf{g}^*$ is a strict NE. Hence, we have $k_{ij} = \underline{k}$ and $g^*_{i'j} = 0, \forall i' \in C/\{i, j\}$ (otherwise i' can switch its link from j to i without decreasing its utility). Since i is a periphery agent, we have $g^*_{ji'} = 1, \exists i' \in C/\{i, j\}$. If $|C| = 3$, then $k_{ji'} = \underline{k}$ and agent i can switch its link from j to i' without decreasing its utility. Therefore, we have $|C| > 3$ and $\bar{g}^*_{i'i''} = 1, \exists i'' \in C/\{i, j, i'\}$. If $k_{ji'} = \bar{k}$, agent j can switch its link from i' to i'' without decreasing its utility, whereas if $k_{ji'} = \underline{k}$, agent i can switch its link from j to i' without decreasing its utility. Both cases contradict the fact that $\mathbf{g}^*$ is a strict NE. It can be thus concluded that $g^*_{ij} = 1$ cannot hold in $\mathbf{g}^*$ and we have $g^*_{ji} = 1$. As a result, j should belong to the same group as i with $k_{ij} = \underline{k}$.

Now consider another agent $j' \in C/\{i, j\}$. If $g^*_{j'j} = 1$, then j' can switch its link from j to i without decreasing its utility, which leads to a contradiction. Therefore, we have $g^*_{jj'} = 1, \forall j' \in C/\{j\}$ and the component forms a star topology. Also, if there is an agent $j'' \in C$ who is not from the same group as j, then j can always increase its utility by removing its link with j'' since $f(x) < \bar{k}$. Hence, this statement follows.

(iii) When $f(x) > \bar{k}$, it is still true that agents from the same group belong to the same component. Also, the network should be connected with a unique component existing under $\mathbf{g}^*$. It is always true that we can find two agents i and i' from one group N_z such that $g^*_{ii'} = 1$. Using the same argument as that in statement (ii), it is easy to show that $g^*_{ii''} = 1, \forall i'' \in N_z/\{i\}$. Now consider an agent $j \notin N_z$. We have $g^*_{ji'} = 0, \forall i' \in N_z$ (otherwise the condition of a strict NE is violated). Now consider a path $path_{i'j} = ((i', j_1), (j_1, j_2), ..., (j_m, j))$ with $j_1, ..., j_m, j \notin N_z$ and $i' \in N_z$. Obviously, we have $g^*_{i'j_1} = 1$. Hence, i' can switch its link from j_1 to j without decreasing its utility, which again violates the fact that $\mathbf{g}^*$ is a strict NE. It can be thus concluded that for each $j \notin N_z$, $g^*_{i'j} = 1, \exists i' \in N_z$ and $g^*_{jj'} = 0, \forall j' \notin N_z$. Therefore, this statement follows. ■

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