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Abstract Markets typically have many ways of learning about quality, with two of the most important being reputational forces and certification, and these types of learning often interact with and influence each other. This paper is the first to consider markets where learning occurs through these different sources simultaneously, which allows us to investigate the rich interplay and dynamics that can arise. Our work offers four main insights: (1) Without certification, market learning through reputation alone can get "stuck" at inefficient levels and high-quality agents may get forced out of the market. (2) Certification "frees" the reputation of agents, allowing good agents to keep working even after an unfortunate string of bad signals. (3) Certification can be both beneficial and harmful from a social perspective, so a social planner must choose the certification scheme carefully. In particular, the market will tend to demand more certification than socially optimal because the market does not bear the certification costs. (4) Certification and reputational learning can act as complementary forces so that the social welfare produced by certification can be increased by faster information revelation.

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1 Introduction

A market typically has several sources from which to learn about the quality of an agent, such as the agent's prior work history, or reports from trusted outside sources like a certification board. An agent's work results provide a steady, but noisy, flow of information to the market, allowing the agent's reputation to be continuously updated. Certification boards also exist in many industries such as healthcare and accounting, and certification verifies that a minimum level of quality has been met. The previous literature has explored markets where quality is learned through each source of information separately, but the combined effects of the two have not yet been well analyzed. Given that these different informational sources often coexist, important questions are raised regarding how they can, and should, interact in a marketplace. As an example, physicians can become board certified to demonstrate proficiency in a medical specialty, and patients also learn about physicians from the experiences of previous patients. While certification helps increase a physician's reputation, it also places significant costs on the physician. In such a market, will the physician choose to perform board certification immediately after entering, or only after some time depending on his performance results? And how stringently should the medical board set its certification requirements if it wishes to maximize social welfare? By considering a general model which incorporates both types of learning, this paper is able to address such questions.

Our main results show that certification remains an important component of a marketplace even when learning can be done through other channels. The key observation is that certification provides a very different kind of information to the market, a type of information that is socially beneficial due both to its informativeness as well as its dependability. Without certification, even high-quality agents may experience an unfortunate string of bad reports and so get forced out of the market. Once out of the market, agents can no longer work and thus cannot produce more information that would change the market's opinion. Certification, however, remedies this problem by acting as a safety net, letting these socially beneficial agents continue to work. The type and amount of information revealed by the agent's work history can have a strong effect on the social welfare provided by certification, and we show that contrary to the case without certification, faster information revelation can be socially beneficial. Thus, certification and the agent's work history can act as complements.

In this paper, we focus attention on markets with pricing frictions that result in fixed wages for the agent. With pricing frictions, information revelation can get "stuck" at inefficient levels of reputation,[1] so that the learning via reputation may not actually increase social welfare. Since principals are myopic and have limited incentive to

[1] See for instance the discussion in Sect. 3 of Bar-Isaac and Tadelis (2008).

experiment, the market does not learn in a socially efficient manner. For instance, patients who see physicians for special procedures are unlikely to return and thus do not internalize the value produced through their own observations. Thus, they may refuse to see newer and less known physicians, even though the visit would produce information beneficial to future patients. If wages are fixed for all physicians, these less known physicians may not be able to get employed altogether, even if their true quality was actually very high. We show in Theorem 1 that under certain conditions, reputational learning alone cannot increase social welfare due to this inefficiency of the learning process. Thus, the markets that we analyze are the ones in which certification can be the most important, by reducing asymmetric information and keeping high-quality agents in the market.

While our fixed wage assumption may represent a departure from the literature, which usually assumes completely flexible wages like in Holmström (1999), we argue that fixed wages are a reasonable assumption in many real-world markets where certification is present. For instance, doctors who see Medicare patients are paid according to a fixed fee for service schedule. This payment schedule is decided by the government every year and remains the same for all doctors regardless of their reputations. In this market, certification plays an extremely important role as doctors can voluntarily choose to get board certified in order to verify their expertise. Although certification does not increase the wage a doctor receives, it helps ensures a steady flow of patients by increasing the doctor's reputation. Outside the US, many European markets also have prices for medical services that are completely fixed by the government, and there are similar informational issues.

In addition to a mandated fixed wage, our model is also applicable to markets with binding price floors, such as a minimum wage or union negotiated price. Especially for lower skilled workers, such as florists or custodians, these price floors can often be binding. In such markets, the wage will be "fixed" at the level of the price floor because the agent cannot work for less and the agent does not have enough market power to demand more. Agents still benefit from good reputations because it helps them find work, while agents that develop bad reputations will not get hired. Certification can be quite important to such markets both to guarantee the quality of workers for employers, as well as to boost the reputation of high-quality workers and help them to continue working. The regulation and licensing of lower skilled workers have been on a sharp rise in recent years,[2] so the proper use of certification in these markets represents a very timely issue.

Our model features reputational dynamics related to those in Holmström (1999), with an agent of uncertain quality working for a market of homogenous short-lived principals. The agent knows its own quality, but the market does not and must infer quality through the agent's work history, which generates a stochastic process that is publically observable. The agent's work history thus provides a steady stream of information to the market, continuously updating the agent's reputation. Our model imposes few assumptions on the stochastic process itself, or the agent's initial quality distribution. In contrast, most papers in the reputation literature make much stronger

[2] See for example the New York Times article "Why License a Florist" (http://nyti.ms/1wnLUZk, Kleiner (2014)).

assumptions such as binary quality types or Brownian motion signals[3]. Conditional on its work history, the agent chooses when (or whether) to certify, revealing additional information to the market. Certification has a cost to the agent of k and verifies that true quality lies above some standard $\underline{q}$, but does not reveal the exact quality level. This type of "imprecise" certification is one of the most common in practice, with examples such as board certification for doctors, pass/fail exams for accountants, and security seals for websites.

We first consider a benchmark model without certification and learning only through the agent's work history. Even in this simpler setting, we find surprising results. The key driver of these results is that high-quality agents can be forced out of the market after a string of bad signals, after which learning stops and the agent's reputation has no chance to improve. Since learning may be halted prematurely, having access to the agent's work history alone does not guarantee a socially efficient outcome. In fact, our result shows that for many types of information processes,*work history information does not provide any social benefit at all*. This result is reminiscent of work on "bad reputation" (Ely and Välimäki 2003; Ely et al. 2008) where the ineptness of reputation is due to the myopia of the short-lived principals. But in our model, the failure of reputational forces alone results not because of moral hazard, but because of an informational friction between principals and agents. Principals are not willing to hire once the agent's expected quality falls to the price level, but there is still positive social value from hiring at this because an agent with a low reputation might still be of high quality—hiring has an informational value, but since the principal only interacts once, it does not appropriate this value. Thus, principals would force agents, even agents who face no moral hazard issues, out of the market inefficiently due to their short-sightedness.

Given the ineffectiveness of learning through the work history alone, we then introduce certification into the model to see if welfare can be improved through a combination of the two information sources. The decision to become certified or not at any given moment is a strategic one, which depends on the results of the agent's previous work history. There are many equilibria as in most signaling games. However, we show that all equilibria share a common feature: They are type independent, meaning that all types of agents could get certified will choose to certify after the same work histories. This implies that every equilibrium is characterized by a single certification stopping time strategy that is used by all types of agents regardless of quality. But there are many different stopping times strategies that could constitute an equilibrium, with some requiring the agent to certify very early on, or even immediately, and others requiring the agent to certify later. We characterize the specific equilibria that maximize principal, agent, and social welfare and rank them based on how quickly certification occurs: Agents prefer later certification than is socially optimal, and principals prefer earlier certification than is socially optimal.

After characterizing these three different types of equilibria, we then perform comparative statics to find the optimal certification standard and price level for each type of equilibrium. We find that certification increases welfare if the standards are set

[3] Such as Holmström (1999), Bar-Isaac (2003), and Bonatti (2011).

appropriately, and certification costs are low. In addition, the information produced by the agent's work history and certification act in a complementary fashion, with faster information revelation increasing the welfare generated by certification. However, certification will also decrease welfare if certification costs are very high or the standards are set inappropriately. When costs are too high, certification will destroy more social welfare than it generates. But even when costs are low, certification may not increase welfare if the certification standards are set incorrectly. For instance, in some equilibrium, the market belief will expect too much certification. Such a case can occur in the principal optimal equilibrium, since principals do not internalize certification costs. Because agents are forced to certify at inefficient reputation levels, overall welfare could actually be reduced. Thus, it is important to set certification standards appropriately depending on the specific market beliefs.

2 Literature review

2.1 Reputation papers

Our work is closely related to several papers in the reputation literature, such as Holmström (1999), Mailath and Samuelson (2001), Bar-Isaac (2003), and Ely and Välimäki (2003). All of these papers have some form of reputational mechanism that follows from work history, but ours is the first to allow the agent to certify and send information through this channel. Holmström (1999) presents the classic "signal jamming" model where an agent of unknown quality can exert effort to bias the market's perception. Contingent contracts are not possible and without reputational incentives the agent would exert no effort. Holmström finds that reputation can provide work incentives in the short run while the agent's quality is unknown, but not in the long run once quality does become known. Crucially, there is no exit in this model and so the agent's quality will become known perfectly over time. We show that when there is an exit point, perhaps due to market frictions resulting in fixed prices, then agents may be forced out before their true quality is revealed. In this case certification is necessary as a form of insurance, so that high quality agents can stay in the market and continue working. Mailath and Samuelson (2001) also considers permanent reputations with moral hazard like Holmström, but with firms that know their own qualities. They show that good firms will build up reputations and exert effort in order to separate themselves from low-quality firms. In our paper, there is no moral hazard and good agents will instead certify to separate themselves. Also related is Board and Meyer-ter-Vehn (2013), in which a firm invests in order to improve its quality and reputation, while in this paper the quality is fixed, but can be verified through certification. Finally, Ordoñez (2013) analyzes a situation where managers can take actions such as scapegoating after failures or successes in order to manage their reputations, which is similar to our model where agents can choose to certify in order to increase their reputations.

Unlike these previous papers that do not consider exit, a reputational paper that does focus on exit is Bar-Isaac (2003). This model arrives at the striking conclusion that high-quality agents never exit the market, because staying in the market is a signal of quality that increases reputation. Even an agent who receives a string of bad signals can demonstrate resolve by refusing to quit, which boosts the market's perceptions. Importantly, Bar-Isaac assumes that the wage varies at every moment in time to equal the agent's expected quality, so good agents can internalize the future benefits of reputation and are thus willing to sustain a period of negative payoffs. While flexible wages are reasonable for some markets, we argue that there are also some markets where fixed wages are more reasonable. Flexible wages require the agent to have market power (in fact the agent is extracting full surplus from principals), which can be unrealistic in many types of markets where agents have unknown qualities. Without market power (e.g., Bertrand competition), agents may be unable to internalize the benefits of high reputation and thus unwilling to endure negative flow payoffs. Second, good agents may be forced to sustain numerous periods of negative payoffs in equilibrium, which may not be feasible for markets where they have liquidity constraints. With liquidity constraints, both good and bad agents may be forced to exit at the same time once their reputation drops below cost. Finally, in some markets factors such as market customs and menu costs can make changing wages quite difficult. When wages are fixed, we show opposite results: High-quality agents are forced to exit the market with positive probability and learning may not be socially efficient.

While most papers in the literature show that reputation can increase social welfare, the papers by Ely and Välimäki (2003) and Ely et al. (2008) are noteworthy in showing cases where reputation is potentially harmful. Ely and Välimäki (2003) consider a car repair framework, with both honest and dishonest mechanics. Honest mechanics wish to recommend the best repair for a car, be it a cheap tune-up or an expensive engine repair, whereas dishonest mechanics always want to recommend engine repairs. The mechanic's reputation represents the probability of being honest, which will change over time depending on how many tune-ups or engine repairs are recommended. The paper shows that a reputational mechanism may actually destroy social surplus: Honest mechanics have an incentive to recommend tune-ups even when engine repairs are truly needed in order to boost reputation. But such an action hurts the consumer, who is short lived and so does not internalize the benefit of higher reputation for the mechanic. In equilibrium, the market may break down as consumers are not willing to go to either type of mechanic. Ely et al. (2008) extends this model to a more general framework and shows that similar results will hold in models that feature "temptation" actions for the long-lived player. These "temptations" are actions that boost reputation but are socially inefficient, like recommending tune-ups in place of engine repairs. The assumption of short-lived consumers is critical in both papers, as Ely and Välimäki (2003) shows that a long-lived consumer could devise a mechanism that benefits from the information process. The results in these papers are related to those of our model, which also considers a market of short-lived players. Because of myopia, the market is unable to internalize the benefits of experimentation and so learning may stop early at an inefficient level. But our model does not require moral hazard in the way of "temptations," as agents face no moral hazard in their actions. Even without moral hazard, we show that the market myopia by itself can be sufficient to undercut the value of reputation if prices are fixed and principals unwilling to experiment. The inefficiency thus results from an informational friction rather than a strategic friction.

Finally, there are some related papers that consider how entry costs may be necessary to complement the gains from reputation in markets where there is free entry. Klein and Leffler (1981) shows that if there are moral hazard issues, firms need to earn price premiums in order to incentivize them not to cheat. With free entry, such price premiums cannot be sustained in equilibrium, and so it is important to have fixed entry or investment costs to support these premiums. Atkeson et al. (2012) also consider a reputational model with free entry. In their paper, producers make quality decisions at the time of entry, and these qualities are then fixed afterward. The market learns over time about quality via signals as in our paper, and the government can impose fixed entry costs for entry into the market. These costs do not produce information as in certification, but do allow for higher-quality firms to enter as in Klein and Leffler, leading to greater social welfare. While we do not explicitly model free entry in our paper, certification is costly and can be thought of as an entry barrier to a market. Indeed, we show that in some markets certification will act as a *de facto* license, with non-certifying agents unable to ever find work. However, there are two main differences between certification and the entry costs in these other papers. The first is that certification provides information to the market, and so serves more than just a signaling role since not all agents can pass certification. The second is that certification can also happen after an agent enters the market, with the agent choosing to certify only if it sends bad signals. Thus, very high-quality agents may not need to pay the certification cost if they produce good results, leading to a potentially more efficient outcome than one where all agents must pay an entry cost.

2.2 Certification papers

Our work represents a novel approach to the certification literature, where the agent's certification choice has never been combined with a similar reputational mechanism in a dynamic setting. Much of the focus of the theoretical certification literature has instead been on the certifier's actions and revolves around studying the decisions made by a strategic certifier who can control the type of information that it releases about agents or the payments that it charges (Lizzeri 1999; Stahl and Strausz 2011; Farhi et al. 2013). Some papers also allow the certifier to collude with agents and assign false ratings as in Strausz (2005). In contrast, our paper focuses more on the agent's decision process and how its reputation affects its certification decision and the principals' beliefs. This allows us to analyze the strategic aspects of certification for the agent, and show how even the possibility of certification will affect an agent's reputation. We do not explicitly model the certifier, instead delegating it to the role of the mechanism designer and analyzing the comparative statics of our model with regard to the certification standard. This implicitly assumes that the certifier is not strategic, is always accurate in its judgments, and is not allowed to cheat. Thus, our work is best suited to markets where the certifier is a government agency, which would be trustworthy and sets certification standards in a benevolent fashion. Finally, we note that while papers by Shapiro (1983) and Panzar and Savage (2011) have considered the welfare implications of minimum standards when agents have reputational concerns, the focus of our work is different because these models consider reputation in terms of moral hazard instead of adverse selection, and so there is no learning about the agent's type.

Although our work on combining agent reputation and certification is new from a theoretical perspective, there have been interesting empirical papers published on this subject. For instance, Xiao (2010) tests the value of certification in the childcare industry. In childcare, certification is voluntary and usually chosen by only a small fraction of firms. The paper shows that the social value generated by certification is positive, albeit small. Further, the costs of certification are not negligible, such as administrative and personnel costs. Thus, certification on the whole is a negative for the industry, as the benefits are outweighed by the costs. In our model, we show that this is certainly a possibility when certification is present, in particular when market beliefs are pessimistic and require certification often. In such a situation, the informational value of certification is outweighed by the inherent costs, resulting in lower social welfare.

Jin and Leslie (2003) analyzes the impact of the introduction of government issued hygiene grade cards on restaurant cleanliness in Los Angeles. They find that cleanliness did in fact improve significantly after the government started using these grade cards. Restaurant revenue is significantly affected by the posted scores, with restaurants that received an "A" grade getting a 5.7 percent increase in revenue from mandatory disclosure, while restaurants that received a "B" got only a .7 percent increase, and restaurants that received a "C" got a 1 percent decrease. This provides evidence that consumers do pay attention to this additional source of information, and that market demand was changing as a result. Jin and Leslie (2009) extends this study by considering whether reputational effects alone are adequate to explain this restaurant behavior. The paper assumes that restaurants affiliated with chains have an incentive to free ride on the chain's reputation. Thus, franchised stores tend to have lower hygiene scores than company-owned stores. After the introduction of the grade cards, the authors assume that such incentives would go away, because consumers are able to infer hygiene directly from the posted scores. From the data, the authors find evidence that the introduction of grade cards causes hygiene scores to increase more for non-chain stores than for chain stores, lending support to the idea that chain stores care more about reputation in the absence of grading cards. And in comparing the effect of reputation versus the effect of the grade cards, the authors find that hygiene improvements due to reputation were 70 % as large as hygiene improvements due to the posted grade cards. The significant changes in hygiene after grade cards were introduced thus shows that certification can send a clearer output to the market than mere reputational effects, and that reputation alone may not be sufficient to solve adverse selection problems. Our model echoes this result, by showing that certification can increase social welfare even if the market can learn via the agent's reputation.

3 The model

We consider an infinite horizon continuous time model with a single long-lived agent and a marketplace of principals.[4] The agent has a fixed quality q that is determined at the start of the game according to a commonly known continuous distribution

[4] Alternatively, we can think of the agent as representing a firm and the principals as representing consumers.

$q \sim f_0(q)$, which has a finite mean μ_0. Denote the support of this distribution by D, which can be any measurable non-null set in $\mathbb{R}$. The agent is privately informed about its own quality, but the market must learn this quality over time. We assume the agent has a reservation value of $c < \mu_0$, which represents the agent's disutility of work and outside options. The agent also has a discount rate of ρ. At each moment in time the agent meets a different short-lived principal, who can make an offer to the agent at a fixed price p. We will assume throughout the paper that the price satisfies $p \geq c$ so that agents are always willing to accept an offer.

If the agent accepts the offer, it will work for one infinitesimal instant of time and return output with true quality q. The market's observations (or reports) of q are noisy, and the evolution of these observations follows a càdlàg[5] stochastic process $R_q(t)$. This stochastic process will depend on the agent's true quality q, with a different stochastic process $R_q(t)$ corresponding to each q. We define the agent's *work history* as the history of all previous observations by the market, $\mathcal{H}_t = \left\{R(t')\right\}_{t'=0}^{t}$, which is a continuous set of realizations of the stochastic process up to time t. We also call the set $\mathcal{L} \equiv \left\{R_q(t)\right\}_{q \in D}$ an *information process* for the market. The set $\mathcal{L}$ is common knowledge, so the market knows what kind of stochastic process to expect from each type of agent. Thus, given any history $\mathcal{H}_t$, the market can update its prior beliefs about the agent via Bayes' rule. We define the agent's time t *work history rating* as the market's belief of the agent's quality distribution given only information from the work history, $f_t(q|\mathcal{H}_t)$.

We make some restrictions on the information process $\mathcal{L}$ to ensure that the agent's work history rating will evolve in a sufficiently continuous fashion over time. Let $\mu_t \equiv E\left[q|\mathcal{H}_t\right]$ be the time t expected mean of the agent's quality, and let $\mu_t^{\underline{q}} \equiv E\left[q|\mathcal{H}_t, q < \underline{q}\right]$ be the time t expected mean of the agent's quality given that it lies below some cutoff $\underline{q}$. We say that the information process is *admissible* if for any $q \in D$, the generated paths of μ_t and $\left\{\mu_t^{\underline{q}}\right\}_{\underline{q} \in \mathbb{R}}$ are almost surely right-continuous and upper semi-continuous in time. That is, at any point of discontinuity these means can jump upwards but not downwards. The reason that our continuity requirement needs only to be one-sided will become apparent in the next section. We will only use admissible information processes in this paper.

Note that a restriction on $\mathcal{L}$ entails a restriction on the types of stochastic signals $R_q(t)$ that can be allowed. One way to satisfy our continuity requirement is for the market to only be able to infer a little bit of information from $R(t)$ at any point in time. For instance suppose that for every q, $R_q(t)$ is the diffusion $dR_q(t) = qdt + \sigma_r dZ(t)$, with drift q and variance σ_r^2 (and precision τ_r), and with $Z(t)$ a standard Brownian motion. This type of process would result if the market observes each unit of output with some normal noise, and we note that this is the continuous time non-moral hazard version of Holmström (1999). In this case, the agent's work history rating would evolve continuously in time, and so our admissibility requirement is satisfied. Even more general diffusion processes are admissible, such as $dR_q(t) = f(q, \mathcal{H}_t)dt + g(\mathcal{H}_t)dZ(t)$ so that the drift and variance do not have independent

[5] Right-continuous with left limits.

or stationary increments. However, stochastic processes such as $dR_q(t) = qdt + q\sigma_r dZ(t)$, where the variance depends on the quality, are not admissible because the variance would be learned immediately through Bayes' rule, causing the means to jump downwards with positive probability. The variance may depend on the history of previous signals or any other public information, but not on the agent's true quality.

Stochastic processes with jumps can also be considered. For instance, suppose we have a Poisson process with an arrival rate $\lambda(q)$, where λ is increasing in q. This is a good news Poisson process, where an arrival indicates a positive event like being mentioned in an article or winning an award. Without a signal arrival, the means drift slowly and continuously downwards, and at a signal arrival, the expected mean and all truncated means would jump upwards. Thus, our admissibility condition would be satisfied. More generally the arrival rate could depend on the history as well, $\lambda(q, \mathcal{H}_t)$, as long as it was still increasing in q at all t. And if the stochastic process was a combination of diffusions and good news Poisson processes, the result could still hold. However, if the stochastic process was a bad news Poisson process, where $\lambda(q)$ was decreasing in q, then admissibility would be violated because at a signal arrival the means would jump downwards. So with bad news, the path of μ_t may not be right-continuous and upper semi-continuous. Thus, jump stochastic processes are admissible as long as they indicate good news, but not if they indicate bad news.

Finally, note that the stochastic process of reports about the agent only runs if the agent is actually working. If at a time t the principal does not hire the agent, no output gets produced and the stochastic process is stopped at that value of $R(t)$. Since the agent is not working, no further information gets sent. If the agent does get hired again in the future, then the stochastic process can once again proceed at that time. We will show that this can only happen if the agent passes certification; that is unless the agent certifies he will be kicked out of the market forever.

At every moment of time t, the agent can choose to certify if it has not done so already by paying a one-time certification cost of $k > 0$ and getting certified if it has a quality level of at least $\underline{q}$, which is a fixed exogenous standard. We assume that $\underline{q} \geq p$, which will imply that certified agents never get forced out of the market. We also assume that $k < \frac{p-c}{\rho}$ so that the cost is low enough for the agent to want to choose certification (this inequality implies that the net present value of staying in the market forever is greater than the certification cost). We denote the time t certification status by $\theta_t \in \{[0, t] \cup \phi\}$, where a number represents the time at which that agent became certified and ϕ means that the agent has not yet certified. Once the agent becomes certified, all future principals will know for certain that the agent quality is at least $\underline{q}$. Since this information is permanent and public knowledge to all future principals, an agent only needs to be certified once.

We also assume principals do not know if an agent attempts to certify and fails, so an agent that cannot pass certification has no incentive to certify. Such an assumption is relevant when $\underline{q} > p$. If the market could observe agents that certify and fail, an agent with quality $q < \underline{q}$ may have an incentive to attempt certification to increase its reputation. For instance, if the market believed that attempting certification was more likely by agents with qualities in the range $[p, \underline{q}]$, then attempting certification, even when the agent cannot pass, would increase the agent's reputation.

There are thus two sources of information in our market: The first being the history of observations from work history, $\mathcal{H}_t = \left\{R(t')\right\}_{t'=0}^{t}$, and the second being the certification status of the agent θ_t. Upon observing these sources of information, the market uses Bayes' rule in updating its beliefs about agent quality. Together, these two signals combined with the prior quality distribution will result in a posterior belief distribution of agent quality $f_t(q|\theta_t, \mathcal{H}_t)$, which we call the *reputation* of the agent. This is different from the work history rating that was defined previously because it also takes into account the agent's certification status. Note that the reputation can be calculated from the work history rating by using Bayes' rule together with θ_t on the work history rating $f_t(q|\mathcal{H}_t)$. We will use $F_t(q|\theta_t, \mathcal{H}_t)$ to denote the cdf of the reputation. Note that we sometimes suppress the notation and write $f_t(q)$ and $F_t(q)$ for the reputation and cdf of the reputation, respectively. We also use $F_t^-(q)$ to denote $\lim_{q \nearrow \underline{q}} F_t(q)$.

We assume principals have identical linear utilities given by $U(p, q) = q - p$. Since principals are short lived, they care only about maximizing their current utility. Given the above linear utility function, we assume principals will hire uncertified agents if and only if $p < E[q|\theta_t = \phi, \mathcal{H}_t]$, where the expectation is taken with respect to the agent's time t reputation. Principals will hire an agent that certified at t' if and only if $p \leq E[q|\theta_t = t', \mathcal{H}_t]$.[6] Note that once the agent's expected quality falls below p the agent will not be able to work again unless it certifies, because principals would not be willing to hire at any quality level less than p, so no outputs will be produced and no further information sent. Thus, the market's beliefs will be stuck unless the agent improves its reputation through certification.

If the agent works at time t, it receives a flow payoff of $\pi_t = p - c$. Otherwise, the agent receives a payoff of $\pi_t = 0$. And the agent must pay a one-time cost of k when it chooses to certify, which can be incorporated by subtracting the flow payoffs after certification, changing them to $\pi_t = p - c - \rho k$ (our assumption that $\underline{q} \geq p$ implies that a certified agent will always keep working). Thus, the agent decides whether to certify at time t through maximizing its expected discounted value, $\int_{t'=t}^{\infty} e^{-\rho(t'-t)} E\left[\pi_{t'}|\theta_t, \mathcal{H}_t, q\right] dt'$, taking into account the state variables: the agent's certification status $\theta_t = \phi$, the history of outputs $\mathcal{H}_t$, and the agent's true quality q. The agent's certification strategy can be represented by an optimal stopping time that is measurable with respect to the filtration generated by $R_q(t)$. We denote the stopping time strategy for an agent of type q by $\tau(q)$, which depends on the specific history of signals $\mathcal{H}_t$, and we note that $\tau(q) = \infty$ for all $q < \underline{q}$ since these agents cannot pass certification. The market must also have beliefs about the certification strategy of agents of each possible quality, and we represent these beliefs using $\tilde{\tau}(q)$. In equilibrium, these beliefs must be correct so that $\tilde{\tau}(q) = \tau(q)$.

4 Benchmark case: no certification

In order to analyze the welfare benefits of certification, we will first derive some welfare results for a benchmark setting with no certification. This can be considered

[6] For technical reasons, we assume principals who are indifferent will hire certified agents, but not uncertified agents. The distinction ensures that certified agents will be hired regardless of the market's beliefs, while uncertified agents have a well-defined time at which they leave the market.

a specialized case of the above model with $\underline{q} = \infty$. A principal will hire at time t if and only if $p < E[q|\theta_t, \mathcal{H}_t]$, with the expectation taken with respect to the agent's reputation, $f_t(q|\theta_t, \mathcal{H}_t)$. Since the agent will never certify, its reputation is always equal to the work history rating, $f_t(q|\mathcal{H}_t)$. To analyze this model we thus need to determine how the agent's reputation evolves as a function of its work history. Working generates a stochastic process $R_q(t)$, and without certification the market will update its beliefs only through this stochastic process itself. Through Bayes' rule the agent's expected quality level, $\mu_t \equiv E_t\,[q|\mathcal{H}_t]$, will be continuously updated as the market receives information. The reputational dynamics in the benchmark are as follows: The agent starts out with a quality $\mu_0 > p$ (for $\mu_0 \leq p$ no hiring ever takes place) and continues working as long as its expected quality stays higher than p, but the first time that its expected quality drops to or below p it will be forced to stop. At this point, no principals are willing to hire the agent, and thus, the stochastic process is stopped forever causing no future principals to hire either (Fig. 1).

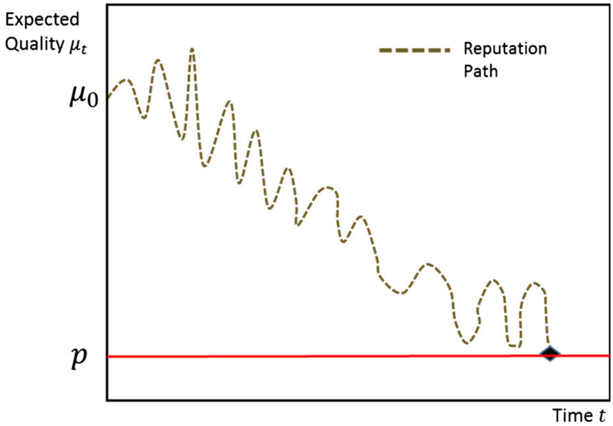

Fig. 1 *Reputational dynamics in benchmark*. The agent starts out with an expected quality greater than the price, and the expected quality then evolves according to the stochastic process generated by the outputs. The agent continues getting hired until its expected quality falls to the price, at which point it stops working forever. The probability that the agent stops working depends on its true quality and the price and may be strictly positive even for very high-quality agents

Given this characterization of the dynamics, once the agent stops working it can never start again. Thus, the probability that the agent is still working at time t, denoted by $P\,(S_t|q, p, \mathcal{L})$, is the same as the probability that the hitting time of μ_t against the price p or any lower value is greater than t. Since we only consider admissible information processes, at the first time that μ_t falls below p it must be that $\mu_t = p$. This is due to the fact that the agent's expected quality can never jump downwards. Let $t^* \equiv \inf\{t|\mu_t = p\}$ denote this (stochastic) hitting time.[7] So $P\,(S_t|q, p, \mathcal{L})$, the probability that the agent is still working at time t, is equal to the probability that $t^* > t$. Now we can write out the *ex ante* expected social surplus (sum of principal and agent surplus) for a price p as:

[7] Note that when $p \geq \mu_0$, we have $t^* = 0$ and the agent is never hired.

$$W(p,\mathcal{L}) = \int_{-\infty}^{\infty}\left[\int_{0}^{\infty} e^{-\rho t}\,(q-c)\,P\,(S_t|q,p,\mathcal{L})\,dt\right] f_0(q)dq$$

For an agent with quality q, the social welfare at every moment will be given by $q-c$ if the agent is hired and 0 otherwise. Thus, hiring is socially optimal if and only if $q \geq c$. There are two sources of inefficiency in our model: Perceived quality may be above p even though true quality is below c so that principals are hiring when they should not (bad agents are working), or perceived quality is below p even though true quality is above c so that principals are not hiring when they should (good agents are not working).

Although in general the working probability may not have an analytic expression, and so the social surplus cannot be directly computed, we can nonetheless still prove results about welfare. We show a perhaps surprising result: For all information processes that are admissible, the *ex ante* social welfare will be less than if no information were generated at all. That is, the information provided through the reputational mechanism can actually improve social welfare.

We define a *blind process* to be any information process such that $R_q(t) = R_{q'}(t), \forall q, q' \in D$. Under a blind process, the information provided to the market is completely uninformative of the agent's true quality, and so no learning occurs. At all times t, the agent's work history reputation is thus the same as the initial prior, $f_t(q|\mathcal{H}_t) = f_0(q)$.

Theorem 1 (Blind Boundedness) *Let $\mathcal{L}^b$ denote any blind process. If $\mathcal{L}$ is an admissible information process, then $W(p,\mathcal{L}) \leq W\left(p,\mathcal{L}^b\right)$ for all $p \in \mathbb{R}$.*

Proof See Appendix. □

The intuition for this theorem is that since the market is forcing out agents when their expected quality hits p, the social welfare impact must be weakly negative for any $p \geq c$ because beliefs are correct, and so the expected quality of an agent that leaves *must* be higher than the social cost of that agent. If, at the time the agent is getting kicked out, we could instead choose to let that agent stay in and work forever, we would wish to do so. Then since under the blind process no agent ever gets kicked out, the blind process must be weakly better than any other admissible information process. So all admissible information processes give a social welfare that is bounded above by the blind process.

It is important to emphasize that the principals are *short lived*, which creates a source of inefficiency in the marketplace because principals do not internalize the social benefits of experimentation. Once the agent's expected quality level reaches the price, it would still be socially beneficial for the agent to work, because working sends information to the market and this information is valuable. Unlike a short-lived principal, a long-lived principal that could make future purchases would wish to continue hiring the agent because there is a chance the agent's reputation could improve and thus a positive value from experimentation.

In addition, from the proof we can see that the social welfare for any information process is the same as for a blind process if $p = c$, but can be strictly less if $p > c$. This implies that for *any* admissible information process, the socially optimal price is equal to the agent's reservation value. When price is exactly equal to the reservation value, there is equal social welfare from letting agents with $\mu_t = p$ stay and work forever, or leave immediately. But when $p > c$ it is strictly better to have agents that are being kicked out stay in because they are still generating positive flow payoffs in expectation. Thus, it is socially optimal to set the price as low as possible in order to make the exiting process more efficient. Note that if it was possible to subsidize agents to work below their reservation values, this would increase social welfare because an agent with expected quality equal to c still generates positive value to society through the information it provides by working.

5 General model

5.1 Belief updating with certification

Having shown the inefficiency of information revelation when certification is not possible, we now turn to the general model with both the work history and certification. If certification is available, agents will have a decision to make regarding the exact time to undergo certification. For an agent whose true quality is above $\underline{q}$ and has not undergone certification before time t, its expected discounted value if it chooses certification at time t will be $\int_t^{\infty} e^{-\rho(t'-t)} E\left[\pi_{t'} | \theta_t = t, \mathcal{H}_t, q\right] dt'$. We can analyze $\pi_{t'}$ by solving the principals' problem. Principals update the agent's reputation with Bayes' rule, and they will hire an uncertified agent if the expected quality level is above the price given the certification status and history of outputs, or $p < E[q_t | \theta_t = \phi, \mathcal{H}_t]$. Recall that they will hire an agent that certified at time t' if $p \leq E[q_t | \theta_t = t', \mathcal{H}_t]$, so principals will hire certified agents when indifferent but not uncertified agents.

The introduction of certification will result in posterior beliefs that are truncated distributions, even for agents that do not get certified. Suppose for instance that in equilibrium all types of agents with quality at least $\underline{q}$ are expected to certify at time 0. Given this fact, an agent that becomes certified at time 0 will be believed to have a quality higher than $\underline{q}$. If the prior distribution is $f_0(q)$, then the posterior distribution given certification will be $f_0(q)$ truncated to be over the interval $[\underline{q}, \infty)$. Since all types of agents in this quality region choose to certify, the relative density inside the support is unchanged. Likewise, the reputation for an agent that does not certify will also be updated. The market will believe that the agent has a quality less than $\underline{q}$, and so the posterior becomes $f_0(q)$ truncated to be over the region $(-\infty, \underline{q})$. An agent that certifies will continue to work forever (since we have assumed $\underline{q} \geq p$), and an agent that does not certify will stop working if the truncated mean drops below the price.

The key with certification, and the reason that it can improve welfare, is that high-quality agents who certify will be able to keep working even if they send an unlucky string of bad signals. Without certification such agents would instead be inefficiently kicked out of the market. In this way, certification is "freeing" the reputation of good agents which could otherwise get "stuck." Another way to look at it is from the perspective of the agents that are getting kicked out because they do not certify. Since their posterior gets truncated downwards, the expected mean of such agents can fall below the price level. Thus, unlike the benchmark case where agents got kicked out when $\mu_t = p \geq c$, here we can kick out agents with an expected mean less than c, and so kicking out these agents is socially efficient. This aspect of certification is welfare improving, but since certification also has a cost, welfare may decrease as well if certification is not implemented correctly.

5.2 Agent's certification strategy

We now analyze the agent's optimal certification stopping time strategy $\tau(q)$. In general this will also depend on what the market's beliefs $\tilde{\tau}(q)$ are, but we will show that for any beliefs the agent's strategy will have a common structure.

Throughout this section it is important to keep in mind two assumptions that we are making. For convenience, we assume that the certification standard $\underline{q} \geq p$, which implies that the agent will never stop working if it certifies. Using this assumption reduces the set of equilibria allowing for greater tractability, and implies that once certified the principals no longer need to doubt the agent's quality. With this assumption, the payoff after certifying is deterministic and given by $\frac{p-c}{\rho} - k$. For technical reasons, we assume that even if the agent's reputation had zero density on $[\underline{q}, \infty)$, and it passes certification, then the market will still be willing to hire the agent forever. This assumption matters only off the equilibrium path, where it ensures that the certification incentives for an agent remain the same. The second important assumption is that $k < \frac{p-c}{\rho}$, so that the certification cost is low enough for the agent to be able to benefit from certification. This can also be thought of as a restriction on the price, specifically that $p > k\rho + c$. This condition then means that the price is high enough to cover the cost of certification. Thus, an agent that is being forced out of the market would choose to certify and continue working instead of exiting forever. Without this assumption, the only possible equilibria would involve no certification.

We show in the following theorem that under our assumptions the only possible equilibria are *type independent equilibria*, where after any given work history either all types of agents with quality above $\underline{q}$ will choose to certify, or none will. Thus, in equilibrium all types of agents will use the same stopping time strategy τ^* and get certified after the same work histories.[8]

Theorem 2 (Type Independence) *In each equilibrium ε, all types of agents with quality $q \geq \underline{q}$ have the same certification stopping time strategy τ^*_ε.*

This type independence result arises from the fact that all agents who are able to certify face the same costs and benefits from certification. Note that since reputation depends only on the observed history, fixing a work history also fixes the agent's reputation independently of its true quality for any (not necessarily type independent) market strategy beliefs $\tilde{\tau}(q)$. Given the agent's reputation, either it can still work without having to certify, or it must certify to work. In the first case, since agents do not receive a higher flow payoff once certified, they would like to delay certification in

[8] Recall that the agent's certification strategy τ^* represents a certification time given the work history. Therefore, saying that all agent's use the same certification strategy is equivalent to saying that they certify after the same work histories.

order to delay the certification cost. Thus, no types of agents will choose to certify, and instead wait until their reputation falls lower. In the second case, since all agents prefer to certify rather than remain unemployed (given our assumption that the certification cost is sufficiently low), all agents would choose to certify regardless of their true quality.

This result does not mean that an agent's quality is irrelevant. For instance, agents with higher qualities may be more likely to send better signals and have better work histories. Since their reputations are more likely to be higher *ex ante*, they will be less likely to need to certify. Thus, the *ex ante* probability that an agent certifies at a given time t will in general depend heavily on its true quality. The type independence result says instead that *given* a history of signals, agents of all types will make the same certification decision. But the probability distribution over the history of signals will vary depending on agent quality.

Due to our type independence result, we will maintain the following notation in this paper: For any time t, let μ_t^{NC} represent the mean of the non-certifying agents and μ_t^{C} represent the mean of certifying agents, if the market were to believe that all types of agents with quality $q \geq \underline{q}$ would choose to certify at that t. Formally, $\mu_t^{NC} \equiv E\left[q|\mathcal{H}_t, q < \underline{q}\right]$ and $\mu_t^{C} \equiv E\left[q|\mathcal{H}_t, q \geq \underline{q}\right]$. These two expectations will be helpful for characterizing the equilibria of our model. We also keep with our previous notation by letting $\mu_t \equiv E_t\left[q|\mathcal{H}_t\right]$ be the agent's expected quality using only information from the work history. This is equal to the agent's expected quality given that it has not certified, $E_t\left[q|\theta_t = \phi, \mathcal{H}_t\right]$, if the market were to believe that no agents would certify before time t (i.e., $\tilde{\tau}^* > t$).

Now that we have shown the equilibria are all type independent, we characterize the possible equilibrium optimal stopping time strategies τ^*. The corollary below shows that the agent's equilibrium certification time is related to the agent's work history and reputation through the values of μ_t^{NC} and μ_t. If $\mu_t^{NC} \leq p$, then agent's that do not certify if the market expects all types of agents to certify cannot keep working, and so all types of agents will choose to certify with these market strategy beliefs. Further, if $\mu_t \leq p$ then no type of agent can keep working without certifying regardless of market strategy beliefs, and so all types of agents that can certify will certify for any market strategy beliefs.

Corollary *Fix any time t and history $\mathcal{H}_t$ such that $\mu_{t'} > p$ for all $t' < t$. Suppose that τ^* is an equilibrium stopping time strategy such that $\tau^* \geq t$ given this history. Then it is possible for $\tau^* = t$ if and only if $\mu_t^{NC} \leq p$. Further, it must be the case that $\tau^* = t$ if and only if $\mu_t = p$.*

Proof This follows from the Proof of Theorem 2. □

Note that there are some histories in which no types of agents certifying or all types of agents certifying are both supportable in equilibrium. The above corollary implies that as long as $\mu_t > p$ no agents certifying is supportable in equilibrium, and as long as $\mu_t^{NC} \leq p$, all types of agents certifying is supportable in equilibrium. The first condition indicates that if no agents are believed by the market to certify, all types of agents can keep working without certifying. The second condition indicates that if all types of agents with qualities in $\left[\underline{q}, \infty\right)$ are believed to certify, then agents that do not certify will be forced to stop working. As long as $\mu_t < p \leq \mu_t^{NC}$, we could have either type of behavior in equilibrium at time t. Figure 2 below highlights the range of possible equilibrium if the work histories follow Brownian motions.[9]

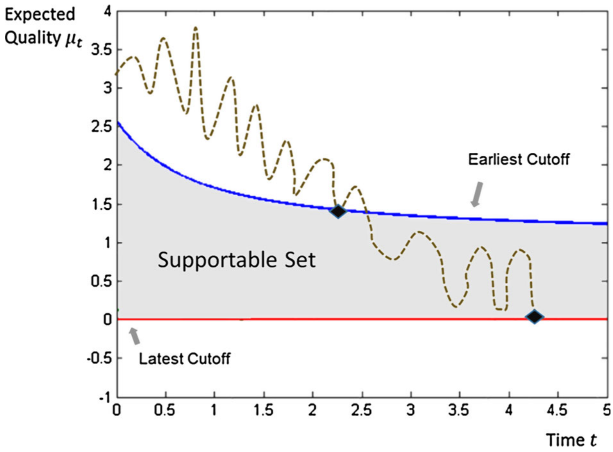

Fig. 2 *Range of possible equilibria* The agent starts with an expected quality above the price of $p = 0$, and continues getting hired without certifying until its expected quality falls into the shaded region. Within this shaded region, if the market expects the agent to certify, then the agent will be forced to certify or exit. But if the market expects the agent not to certify, then the agent will choose not to certify. The higher curve represents the earliest supportable certification time for the agent and the lower curve represents the latest time. Above the higher curve, certification is not supportable in equilibrium, and at or below the lower curve, certification must occur

5.3 Equilibrium characterization

As we have shown, and as is common in signaling games, many different equilibria are possible. Thus, we narrow down the set of equilibria by focusing on the specific equilibria that give the highest payoffs to the different players in our model. In order to find out which equilibrium maximizes principal, agent, or social surplus, we find the specific equilibrium beliefs and strategies that will result in the highest payoffs to each party. Theorem 2 implies that every equilibrium features a single stopping time strategy used by all types of agents. We denote the agent optimal, socially optimal, and principal optimal equilibrium stopping times by τ_a^*, τ_s^*, and τ_p^*, respectively. The next section characterizes the optimal equilibria for these three different cases. We show that the different types of optimal equilibria can be ranked according to the

[9] With Brownian motion signals and a normal prior, the posterior will always be normal and so the expected mean μ_t will be strictly increasing in the truncated mean μ_t^{NC}. Thus, we can transform the condition $\mu_t \leq p \leq \mu_t^{NC}$ into just a condition on p and μ_t, allowing us to depict the stopping times explicitly via cutoffs for the expected mean as in Fig. 2.

equilibrium certification times. Theorem 3 summarizes our main findings: We see that principals always prefer earlier certification than is socially optimal and that agents always prefer later certification than is socially optimal.

Theorem 3 *The optimal stopping times for the principal, agent, and social welfare cases must satisfy*

$$\tau_p^* \leq \tau_s^* \leq \tau_a^*$$

Proof The result follows from a comparison of the stopping times in Propositions 1, 2, and 3 below. □

5.3.1 Agent optimal equilibrium

We start with characterizing the best equilibrium from the agent's point of view. Since the agent is long lived, the notion of optimality we will use is the one that maximizes the agent's expected value at time 0, given by $\int_0^\infty e^{-\rho t}(p-c)\,dt - E_{\tau_a^*}\left[e^{-\rho\tau_a^*}k\right]$ if $q \geq \underline{q}$ or $E_{\tau_a^*}\left[\int_0^\infty e^{-\rho t}(p-c)\,dt\right]$ if $q < \underline{q}$. In general, the agent's expected discounted value will depend on its own quality level, but we show that the specific equilibrium that maximizes the agent's value does not.

Proposition 1 *The equilibrium that maximizes the agent's payoff is for* $\tau_a^* = \inf\{t|\mu_t \leq p\}$.

Note that τ_a^* is the absolute latest that the agent can certify in equilibrium because at this point the agent must stop working no matter what the market certification beliefs are. This proposition thus shows that the best equilibrium from the agent's perspective will always delay certification for as long as possible, with certification occurring just as it would be forced to exit the market. Since agents bear the full cost of certification and do not directly benefit in terms of flow payoffs, they are much less willing to certify than principals would want them to. In this equilibrium the beliefs expect certification at the latest possible time, and so we will call them *optimistic* beliefs.

If $\mu_o \leq p$ then agents cannot work initially and must certify to get hired. In this case certification act as a *de facto license*, with non-certifying agents never being able to work. Certification thus functions as an entrance key into the market itself. But as long as $\mu_0 > p$, in any agent optimal equilibrium certification will not be a de facto license. Instead the equilibrium will feature *delayed certification*, with agents working and sending information initially before needing to certify if their reputation drops too low. In this case the particular information process will make a big difference on the timing of the agent's certification decision, as well as the overall social welfare that is generated by certification.

5.3.2 Social welfare optimal equilibrium

Next, we characterize the equilibrium beliefs that maximize the total social welfare for the market. We will use the same notion of *ex ante* social welfare that was defined in Sect. 4, meaning that we want to maximize the working probability of good agents while minimizing the working probability of bad agents. The formula is given by:

$$W = \int_{\underline{q}}^{\infty} \left[E \left[\int_0^{\infty} e^{-\rho t} \left(q - c \right) dt - e^{-\rho \tau_s^*} k | q \right] \right] f_0(q) dq$$
$$+ \int_{-\infty}^{\underline{q}} \left[E \left[\int_0^{\tau_s^*} e^{-\rho t} \left(q - c \right) dt | q \right] \right] f_0(q) dq$$

Early certification is beneficial because it verifies that an agent's quality is high more quickly. However, since certification is also costly, we may want agents to delay certification if their reputation is sufficiently high, since that saves the discounted flow cost of certification, as well as keeps the option value of certifying for the future. The following proposition characterizes a *necessary condition* for certification to be socially optimal: The agent should certify only if its reputation is lower than the following cutoff.

Proposition 2 *At the socially optimal stopping time τ_s^*, if the expected quality of the agent is strictly higher than the price, $\mu_{\tau_s^*} > p$, then the following condition must also be satisfied:*

$$\mu_{\tau_s^*}^{NC} \leq c + \rho k - \frac{\rho k}{F_{\tau_s^*}^{-}\left(\underline{q}\right)} \tag{1}$$

We note that this bound can also be a sufficient condition for some information processes. With a blind process (1) gives the exact value of the truncated mean below which it becomes optimal to have the agent certify. Certification is socially beneficial because it gets rid of "bad" agents with qualities less than c, but it is socially harmful because it can get rid of "good" agents with qualities above c but below $\underline{q}$ and it also carries a cost of k. Certification at any time t must balance these two aspects together, and (1) gives a condition on how low the mean of non-certifying agents (i.e., the agents that would be kicked out by certification) must be for certification to be beneficial.

However, it is possible for (1) not to be satisfied at any $t \leq t^* = \inf\{t|\mu_t \leq p\}$. In this case we would have $\tau_s^* = t^*$, because when the expected quality falls below the price, the unique equilibrium is for an agent to certify. Consequently, the agent optimal stopping time and the socially optimal stopping time would coincide. In general, however, the socially optimal stopping time can be strictly earlier than the agent optimal stopping time due to the beneficial information that certification provides.

5.3.3 Principal optimal equilibrium

Finally, we analyze the equilibrium that maximizes the utility of the principals. The right notion of maximizing principal utility is a bit tricky since each principal is short lived and only cares about itself. The equilibrium that gives the time t principal the highest payoff will in general be different than the equilibrium that gives the time t' principal the highest payoff. One way to define principal welfare would be to aggregate all principal utilities *ex ante*, similar to the way we computed the social welfare

$$PW = \int_{\underline{q}}^{\infty} E\left[\int_0^{\infty} e^{-\rho t}\,(q-p)\,dt|q\right] f_0(q)dq \\ +\int_{-\infty}^{\underline{q}} E\left[\int_0^{\tau_s^*} e^{-\rho t}\,(q-p)\,dt|q\right] f_0(q)dq$$

For such a definition of principal welfare, the result of Theorem 3 still holds. However, such coordination among principals may seem unrealistic given that all the principals are short lived. We resolve this tension in the following way: Principals that arrive earlier should have a greater say in the equilibrium than principals who arrive later, because presumably the later principals may not be able to interact with or even know of the agent at earlier times. Thus, we define a *principal optimal equilibrium* to be the equilibrium in which the agent certifies at the first (stochastic) time t such that given the agent's reputation, the principal at t prefers hiring the agent only if it could certify over always hiring the agent.[10] Formally, the agent certifies at the first t such that $\left(\mu_t^C - p\right)\left(1 - F_t^-\left(\underline{q}\right)\right) \geq \mu_t - p$. We show in the proof of the following proposition that such an equilibrium always exists and is unique. In this equilibrium, the earlier principals have more influence over the agent, with later principals having an influence only if the agent has not certified early on. This notion of principal optimality is not the same as the net present value of principal surplus defined above, since the myopic time t principal does not consider the welfare of future principals.

Proposition 3 *The principal optimal equilibrium stopping time is*

$$\tau_p^* = inf\{t|\mu_t^{NC} \leq p\} \tag{2}$$

This Proposition gives a cutoff in terms of μ_t^{NC}, with principals wanting the agent to certify the first time its truncated expected quality falls below the price. Certification is good for principals because it gets rid of "bad" agents with qualities less than p, but it may also hurt principals because it can get rid of "good" agents with qualities between p and $\underline{q}$. The cutoff $\mu_t^{NC} = p$ is the quality such that these two effects exactly balance out. We see from (2) that the best equilibria for principals are those where the certification strategy beliefs τ^* expect agents to certify at the highest possible truncated quality that is still an equilibrium, since $\mu_t^{NC} \leq p$ is necessary for certification to occur. Since this is the highest possible supportable value, we call such types of beliefs *pessimistic beliefs*. Because principals do not bear the cost of certification, they want agents to certify at the fastest possible time that is supportable in an equilibrium. We will denote the principal optimal equilibrium stopping time by τ_p^*.

Note that if $\underline{q} = p$ then $\mu_t^{NC} \leq p$ for all t, and so principals will want certification by all types of agents immediately. In such a case, certification acts as a de facto license because any agent that does not certify will be immediately excluded from the market and unable to work at all. With a de facto license, the actual information process being

[10] If $\underline{q} = p$, then principals would always wish to hire the agent only if it could certify. However, if $\underline{q} > p$, and if the principal believes it is very likely that the agent's quality falls between $(p, \underline{q})$, the principal may prefer to hire all types of agents instead of hiring only when the agent's quality was above $\underline{q}$.

used by the market is irrelevant since agents certify before any learning takes place, and they then stay in the market forever regardless of the signals that are sent.

However, for $\underline{q} > p$ it could be possible that $\mu_0^{NC} > p$, and so principals may not want agents to certify immediately. In this case, if the agent's initial reputation is high enough, then the principals will want the agent to work for a while and only certify if its reputation drops too low, with the specific threshold given by Eq. (2). Thus, the equilibrium will be a delayed certification equilibrium. In this type of equilibrium, the information process itself matters because it affects how quickly the agent's reputation changes.

Putting together all our results for the optimal certification times from Theorem 3 allows us to show on the graph below what the optimal stopping time thresholds are for the special case of Brownian motion reports. Figure 3 below highlights the different optimal certification thresholds, which assume the same type of Brownian motion as in Fig. 2. The principal optimal threshold is the highest supportable threshold, the agent optimal threshold is the lowest supportable threshold, and the socially optimal threshold is in between.

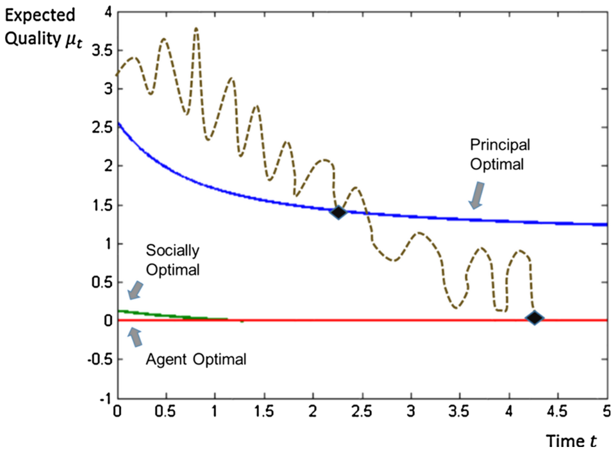

Fig. 3 *Optimal certification cutoffs*. This figure shows several thresholds that represent optimal certification cutoff qualities for principals and agents, as well as the socially optimal cutoff quality. The principal and agent thresholds represent optimal stopping rules: The agent should certify the first time that its expected quality hits each threshold. The socially optimal threshold represents a necessary condition: The agent should certify only if its expected quality is below the threshold. The principal optimal threshold lies far above the other two, whereas the socially optimal and agent optimal thresholds are close together and coincide after a while. The first point shows the optimal certification time from the principal's perspective, and the second point shows the optimal certification time from the agent's perspective, as well as the socially optimal certification time

6 Optimal certification standards

6.1 Certification standards for socially optimal beliefs

Now that we have characterized what can happen in the various types of equilibrium, we will perform some comparative statics in order to analyze the socially optimal certification standards and prices. This analysis will tell us how certification should be implemented depending on what type of equilibrium is expected. Thus, we are now analyzing a mechanism design framework, where the designer chooses the $\underline{q}$ and p in order to maximize social welfare. In doing so we maintain our assumption that the certification standard must be set higher than the price, $\underline{q} \geq p$. However, we no longer require that $\frac{p-c}{\rho} < k$, so it is possible to set the price sufficiently low that agents would never wish to get certified. Note also that the price can be chosen such that $\frac{p-c}{\rho} = k$, in which case the agent is indifferent between certifying and leaving the market[11]. Since there are many possible equilibrium, and the optimal standard and price will differ across equilibrium, we focus specifically on the socially optimal, agent optimal, and principal optimal equilibria that we analyzed above.

We start by analyzing socially optimal equilibrium because it is the most straightforward case. We assume that for any choice of $p, \underline{q}$ by the designer, the market will hold the socially optimal beliefs that were discussed in the previous section. Since these beliefs are part of an equilibrium, the agent will play according to these beliefs. With this restriction on the resulting game equilibrium, we find the values of $p, \underline{q}$ that maximize *ex ante* social welfare.

Although finding the socially optimal equilibrium stopping time strategy is challenging in general, characterizing how the price and standard should be set is much simpler. The reason is that, unlike in the principal and agent optimal cases, the beliefs will always be chosen in a socially optimal fashion, and we do not need to alter the price and standard in order to counteract possible "bad beliefs" that expect certification at inefficient reputation levels. Thus, it is socially optimal to set the standard and price as low as possible (but still above c) to allow as many good firms to certify as necessary. Let $p^* = c + k\rho$ and $\underline{q}^* = p$, and let $W^*(k, p^*, \underline{q}^*, \mathcal{L})$ be the corresponding *ex ante* social welfare given this certification standard and price assuming a socially optimal equilibrium. This corresponds to the lowest possible price that still allows for certification, and the lowest possible standard given this price. The next theorem states that the price and standard should be set at these lowest possible levels as long as the resulting social welfare is higher than that without certification.

Theorem 4 *Assuming that a socially optimal equilibrium will occur, the price and certification standard that maximize ex ante social welfare are given by:*

1. $p = c + k\rho$ *and* $\underline{q} = p$ *if* $W^* \geq \frac{\mu_0 - c}{\rho}$.
2. $p = c$ *and* $\underline{q} = \infty$ *if* $W^* \leq \frac{\mu_0 - c}{\rho}$.

[11] When this equation holds with equality, there may now be equilibria that are type dependent, because agents are indifferent between certifying and exiting the market, so some types could choose to certify and other types could choose to exit. But all the type independent equilibria characterized by the corollary to theorem 2 will still exist as well, and they will generate higher social welfare than the corresponding type dependent equilibria. This is because it is socially optimal for an agent with quality $q \geq \underline{q}$ to certify instead of exiting the market, so the type dependent equilibria that feature some types exiting will be suboptimal.

6.2 Socially optimal certification standards for pessimistic beliefs

We now characterize the social welfare maximizing certification standards and prices for the pessimistic beliefs defined in Sect. 5.3.3, which result in a principal optimal equilibrium. Unlike with the socially optimal beliefs, these beliefs are not aimed at maximizing the social welfare and so we may need to manipulate the standards and prices in order to increase welfare. We analyze the comparative statics of our model for the parameters $\underline{q}$ and p, assuming that given any choice of the parameters a principal optimal equilibrium will occur. We present a perhaps surprising result: Certification should only be implemented in a principal optimal equilibrium as a de facto license. In this equilibrium, the principals should never get to learn from an agent's work history at all before certification occurs.

The next theorem tells us exactly what the optimal certification standards and prices are as a function of the certification cost. When the certification cost is sufficiently high, certification should not be allowed, and so we end up with our benchmark model. When the certification cost is low, certification should be allowed and implemented as a de facto license where agents certify immediately. In no situation is it optimal to allow delayed certification, where agents do not certify immediately, but only after working and sending some signals.

Theorem 5 *Assuming a principal optimal equilibrium will occur, the price and certification standard that maximize ex ante social welfare are given by:*

1. $p = c + k\rho$ *and* $\underline{q} = p$ *if* $k \leq \frac{\mu_0^c - c}{\rho} - \frac{\mu_0 - c}{\rho(1 - F_0^-(c+\rho k))}$.
2. $p = c$ *and* $\underline{q} = \infty$ *if* $k \geq \frac{\mu_0^c - c}{\rho} - \frac{\mu_0 - c}{\rho(1 - F_0^-(c+\rho k))}$.

When $\underline{q} = p$, principals will want certification immediately, and when $\underline{q} = \infty$ no agents will ever be able to certify. Thus, there is no case where agents are allowed to certify but not forced to certify immediately. The reason why delayed certification is never optimal is that if it were implemented the pessimistic beliefs would require certification by agents very early and often, and in fact so early and often that certification destroys social welfare. To understand this intuitively, note that under pessimistic beliefs the mean of a non-certifying agent will be exactly equal to the price level. Thus, we have the same result as in the benchmark, where we are always kicking out agents who have expected qualities above c. And since $p > c$ in order to allow for certification, kicking out these agents will strictly reduce social welfare. Even worse, high-quality agents are also forced to pay for the cost of certification, destroying additional social value. Since the myopic principals would want the agent to certify at inefficient reputation levels, the value of certification gets entirely undercut.

Because certification should act as a de facto license under these types of beliefs, learning through the work history will be irrelevant and does not affect the social welfare. Agents that do not certify can never work, and agents that do certify never exit, so it does not matter what type of information the work history is sending. For any information process, the ex ante social welfare will be the same. When certification costs are low enough, de facto licenses will provide a higher social welfare than the no certification benchmark and thus should be implemented. In fact, for $k = 0$ we can achieve the first best outcome through setting $\underline{q} = p = c$, which results in all socially beneficial agents working forever, and all other agents exiting immediately. And as $k \to 0$, we can asymptotically achieve first best by setting the price and certification standard lower and lower. However, if certification costs are too high, then too much social welfare is lost through certification itself, and it is thus better to not allow for certification.

6.3 Socially optimal certification standards for optimistic beliefs

Now we will analyze the agent optimal equilibrium that features the optimistic beliefs mentioned earlier in Sect. 5.3.1. Since optimistic beliefs never result in de facto licenses if $\mu_0 > p$, the social welfare generated by these beliefs can be heavily dependent on the specific information process that the market has access to. In order for certification to occur, the price needs to be high enough to compensate for the cost of certification, so we need $p \geq c + k\rho$ or the result is no certification. Given that agents with qualities higher than $c+k\rho$ contribute positively by certifying instead of exiting, the certification standard $\underline{q}$ should be set equal to p no matter what the optimal p is. The inequality $p \geq c + k\rho$ need not be binding, because it may be optimal for the price to be raised so that certification occurs sooner. For instance, if the certification cost k is very low, then it is optimal to have p higher so that we can get certification earlier.

Also note that a de facto license can be implemented if the price is set at $p \geq \mu_0$, since then principals would never buy from non-certifying agents. The optimal way to implement a de facto license is to set $p = \underline{q} = \max(c + \rho k, \mu_0)$, for similar reasons as in the principal optimal case. However, a de facto license would result in all types of agents with qualities less than μ_0 but greater than c getting forced out of the market. Thus, it may be better not to have a de facto license, but delayed certification instead.

In general then, for k sufficiently low it will be optimal to have $\underline{q} = p \in [c+\rho k, \mu_0]$, where the exact value depends on the information process itself. This results in either delayed certification if $p < \mu_0$ or a de facto license if $p \geq \mu_0$. If k is too high, then we should set $p = c$ and $\underline{q} = \infty$ to ensure that no certification occurs. The exact cutoff value for k, as well as the exact values of the price and certification standard, will depend on the information process because that will influence the social welfare of any delayed certification scheme.

Given that delayed certification can be desirable in an agent optimal equilibrium, one may think that the social welfare under optimistic beliefs will always be higher than the social welfare under pessimistic beliefs, where delayed certification could never be optimal. However, optimistic beliefs also have a downside – de facto licenses are now much harder to implement because the price needs to be set at a very high level to do so. Under pessimistic beliefs, we could asymptotically get first best as $k \to 0$ by implementing a de facto license. But that is not the case with optimistic beliefs, as the next theorem shows. With optimistic beliefs, a de facto license cannot achieve first best, and delayed certification cannot achieve first best either for any positive k[12]. Given any value of k, we define $W_a(k, \mathcal{L})$ as the optimal *ex ante* social welfare in the

[12] When $k = 0$, agents do not mind certifying at an expected quality above the price, and so first best can be achieved in an agent optimal equilibrium.

agent optimal equilibrium. That is, this is the highest social welfare that can result in an agent optimal equilibrium from any choice of p and $\underline{q}$.

Theorem 6 *Suppose that the initial quality distribution $f_0(q)$ has positive density on the interval $[c-\epsilon, c+\epsilon]$ for some $\epsilon > 0$. Then for all $k > 0$, the value of the ex ante social welfare in an agent optimal equilibrium is strictly bounded away from the first best value,* $\sup_{k>0} W_a(k, \mathcal{L}) < \int_c^\infty \frac{q-c}{\rho} f_0(q) dq$.

Proof See Appendix. □

This theorem shows that given any information process, the *ex ante* social welfare is strictly bounded away from the optimal as costs become low, as long as it is initially possible for the agent to have a quality close to c. However, we note that if the information process itself become very informative, then the optimal social welfare can be asymptotically achieved. Define a *fully revealing* information process to be such that almost surely $\tau_a^* < \infty$ for all types of agents with $q < c$. This means that bad agents will for sure be forced to certify at some point, and thus will always get kicked out of the market in the long run. Such a property holds as long as the information process allows bad agents to be distinguished from good agents over time. For fully revealing information processes, faster information revelation will allow for the first best social welfare to be asymptotically achieved when certification costs are low. Formally, we say that an information process $\mathcal{L}'$ is *faster* than another process $\mathcal{L}$ if for some $n > 1$, $R'_q(t) = R_q(nt)$ for all t, q. The parameter n tells us how much faster one process is at sending information than another. For some information processes, faster information has a natural interpretation. For example, with the Brownian motion $dR_q(t) = qdt + \sigma_r dZ(t)$, a faster information process would correspond to dividing the variance of the Brownian motion by n. For Poisson processes, faster information would correspond to multiplying the arrival rates by n.

The reason that faster information revelation is helpful is with certification is because low-quality agents will get kicked out of the market very quickly, while high-quality agents can still stay in.[13] As the following proposition shows, first best can be asymptotically achieved if the certification costs become very low and the information arrives at a very fast speed. Thus, information revelation and certification can act in a complementary fashion, with more information increasing the welfare provided by certification.

Proposition 4 *Suppose that $\mathcal{L}$ is a fully revealing information process, and let $\{\mathcal{L}'_n\}$ be a sequence of information processes that are faster than $\mathcal{L}$ by the factor n. Then we have*

$$\lim_{n\to\infty} \sup_{k>0} W_a\left(k, p, \underline{q}, \mathcal{L}'_n\right) = \int_c^\infty \frac{q-c}{\rho} f_0(q) dq$$

Proof See Appendix. □

[13] Note that without certification, faster information cannot increase welfare as shown by Theorem 1.

On the other hand, if certification costs are high and a delayed certification scheme were attempted, faster information revelation could actually lower social welfare. When costs are high, the price and certification standard may need to be much higher than c to get agents to certify. But setting high certification standards will make certification kick out agents less efficiently than before because exiting agents will have higher quality levels. A faster information process compounds this problem by getting to the kick out point faster, thus lowering social welfare. In this case, it may be better not to implement certification.

7 Conclusion

This paper analyzed how two separate avenues of information, certification and work history, can interact to affect learning about the quality of an agent. We showed that the information provided by the work history alone cannot raise social surplus, making certification necessary even when the market can learn through the agent's work history. With certification, all equilibria will feature type independent certification strategies, and the various equilibria that maximize principal, agent, and social welfare can be ordered according to the equilibrium certification stopping time strategies. Principals will prefer the agent to certify at the earliest possible time, and thus, delayed certification by the agent will not be socially efficient. With pessimistic beliefs, only a de facto license type of certification can be socially beneficial. On the other hand, in the agent optimal case, the agent will delay certification for as long as possible. This is suboptimal if the certification costs are low, and it is socially beneficial to have the agent certify quickly. In this case, faster information revelation can increase social welfare, creating a complementarity between the reputational forces and certification.

There are several possible extensions for future research that could have interesting implications. One important case is to allow for variable prices that depend partially or even wholly on the agent's reputation. Such a change complicates the exit decision for the agent, because it may wish to keep working even if the price falls below its own reservation value. In extreme cases such as Bar-Isaac (2003), higher-quality agents may never exit, which means that there is no loss in efficiency in the long run. Thus, certification would only be useful to flush bad agents out of the market sooner, and so certification would be less valuable than in the current model.

Another interesting extension is to allow for moral hazard. For instance, the agent could increase the quality of its work by exerting effort, which would also send more positive information to the market. The agent may thus choose greater effort to increase its reputation instead of certifying. And after the agent does certify, its work incentives may drop quite significantly. Principals would anticipate this and therefore prefer the agent to wait longer instead of certifying quickly. So with moral hazard, delayed certification could now be socially beneficial in a principal optimal equilibrium. Likewise, certification times in socially optimal equilibrium may get delayed as well. Certification may thus become less beneficial with moral hazard, because it undercuts an agent's incentive to signal through effort.

8 Appendix

8.1 Proof of Theorem 1

First note that if the price is less than c, the agent would refuse to accept any offers, and so the social welfare is equal to 0 regardless of the information process. Thus, we only need to consider prices $p \geq c$.

Note that under any blind process, the agent's expected quality is never updated, and so since we assume that $\mu_o > p$ the agent will never stop working. The *ex ante* social surplus can thus be calculated as $\frac{\mu_0 - c}{\rho}$. For a general information process the market continues to hire any agent until its expected quality drops below p, and for an admissible process this happens at the first time that $\mu_t = p$. We can write out the *ex ante* expected social welfare for any information process as

$$\begin{aligned} W(p, \mathcal{L}) &= \int_{-\infty}^{\infty} \left[\int_0^{\infty} e^{-\rho t} (q - c) \, dt \right] f_0(q) \, dq - E_{q,t^*} \left[\int_{t^*}^{\infty} e^{-\rho t} (q - c) \, dt \right] \\ &= \int_{-\infty}^{\infty} \left[\int_0^{\infty} e^{-\rho t} (q - c) dt \right] f_0(q) \, dq - E_{t^*} \left[\int_{t^*}^{\infty} E_q \left[e^{-\rho t} (q - c) | t^* \right] dt \right] \\ &= \frac{\mu_0 - c}{\rho} - \frac{p - c}{\rho} E_{t^*} \left[e^{-\rho t^*} \right] \leq \frac{\mu_0 - c}{\rho} \end{aligned}$$

□

8.2 Proof of Theorem 2

Suppose the market believes that agents are following some (not necessarily type independent) certification strategy $\tilde{\tau}(q)$. Consider any time t' and history $\mathcal{H}_{t'}$, and let the expected quality for an agent that has not certified by t' be given by $\mu_{t'}^N \equiv E[q | \mathcal{H}_{t'}, \theta_{t'} = \phi]$. Note that for any market strategy beliefs, the value of $\mu_{t'}^N$ depends only on the work history $\mathcal{H}_{t'}$ and certification status $\theta_{t'}$, and not the agent's true quality. Thus, fixing the work history and certification status, all agents will have the same $\mu_{t'}^N$ regardless of their true quality. There are two possible cases: $\mu_{t'}^N > p$ or $\mu_{t'}^N \leq p$, and we show that for both cases, either all types of agents with qualities $q \geq \underline{q}$ will choose to certify at t' or no types of agents will choose to certify at t'.

First consider the case where $\mu_{t'}^N > p$. Then agents can still work even without certifying. The payoff of an agent that chooses to certify is given by $\frac{p - c}{\rho} - k$. This payoff is identical for all agents with quality $q \geq \underline{q}$ because of our assumption that $\underline{q} \geq p$, and so certified agents will never stop working. Now consider the alternate strategy of waiting until the time $\hat{t} \equiv \inf \left\{ t | \mu_t^N \leq p \right\}$ and then certifying. This strategy gives a payoff of

$$\int_0^{\hat{t}} e^{-\rho t} (p - c) \, dt - \left(e^{-\rho \hat{t}} \right) \left(\frac{p - c}{\rho} - k \right)$$

This alternate strategy gives a payoff higher than certifying immediately by $(1 - e^{-\rho \hat{t}}) * k$. So certifying at time t' is not optimal, and with this $\mu_{t'}^N$ all types of agents would choose not to certify.

Next consider the case where $\mu_{t'}^N \leq p$. An agent that does not certify will not be able to work at time t', and so receives a maximum payoff of

$$(1-\rho dt)\left(\frac{p-c}{\rho}-k\right)$$

This is the payoff the agent would receive if it certified at time $t'+dt$. Since no observations are made, the mean at a later time cannot be greater than p unless the agent were to certify at some time $t'+dt$. If the agent instead chooses to certify immediately at time t', it would get a payoff of

$$\frac{p-c}{\rho}-k > (1-\rho dt)\left(\frac{p-c}{\rho}-k\right)$$

Thus, certifying in the current time step would increase the payoff by $\rho dt\left(\frac{p-c}{\rho}-k\right)$. Therefore, all types of agents would choose to certify given this value of $\mu t'\ N$.

Since all agents with quality $q \geq \underline{q}$ would choose the same certification decision for any value of $\mu_{t'}^N$, every equilibrium must feature all agents with quality $q \geq \underline{q}$ utilizing the same certification strategy τ_ε^*. □

8.3 Proof of Proposition 1

Consider any equilibrium that requires the agent to certify at a time t' where $\mu_{t'} > p$. Now let us compare the payoffs to the agent against the equilibrium where the agent certifies at the first time t such that $\mu_t \leq p$. In the second equilibrium, at time t' the agent would instead delay certification until the time $t^* = \inf\{t|\mu_t = p\}$. Since the payoff from certification is the same regardless, the agent would be able to improve its payoff by the amount $(e^{-\rho t'} - e^{-\rho t^*})k$. Thus, no equilibrium that requires the agent to certify at a $\mu_t > p$ can be agent optimal. □

8.4 Proof of Proposition 2

We fix an arbitrary equilibrium and compute the social welfare flow payoff difference at the equilibrium certification time τ^* between certification and no certification. Without certification, the social welfare flow payoff is given by

$$E_{\tau^*}[q-c] = \int_{-\infty}^{\infty}(q-c)\, f_{\tau^*}(q)dq$$

With certification this would become

$$E_{\tau^*}[q-c] = \left(1-F_{\tau^*}^{-}\left(\underline{q}\right)\right)\int_{\underline{q}}^{\infty}(q-c)\, f_{\tau^*}(q|q \geq \underline{q})dq = \int_{\underline{q}}^{\infty}(q-c)\, f_{\tau^*}(q)dq$$

Thus, the difference in the two expectations is given by

$$\lim_{q' \nearrow \underline{q}} \int_{-\infty}^{q'} (q - c)\, f_{\tau^*}(q) dq$$

Certification has a higher flow payoff if and only if the difference in expectations plus the flow cost of certification is less than zero:

$$\lim_{q' \nearrow \underline{q}} \int_{-\infty}^{q'} (q - c)\, f_{\tau^*}(q) dq + \rho k \left(1 - F_{\tau^*}^{-}\left(\underline{q}\right)\right) \leq 0$$

Or equivalently $\left(\mu_{\tau^*}^{NC} - c - \rho k\right) F_{\tau^*}^{-}(\underline{q}) \leq -\rho k$, which leads to (1).

Now suppose for the sake of contradiction that (1) does not hold at the certification time τ^*. We will consider the following strategy that can be an equilibrium, and we show that it provides a higher social welfare than certifying at time τ^* if (1) does not hold at τ^*. Suppose instead that the agent keeps working until 1: the first $t > \tau^*$ such that (1) holds, or until 2: the first $t > \tau^*$ such that $\mu_t \leq p$, and then the agent certifies.

Note that the agent keeps working without certifying as long as the social welfare flow payoff from not certifying is greater than the flow payoff from certifying at τ^*. Once the agent certifies, the flow payoffs are the same as with certifying at τ^*. Thus, under this alternate strategy the flow payoffs can never be less than under certifying at τ^*. Therefore, the total social welfare generated must be higher as well. □

8.5 Proof of Proposition 3

The short run time t principal's utility is given by $\mu_t - p = E_t[q - p]$, so the principal prefers certification if and only if this expectation with certification is higher than the expectation without certification. Without certification, we have

$$E_t\left[q - p\right] = \int_{-\infty}^{\infty} (q - p)\, f_t(q) dq$$

With certification this would become

$$E_t\left[q - p\right] = \left(1 - F_t^{-}\left(\underline{q}\right)\right) \int_{\underline{q}}^{\infty} (q - p)\, f_t(q | q \geq \underline{q}) dq = \int_{\underline{q}}^{\infty} (q - p)\, f_t(q) dq$$

Thus, the difference in the two expectations is given by

$$\lim_{q' \nearrow \underline{q}} \int_{-\infty}^{q'} (q - p)\, f_t(q) dq$$

Certification is preferred if and only if the above term is less than zero.

$$\int_{-\infty}^{p} (q - p)\, f_t(q) dq + \lim_{q' \nearrow \underline{q}} \int_{p}^{q'} (q - p)\, f_t(q) dq \leq 0$$

This means that the benefit of removing bad agents (qualities below p) from the market outweighs the costs of removing the good agents (qualities above p). Or equivalently:

$$\left(\mu_t^{NC} - p\right) F_t^-(\underline{q}) \leq 0$$
$$\mu_t^{NC} \leq p$$

This results in Eq. (2) in the Proposition. Since at each time t, that time t principal wishes for the agent to certify if and only if this equation holds, the resulting certification strategy will feature the agent certifying at the first moment that this equation holds. By the corollary to theorem 2, we know that such a certification strategy can be an equilibrium. □

8.6 Proof of Theorem 4

First note that if $p < c + \rho k$ there can be no certification in equilibrium. The optimal way to implement no certification is to set $p = c$, $\underline{q} = \infty$ from Theorem 1. This results in a social welfare of $\frac{\mu_0 - c}{\rho}$ for any admissible information process. Next, suppose that we wish to allow certification in equilibrium. Thus, we need to set $p \geq c + \rho k$. We show that for any $p \geq c + \rho k$ and any $\underline{q} \geq p$, we can achieve at least as high of a welfare by setting $\underline{q} = p = c + \rho k$. Given the first set of parameters, denote the socially optimal certification stopping time by τ_s^*. But under the second set of parameters, τ_s^* can also be implemented because $\mu_{\tau_s^*}^{NC} \leq p$ will also hold at any τ_s^* (recall that $\mu_t^{NC} \leq p$ for all t if $\underline{q} = p$), and $\mu_t > p$ for all $t < \tau_s^*$. In addition, once implemented the social welfare provided by certification will be at least as high, because all types of agents with qualities $\underline{q} > c + \rho k$ contribute positively to social welfare by certifying instead of exiting. Thus, the social welfare with $\underline{q} = p = c + \rho k$ must be at least as high as with any other standard. □

8.7 Proof of Theorem 5

This proof will proceed in several steps. Note that certification can be broken up into three possible types: immediate certification at $t = 0$, delayed certification that takes place at some $t > 0$, and no certification for all times. Which type of certification results will depend on the specific values of $\underline{q}$ and p. We prove that under pessimistic principal beliefs, delayed certification is never optimal. Then, we characterize the social welfare generated by immediate and no certification. We prove that immediate certification and no certification can both be optimal depending on how high the certification cost is.

First we show that implementing delayed certification is never socially optimal. If the price is set lower than $c + \rho k$, then certification can never occur because agents would prefer to exit than certify. Thus, for any type of certification to be implemented, we must have $p \geq c + \rho k$. Next, note that under pessimistic principal beliefs, the certification standard $\underline{q}$ must be set high enough such that $\mu_0^{NC} > p$ because otherwise agents would be expected and thus forced to certify immediately in a principal optimal equilibrium by Proposition 3. In particular, this requires that $\underline{q} > p$.

Now we analyze the social welfare generated by any delayed certification scheme, and we show that the welfare is strictly less than under no certification. Let $\hat{t} = \inf \left\{t | \mu_t^{NC} \leq p\right\}$. In a principal optimal equilibrium, $\hat{t}$ is the time at which certification would occur. Note that admissibility implies that at t, the truncated expected mean $\mu_{\hat{t}}^{NC} = p$. For any p and $\underline{q}$ that satisfy the above conditions, we can compute the resulting social welfare as:

$$\begin{aligned} W(p) &= \int_{-\infty}^{\infty} \left[\int_0^{\infty} e^{-\rho t} (q-c)\, dt\right] f_0(q)\, dq - E_{\hat{t}}\left[ke^{-\rho \hat{t}}\left(1 - F_{\hat{t}}^{-}\left(\underline{q}\right)\right)\right. \\ &\quad \left. + F_{\hat{t}}^{-}\left(\underline{q}\right) \int_{\hat{t}}^{\infty} e^{-\rho t}\left(\mu_{\hat{t}}^{NC} - c\right) dt\right] \\ &= \frac{\mu_0 - c}{\rho} - \left(kE_{\hat{t}}\left[e^{-\rho\hat{t}}\left(1 - F_{\hat{t}}^{-}\left(\underline{q}\right)\right)\right] + \frac{p-c}{\rho} E_{\hat{t}}\left[e^{-\rho \hat{t}} F_{\hat{t}}^{-}\left(\underline{q}\right)\right]\right) \\ &\leq \frac{\mu_0 - c}{\rho} \end{aligned}$$

This proves that the social welfare of any delayed certification scheme is bounded above by setting $p = c$ and $\underline{q} = \infty$, which results in no certification. From the blind boundedness theorem, we know that the payoff of such a scheme is exactly $\frac{\mu_0 - c}{\rho}$.

Now, fixing a $\underline{q}$, the welfare provided by a de facto license is given by the expression $\int_{\underline{q}}^{\infty} (\frac{q-c}{\rho} - k) f_0(q)\, dq$. Given a k, the optimal certification standard is $\underline{q} = c + \rho k$. The reason is that any agent that certifies will give a social welfare of $\frac{q-c}{\rho} - k$, and this is the quality where this expression is equal to zero. Any agent with a quality higher than this amount contributes positively to welfare by certifying. Since we require that $p \leq \underline{q}$ and we need $p \geq c + \rho k$ for certification to occur, this implies that we need to set $p = c + \rho k$ in order to implement immediate certification. Thus, the highest *ex ante* surplus generated by any immediate certification scheme is $\int_{c+\rho k}^{\infty} (\frac{q-c}{\rho} - k) f_0(q)\, dq = \left(\frac{\mu_0^C - c}{\rho} - k\right)\left(1 - F_0^{-}(c + \rho k)\right)$.

Thus, to see whether immediate certification is better, or whether no certification is better, we need to see which of the two surpluses is higher. This depends on the value of k, and specifically the cutoff value will be given by $k^* = \frac{\mu_0^C - c}{\rho} - \frac{\mu_0 - c}{\rho\left(1 - F_0^{-}(c+\rho k)\right)}$. □

8.8 Proof of Theorem 6

First note that if either no certification or a de facto license is implemented, the social welfare will always be bounded away from the social optimal. With no certification, the welfare always equals the benchmark welfare for any k, and with a de facto license, the price and standard have to be set to at least μ_0. The social welfare of the de facto license is thus equal to $\int_{\mu_0}^{\infty} (\frac{q-c}{\rho} - k) f_0(q)\, dq$, which is bounded away from the first best welfare, $\int_c^{\infty} (\frac{q-c}{\rho}) f_0(q)\, dq$, for any value of k.

Thus, in order to get asymptotic efficiency as $k \to 0$, we need to do it through a delayed certification scheme. We now show that the social welfare of any delayed certification scheme will also be bounded away from first best. First note that if the standard $\underline{q} \nrightarrow c$, then first best cannot be achieved. The reason is that before certification occurs, we are losing welfare from letting bad agents work, and after certification occurs we also lose welfare since not all good agents are working. Then assume that $p, \underline{q} \to c$. Fix any path of the expected mean for the agent. Let $t_c^* = \inf\{t | \mu_t \leq p; p = c\}$ be the stopping time of this path in the limit as the price approaches c. In order to achieve first best as $k \to 0$, we must have $t_c^* = 0$ or else bad agents will be working for some stretch of time. But for any admissible information process this is impossible since $\mu_0 > p$. Thus, t_c^* is strictly above 0, and so delayed certification cannot achieve first best. □

8.9 Proof of Proposition 4

From the proof of Theorem 6, we see that immediate certification and no certification cannot asymptotically achieve first best as the information speed increases, because the speed of the reputational mechanism does not affect social welfare in either case. So we must show that the social welfare of a delayed certification scheme approaches first best. We wish to show that as the speed becomes very high, $t_c^*(q) \to 0 \, \forall q < c$, because doing so means that all agents who have socially inefficient qualities will be kicked out extremely quickly, and the good agents will be able to stay in forever (perhaps paying the certification cost that asymptotically approaches 0). Since the process is fully revealing, almost surely $t_c^*(q) < \infty$ for agents with quality $q < c$. Then as n gets large, agents will be kicked out at time $\frac{t_c^*(q)}{n}$ instead, which approaches 0 for all finite $t_c^*(q)$. Thus, $t_c^*(q) \to 0 \, \forall q < c$ and so delayed certification asymptotically achieves the first best outcome. □

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