A New Look at Multi-user Power Control Games

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Abstract—**This paper considers the problem of how to allocate power among competing users sharing a frequency-selective interference channel. We model the interaction between selfish users as a non-cooperative game and analyze their strategic behavior. As opposed to the existing iterative water-filling algorithm, this paper introduces the Stackelberg equilibrium and shows the existence of this equilibrium for the investigated non-cooperative game. We model the two-user case as a bi-level programming problem and derive the necessary optimality conditions. It is analytically shown that a user can improve its performance if it knows the channel state information and the response strategy of the competing user. Due to computationally prohibitive nature of the optimal solution, a practical low-complexity approach is proposed based on the intuition gained from the necessary conditions. Numerical simulations verify the performance improvements.**

Keywords—**interference channel, power control, Stackelberg equilibrium , non-cooperative game theory**

I. INTRODUCTION

The multi-user power control problem in frequency-selective interference channels was investigated from the game theoretic point of view [1]-[7]. In these multi-user power control games, users are modeled as players with individual goals and strategies. They are competing and cooperating with each other until they agree on an acceptable resource allocation outcome. Existing research can be categorized into two types, *non-cooperative* games and *cooperative* games.

First, the formulation of the multi-user environment as a non-cooperative game has appeared in several recent works [1] [2]. An iterative water-filling (IW) algorithm has been proposed to mitigate the mutual interference and optimize the performance without the need for a central controller [1]. Users deploying the IW algorithm are assumed to be myopic in the sense that they try to maximize their achievable rate at every decision stage until a Nash equilibrium is reached.

Second, there also have been a number of related works studying dynamic spectrum management in the setting of cooperative games [3]-[7]. The Optimal Spectrum Balancing algorithm [4], the Iterative Spectrum Balancing algorithm [5][6], and the autonomous spectrum balancing technique [7] are proposed to achieve near-optimal performance. These works focus on cooperative games, because it has been well-known that the IW algorithm may lead to Pareto-inefficient solutions [8], i.e. selfishness is detrimental in the interference channel.

In short, previous research mainly concentrates on studying the existence and performance of Nash equilibrium in non-cooperative games and developing efficient algorithms to cooperatively approach the Pareto boundary. However, an important intrinsic dimension of this information-decentralized multi-user interaction still remains unexplored. Prior research does not consider the users' availability of information about other users. How should a selfish user behave if it gets the information about competing users? Can it achieve a better performance rather than adopting the IW algorithm? It is important to look at these scenarios in order to access the significance of information availability from the users' viewpoint and shows why the selfish users have incentives to learn their environment and adapt their strategies [9]. A "clever" user with more information in this non-cooperative game should be able to gain more benefits [10].

In this paper, we discuss how rational users should behave in non-cooperative power control games. As opposed to previous approaches considering myopic users [1], we focus on the strategic behavior of the selfish user with additional information about its competing users. We explicitly show that a strategic user can gain more benefits if it takes its competitors' information and strategies into account. The concept of Stackelberg equilibrium is introduced in order to characterize the strategic behavior of a user by considering the response of its competing users. Particularly, for the two-user case, we formulate a bi-level programming problem and derive the necessary optimality conditions. Inspired by the optimality conditions, we provide a low-complexity solution of the original intractable non-convex optimization problem.

This paper is organized as follows. Section II presents the non-cooperative game model and introduces the concept of Stackelberg equilibrium. In Section III, using a simple two-user example, we define the strategic behavior of a user to be a bi-level programming problem, and derive the necessary optimality conditions. Section IV discusses the complexity of the optimal solution and proposes a practical sub-optimal approach. Simulations show that a strategic user can achieve substantial performance improvement compared to the case in which users are myopic. Conclusions are drawn in Section V.

II. SYSTEM MODEL

A. System Description

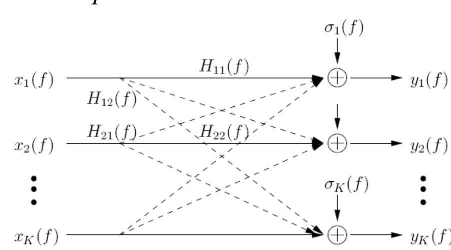

Fig. 1. Gaussian interference channel model.

Fig. 1 illustrates a frequency-selective Gaussian interference channel model. There are K transmitters and K receivers in the system. Each transmitter and receiver pair can be viewed as a player (or user). The transfer function of the channel from transmitter i to receiver j is denoted as $H_{ij}(f)$, where

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$0 \le f \le F_s$. The noise power spectral density (PSD) that receiver k experiences is denoted as $\sigma_k(f)$. Denote the player k's transmit PSD as $P_k(f)$. For user k, the transmit PSD is subject to its power constraint:

$$\int_0^{F_s} P_k(f)df \le \mathbf{P_k}. \quad (1)$$

For a fixed $P_k(f)$, if treating interference as noise, user k can achieve the following data rate:

$$R_k = \int_0^{F_s} \log_2\left(1+\frac{P_k(f)|H_{kk}(f)|^2}{\sigma_k(f)+\sum_{j\ne k} P_j(f)|H_{jk}(f)|^2}\right)df. \quad (2)$$

The payoffs for the players are the respective achievable data rates and their strategies are to determine their own transmit PSD. To fully capture the performance tradeoff in the system, the concept of a rate region is defined as

$$\mathcal{R} = \left\{(R_1,\cdots,R_K) : \exists(P_1(f),\cdots,P_K(f)) \; satisfying \; (1) \; and \; (2)\right\}. \quad (3)$$

Because of the non-convexity in the capacity expression, the complexity of optimal solutions in finding the rate region is prohibitively high. Existing works aim to efficiently approach the Pareto boundary of the rate region and provide near-optimal performance via the cooperation among users [4]-[7]. On the other hand, the interference channel can be modeled as a non-cooperative game among multiple competing users. Instead of solving the optimization problem globally, the IW algorithm models the users as selfish myopic decision makers [1]. This means that they optimize their transmit PSD by water-filling and compete to increase their transmission data rates with the sole objective of maximizing their own performance regardless of all the others. Under a wide range of realistic conditions [1][12], the existence and uniqueness of the Nash equilibrium is proved and can be obtained by the IW algorithm. This gives substantial performance improvements over static spectrum management algorithms.

We still concentrate on the non-cooperative game setting in this paper. However, unlike the IW algorithm, in which users are assumed be myopic, i.e., they shortsightedly their update actions without considering the long-term impacts of taking these action, we study the problem of how a strategic user should behave rather than taking myopic action.

B. Stackelberg Equilibrium

Game theory studies the interaction of rational players. Let $\mathcal{G} = [\mathcal{K},\{\mathcal{A}_k\},U_k]$ represent a game where $\mathcal{K}=\{1,\cdots,K\}$ is the set of players, $\mathcal{A}_k$ is the set of actions available to user k, and U_k is the utility of user k. In our power control game, $\mathcal{A}_k$ is the transmit PSD satisfying the constraint in (1), and U_k is user k's achievable rate R_k. Recall that the Nash equilibrium is defined to be any $\left(a_1^*,\cdots,a_K^*\right)$ satisfying

$$U_k\left(a_k^*,a_{-k}^*\right) \ge U_k\left(a_k,a_{-k}^*\right) \; for \; all \; a_k \in \mathcal{A}_k \; and \; k=1,\cdots,K, \quad (4)$$

where $a_{-k}^* = \left(a_1^*,\cdots,a_{k-1}^*,a_{k+1}^*,\cdots,a_K^*\right)$ [11].

The Nash equilibrium is the best response only in a competitive optimality sense [1]. The Stackelberg equilibrium is a best response when a hierarchy exists between users [11], i.e. one of agents is the *leader* and the remaining ones are *followers*. Stackelberg equilibrium prescribes an optimal strategy for the leader if its followers always react by playing their Nash equilibrium strategies in the smaller sub-game. Let $NE(a_k)$ be the Nash equilibrium strategy of the remaining players if player k chooses to play a_k. The strategy profile $\left(a_k^*, NE\left(a_k^*\right)\right)$ is a Stackelberg equilibrium with user k leading iff

$$U_k\left(a_k^*,NE\left(a_k^*\right)\right) \ge U_k\left(a_k,NE(a_k)\right), \forall a_k \in \mathcal{A}_k. \quad (5)$$

The following theorem establishes the existence of the Stackelberg equilibrium in this multi-user power control game.

Theorem 1: In realistic channel settings, e.g., arbitrary symmetric interference environment and diagonally dominant asymmetric channel with any number of users [12], the Stackelberg equilibrium always exists.

Proof: First, we can show that R_k is bounded because

$$0 \le R_k \le \int_0^{F_s} \log_2\left(1+\frac{P_k^*(f)|H_{kk}(f)|^2}{\sigma_k(f)}\right)df, \quad (6)$$

where $P_k^*(f) = \left(\lambda - \sigma_k(f)/|H_{kk}(f)|^2\right)^+$ is the water-filling solution, $(x)^+ = \max(0,x)$, and λ is a constant satisfying the constraint in (1) with equality.

Second, we know from [12] that under a wide range of realistic channel, e.g., arbitrary symmetric interference environment and diagonally dominant asymmetric channel with any number of users, the existence and uniqueness of Nash equilibrium is always guaranteed. In other words, whatever form of $P_k^*(f) \in \mathcal{A}_k$ user k chooses, the remaining users will regard user k's transmit PSD as background noise PSD, i.e. $\tilde{\sigma}_j(f) = \sigma_j(f) + |H_{jk}(f)|^2 P_k(f), j \ne k$, and the convergence to a unique Nash equilibrium always holds, i.e. a single $NE(a_k)$ exists for $\forall a_k = P_k^*(f) \in \mathcal{A}_k$.

To summarize, since R_k is bounded, and for $\forall a_k \in \mathcal{A}_k$, the remaining players' action will always lead to a unique Nash equilibrium, we have

$$0 \le U_k\left(a_k^*,NE\left(a_k^*\right)\right) \le \int_0^{F_s} \log_2\left(1+\frac{P_k^*(f)|H_{kk}(f)|^2}{\sigma_k(f)}\right)df, \quad (7)$$

Therefore, there exist $a_k^* \in \mathcal{A}_k$ such that $U_k\left(a_k^*,NE\left(a_k^*\right)\right) = \sup_{a_k \in \mathcal{A}_k}\left\{U_k\left(a_k,NE(a_k)\right)\right\}$. We can conclude that the Stackelberg equilibrium always exists for this power control game. ■

In our problem, the requirement of hierarchic actions can be removed if the strategic user knows that all the other users use the IW algorithm. Note that we assume only one strategic user exists in this game. The strategic user will always regard itself as the leader and perform the Stackelberg-strategy, and the others will act their best responses until converging to the equilibrium.

III. Problem Formulation

A. A Bi-level Programming Formulation

The Stackelberg equilibrium applied to the two-user power control game can be represented by a bi-level mathematical problem [13], in which the strategic user acts as the leader and the other user behaves as the follower. The leader chooses an appropriate transmit PSD to maximize its own benefits by considering the response of its follower, who always reacts to the given transmit PSD of the leader by water-filling over the entire frequency band. The problem can be formulated as

$$\begin{aligned}
&\max_{P_1(f),P_2(f)} \int_0^{F_s} \ln\left(1+\frac{P_1(f)}{N_1(f)+\alpha_2(f)P_2(f)}\right) df && (a)\\
&s.t. \int_0^{F_s} P_1(f)\, df \le \mathrm{P}_1 && (b)\\
&\quad P_1(f) \ge 0 && (c)\\
&\quad P_2(f) = \arg\max_{P_2'(f)} \int_0^{F_s} \ln\left(1+\frac{P_2'(f)}{N_2(f)+\alpha_1(f)P_1(f)}\right) df && (d)\\
&\quad s.t.\ P_2'(f) \ge 0 && (e)\\
&\quad\quad \int_0^{F_s} P_2'(f)\, df \le \mathrm{P}_2 && (f)
\end{aligned} \quad (8)$$

where $N_1(f)=\sigma_1(f)/|H_{11}(f)|^2$, $\alpha_1(f)=|H_{12}(f)|^2/|H_{22}(f)|^2$, $N_2(f)=\sigma_2(f)/|H_{22}(f)|^2$, $\alpha_2(f)=|H_{21}(f)|^2/|H_{11}(f)|^2$. The sub-problem (8.a)-(c) is called the *upper-level problem* and (8.d)-(f) corresponds to the *lower-level problem*.

The bi-level programming formulation is different from the IW approach. By letting $P_1(f)$ and $P_2(f)$ to be the individual transmit PSD of the IW algorithm, we can see that the Nash equilibrium actually gives the lower bound of the problem (8). By taking the opponent's reaction into account, the user can improve the myopic behavior of the IW approach and improve its performance. Note that in order to achieve the Stackelberg equilibrium, the complete information of the game is indispensable, which includes the other user's channel condition, $N_2(f)$ and $\alpha_2(f)$, and best response strategy. Possible ways of acquiring such information include channel state estimation and learning [9][16].

Noting that the lower-level problem is a standard convex programming problem, KKT conditions are necessary and sufficient for the lower-level problem to achieve the optimum. Therefore, we can replace the lower-level problem by its KKT conditions, leading to the single-level reformulation:

$$\begin{aligned}
&\max_{P_1(f),P_2(f),\lambda_2(f),K_2} \int_0^{F_s} \ln\left(1+\frac{P_1(f)}{N_1(f)+\alpha_2(f)P_2(f)}\right) df\\
&s.t.\ \int_0^{F_s} P_1(f)\, df \le \mathrm{P}_1,\ \int_0^{F_s} P_2(f)\, df = \mathrm{P}_2\\
&\quad P_1(f)\ge 0,\ \lambda_2(f)\ge 0,\ P_2(f)\ge 0,\ K_2>0\\
&\quad \lambda_2(f)P_2(f)=0\\
&\quad P_2(f)=\frac{1}{K_2-\lambda_2(f)}-N_2(f)-\alpha_1(f)P_1(f)
\end{aligned} \quad (9)$$

Note that here we assume that the myopic user will always choose to transmit at its maximum power, i.e. $\int_0^{F_s} P_2(f)\, df = \mathrm{P}_2$ and $K_2>0$. In the following, we will investigate (9). However, the above mathematical problem is not easy to solve because of the non-convexities that occur in the Lagrangian constraints of the lower-level problem. Therefore, we study the necessary optimality conditions first, and then develop a sub-optimal solution using intuition gained from the derived conditions.

B. Necessary Conditions of Optimality

Although the problem in (9) is non-convex, the KKT conditions are still necessary for the optimal solution [14]. The Lagrangian function of (9) can be written as a function of $P_1(f),P_2(f),\lambda_2(f),K_2$:

$$\begin{aligned}
&\mathcal{L}\left(P_1(f),P_2(f),\lambda_2(f),K_2,\boldsymbol{\mu}(f),K_1',K_2',K_3'\right)=\\
&\int_0^{F_s}\ln\left(1+\frac{P_1(f)}{N_1(f)+\alpha_2(f)P_2(f)}\right)df-K_1'\left(\int_0^{F_s}P_1(f)\,df-\mathrm{P}_1\right)\\
&+\int_0^{F_s}\mu_1(f)P_1(f)df+\int_0^{F_s}\mu_2(f)\lambda_2(f)df+\int_0^{F_s}\mu_3(f)P_2(f)\,df\\
&+\int_0^{F_s}\mu_4(f)\lambda_2(f)P_2(f)\,df+K_2'\left(\int_0^{F_s}P_2(f)\,df-\mathrm{P}_2\right)+K_3'K_2\\
&+\int_0^{F_s}\mu_5(f)\left[P_2(f)-\frac{1}{K_2-\lambda_2(f)}+N_2(f)+\alpha_1(f)P_1(f)\right]df
\end{aligned} \quad (10)$$

where $\mu_1(f),\mu_2(f),\mu_3(f),\mu_4(f),\mu_5(f),K_1',K_2'$, and K_3' are Lagrangian multipliers. Table I shows the relationship between the constraints and the multipliers.

Constraints of the primal problem	Multipliers
$\int_0^{F_s} P_1(f)\,df \le \mathrm{P}_1$	$K_1' \ge 0$
$P_1(f)\ge 0$	$\mu_1(f)\ge 0$
$\lambda_2(f)\ge 0$	$\mu_2(f)\ge 0$
$P_2(f)\ge 0$	$\mu_3(f)\ge 0$
$\lambda_2(f)P_2(f)=0$	$\mu_4(f)$
$\int_0^{F_s} P_2(f)\,df = \mathrm{P}_2$	K_2'
$K_2>0$	$K_3'\ge 0$
$P_2(f)=\frac{1}{K_2-\lambda_2(f)}-N_2(f)-\alpha_1(f)P_1(f)$	$\mu_5(f)$

Table I. Lagrangian multipliers for the problem in (9).

Taking the derivative with respect to the primal variables in (9) gives part of the necessary KKT conditions:

$$\begin{aligned}
&\frac{1}{N_1(f)+\alpha_2(f)P_2(f)+P_1(f)}=K_1'-\mu_1(f)-\alpha_1(f)\mu_5(f) && (a)\\
&\mu_2(f)+\mu_4(f)P_2(f)-\mu_5(f)\cdot\frac{1}{\left(K_2-\lambda_2(f)\right)^2}=0 && (b)\\
&\frac{\alpha_2(f)P_1(f)}{\left(N_1(f)+\alpha_2(f)P_2(f)\right)\left(N_1(f)+\alpha_2(f)P_2(f)+P_1(f)\right)} && (c)\\
&\qquad=\mu_3(f)+\mu_4(f)\lambda_2(f)+\mu_5(f)+K_2'\\
&\int_0^{F_s}\mu_5(f)\cdot\frac{1}{\left(K_2-\lambda_2(f)\right)^2}df=0 && (d)
\end{aligned} \quad (11)$$

In the four equalities above, Eq. (11.a) describes the summation of the overall PSD level experienced by the user 1, which is a flat water-level in the IW algorithm. Eq. (11.c) gives user 1's signal to interference-and-noise ratio (SINR), i.e. $P_1(f)/\left(N_1(f)+\alpha_2(f)P_2(f)\right)$, at the optimum. Eq. (11.b) and (d) provide additional constraints over the primal and dual variables. Note that (11.d) holds because we always have $K_3'=0$. The remaining parts of the necessary KKT conditions are given by the constraints of the primal and dual variables in Table I, and complementary slackness [14].

Based on the necessary conditions, some key remarks can be made. The details of the derivation are omitted here.

Remark 1 : The Nash equilibrium achieved by the IW algorithm may not satisfy the necessary KKT conditions. Therefore, it may not solve the problem in (8).

Remark 2 : Non-Nash-equilibrium strategies may satisfy the necessary optimality conditions and solve the problem in (8), because it is possible that there exist primal and dual variables with $\mu_5(f) \neq 0, \exists f \in [0, F_s]$, which form non-Nash-equilibrium strategies and satisfy (11).

Remark 3 : If $P_1(f) > 0$ and $\lambda_2(f) > 0$, it always holds:

$$\frac{1}{N_1(f) + P_1(f)} = K_1' . \tag{12}$$

Note that $\lambda_2(f) > 0$ leads to $P_2(f) = 0$. In other words, for the strategic user, waterfilling gives the optimal power allocation within the interference-free frequency band.

Remark 4: For a non-Nash-equilibrium strategy to satisfy the necessary conditions, it is impossible to have only these two power allocation patterns, $P_1(f) > 0, \lambda_2(f) > 0 (\Rightarrow P_2(f) = 0)$ and $P_1(f) = 0, P_2(f) > 0$, over $[0, F_s]$, i.e. user 1 can still adjust its power allocation and increase its achievable rate.

IV. A LOW-COMPLEXITY SOLUTION

A. Optimal Solution

Since the optimization problem in (8) is non-convex, it generally can only be solved through an exhaustive search. A possible exhaustive search is to divide the whole frequency band into $N = F_s / \Delta_f$ bins. Define user *k*'s transmit power in the *i*-th frequency bin to be s_k^i and the granularity in the transmit PSD as Δ_P. The value of s_k^i can now be limited to the set $\{0, \Delta_P, \cdots, \mathbf{P_k}\}$. By performing an exhaustive search of the all possible combinations, the optimum could be found. Therefore, such a exhaustive search in $(s_k^1, \cdots, s_k^N)$ has a overall complexity of $\mathcal{O}((\mathbf{P_k}/\Delta_P)^N)$. Generally speaking, in order to approximate the optimal solution, we need to divide the frequency band into small bins, i.e., N will be sufficiently large. Therefore, to reduce the computation complexity, we propose a sub-optimal approach in the following subsection based on the necessary optimality conditions.

B. A Low-Complexity Sub-optimal Approach

From the necessary conditions in (11), we have

$$R_1 = \int_0^{F_s} \ln\left|1 + \frac{\mu_3(f) + \mu_4(f)\lambda_2(f) + \mu_5(f) + K_2'}{\alpha_2(f)\left[K_1' - \mu_1(f) - \alpha_1(f)\mu_5(f)\right]}\right| df . \tag{13}$$

For the integration over $[0, F_s]$ in (13) to be large, we expect that $\mu_5(f) > 0$ when $\alpha_2(f)$ is small and $\alpha_1(f)$ is large. In other words, user 1 should allocate its power in such that, at the frequency band it occupies, the maximal rate is achieved with minimal noise and interference of user 2, i.e. $N_1(f)$ and $\alpha_2(f)$ are small. On the other hand, if user 2 to avoid some frequency channels, we expect that, in those channels, user 2 experiences weak channel condition and strong noise and interference, i.e. $N_2(f)$ and $\alpha_1(f)$ are large.

ALGORITHM 1 SUBOPTIMAL POWER ALLOCATION STRATEGY[1]

input: $N_1(f), N_2(f), \alpha_1(f), \alpha_2(f), \mathrm{P}_1, \mathrm{P}_2$

initialization: $K^1 = \mathrm{P}_1, K^2 = 0, R_1' = 0, N = F_s/\Delta_f, \mathcal{F}_1 = \varnothing,$
$\mathcal{F}_2 = \{1, 2, \cdots, N\}, \mathcal{F} = \{1, 2, \cdots, N\}, flag = 1$

procedure:
Calculate $P_1^{nash}(f)$ and R_1^{nash} of the IW algorithm
while $flag = 1$ **do**
1) $M(f_s) = \dfrac{N_2(f_s) + \alpha_1(f_s)K^1}{N_1(f_s) + \alpha_2(f_s)K^2}, \forall f_s \in \mathcal{F}_2$
2) $f_s^{\max} = \arg\max_{f_s \in \mathcal{F}_2} M(f_s), \mathcal{F}_1' = \{f_s^{\max}\} \cup \mathcal{F}_1,$
$\mathcal{F}_2' = \mathcal{F}_2 \setminus \{f_s^{\max}\}, f_s'^{\max} = \arg\max_{f_s \in \mathcal{F}_2'} M(f_s)$
3) $P_1(f) = waterfilling(\mathbf{P_1}, \mathcal{F}_1', N_1(f_s))$,
$P_2(f) = waterfilling(\mathbf{P_2}, \mathcal{F}, N_2(f) + \alpha_1(f)P_1(f))$
4) $R_1'' = rate(P_1(f), N_1(f) + \alpha_2(f)P_2(f))$,
$R_2' = rate(P_2(f), N_2(f) + \alpha_1(f)P_1(f))$
if $R_1'' \geq R_1'$
$\mathcal{F}_1 = \mathcal{F}_1', \mathcal{F}_2 = \mathcal{F}_2', K^1 = P_1(f_s^{\max}) \cdot |\mathcal{F}_1| / (|\mathcal{F}_1| + 1),$
$K^2 = P_2(f_s'^{\max}) \cdot (|\mathcal{F}_2| + 1) / |\mathcal{F}_2|, R_1' = R_1''$
end if
if $R_1'' < R_1'$ or $\mathcal{F}_2' = \varnothing$
$flag = 0$
end if
end while
if $R_1^{nash} < R_1'$
return $P_1^{nash}(f)$ and R_1^{nash}
else
return $P_1(f) = waterfilling(\mathbf{P_1}, \mathcal{F}_1, N_1(f_s))$ and R_1'
end if
end procedure

Based on the arguments above, we develop a sub-optimal power allocation strategy and summarize it in Algorithm 1. We propose a metric $M(f_s)$, which is defined as the ratio between the noise and interference PSD that the strategic user and its competing user experience at frequency bin $f = f_s$. The value of $M(f_s)$ reflects the incentive of the strategic user to occupy the frequency bin $f = f_s$. The basic idea is to rank the frequency bins based on this metric. Initially, user 1 owns no frequency bins and all the bins belong to user 2. According to Remark 3, user 1 water-fills its allocated frequency bins. It continues moving the frequency bin with the largest value of $M(f_s)$ from user 2 to user 1 until no rate improvement in R_1 can be achieved. This procedure is proposed based on the observation in Remark 4 that for sufficiently large P_1, initially, $\exists f' \in \mathcal{F}_1$, $f'' \in \mathcal{F}_2$ satisfying $P_1(f') > 0, P_1(f'') = 0, P_2(f'') > 0$ and $N_1(f') + \alpha_2(f')P_2(f') + P_1(f') > N_1(f'') + \alpha_2(f'')P_2(f'') \Rightarrow \lambda_2(f') > 0$, adjusting $P_1(f)$ might further improve the performance.

If the achievable rate of the above procedure is less than the IW approach, user 1 will choose the Nash-equilibrium strategy, which guarantees that the performance of Algorithm 1 is no

[1] $waterfilling(\mathbf{P}, \mathcal{F}, N(f))$ denotes the water-filling transmit PSD with power constraint $\mathbf{P}$ in frequency set $\mathcal{F}$ treating $N(f)$ as noise, and $rate(P(f), N(f))$ denotes the achievable rate of transmit PSD $P(f)$ with respect to the noise PSD $N(f)$.

worse than the IW algorithm. The complexity of Algorithm 1 is only $\mathcal{O}\left(2F_s / \Delta_f\right)$, which reduces the complexity by a factor of $\mathcal{O}((\mathbf{P_k} / \Delta_P)^{(F_s / \Delta_f)} / (2F_s / \Delta_f))$ compared with the optimal solution, which is considerably large if $\Delta_f \to 0$ and $\Delta_P \to 0$.

C. Simulation Results

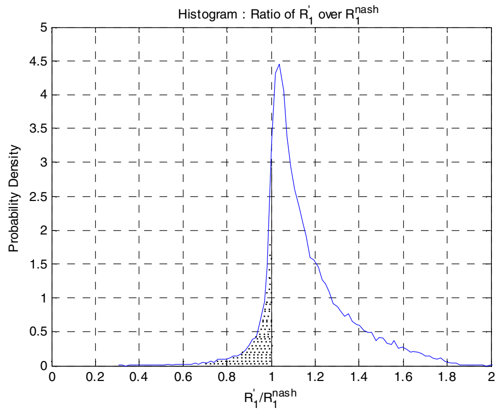

Fig. 2. Histogram for the ratio of R_1' / R_1^{nash}.

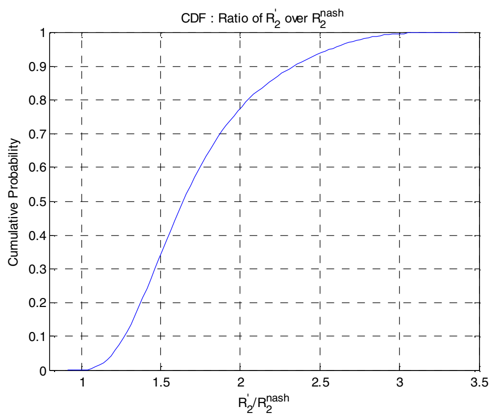

Fig. 3. cdf for the ratio of R_2' / R_2^{nash}.

We evaluate the performance of the proposed sub-optimal algorithm by comparing with the IW algorithm. We simulate a wireless system with 200 sub-carriers over the 10-MHz band. We assume that $\mathrm{P}_1 = \mathrm{P}_2 = 100$ and $\sigma_1(f) = \sigma_2(f) = 0.01$. To evaluate the performance, we tested 3×10^5 sets of frequency-selective fading channels where the Nash equilibrium exists, which are simulated using a four-ray Rayleigh model with the exponential power profile and 100 ns root mean square delay spread [15]. The simulated power of each ray is decreasing exponentially according to its delay. The total power of all rays of $H_{11}(f)$ and $H_{22}(f)$ is normalized as one, and that of $H_{12}(f)$ and $H_{21}(f)$ is normalized as 0.5.

Fig. 2 shows the histogram of the ratio of R_1' over R_1^{nash}. If the ratio is larger than one, Algorithm 1 provides a performance strictly better than the IW algorithm. We can see from the curve that Algorithm 1 achieves a higher rate R_1' than R_1^{nash} most of the time. It is because Algorithm 1 mitigates the interference by explicitly considering the other user's rational response. On the other hand, there is a small probability of approximately 14% (the shaded area in Fig.2) that the rate R_1' is smaller than R_1^{nash}. Note that in these cases, Algorithm 1 returns the same power allocations as the IW algorithm, which ensures a solution no worse than the IW algorithm. The average improvement of Algorithm 1 over the IW algorithm is 16.43%.

The ratio between user 2's achievable rate R_2' and R_2^{nash} in IW algorithm is shown in Fig. 3. It is surprising to find that, in very few cases with only a probability of 0.05%, Algorithm 1 will result in a rate R_2' smaller than R_2^{nash}. The average rate improvement for user 2 is 74%, which is significantly higher than that of user 1. This is because user 1 plays the Stackelberg equilibrium strategy that mitigates the interference to user 2. However, if user 1 plays the Nash strategy, user 2's achievable rate will be reduced immediately after user 1 updates its $P_1(f)$.

V. Conclusion

This paper considers the strategic behavior in determining the transmit power PSD for selfish users sharing a frequency-selective interference channel. We introduce the concept of Stackelberg equilibrium to model the two-user non-cooperative case as a bi-level programming problem, and derive the necessary optimality conditions. We show that a strategic user will avoid shortsighted Nash-strategy and improve its performance if it has the knowledge of the channel state information and best-response strategy of the competing user. A low-complexity sub-optimal approach is proposed and numerical results show substantial performance improvements.

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