Congestion Game Modeling for Brokerage based Multimedia Resource Management

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Abstract— **We introduce the concept of resource brokers, which enables efficient and fair management of the available network resources for multimedia users in large networks while reducing the complexity of a central resource allocation authority. To manage the available resources, the resource brokers deploy axiomatic bargaining solutions from economic game theory in order to explicitly consider the utility impact for different resource allocation schemes. We focus on the Kalai-Smorodinsky bargaining solution because it can successfully model relevant autonomous utility-aware fairness policies for multimedia users. Based on the interpretations of the bargaining solutions, we can model the proposed resource allocation scheme as a utility-driven congestion game, thereby ensuring that the presented resource management game will reach a steady-state after a finite (small) number of changes across resource brokers (i.e., at least one Nash equilibrium exists).**

I. Introduction

In this paper, we consider a decentralized and hierarchical resource management scheme for non-collaborative multi-user networked multimedia applications. To achieve this goal, we use the concept of (intermediate) brokers, which has been successfully deployed in several applications (e.g., [1]–[3]). The deployed resource management scheme is briefly explained as follows. First, the central authority, referred to as the central resource manager (CRM) in this paper, mitigates the complexity associated with administrating the resources for a large number of users by partitioning the whole network into several sub-networks, which are controlled by resource brokers (RBs). When the CRM distributes the total resources, it can deploy several fairness policies such as maximum fair resource allocation, proportional resource allocation, etc. Secondly, the RBs control their portion of the total resources by deploying different utility-aware fairness policies that explicitly consider the utilities of the users. Finally, end-users dynamically associate themselves with a sub-network based on their possibly achievable utilities. Since the goal of each end-user is to increase its video quality, the end-users prefer a sub-network that guarantees higher quality. Such a resource management scheme may be used in many networks aimed at supporting multimedia users (e.g., IEEE 802.11e WLAN).

Game theoretical approaches have been proposed to resolve resource allocation issues for various networks in a distributed and scalable manner [4]–[7]. However, prior research has not considered the resulting impact on the multimedia quality for various content-aware and delay-sensitive streaming applications. Video users can especially benefit from an efficient resource allocation as they require a high amount of resources (e.g., bandwidth) in a timely manner (given a delay constraint). Moreover, since multimedia is loss-tolerant (i.e., graceful degradation can be obtained), different resource-quality tradeoffs can be performed during this resource allocation, depending on the content characteristics. Distributed resource allocation schemes also need to consider the non-collaborative behavior of the users. Unlike conventional resource management policies, which manage the resources without considering the actual benefit in terms of utility derived by the users, we propose a distributed allocation approach based on the axiomatic bargaining solutions from well-suited game-theoretic concept [8]. Even though several bargaining solutions exist in the literature, we consider in this paper one bargaining solution that effectively model the interaction of different non-collaborative video users by the RBs: the Kalai-Smorodinsky Bargaining Solution (KSBS) [9]. As we will show, the KSBS guarantees the *same quality drop* from each user's maximum achievable quality.

To model the evolution of the proposed system over time, we interpret the characteristics of bargaining solutions in terms of multimedia quality, and use them to model the proposed resource management game as a utility-driven congestion game [10]. The primary advantage for modeling the considered resource allocation scheme as a congestion game is that there exists at least one Nash equilibrium. This implies that the decentralized non-collaborative interaction of users trying to maximize their achievable utility does eventually converge to a stationary distribution over the sub-networks without the specific control of the CRM.

This paper is organized as follows. In Section II, we propose the brokerage based resource management, and the utility-driven congestion game based on the KSBS is derived in Section III. In Section IV, we discuss the speed of convergence to an equilibrium point and simulation results are provided in Section V. The conclusions are drawn in Section VI.

II. Broker-based Resource Management

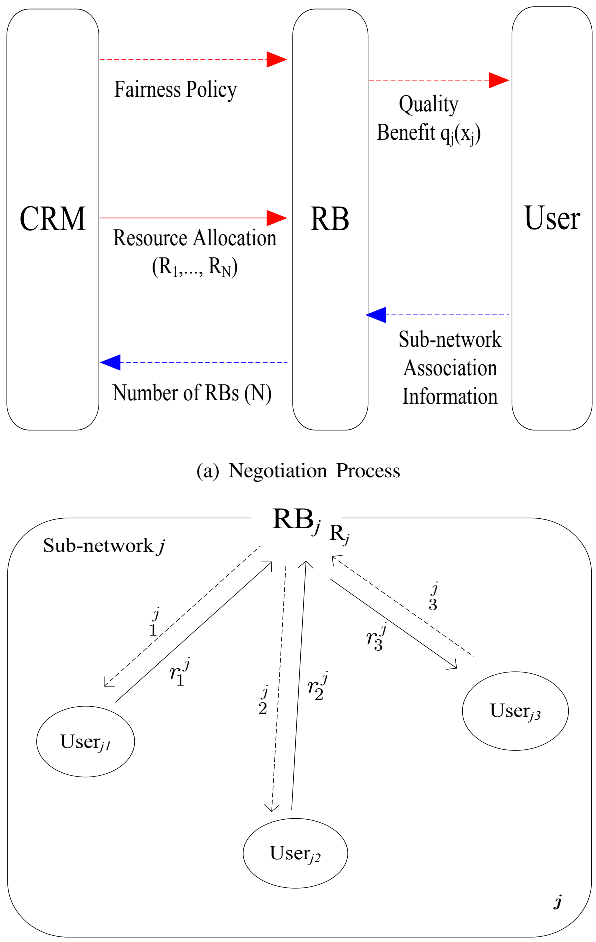
We consider a novel decentralized resource management mechanism that provides scalable and flexible resource allocation based on different utility-driven fairness policies. In the considered networks (e.g., a wireless LAN, wireless cellular or overlay network relaying on the Internet infrastructure), different network entities, which are a central authority, referred as the CRM, multiple RBs, and multimedia users. The CRM controls the total available resources but it does not directly interact with the final users. Instead, the CRM allocates the total resources to multiple RBs based on a pre-determined fairness policy. The RBs take turns to allocate the available resources to multimedia users based on their own fairness policies.

The hierarchic resource management process is illustrated in Fig. 1. In Fig. 1 (a), the CRM allocate total resources R_{MAX} to N RBs $(R_1, \dots, R_N)$, hence, $\sum_{j=1}^{N} R_j \leq R_{MAX}$. Users then switch sub-networks (denoted by sub-network association information) according to the quality benefits provided by the RBs. After the negotiation process, RB j deploys the bargaining solution $\mathcal{F}^j$ to divide the available resources R_j to users (Fig. 1 (b)). Each RB j allocates its resource R_j to the users in its sub-network C_j. The allocated resources to the users are denoted by $r_1^j, \dots, r_{|C_j|}^j$, where $|C_j|$ denotes the number of users in sub-network C_j and $\sum_{i=1}^{|C_j|} r_i^j \leq R_j$. The external information γ_i^j is the desired maximum quality of user i in sub-network C_j. Several advantages for this hierarchical brokerage based resource management and their implementations are summarized below:

1) CRM Level: The role of the CRM is to allocate the total available resources to RBs based on fairness policies and the declared information by RBs (e.g., multimedia application characteristics). The steps involved in implementing the dynamic resource allocation at the CRM side are:

- Session Initialization: The CRM gathers basic information in a network such as the number of total RBs. The RBs then declare the corresponding information, called *external information*, to the CRM.
- Gathering Information from RBs: The CRM gathers the external information from RBs $\boldsymbol{\Psi} = (\boldsymbol{\psi}_1, \dots, \boldsymbol{\psi}_N)$, which depends on the deployed resource allocation fairness policy $\mathcal{G}$ of the CRM.
- Allocating Available Resources to RBs: The CRM decides the resource allocation $(R_1, \dots, R_N)$ based on the gathered external information from RBs and the deployed fairness criteria $\mathcal{G} : \boldsymbol{\Psi} \to \mathbb{R}_+^N$ defined as
$$\mathcal{G}(\boldsymbol{\psi}_1, \dots, \boldsymbol{\psi}_N) = (R_1, \dots, R_N),$$
where $\sum_{j=1}^{N} R_j \leq R_{MAX}$.

However, the CRM does not need to explicitly consider the characteristics and resource requirement of all the multimedia users in its network. Hence, this hierarchical resource management can successfully reduce the complexity of managing numerous end-users by the central authority. This hierarchical brokerage system enables efficient and scalable management of a large number of simultaneous users by the same CRM.



(a) Negotiation Process

(b) Resource Allocation Process

Fig. 1. Proposed brokerage based resource management system

2) RB Level: An RB can provide adaptive admission control that explicitly considers the utility (multimedia quality) of the multimedia users. This feature can be implemented by deploying bargaining solutions. Bargaining solutions are useful to guarantee a certain level of multimedia quality in a sub-network. Note that RBs use the *quality benefit* to "advertise" the *guaranteed* quality provided by their sub-networks, as multimedia users try to maximize their quality.

As opposed to the role of the CRM, the resource allocation process in RBs is performed as follows.

- Session Initialization: Based on obtained resource allocation $(R_1, \dots, R_N)$ from the CRM, every RB calculates its quality benefits for each possible number of users in its sub-network. This information is then broadcasted to all users, since users need to know the benefits derived by switching sub-networks. We denote this quality benefit by $q_j(x_j)$ for $j = 1, \dots, N$, where x_j represents the number of users in sub-network C_j.
- Negotiation Processes (Updating and Broadcasting Sub-network Status): Based on the quality benefits advertised by the RBs, the multimedia users associate themselves with a specific RB that maximizes

the quality. Since users can switch sub-networks, the number of users in a sub-network can change over time, and thus, the quality benefit is also varying. Therefore, RBs keep track of the number of users in their sub-networks while providing its sub-network status to users.

- Gathering Information from Users: After the negotiation processes are finished, RB j polls users in its sub-network C_j, and gathers the external information $(\gamma_1^j, \ldots, \gamma_{|C_j|}^j)$ from users, which depends on its deployed resource allocation fairness policy $\mathcal{F}^j$ (i.e., bargaining solutions).
- Allocating Resources to Users: RB j decides the resource allocation $(r_1^j, \ldots, r_{|C_j|}^j)$ based on the gathered external information from users $(\gamma_1^j, \ldots, \gamma_{|C_j|}^j)$ in its sub-network C_j, and the deployed fairness policy $\mathcal{F}^j : \mathbf{\Gamma}^j \to \mathbb{R}_+^{|C_j|}$ defined as

$$\mathcal{F}^j(\gamma_1^j, \ldots, \gamma_{|C_j|}^j) = (r_1^j, \ldots, r_{|C_j|}^j),$$

where $\sum_{i=1}^{|C_j|} r_i^j \leq R_j$.
- Polling Users: Based on the determined resource allocation $(r_1^j, \ldots, r_{|C_j|}^j)$, RBs poll users in their sub-network.

3) Multimedia User Level: Every autonomous and rational user selects to join the sub-network of the RB that can provide the maximum guaranteed quality. Hence, users only consider the quality benefit (i.e., the expected gain in terms of quality provided by a sub-network) when they join a sub-network. In addition, users can switch to another sub-network if it can provide an increased quality benefit compared to the current one, and they can keep switching sub-networks until they cannot obtain increased quality benefit (i.e., a Nash equilibrium).

To maximize its achievable quality, a user needs to select the sub-network leading to the highest quality benefit. The following steps describe how this selection is performed:

- Session Initialization: Every user broadcasts its utility function parameters to every RB, and then, listens to the quality benefit in each sub-network.
- Determining Sub-network Groups: Users choose the sub-network that guarantees the largest quality benefit, i.e., user i currently in sub-network C_j will stay in, or switch to, sub-network $C_{j'}$ such that

$$j' = \arg \max_{j' \in \{1, \ldots, N\}} \{q_{j'}(x_{j'})\} > q_j(x_j).$$

- Video Transmission: Users start to transmit video data as soon as they decide to stop switching RBs (i.e. at a Nash equilibrium or when they decide that the additional quality benefit is not significant). Various algorithms can be adopted for video streaming. However, this goes beyond the scope of this paper. The interested reader is referred to [11] for a review of this topic.

Note that like an enterprise-network, users truthfully exchange their information only with RBs not the CRM.

III. Utility-Driven Congestion Game

In this section, we model the previously discussed resource management scheme as a utility-driven congestion game. We begin by reviewing fundamental concepts of congestion games.

A. Congestion Game Definitions

Congestion games were defined by Rosenthal [12] and the following definitions can be found in [10].

Definition *1 (Congestion Model):* A congestion model is a tuple $\langle M, A, (\Omega_i)_{i \in M}, (c_f)_{f \in A} \rangle$, where M is a nonempty finite set of players, and A is a nonempty finite set of facilities. For each player $i \in M$, its collection of pure strategies Ω_i is a nonempty finite family of subsets of A. For each facility $f \in A$, $c_f : \{1, \ldots, n\} \to \mathbb{R}$ is the benefit function of facility f, with $c_f(k), k \in \{1, \ldots, n\}$, the benefits to each of the players of facility f if there is a total of k players.

Definition *2 (Congestion Game):* A congestion game G is a tuple $G = \langle M, (\Omega_i)_{i \in M}, (u_i)_{i \in M} \rangle$, where M and $(\Omega_i)_{i \in M}$ are as in Definition 1 and for $i \in M$, $u_i : \Omega \to \mathbb{R}$ is defined by $u_i(\sigma) = \sum_{f \in \sigma_i} c_f(n_f(\sigma))$, where for each set of strategies $\sigma = (\sigma_1, \ldots, \sigma_n) \in \Omega = \times_{i \in M} \Omega_i$, and each $f \in A$, $n_f(\sigma) = |\{i \in M : f \in \sigma_i\}|$ is the number of players of facility f if the players choose σ.

Note that the players and the facilities included in the definitions can be considered as the users and the sub-networks, respectively, in this paper. Moreover, the benefit function $c_f(k)$ is *only* a function of the number of players in the facility f in the above definition.

B. Axiomatic Bargaining Solutions and Interpretations

As we discussed in Section II, RBs use axiomatic bargaining solutions for the resource management. Specifically, we focus on the KSBS, which can provides the desired relationship between the utilities of autonomous and rational multimedia transmitting users. [1] (see e.g., [8], [14] for more details on the basics of bargaining solutions).

In axiomatic bargaining theory, a solution is selected out of the set of possible resource allocation choices that satisfies a set of rational and desirable axioms. Different bargaining solutions are differentiated by their unique fairness axioms. More details on general axiomatic bargaining theory can be found in [8]. A bargaining solution is a function $F : \mathcal{B} \to \mathbb{R}_+^n$, with $F(\mathbf{S}, \mathbf{d}) \in \mathbf{S}$, where $\mathcal{B}$ denotes the set of all bargaining problems $(\mathbf{S}, \mathbf{d})$. $\mathbf{S}$ is the feasible utility set, and $\mathbf{d} \in \mathbf{S}$ is the disagreement point [8]. In this paper, the disagreement point is the origin (i.e., $\mathbf{d} = \mathbf{0}$) since this corresponds to the zero utility case, where a user does not join any sub-network because its minimum quality requirement is not satisfied. Based on the axioms of the KSBS [9], the KSBS $\mathbf{X}^* =$

[1] The Nash bargaining solution (NBS), which is the other well-known bargaining solution, cannot be efficiently used for non-collaborative multimedia applications as it maximizes the sum of qualities from all rather than focus on the individual quality of each user [13].

$F(\mathbf{S}, \mathbf{d} = \mathbf{0})$ can be expressed as [13]

$$\mathbf{X}^* = \max_{\mathbf{X}} \left\{ \mathbf{X} \in \mathbf{S} \;\middle|\; \frac{X_1}{X_M^1} = \cdots = \frac{X_n}{X_M^n} \right\}, \quad (1)$$

where $X_i > 0$ for all i and X_i is a utility of player i, and $X_M^i = \max_{X_i \in \mathbf{S}, X_i \geq 0} X_i$. The point $(X_M^1, \ldots, X_M^n)$ is called the *ideal point*. Note that the KSBS $\mathbf{X}^*$ can be obtained as the intersection point of the bargaining set and the line joining the disagreement point and the ideal point. An illustrative example is shown in Fig. 2.

As already discussed in [13], to consider multimedia applications, we define *utility* function $U_i(\cdot)$ for user i as

$$U_i(r_i) = 255^2 / D_i(r_i),$$

where r_i denotes allocated rate and $D_i(r_i)$ represents the distortion of multimedia contents measured as the mean square error (MSE). Note that for video compression applications, the utility function $U_i(r_i)$ is an increasing concave function with respect to the allocated rate r_i [15]. By taking the logarithm of the utility function, we can derive a widely used quality measure for video quality, Peak Signal to Noise Ratio (PSNR), defined as

$$PSNR_i = 10 \log_{10} \frac{255^2}{D_i(r_i)} = 10 \log_{10} U_i(r_i),$$

where $PSNR_i$ denotes the PSNR for user i.

The resource allocation based on the KSBS can be interpreted as follows in terms of PSNR [13].

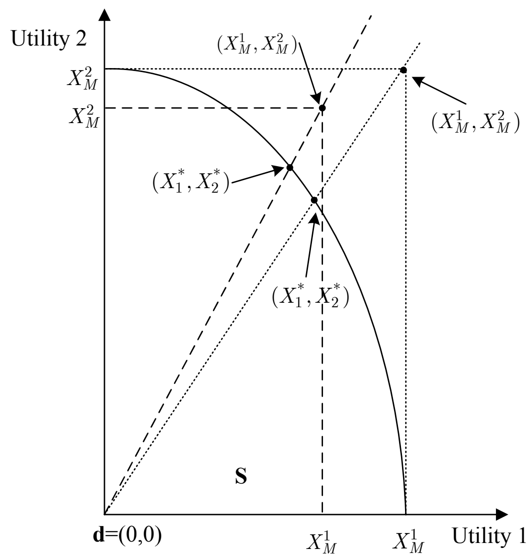


Fig. 2. Illustrative examples for the KSBS. The KSBS is (X_1^*, X_2^*) if there are no desired maximum utilities from users (i.e., the KSBS is determined by the maximum achievable utilities (X_M^1, X_M^2) in $\mathbf{S}$). If each user has its desired maximum achievable utility $(\hat{X}_M^1, \hat{X}_M^2)$, then the KSBS is $(\hat{X}_1^*, \hat{X}_2^*)$ in $\mathbf{S}$. Note the two KSBSs are different since the ideal points are different (i.e.,$(X_M^1, X_M^2) \neq (\hat{X}_M^1, \hat{X}_M^2)$).

Since the KSBS $\mathbf{X}^*$ satisfies (1), it can be shown that the quality drop from the maximum achievable quality of every user is the same by taking the logarithm in (1). Hence, if the KSBS is deployed for the utility-driven fairness policy in sub-network C_j managed by a RB j, then the quality drop from the maximum achievable quality of every user in the sub-network is the same, i.e.,

$$\triangle Q_1 = \cdots = \triangle Q_{|C_j|},$$

where $\triangle Q_i \triangleq (Q_M^i - Q_i^*)$ represents the quality decrease (or drop) from user i's maximum achievable quality. $Q_M^i = 10 \log_{10} X_M^i$ is the maximum achievable PSNR for user i, and $Q_i^* = 10 \log_{10} X_i^*$ is achievable PSNR determined by the KSBS. This is the key property for modeling as the congestion game, which will be discussed in the next section.

C. Bargaining-based Congestion Game Modeling

For video compression, achieving a PSNR level higher than a certain quality threshold (e.g., 35dB PSNR) is not meaningful, because it does not impact the visual quality. Hence, we assume that each user has its own *desired* maximum quality level at which it will prefer to operate. Based on this assumption, the KSBS has the property that the quality drop from the desired maximum quality level is also the same, which is shown in the following proposition. This is illustrated in Fig. 2.

Proposition 1: If the KSBS is deployed for the utility-driven fairness policy in a RB and each user has its own desired maximum utility, then the quality drop from the desired maximum quality of every user in the sub-network of an RB is the same.

Proof: Let $\hat{\mathbf{X}}_M = (\hat{X}_M^1, \ldots, \hat{X}_M^{|C_j|})$ be the desired maximum utility for every user in the sub-network of the RB j. Since the desired maximum utility cannot exceed the achievable maximum utility for user i the ideal point becomes the desired maximum utility $\hat{\mathbf{X}}_M$ and the corresponding KSBS $\hat{\mathbf{X}}^* = (\hat{X}_1^*, \ldots, \hat{X}_{|C_j|}^*) \in \mathbf{S}$ satisfies

$$\hat{X}_1^* / \hat{X}_M^1 = \cdots = \hat{X}_{|C_j|}^* / \hat{X}_M^{|C_j|},$$

where $\hat{X}_i^* > 0$ for all i as (4). By taking the logarithm, we have

$$\triangle \hat{Q}_1 = \cdots = \triangle \hat{Q}_{|C_j|}.$$

Note that $\triangle \hat{Q}_i \triangleq (\hat{Q}_M^i - \hat{Q}_i^*)$ represents the quality drop from user i's *desired* maximum quality. ■

Proposition 2: If the KSBS is deployed as utility-driven fairness policy in RBs with desired maximum utilities, then the proposed resource allocation scheme is a congestion game.

Proof: To model the proposed resource management game as a congestion game, we construct the proposed resource allocation scheme as follows:

1. We can consider N RBs as N facilities of a congestion game.
2. We can consider n users as n players in a congestion game.
3. We can consider either quality drop for the KSBS or achievable quality for the EBS as a benefit for a user by joining a sub-network of an RB.

The first and the second conditions for the congestion game are obvious based on the definition of the congestion game. We focus on the third condition. As we discussed in Section III-A, the benefit function should be only a function of the number of users to be modeled as a congestion game. We can design a *quality benefit function* in this problem as a benefit function for a congestion game.

If all RBs deploy the KSBS as a utility-driven fairness policy, the quality drop of users is the same in a sub-network. Even though the amount of quality drop can be varied by the characteristics of users in a sub-network, the sub-network can determine the maximum quality drop for a certain number of users by considering all possible combinations. Therefore, the maximum quality drop for a user i in the RB j can be considered as a benefit function $q_j(x_j)$, expressed as

$$u_i(\sigma) = c_{\sigma_i}(n_{\sigma_i}(\sigma)) = c_j(x_j) = q_j(x_j),$$

where $x_j = n_j(\sigma)$ denotes the number of users in sub-network C_j for the set of strategies σ. The quality benefit function $q_j(x_j)$ for RB j can be expressed as

$$q_j(x_j) = Q_i(x_j) - \hat{Q}^i_{MAX} \text{ for all } i \in C_j, \qquad (2)$$

where $Q_i(x_j)$ denotes the *minimum* achievable quality for user i if there are only x_j users including the user i in sub-network C_j. Since $Q_i(x_j)$ is the minimum achievable quality for all possible x_j users, it can be interpreted as the guaranteed achievable quality by sub-network C_j. Note that $q_j(x_j) = -\triangle\hat{Q}_i$. Since the desired maximum utilities are determined by users and they are fixed, users only consider the quality benefits by sub-networks. In other words, each user chooses the sub-network guaranteeing the smallest quality drop (i.e., the largest quality benefit).

We observed that the quality benefit functions for the KSBS fairness policy in (2) is only a function of the RB and the number of users in the sub-network of that RB. Moreover, the quality benefit function $q_j(x_j)$ is non-increasing function, and thus, users prefer larger quality benefit. Therefore, the proposed resource allocation scheme with the KSBS can be modeled as a congestion model $\langle M, A, (\sigma_i)_{i\in M}, (q_j)_{j\in A}\rangle$, where $M = \{1, \ldots, n\}$ and $A = \{1, \ldots, N\}$, and hence, the corresponding congestion game is $G = \langle M, (\sigma_i)_{i\in M}, (q_{\sigma_i})_{i\in M}\rangle$. ■

Since the proposed resource management scheme based on the KSBS is a congestion game, it has a Nash equilibrium as a direct consequence of [10]. Hence, we can conclude that the proposed utility-driven resource management based on the KSBS has at least one pure Nash equilibrium, i.e., stationary users distribution.

IV. Speed of Convergence to the Nash Equilibrium

In this section, we investigate the speed of convergence for the utility-driven congestion game. From the previous section, we already know that there should be at least one pure Nash equilibrium. Hence, the next question is how fast a Nash equilibrium is reached. Note that the speed of convergence is especially important for delay-constraint multimedia applications. The proposed system is assumed to be implemented based on *Elementary Stepwise System (ESS)*, where each user decides their sub-networks sequentially [16]. The switching orders can be determined based on currently derived multimedia quality, as the improvement of visual impact highly depends on it. Hence, users currently deriving lower quality in RBs may decide to switch sub-networks earlier. If multiple users are allowed to switch sub-networks simultaneously, an equilibrium distribution of users cannot be ensured due to problems of repeatedly switching resource brokers, which implies that no user can start to transmit their data. This issue has been discussed in several works, e.g., [17]–[19]. Hence, we assume that only one user can switch at time, and this assumption forces fast convergence to a Nash equilibrium. The following theorem provides a bound for the speed of convergence.

Theorem 3: If every RB deploys the KSBS as a fairness policy, then the required number of sub-network switches for the n users to reach a Nash equilibrium is at most $n-1$.

Proof: Let $\mathbf{Q}$ be the $n \times N$ matrix of all quality benefits for n users and N RBs, defined as

$$\mathbf{Q} = [q_j(x) : 1 \le j \le N, 1 \le x \le n],$$

where $q_j(x_j)$ is defined in (2). Note that $q_j(1) = 0$ for all $j = 1, \ldots, N$ since there is no quality drop if there is only one user in a sub-network. Furthermore, notice that $q_j(x_j)$ is a non-increasing function of the number of users x_j in sub-network C_j since the available resource for an RB is fixed and it is shared with users in the sub-network. Let $\mathbf{q}_k$ be the set of quality benefits from all RBs after k switches of users, called *quality benefit status*. It is defined as

$$\mathbf{q}_k = \{q_1(y_1), \ \ldots, \ q_N(y_N)\}, \qquad (3)$$

and $\mathbf{q}_k$ can be transmitted from RBs to users. Note that (3) implies y_j users are in sub-network C_j, and the corresponding quality benefit is $q_j(y_j)$ for all y_j such that $\sum_{j=1}^{N} y_j \le n$, and $0 \le y_j \le n$. Since every user is rational, it is trying to switch to the sub-network which can provide it a higher quality benefit as opposed to the current one. Therefore, a user i in sub-network C_w chooses sub-network C_v if and only if

$$q_w(y_w) < q_v(y_v{+}1) = \max\{q_1(y_1{+}1), \ldots, q_N(y_N{+}1)\}. \qquad (4)$$

After $(k+1)$ switches (i.e., one more switch of the user i after k switches), the quality benefit status $\mathbf{q}_{k+1}$ is expressed as

$$\mathbf{q}_{k+1} = \{q_1(z_1), \ldots, q_v(z_v), \ldots, q_w(z_w), \ldots, q_N(z_N)\},$$

where $z_j = y_j$ for all j except w and v. Since the user i switches from sub-network C_w to sub-network C_v, $z_v = y_v + 1$ and $z_w = y_w - 1$. And $\sum_{j=1}^{N} z_j \le n$ and $0 \le z_j \le n$ for $j = 1, \ldots, N$ since there is no change of the total number of users.

Another user i' in sub-network C'_w after $(k{+}1)$ switches chooses the sub-network C'_v if and only if

$$q_{w'}(z_{w'}) < q_{v'}(z_{v'}{+}1) = \max\{q_1(z_1{+}1), \ldots, q_N(z_N{+}1)\}.$$

Note that $q_{v'}(z_{v'}+1) \le q_v(y_v+1)$ since

$$\begin{aligned}
&q_{v'}(z_{v'}+1)\\
&\quad = \max\{q_1(z_1+1), \ldots, q_v(z_v+1),\\
&\qquad\qquad \ldots, q_w(z_w+1), \ldots, q_N(z_N+1)\} \quad (5)\\
&\quad = \max\{q_1(y_1+1), \ldots, q_v(y_v+2),\\
&\qquad\qquad \ldots, q_w(y_w), \ldots, q_N(y_N+1)\} \quad (6)\\
&\quad \le \max\{q_1(y_1+1), \ldots, q_v(y_v+1),\\
&\qquad\qquad \ldots, q_w(y_w), \ldots, q_N(y_N+1)\}\\
&\quad = q_v(y_v+1).
\end{aligned}$$

The inequality for (5) and (6) follows from the fact that $q_j(x_j)$ is non-increasing function, and the solution for (6) follows from (4), i.e., $q_w(y_w) < q_v(y_v+1)$. Therefore, we conclude that if a user switches to the sub-network which provides the largest quality benefit after k switches, its switch decreases the maximum value of the quality benefit status $\mathbf{q}_{k+1}$. Note that this process stops after l switches if

$$\min \mathbf{q}_l \ge \max \mathbf{q}_{l+1}.$$

Hence, we can interpret this problem as simply filling the unoccupied n largest $q_j(x_j)$ in matrix $\mathbf{Q}$. List the elements of the matrix $\mathbf{Q}$ from the largest value to the smallest value, which can be expressed as

$$q_1(1),\, q_2(1),\, \ldots,\, q_N(1),\, q_j(x_j),$$

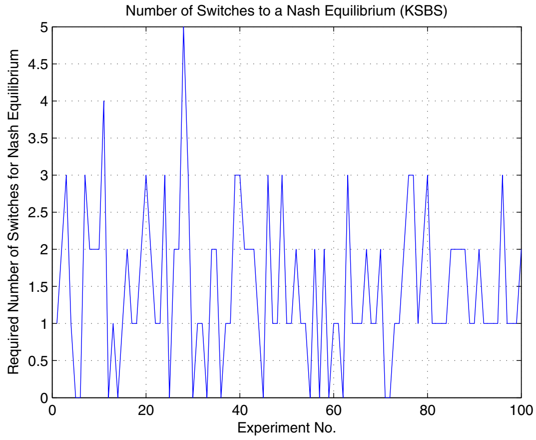
for all $j = 1, \ldots, N$, $x_j = 2, \ldots, n$. Note that $q_j(1) = 0$ for all $j = 1, \ldots, N$. Now the original problem is filling the unoccupied largest elements by switching from smaller elements. Since at least one sub-network must be filled with users, one of $q_j(1)$ for $j = 1, \ldots, N$ is occupied. Hence, n largest elements must be occupied by at most $n-1$ switches. ■

From *Theorem* 3, we conclude that the required number of sub-network switches for users to reach a Nash equilibrium has the upper bound of $(n-1)$, which is linear to the number of users. Therefore, we know that users need to wait for at most $(n-1)$ switches before then can actually transmit multimedia sequences. These results can be extended to the case, where new n' users participate in this network after a Nash equilibrium is already established. Since the network is already at a Nash equilibrium, the only required steps to reach another new Nash equilibrium with n' users are to switch to the sub-networks that provide higher quality benefit, which requires at most n' switches. Therefore, it is also concluded that a new Nash equilibrium is achieved in one switch when a new user is joining the network that is already in a Nash equilibrium.

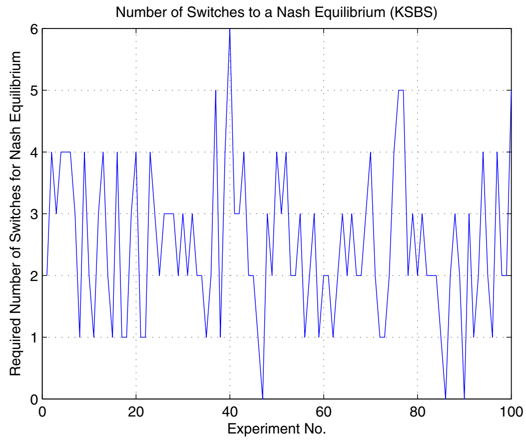
V. Simulation Results

A. Required Number of Switches for a Nash Equilibrium

We present simulation results that show the convergence (i.e., Nash equilibrium) of user's distribution over several sub-networks with finite number of switches (N_s). We assumed that there are 10 users ($n = 10$) transmitting video sequences, and there are 3 sub-networks ($N = 3$). The total resources are *equally* divided in scenario I, and *proportionally* divided given a fixed proportion (5:3:2) in scenario II. To analyze the average number of switches (AVG), we uniformly distribute all users to all available sub-networks, and then, count the number of switches before reaching a Nash equilibrium. This simulation is repeated 100 times. Moreover, for the worst-case (WC) analysis, we assume that all users are initially located in one of sub-networks.



(a) Scenario I AVG



(b) Scenario II AVG

Fig. 3. Number of switches for Nash equilibrium.

Scenario		N_s	Distribution
I	AVG	1.47	(3.34, 3.32, 3.34)
	WC	7	(4, 3, 3)
II	AVG	2.49	(5, 3, 2)
	WC	8	(5, 3, 2)

TABLE I

Required number of switches for a Nash equilibrium and distribution of users.

The simulation results for the number of switches are shown in Fig 3 and they are summarized in Table I. As shown in Table I, the average required number of switches to reach a Nash equilibrium is approximately 1.4

and 2.5 for AVG in each scenario respectively. These are which are 15.6% and 27.8% compared to the bound (i.e., $n-1=9$). Moreover, the required number of switches for WC in each scenario are also bounded. Therefore, users' interactions eventually converge to a stable distribution (i.e., Nash equilibrium) over sub-networks with finite number of switches in any scenarios. Moreover, the user's distribution is the same as the resource distribution over sub-networks in these simple simulation scenarios.

B. Resource Broker Switching Criteria and Delay

In this section, we analyze the relationship between the required number of sub-network switches and the switching criteria. Multimedia applications having stringent delay-constraints will change their strategies (i.e., switch RBs) only if there is a significant quality benefit. This is because switching sub-networks increases the waiting time to transmit data, and it is critical for stringent delay-constraint applications. Hence, each user can have its own quality threshold to determine whether it is worthwhile to switch sub-networks. For example, if users try to maximize the achievable quality without considering delay-constraints, they set their quality threshold as 0, i.e., users will switch sub-networks as long as the sub-networks provide higher quality benefit. However, if users are transmitting stringent delay-constraint multimedia applications, users need to set their thresholds higher to minimize the waiting time to reach a Nash equilibrium. We assume that users in a sub-network have the same quality threshold in this simulation. Simulation results are shown in Table II. Note that the percentage values are obtained based on the average numbers of sub-network switches at $\delta=0$.

Threshold	$\delta = 0$	$\delta = 0.5$ [dB]	$\delta = 1$ [dB]
Average Number (%)	1.42 (100%)	1.23 (86.6%)	1.02 (71.8%)
Threshold	$\delta = 1.5$ [dB]	$\delta = 2$ [dB]	$\delta = 2.5$ [dB]
Average Number (%)	0.71 (50.0%)	0.43 (30.3%)	0.30 (21.1%)

TABLE II
QUALITY THRESHOLDS (δ) AND AVERAGE REQUIRED NUMBER OF SWITCHES FOR A NASH EQUILIBRIUM

We observe that the average number of sub-network switches required to reach a Nash equilibrium decreases as the thresholds increase. It implies that if users have higher quality thresholds, then they can reduce their waiting time for transmitting multimedia data. This is a tradeoff between achievable quality and delay.

C. Comparison of Different Resource Management Policies

In this section, we investigate the multimedia quality which can be derived by one user as more users join the sub-networks.

In this experiment, users have compressed and encoded video sequences, and transmit the encoded bitstream using the state-of-the-art wavelet video coder in [20]. A single video file has 1000 seconds duration, which was obtained by concatenating 100 times of the same typical MPEG test sequences. Users receive the announced quality benefit and decide one of the sub-networks that they join. Once users join the sub-network, users declare the required information for the resource management to their RBs. Then, the RBs decide TXOPs that can be allocated to each user based on their resource management policies and notify the determined TXOPs to users. The alternative resource management policy is equal resource allocation (ERA), which allocates the equal amount of resources to users associated in each RB. Users having multiple sets of Traffic Specification (TSPEC) select one of TSPEC parameters that can be fit to the allocated transmission rates, and start to transmit their bitstream. If additional users join a sub-network, the above process is repeated and new TXOP allocation is allocated to users. Then, users adaptively select another set of TSPEC parameters and transmit their bitstream. The results are shown in Fig. 4.

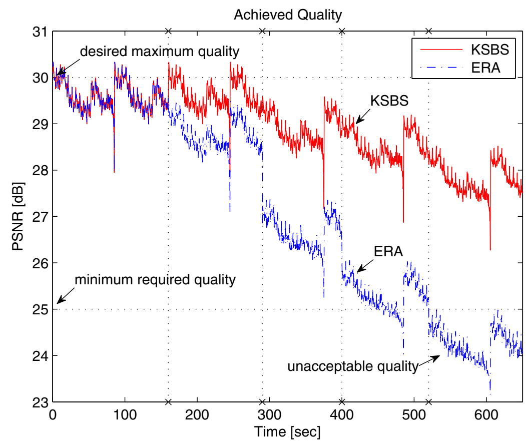


Fig. 4. Derived quality of *Mobile* user over time. Both results present that the derived quality of *Mobile* user over time as users join sub-networks. Vertical lines represent the time stamps when users switch or join sub-networks.

Fig. 4 shows that the derived quality of one user transmitting *Mobile* sequences as users join sub-networks. Fig. 4 shows the derived quality of the user when new users join the sub-network based on the different resource management policies. As more users join the sub-network, the derived quality decreases in both policies. Since the ERA simply divides the resources by the number of users in the sub-network and does not consider the multimedia characteristics, the quality can be derived below the minimum required quality (25dB in this simulation). However, the KSBS can explicitly consider the multimedia characteristics including the minimum required quality, it can ensure the minimum required quality.

VI. CONCLUSION

In this paper, we discuss a brokerage based decentralized resource management scheme for multi-user multimedia transmission over networks. The resource brokers enable efficient and fair management of the available network resources for large networks while reducing the complexity of the central authority. In order to address the autonomous behavior of multimedia users streaming video over the networks, resource brokers deploy the axiomatic bargaining solution, KSBS, which explicitly consider the utility impact for different resource allocation schemes. Moreover, based on the interpretations of the bargaining solutions in terms of multimedia quality, we can model the proposed resource management scheme as a utility-driven congestion game, which guarantees convergence to a Nash equilibrium. Simulation results show that users' interaction converges to a stable distribution with finite number of switches.

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