Achieving Coordination in Random Access Networks Without Explicit Message Passing

Jaeok Park and Mihaela van der Schaar

***Abstract*— We propose a class of medium access control (MAC) protocols that utilize users' own transmission decisions and feedback information from the past slots. We consider an idealized slotted Aloha system and formulate the problem of a protocol designer who cares about the total throughput, the short-term fairness, and the complexity of protocols. A solution to the protocol designer's problem is provided with two users, and an approximate solution with three or more users. We use numerical methods to obtain optimal protocols that solve the protocol designer's problem, compare the total throughput of optimal protocols with that of other protocols proposed in the literature, and analyze a trade-off between throughput and fairness. The results show that by utilizing information obtained in the previous slot, users can achieve some degree of coordination without explicit message passing, which leads to high total throughput.**

I. INTRODUCTION

In wireless communication networks, multiple users often share a common channel and contend for access. In a scenario where transmissions of packets by more than one user result in a collision, there is a need for medium access control (MAC) protocols to coordinate transmissions by users. In this paper, we consider an idealized slotted Aloha system and propose a particular class of protocols that utilize users' own transmission decisions and feedback information from the past slots. The proposed protocols allow users to coordinate their access (at least partially) without explicit message passing.

Early work on slotted Aloha focused on exponential backoff (EB) protocols and analyzed the stability and performance of EB protocols [1]–[4]. Recently, the framework of game theory has been used to analyze slotted Aloha models in which users determine their transmission probabilities [5]–[8]. In [5], the strategy, or the decision rule, for a user is simply its transmission probability used over time to attain its desired throughput. In [6], the number of users contending for the channel varies over time, and users know the number of users currently in the system. The decision rule for a user in [6] is its transmission probability as a function of the number of users.

Altman et al. [7] assume that information on the number of users in the system is unavailable to users and that newly arrived packets are always transmitted. The decision rule is the transmission probability for backlogged packets. Ma et al. [8] define two states for a user, a free state and a backlogged state, and relax the assumption of [7] that newly arrived packets are always transmitted. The decision rule for a user in their model is two transmission probabilities used in each state.

In a repeated game, a strategy for a player is a mapping from the set of possible histories to the set of (mixed) actions [9]. Using this idea, we can define a decision rule for a user as a mapping from the set of possible local information to the set of transmission probabilities. We assume that users receive immediate feedback on the transmissions by other users at the end of each slot. In each slot, a user knows its own transmission decision and obtain feedback information on others' transmission decisions. Hence, the local information of a user includes its own transmission decisions and feedback information in the past.

Local information may include information obtained through message exchanges or other devices. For example, in token ring networks [10], users know whether they have a token or not, and this information can be used to determine their transmission probabilities. Another example is a correlation device as in [11]. With the presence of a correlation device, users can adjust their transmission probabilities depending on random signals generated by the correlation device. In this paper, we preclude any kind of message passing other than users' receiving feedback information on the transmission decisions by other users. This assumption can be justified by the distributed nature of wireless networks, where communication schemes incur large overhead costs. Another reason for making this assumption is that analysis based on no message passing will provide a benchmark case against which various schemes allowing message exchanges can be compared.

The remainder of this paper is organized as follows. We describe the considered slotted Aloha model in Section II and formulate the problem of a protocol designer in Section III. Section IV provides analytical results while Section V analyzes the performance of the proposed protocol using numerical methods. We conclude the paper in Section VI.

II. MODEL

We consider an idealized slotted Aloha system as in [8]. Users (pairs of transmitter-receiver nodes) share a communication channel though which they transmit packets. The total number of users is N, and the set of users is denoted by $\mathcal{N} = \{1, \dots, N\}$. We assume that the number of users is fixed over time and known to users.

Time is slotted, and slots are synchronized. We label slots by $t = 1, 2, \dots$. Packets are of the same size, and each packet

J. Park is with the Department of Economics, University of California, Los Angeles, CA 90095, USA jpark31@ucla.edu

M. van der Schaar is with the Department of Electrical Engineering, University of California, Los Angeles, CA 90095, USA mihaela@ee.ucla.edu

requires one slot for transmission. A user always has a packet to transmit and makes a decision on whether to transmit or not in every slot [5] [8]. Hence, the set of actions available to a user is $A = \{T, W\}$, where T stands for "transmit" and W for "wait." We denote the action of user i by $a_i \in A$ and an action profile or *outcome* by $\mathbf{a} = (a_1, \ldots, a_N)$. The set of outcomes is denoted by $\mathcal{A} \triangleq A^N$.

A packet is successfully transmitted if it is the only transmission in the slot. If there is more than one transmission, a collision occurs. If the transmission of a packet results in a collision, it is retransmitted in some later slot until it is successfully received. We assume that at the end of each slot users obtain *feedback information* on whether there is no transmission or at least one transmission by other users in the slot. Let $Z \triangleq \{0, 1\}$ be the set of feedback information for a user and $z_i \in Z$ be the feedback information of user i. Then $z_i = 0$ if $a_j = W$ for all $j \in \mathcal{N} \setminus \{i\}$ and $z_i = 1$ otherwise. That is, each user receives binary feedback information on the number of transmissions by other users. There are four possible action-feedback pairs for a user: $(W, 0)$, $(W, 1)$, $(T, 0)$, and $(T, 1)$.

We define the *local information* of user i in slot t as all information that user i has at the beginning of slot t. Without explicit message passing, it consists of the action-feedback pairs of user i from slot 1 to slot $t-1$. The *m-period local information* of user i in slot t is given by

$$L_i^t = (a_i^{t-m}, z_i^{t-m}; a_i^{t-m+1}, z_i^{t-m+1}; \ldots; a_i^{t-1}, z_i^{t-1}), \quad (1)$$

where we set $(a_i^t, z_i^t) = (W, 0)$ for $t \leq 0$ as a default. Let $\mathcal{L}_m \triangleq (A \times Z)^m$ be the set of all possible m-period local information, for $m = 0, 1, \ldots$. A *stationary decision rule for user i based on m-period memory* is defined to be a mapping

$$f_i : \mathcal{L}_m \to [0, 1]. \quad (2)$$

$f_i(L_i)$ gives the transmission probability for user i when user i has m-period local information $L_i \in \mathcal{L}_m$. We use $\mathcal{F}_m$ to denote the set of stationary decision rules based on m-period memory. The set of all stationary decision rules based on finite memory is denoted by $\mathcal{F} \triangleq \cup_{m=0}^{\infty} \mathcal{F}_m$.

We define a *protocol* as a decision rule profile $\mathbf{f} \triangleq (f_1, \ldots, f_N) \in \mathcal{F}^N$. Given a protocol $\mathbf{f}$, let m_i be the minimum length of memory required to implement the decision rule f_i. Then m_i takes a value in $\mathbb{N}_+ \triangleq \{0, 1, \ldots\}$. We take the maximum of m_i across users to obtain $m^*(\mathbf{f}) \triangleq \max\{m_1, \ldots, m_N\}$. Then $m^*(\mathbf{f})$ is the minimum length of memory required to implement the protocol $\mathbf{f}$, and we say that the protocol $\mathbf{f}$ is *based on $m^*(\mathbf{f})$-period memory*. Intuitively, a protocol is simpler when it is based on shorter memory, and thus we call $m^*(\mathbf{f})$ the *complexity level* of the protocol $\mathbf{f}$.

III. PROBLEM FORMULATION

We approach the system problem from a protocol designer's perspective. That is, we consider the problem of the protocol designer who prescribes decision rules in $\mathcal{F}$ to users in order to achieve his objective. By requiring the protocol designer to consider decision rules in $\mathcal{F}$, we impose stationarity on decision rules. This requirement is reasonable in a distributed system where users do not maintain common labels for slots. To implement stationary decision rules, it suffices that the system is slotted and users know when slots start. Moreover, stationary decision rules can be easily used in an environment where users enter and leave the system over time.

In this paper, we mainly consider protocols based on one-period memory. Considering the large memory spaces of computing devices, one may find that the restriction on decision rules to be based only on one-period memory is too restrictive. However, decision rules based on one-period memory are easy to follow and robust to variations on memory and computation constraints. Suppose that the protocol designer is uncertain about the memory and computation capacities of individual users. If a failure to follow the prescribed decision rule by a single user results in a total breakdown of the system, then the protocol designer wants to provide a simple protocol to ensure that every user can follow it. Moreover, analysis with decision rules based on one-period memory is meaningful in that the performance of protocols based on one-period memory provides a lower bound on that of more complicated protocols based on longer memory.

A decision rule for user i based on one-period memory can be expressed as $f_i : (A \times Z) \to [0, 1]$. $f_i(a_i, z_i)$ is the transmission probability of user i when it took action a_i and received feedback information z_i in the previous slot. Suppose that users follow a protocol $\mathbf{f}$ based on one-period memory. Then a Markov model can be constructed where the state space of the Markov chain is taken to be $\mathcal{A}$. If $\mathbf{f}$ is chosen so that the induced Markov chain is irreducible, then there exists a unique stationary distribution π on $\mathcal{A}$, independent of the initial distribution [12]. The throughput of user i is defined by

$$\tau_i(\mathbf{f}) \triangleq \pi(\mathbf{a}^i), \quad (3)$$

where $\mathbf{a}^i \in \mathcal{A}$ is the outcome in which only user i transmits. Total throughput is defined by

$$\tau(\mathbf{f}) \triangleq \sum_{i=1}^{N} \tau_i(\mathbf{f}). \quad (4)$$

We assume that protocols cannot be specialized for individual users due to the anonymity of users, and thus the protocol designer is bound to prescribe the same decision rule to all users. We partition the set of outcomes $\mathcal{A}$ into $(N+1)$ sets according to the number of transmissions in outcomes. That is, we express $\mathcal{A} = \mathcal{A}_0 \cup \cdots \cup \mathcal{A}_N$ where $\mathcal{A}_k$ is the set of outcomes with k transmissions, for $k = 0, 1, \ldots, N$. When every user uses the same decision rule, we can take the set of Markov states as $\{\mathcal{A}_0, \ldots, \mathcal{A}_N\}$ instead of $\mathcal{A}$. For the moment, let us assume that the protocol designer's problem is to find a decision rule f in $\mathcal{F}_1$ that maximizes *one-step* transition probabilities to $\mathcal{A}_1$, in which a successful transmission occurs, when followed by every user.

First, suppose that the outcome in the previous slot is in $\mathcal{A}_0$, i.e., the channel was idle. Then every user transmits with probability $f(W,0)$. If every user uses the same transmission probability, say p, then the probability of success is given by $Np(1-p)^{N-1}$, and this expression is maximized at $p = 1/N$. Hence, we set $f(W,0) = 1/N$ to maximize the one-step transition probability from $\mathcal{A}_0$ to $\mathcal{A}_1$.

Next, suppose that the outcome in the previous slot is in $\mathcal{A}_1$, i.e., there was a successful transmission. Then one user transmits with probability $f(T,0)$ while $(N-1)$ users with $f(W,1)$. If we choose $f(T,0) = 1$ and $f(W,1) = 0$, then a success is guaranteed in the current slot. However, this allows an initially successful user to "capture" the channel for all the subsequent slots. We assume that the protocol designer wants to prevent this phenomenon and to bound the expected number of slots with consecutive successes from fairness considerations. This requirement leads to the following constraint:

$$f(T,0) \leq 1-\theta. \tag{5}$$

Since $0 \leq f(T,0) \leq 1$, we require that $0 \leq \theta \leq 1$. When (5) is imposed, the expected number of slots with consecutive successes is bounded by $1/\theta$ when $\theta > 0$. We call θ the *short-term fairness parameter* because as θ is larger, a capture by a user lasts shorter on average.

Lastly, suppose that the outcome in the previous slot is in $\mathcal{A}_2, \ldots, \mathcal{A}_N$, i.e., there was a collision. The transmission probability that has not been specified is $f(T,1)$. With transmission probabilities chosen so far, i.e., $f(W,0) = 1/N$, $f(W,1) = 0$, and $f(T,0) = 1-\theta$, a transition from a success state to a collision state is not possible, and from an idle state, $\mathcal{A}_2$ is most likely among $\mathcal{A}_2, \ldots, \mathcal{A}_N$. Hence, we choose $f(T,1)$ to maximize the one-step transition probability from $\mathcal{A}_2$ to $\mathcal{A}_1$. Since there are two users who transmit with $f(T,1)$ while others wait, the one-step transition probability is maximized at $f(T,1) = 1/2$. The discussion so far provides an approximate solution to the problem of maximizing one-step transition probabilities to a success state, which we denote by $\tilde{f}$ where $\tilde{f}(W,0) = 1/N$, $\tilde{f}(W,1) = 0$, $\tilde{f}(T,0) = 1-\theta$, and $\tilde{f}(T,1) = 1/2$.

Fixing the short-term fairness parameter at θ and considering stationary decision rules based on one-period memory, we can formulate the problem of the protocol designer who seeks to maximize total throughput. The protocol designer's problem (PDP) is written as

$$\text{(PDP}(\theta,1)) \qquad \hat{\tau}(\theta) = \max_{\mathbf{f} \in \mathcal{F}_1^N} \tau(\mathbf{f}) \tag{6}$$

subject to

$$f_1 = \cdots = f_N, \tag{7}$$

$$f_i(T,0) \leq 1-\theta, \text{ for all } i \in \mathcal{N}. \tag{8}$$

(7) is the symmetry constraint that the protocol designer cannot differentiate users. We use f^* to denote a decision rule that solves (PDP(θ,1)) when used by every user.

Given a protocol $\mathbf{f}$, we define

$$\theta_i = 1 - f_i(T,0), \tag{9}$$

for $i \in \mathcal{N}$. Then the expected number of slots with the consecutive successes of user i is bounded by $1/\theta_i$. We take the minimum of θ_i to obtain

$$\theta^*(\mathbf{f}) \triangleq \min\{\theta_1, \ldots, \theta_N\}, \tag{10}$$

and call $\theta^*(\mathbf{f})$ the *short-term fairness level* of the protocol $\mathbf{f}$. Then the protocol designer can evaluate a given protocol $\mathbf{f}$ in three aspects:

1) Throughput $\tau^*(\mathbf{f}) \triangleq (\tau_1(\mathbf{f}), \ldots, \tau_N(\mathbf{f}))$,
2) Short-term fairness $\theta^*(\mathbf{f})$, and
3) Complexity $m^*(\mathbf{f})$.

Criteria to evaluate throughput can include efficiency (i.e., total throughput) and equity (i.e., uniformity of individual throughput).

To formulate the problem of the protocol designer at a general level, we assume that the protocol designer has preferences over throughput, short-term fairness, and complexity, which are represented by a utility function U defined on $[0,1]^N \times [0,1] \times \mathbb{N}_+$. Then the protocol designer's overall problem (PDOP) can be written as

$$\text{(PDOP)} \quad \max_{\mathbf{f} \in \mathcal{F}^N} U(\tau^*(\mathbf{f}), \theta^*(\mathbf{f}), m^*(\mathbf{f})). \tag{11}$$

In this formulation, (PDP(θ,1)) can be interpreted as a *reduced* problem of (PDOP) when the protocol designer desires to yield equal throughput for every user and has a strong preference for the short-term fairness level of θ and the complexity level of 1.

If users can manipulate a given decision rule to increase their own throughput, then the protocol designer needs to take the incentives of users into consideration. A protocol is *incentive compatible* if no user can increase its throughput by choosing a different decision rule than the one prescribed by the protocol provided that other users follow the prescribed decision rules. Formally, the incentive compatibility (IC) constraint can be expressed as follows and can be incorporated as an additional constraint to the protocol designer's problems, (PDP(θ,1)) and (PDOP), when he faces users who may disobey protocols.

$$\text{(IC)} \qquad \tau_i(f_i, \mathbf{f}_{-i}) \geq \tau_i(g_i, \mathbf{f}_{-i}), \quad \text{for any } g_i \in \mathcal{F}, \text{ for all } i \in \mathcal{N}, \tag{12}$$

where $\mathbf{f}_{-i} \triangleq (f_1, \ldots, f_{i-1}, f_{i+1}, \ldots, f_N)$.

IV. ANALYTICAL RESULTS

In this section, we provide analytical results on the optimal protocol and the optimal value of (PDP(θ,1)). When there are only two users, the protocol designer can find an incentive compatible optimal protocol based on one-period memory that achieves maximum total throughput 1.

Theorem 1: With $N = 2$, $\hat{\tau}(\theta) = 1$ for any $\theta \in [0,1]$.

Proof: Consider a decision rule $\hat{f} \in \mathcal{F}_1$ defined by $\hat{f}(W,0) = \hat{f}(T,1) = 1/2$, $\hat{f}(W,1) = 1$, and $\hat{f}(T,0) = 0$. Note that $\hat{f}$ satisfies (8) for any $\theta \in [0,1]$. The transition probability matrix on $\mathcal{A} =$

$\{(W,W),(W,T),(T,W),(T,T)\}$ when both users use $\hat{f}$ is given by

$$\mathbf{P} = \begin{bmatrix} \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \end{bmatrix}. \quad (13)$$

From the structure of $\mathbf{P}$, we can see that (W,W) and (T,T) are *transient* states while (W,T) and (T,W) are *ergodic* states [12]. Once an ergodic state is reached, (W,T) and (T,W) alternate. Thus, $\tau_1(\hat{\mathbf{f}}) = \tau_2(\hat{\mathbf{f}}) = 1/2$ and $\tau(\hat{\mathbf{f}}) = 1$ where $\hat{\mathbf{f}} \triangleq (\hat{f}, \hat{f})$. Since $\tau(\mathbf{f}) \leq 1$ for all $\mathbf{f}$, $\hat{\mathbf{f}}$ attains the maximum of (PDP(θ,1)). ■

Theorem 2: With $N = 2$, $\hat{\mathbf{f}}$ is incentive compatible.

Proof: Suppose that user j uses $\hat{f}$. Since $\hat{f}(W,1) = 1$, user $i \neq j$ cannot have two consecutive successes. Hence, $\tau_i(f_i, \hat{f}) \leq 1/2$ for any $f_i \in \mathcal{F}$. Since $\tau_i(\hat{\mathbf{f}}) = 1/2$, it follows that $\hat{\mathbf{f}}$ is incentive compatible. ■

Theorem 1 shows that channel sharing between two users can be achieved without communication when they use the decision rule $\hat{f}$. Initially, they contend with each other with transmission probability 1/2. Once a user succeeds, they take a turn by alternating between T and W. This perfect channel sharing scheme is no longer possible with three or more users. If three or more users use $\hat{f}$, then a success can last only one slot because it will be followed by a collision for sure, and as a result the system will be in a collision state most of the time. Hence, with three or more users, the approximate solution $\tilde{f}$ found in Section III is more relevant than the optimal solution with two users, $\hat{f}$. The next theorem provides a lower bound on the maximum value of (PDP(θ,1)) by deriving the expression for $\tau(\tilde{\mathbf{f}})$ where $\tilde{\mathbf{f}} \triangleq (\tilde{f}, \dots, \tilde{f})$.

Theorem 3: Suppose $\theta > 0$ in (8). Define

$$q_k = C_k^N (1/N)^k (1 - 1/N)^{N-k}, \quad (14)$$

for $k = 0, \dots, N$. Define recursively from $k = N$ down to 2 by $H_k(k) = 1$ and

$$\begin{aligned} H_{k'}(k) &= \frac{C_k^{k+1}}{2^{k+1}-1} H_{k'}(k+1) + \frac{C_k^{k+2}}{2^{k+2}-1} H_{k'}(k+2) \\ &\quad + \cdots + \frac{C_k^{k'}}{2^{k'}-1} H_{k'}(k'), \end{aligned} \quad (15)$$

for $k' = k+1, \dots, N$. Also, define

$$G_k = \frac{2^k}{2^k - 1} \sum_{j=k}^{N} H_j(k) q_j, \quad (16)$$

for $k = 2, \dots, N$, and

$$G_1(\theta) = \frac{1}{\theta} \left(1 - q_0 - \sum_{k=2}^{N} \frac{G_k}{2^k} \right). \quad (17)$$

Then

$$\hat{\tau}(\theta) \geq \frac{G_1(\theta)}{1 + G_1(\theta) + G_2 + \cdots + G_N}. \quad (18)$$

If $\theta = 0$, then $\hat{\tau}(0) = 1$.

Proof: The lower bound in the theorem is total throughput attained at $\tilde{\mathbf{f}}$. Let $P(k'|k)$ be the transition probability from $\mathcal{A}_k$ to $\mathcal{A}_{k'}$ when every user uses $\tilde{f}$. The transition probabilities are given by

$$P(k'|0) = q_{k'} \quad \text{for } k' = 0, \dots, N, \quad (19)$$

$$P(k'|1) = \begin{cases} \theta & \text{for } k' = 0 \\ 1 - \theta & \text{for } k' = 1 \\ 0 & \text{for } k' = 2, \dots, N, \end{cases} \quad (20)$$

$$P(k'|k) = \begin{cases} \frac{C_{k'}^k}{2^k} & \text{for } k' = 1, \dots, k \\ 0 & \text{for } k' = k+1, \dots, N \end{cases}, \quad (21)$$

for $k = 2, \dots, N$.

If $\theta = 0$, then $\mathcal{A}_1$ is the unique ergodic state, and thus $\tau(\tilde{f}) = 1$ implying $\hat{\tau}(0) = 1$. If $\theta > 0$, then every state of the Markov chain is positive-recurrent since $P(0|k) > 0$ for all $k = 0, \dots, N$ and $P(k'|0) > 0$ for all $k' = 0, \dots, N$. We denote the unique stationary distribution by $(\pi_k)_{k=0}^N$ where π_k is the probability of $\mathcal{A}_k$ in steady state. Using the stationarity condition $\pi'_k = \sum_{k=0}^N P(k'|k)\pi_k$ for $k = 0, \dots, N$ (one of them redundant), we obtain $\pi_k = G_k \pi_0$ for $k = 1, \dots, N$. Imposing the probability condition $\sum_{k=0}^N \pi_k = 1$, we get

$$\pi_1 = \frac{G_1(\theta)}{1 + G_1(\theta) + G_2 + \cdots + G_N}, \quad (22)$$

which is total throughput at the approximate solution. ■

G_2 through G_N are independent of θ, and G_1 is decreasing in θ. This implies that the lower bound is decreasing in the short-term fairness parameter θ, leading to a trade-off between throughput and fairness. Since $G_1 \to \infty$ as $\theta \to 0$, total throughput can be made arbitrarily close to 1 by choosing θ sufficiently small, and total throughput is 1 when $\theta = 0$, which allows the capture of the channel by a user for an infinite number of slots.

V. NUMERICAL RESULTS

Since the objective function of (PDP(θ,1)) is derived from the stationary distribution of a Markov chain, it is difficult to express it analytically, and thus we rely on numerical methods to compute f^* for $N \geq 3$. Table I and Fig. 1 show optimal decision rules for (PDP(θ,1)) where $\theta = 0.1$. We can see that the approximate solution is quite close to the optimal solution. As a result, the lower bounds found in Theorem 3 are tight as shown in Table II and Fig. 2.

Table II and Fig. 2 also make a comparison of total throughput under different protocols. A two-state protocol is proposed in [8] where users use different transmission probabilities depending on whether they are in a free state or in a backlogged state. The total throughput under η-short-term fairness is given in (6) of [8]. We choose $\eta = 1/\theta = 10$ so that the expected numbers of slots with consecutive successes are the same with (8) and η-short-term fairness. The total throughput of the two-state protocol can be obtained by a decision rule based on one-period memory f_{two} where $f_{two}(T,0) = 1$ and $f_{two}(W,0) = f_{two}(W,1) = f_{two}(T,0) = 1 - \sqrt[N-1]{1 - \frac{1}{\eta}}$. Since f_{two} does not fully utilize information from the previous slot, there is a reduction

TABLE I
OPTIMAL DECISION RULES FOR (PDP(0.1,1))

N	$f^*(W,0)$	$f^*(W,1)$	$f^*(T,0)$	$f^*(T,1)$
3	0.339	0	0.9	0.491
4	0.258	0	0.9	0.486
5	0.207	0	0.9	0.483
10	0.105	0	0.9	0.479
15	0.070	0	0.9	0.478
20	0.053	0	0.9	0.477

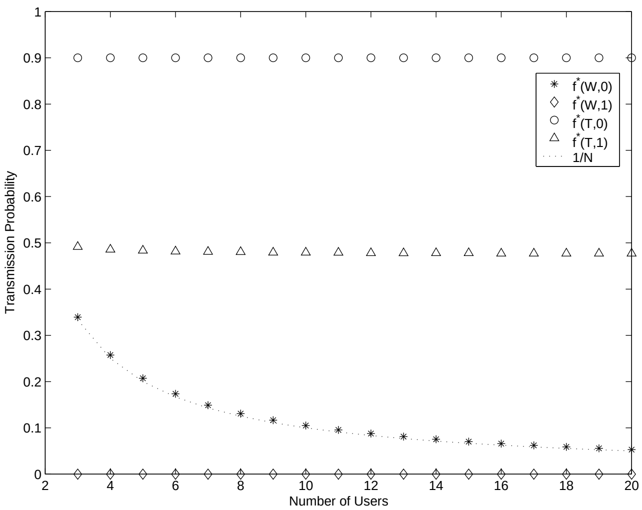

Fig. 1. Optimal decision rules for (PDP(0.1,1))

in obtained total throughput compared to that obtained using f^*. If users do not use information from the past at all, i.e., $m^* = 0$, then they transmit with the same transmission probability in every slot. With (7), the probability of success is maximized when users use $f_{one} \equiv 1/N$. This yields the total throughput of $(1 - 1/N)^{N-1}$, which converges to $1/e \approx 0.368$ as $N \to \infty$.

Fig. 3 illustrates a trade-off between total throughput and the short-term fairness parameter at f^* and $\tilde{f}$ with $N = 10$. A divergence between the two decision rules occurs for $\theta > 0.5$ because the structure of f^* changes in that region so that $f^*(W,1) > 0$. With both decision rules, higher total throughput can be achieved with a smaller short-term fairness parameter. The key feature of protocols based on one-period memory is their ability to correlate successful users in the previous slot and in the current slot. The degree of correlation is determined by θ. When θ is close to 1, this correlation loses its force, and thus utilizing information from the previous slot does not help to increase throughput.

VI. CONCLUSION

We have proposed a class of distributed MAC protocols that can be implemented by users based on their local information. Without explicit message passing, the local information of a user consists of its own transmission decisions and feedback information on others' transmission decisions in the past. By utilizing one-period local information, users can divide themselves into two groups unless all of them

TABLE II
COMPARISON OF TOTAL THROUGHPUT UNDER DIFFERENT PROTOCOLS

N	$\mathbf{f}^*$	$\tilde{\mathbf{f}}$	$\mathbf{f}_{two}$	$\mathbf{f}_{one}$
3	0.8200	0.8199	0.5808	0.4444
4	0.8140	0.8139	0.5541	0.4219
5	0.8105	0.8104	0.5391	0.4096
10	0.8040	0.8038	0.5116	0.3874
15	0.8020	0.8017	0.5030	0.3806
20	0.8009	0.8007	0.4988	0.3774

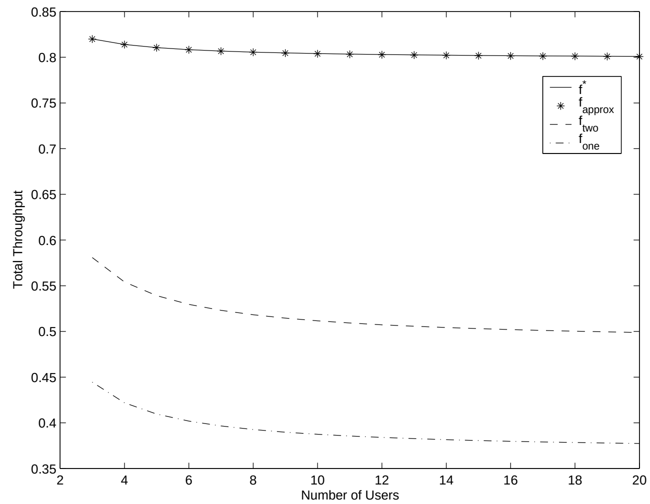

Fig. 2. Total throughput under different protocols ($\mathbf{f}_{approx} = \tilde{\mathbf{f}}$)

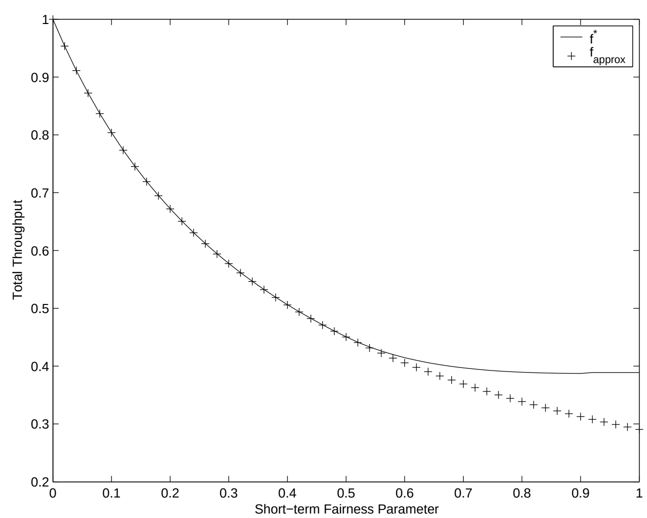

Fig. 3. Trade-off between throughput and fairness with $N = 10$ ($\mathbf{f}_{approx} = \tilde{\mathbf{f}}$)

took the same action in the previous slot. They can increase the probability of successful transmission by choosing different transmission probabilities depending on the group they belong to. As shown in Section V, the utilization of this information leads to high total throughput compared to the case where no or limited information is used.

Our framework is general in that local information can include any information that users have. For example, it can contain information about channel conditions, the priorities of packets, or the actions or characteristics of other users. Therefore, the framework can be used to analyze more complex scenarios in which local information contains more than past transmission decisions and feedback information. In the case where local information expands due to the complications of the system, we can use the framework to develop protocols that perform well facing a specific complication under consideration. If the amount of information in local information increases because of message passing, then the framework can be used to find a protocol that makes the best use of additional information and to analyze the value of additional information to the system and users.

References

[1] D. J. Aldous, "Ultimate instability of exponential back-off protocol for acknowledgement-based transmission control of randon access communication channels," *IEEE Trans. Inf. Theory*, vol. IT-33, no. 2, pp. 219–223, Mar. 1987.

[2] J. Goodman, A. G. Greenberg, N. Mardras, and P. March, "Stability of binary exponential backoff," *JACM*, vol. 35, no. 3, pp. 579–602, Jul. 1988.

[3] D. G. Jeong and W. S. Jeon, "Performance of an exponential backoff scheme for slotted-Aloha protocol in local wireless environment," *IEEE Trans. Veh. Technol.*, vol. 44, no. 3, pp. 579–602, Aug. 1995.

[4] B.-J. Kwak, N.-O. Song, and L. E. Miller, "Performance analysis of exponential backoff," *IEEE/ACM Trans. Netw.*, vol. 13, no. 2, pp. 343–355, Apr. 2005.

[5] Y. Jin and G. Kesidis, "Equilibria of a noncooperative game for heterogeneous users of an Aloha network," *IEEE Commun. Lett.*, vol. 6, no. 7, pp. 282–284, Jul. 2002.

[6] A. B. MacKenzie and S. B. Wicker, "Stability of slotted Aloha with multipacket reception and selfish users," *Proc. INFOCOM*, 2003.

[7] E. Altman, R. El Azouzi, and T. Jiménez, "Slotted Aloha as a game with partial information," *Comput. Networks*, vol. 45, no. 6, pp. 701–713, Aug. 2004.

[8] R. T. Ma, V. Misra, and D. Rubenstein, "An Analysis of Generalized Slotted-Aloha Protocols," *IEEE/ACM Trans. Netw.*, to be published.

[9] G. Mailath and L. Samuelson, *Repeated Games and Reputations: Long-run Relationships*, New York: Oxford Univ. Press, 2006.

[10] D. Bertsekas and R. Gallager, *Data Networks*, 2nd ed. Saddle River, NJ: Prentice Hall, 1992.

[11] E. Altman, N. Bonneau, and M. Debbah, "Correlated equilibrium in access control for wireless communications," *Proc. NETWORKING*, 2006, pp. 173–183.

[12] C. D. Meyer, *Matrix Analysis and Applied Linear Algebra*, Philadelphia, PA: SIAM, 2000.