This paper was presented as part of the Mini-Conference at IEEE INFOCOM 2011

Incentive Provision Using Intervention

Jaeok Park, Mihaela van der Schaar
Electrical Engineering Department, University of California, Los Angeles (UCLA)

***Abstract*—Overcoming the inefficiency of non-cooperative outcomes poses an important challenge for network managers in achieving efficient utilization of network resources. This paper studies a class of incentive schemes based on intervention, which are aimed to drive self-interested users towards a system objective. A manager can implement an intervention scheme by introducing in the network an intervention device that is able to monitor the actions of users and to take an action that influences the network usage of users. We consider the case of perfect monitoring, where the intervention device can immediately observe the actions of users without errors. We also assume that there exist actions of the intervention device that are most and least preferred by all users and the intervention device, regardless of the actions of users. We derive analytical results about the outcomes achievable with intervention and optimal intervention rules, and illustrate the results with an example based on random access networks.**

***Index Terms*—Game theory, incentive schemes, intervention, network management, random access networks**

I. Introduction

The operation of networks by non-cooperative, self-interested users in general leads to a suboptimal performance [1]. Hence, incentive schemes are needed to overcome the inefficiency of non-cooperative equilibrium, and different forms of incentive schemes have been investigated in the literature. One form of incentive schemes widely studied in economics and engineering is pricing [2]. Pricing can induce efficient use of network resources by aligning private incentives with social objectives. Although pricing has a solid theoretical foundation, the implementation of pricing schemes may be challenged by technical difficulties or policy considerations. Let us consider a wireless Internet service as an example. A service provider can limit access to its network resources by charging an access fee. However, charging an access fee requires a secure and reliable method to process payments, which creates burden on both sides of users and service providers. There also arises the issue of allocative fairness when a service provider charges for the Internet service. In the presence of the income effect, uniform pricing will bias the allocation of network resources towards users with high incomes. Because the Internet can play the role of an information equalizer, it has been argued in a public policy debate that access to the Internet should be provided as a public good by a public authority rather than as a private good in a market [3].

Another method to provide incentives is to use repeated interaction [4]. Repeated interaction can encourage cooperative behavior by adjusting future payoffs depending on current behavior. A repeated game strategy can form a basis of an incentive scheme in which monitoring and punishment burden is decentralized to users (see, for example, [5]). However, implementing a repeated game strategy requires repeated interaction among users, which may not be available. For example, users interacting in a mobile network change frequently in nature.

In this paper, we study yet another form of incentive schemes, which use the idea of intervention [6]. To implement an intervention scheme, a network manager introduces in the network an intervention device that is able to monitor the actions of users and to take an action that influences their network usage. Intervention affects the payoffs of users by directly influencing their network usage, whereas pricing does so by using an outside instrument, money. Thus, intervention schemes have a potential to offer an effective and robust method to provide incentives in that users cannot avoid intervention as long as they use network resources. Moreover, it does not require long-term relationship among users, which makes it applicable to networks with a dynamically changing user population. Also, since intervention usually affects payoffs through measurable physical quantities arising from network usage, the information about the impact of intervention can be easily obtained.

The remainder of this paper is organized as follows. In Section II, we formulate intervention games with perfect monitoring. In Section III, we characterize sustainable action profiles and intervention equilibria. In Section IV, we provide an illustrative example using random access networks. In Section V, we conclude.

II. Intervention Games with Perfect Monitoring

We consider a network where N users and an intervention device interact. The set of the users is denoted by $\mathcal{N} = \{1, \ldots, N\}$. The action space of user i is denoted by $\mathcal{A}_i$, and an action for user i is denoted by $a_i \in \mathcal{A}_i$, for all $i \in \mathcal{N}$. An action profile is represented by a vector $a = (a_1, \ldots, a_N) \in \mathcal{A} \triangleq \prod_{i \in \mathcal{N}} \mathcal{A}_i$. An action profile of the users other than user i is written as $a_{-i} = (a_1, \ldots, a_{i-1}, a_{i+1}, \ldots, a_N)$ so that a can be expressed as $a = (a_i, a_{-i})$. Once an action profile of the users is determined, a signal is realized and observed by the intervention device. In this paper, we consider *perfect monitoring*, where the intervention device observes the actions chosen by the users immediately without errors. After observing the action profile, the intervention device chooses an action a_0 from the set of available actions for it, denoted $\mathcal{A}_0$.[1] Hence, a strategy for the intervention device can be represented by a mapping $f : \mathcal{A} \rightarrow \mathcal{A}_0$, which is called an

[1] In this paper, we restrict attention to pure actions for simplicity, although our formulation can be easily extended to the case where mixed actions are allowed.

intervention rule. The set of all possible intervention rules is denoted by $\mathcal{F}$.

There is a network manager who determines the intervention rule used by the intervention device. The payoffs of the users and the manager are determined by the actions of the intervention device and the users. We denote the payoff function of user i by $u_i : \mathcal{A}_0 \times \mathcal{A} \to \mathbb{R}$ and that of the manager by $u_0 : \mathcal{A}_0 \times \mathcal{A} \to \mathbb{R}$. The payoff of the manager can be regarded as a measure of network performance. We assume that the manager can commit to an intervention rule, for example, by using a protocol embedded in the intervention device. The game played by the manager and the users is called an *intervention game*, and the sequence of events in an intervention game with perfect monitoring can be listed as follows.

1) The manager chooses an intervention rule $f \in \mathcal{F}$.
2) The users choose their actions $a \in \mathcal{A}$ simultaneously, knowing the intervention rule f chosen by the manager.
3) The intervention device observes the action profile a and takes the action $a_0 = f(a) \in \mathcal{A}_0$.

Define a function $v_i : \mathcal{F} \times \mathcal{A} \to \mathbb{R}$ by $v_i(f, a) = u_i(f(a), a)$, for all $i \in \mathcal{N}_0 \triangleq \mathcal{N} \cup \{0\}$. Once the manager chooses an intervention rule f, the users play a simultaneous game, whose normal form representation is given by

$$\Gamma_f = \langle \mathcal{N}, (\mathcal{A}_i)_{i \in \mathcal{N}}, (v_i(f, \cdot))_{i \in \mathcal{N}} \rangle .$$

We predict actions chosen by the users given an intervention rule f by applying the solution concept of Nash equilibrium [7] to the induced game Γ_f.

Definition 1: An intervention rule $f \in \mathcal{F}$ *sustains* an action profile $a^* \in \mathcal{A}$ if a^* is a Nash equilibrium of the game Γ_f, i.e.,

$$v_i(f, a_i^*, a_{-i}^*) \geq v_i(f, a_i, a_{-i}^*)$$

for all $a_i \in \mathcal{A}_i$, for all $i \in \mathcal{N}$. An action profile a^* is *sustainable* if there exists an intervention rule f that sustains a^*.

Let $\mathcal{E}(f) \subset \mathcal{A}$ be the set of action profiles sustained by f. Then the set of all sustainable action profiles is given by $\mathcal{E} = \cup_{f \in \mathcal{F}} \mathcal{E}(f)$. A pair (f, a) is said to be attainable if $a \in \mathcal{E}(f)$. The manager's problem is to find an attainable pair that maximizes his payoff among all attainable pairs, which leads to the following solution concept for intervention games.

Definition 2: $(f^*, a^*) \in \mathcal{F} \times \mathcal{A}$ is an *intervention equilibrium* if $a^* \in \mathcal{E}(f^*)$ and

$$v_0(f^*, a^*) \geq v_0(f, a)$$

for all (f, a) such that $a \in \mathcal{E}(f)$. $f^* \in \mathcal{F}$ is an *optimal intervention rule* if there exists an action profile a^* such that (f^*, a^*) is an intervention equilibrium.

An intervention equilibrium solves the following problem:

$$\max_{f \in \mathcal{F}} \max_{a \in \mathcal{E}(f)} v_0(f, a).$$

An intervention equilibrium can be considered as a subgame perfect equilibrium, with an implicit assumption that the manager can induce the users to choose the best Nash equilibrium for him in case of multiple Nash equilibria. One possible explanation for this is that the manager recommends to the users an action profile sustained by the intervention rule he chooses so that the action profile becomes a focal point [7].

III. Analytical Results

In this section, we derive analytical results about sustainable action profiles and intervention equilibria imposing the following assumption.

Assumption 1: There exist two actions for the intervention device $\underline{a}_0, \overline{a}_0 \in \mathcal{A}_0$ that satisfy for all $i \in \mathcal{N}_0$,

$$u_i(\underline{a}_0, a) \geq u_i(a_0, a) \geq u_i(\overline{a}_0, a) \tag{1}$$

for all $a_0 \in \mathcal{A}_0$, for all $a \in \mathcal{A}$.

$\underline{a}_0$ and $\overline{a}_0$ can be interpreted as the most and least preferred actions of the intervention device, respectively. For any given $a \in \mathcal{A}$, the users and the manager receive the highest (resp. lowest) payoff when the intervention device takes the most (resp. least) preferred intervention action. Assumption 1 allows the intervention device to reward or punish all the users at the same time. Below we define a class of intervention rules.

Definition 3: $f_{\tilde{a}} : \mathcal{A} \to \mathcal{A}_0$ is an *extreme intervention rule with target action profile* $\tilde{a} \in \mathcal{A}$ if

$$f_{\tilde{a}}(a) = \begin{cases} \underline{a}_0 & \text{if } a = \tilde{a}, \\ \overline{a}_0 & \text{otherwise.} \end{cases}$$

With an extreme intervention rule, the intervention device chooses the most preferred action when the users follow the target action profile, and chooses the least preferred action when they deviate. Hence, an extreme intervention rule provides the strongest incentive for sustaining a given target action profile, which leads to the following lemma.

Lemma 1: If $a^* \in \mathcal{E}$, then $a^* \in \mathcal{E}(f_{a^*})$.

Proof: Suppose that $a^* \in \mathcal{E}$. Then there exists an intervention rule f such that $v_i(f, a^*) \geq v_i(f, a_i, a_{-i}^*)$ for all $a_i \in \mathcal{A}_i$, for all $i \in \mathcal{N}$. Then we obtain $v_i(f_{a^*}, a^*) = u_i(\underline{a}_0, a^*) \geq u_i(f(a^*), a^*) \geq u_i(f(a_i, a_{-i}^*), a_i, a_{-i}^*) \geq u_i(\overline{a}_0, a_i, a_{-i}^*) = v_i(f_{a^*}, a_i, a_{-i}^*)$ for all $a_i \neq a_i^*$, for all $i \in \mathcal{N}$, where the first and the third inequalities follow from (1). ■

Let $\mathcal{E}^* = \{a \in \mathcal{A} : a \in \mathcal{E}(f_a)\}$. The following results can be obtained by using Lemma 1.

Proposition 1: (i) $\mathcal{E} = \mathcal{E}^*$.
(ii) If (f^*, a^*) is an intervention equilibrium, then (f_{a^*}, a^*) is also an intervention equilibrium.

Proof: (i) Let $a^* \in \mathcal{E}^*$. Then $a^* \in \mathcal{E}(f_{a^*}) \subset \mathcal{E}$. Hence, $\mathcal{E}^* \subset \mathcal{E}$. The other inclusion $\mathcal{E} \subset \mathcal{E}^*$ follows from Lemma 1.

(ii) Suppose that (f^*, a^*) is an intervention equilibrium. Then by Definition 2, $a^* \in \mathcal{E}(f^*)$ and $v_0(f^*, a^*) \geq v_0(f, a)$ for all (f, a) such that $a \in \mathcal{E}(f)$. Since $a^* \in \mathcal{E}$, we have $a^* \in \mathcal{E}(f_{a^*})$ by Lemma 1. Hence, $v_0(f^*, a^*) \geq v_0(f_{a^*}, a^*)$. On the other hand, since $f_{a^*}(a^*) = \underline{a}_0$, we have $v_0(f^*, a^*) \leq v_0(f_{a^*}, a^*)$ by (1). Therefore, $v_0(f^*, a^*) = v_0(f_{a^*}, a^*)$, and thus $v_0(f_{a^*}, a^*) \geq v_0(f, a)$ for all (f, a) such that $a \in \mathcal{E}(f)$. This proves that (f_{a^*}, a^*) is an intervention equilibrium. ■

Proposition 1 shows that it is without loss of generality to restrict attention to pairs of the form (f_a, a) when we ask whether a given action profile is sustainable and whether there exists an intervention equilibrium. The role of extreme intervention rules is analogous to that of grim trigger strategies in repeated games with perfect monitoring. To generate the set of equilibrium payoffs, it suffices to consider grim trigger strategies that trigger the most severe punishment in case of a deviation. Under Assumption 1, the least preferred intervention action $\overline{a}_0$ plays a similar role to mutual minmaxing [4] in that it provides the strongest threat to deter a deviation.

Proposition 2: (f_{a^*}, a^*) is an intervention equilibrium if and only if $a^* \in \mathcal{E}^*$ and $u_0(\underline{a}_0, a^*) \geq u_0(\underline{a}_0, a)$ for all $a \in \mathcal{E}^*$.

Proof: Suppose that (f_{a^*}, a^*) is an intervention equilibrium. Then $a^* \in \mathcal{E}(f_{a^*})$, and thus $a^* \in \mathcal{E}^*$. Also, $v_0(f_{a^*}, a^*) \geq v_0(f, a)$ for all (f, a) such that $a \in \mathcal{E}(f)$. Choose any $a \in \mathcal{E}^*$. Then $a \in \mathcal{E}(f_a)$, and thus $u_0(\underline{a}_0, a^*) = v_0(f_{a^*}, a^*) \geq v_0(f_a, a) = u_0(\underline{a}_0, a)$.

Suppose that $a^* \in \mathcal{E}^*$ and $u_0(\underline{a}_0, a^*) \geq u_0(\underline{a}_0, a)$ for all $a \in \mathcal{E}^*$. To prove that (f_{a^*}, a^*) is an intervention equilibrium, we need to show that (i) $a^* \in \mathcal{E}(f_{a^*})$, and (ii) $v_0(f_{a^*}, a^*) \geq v_0(f, a)$ for all (f, a) such that $a \in \mathcal{E}(f)$. (i) follows from $a^* \in \mathcal{E}^*$. To prove (ii), choose any (f, a) such that $a \in \mathcal{E}(f)$. By Lemma 1, we have $a \in \mathcal{E}^*$. Then $v_0(f_{a^*}, a^*) = u_0(\underline{a}_0, a^*) \geq u_0(\underline{a}_0, a) \geq v_0(f, a)$, where the first inequality follows from $a \in \mathcal{E}^*$. ■

Proposition 2 shows that the pair (f_a, a) constitutes an intervention equilibrium if a solves

$$\max_{a \in \mathcal{E}^*} u_0(\underline{a}_0, a). \tag{2}$$

The next proposition provides a sufficient condition under which an intervention equilibrium exists.

Proposition 3: Suppose that $\mathcal{A}_i$ is a bounded set in Euclidean space, for all $i \in \mathcal{N}$, and that $u_i(a_0, a)$ is continuous in a, for $a_0 = \underline{a}_0, \overline{a}_0$, for all $i \in \mathcal{N}_0$. Then there exists an intervention equilibrium.

Proof: By Propositions 1(ii) and 2, an intervention equilibrium exists if and only if there exists a solution to the problem (2). Since $u_0(\underline{a}_0, a)$ is continuous in a, the result follows if we show that the constraint set $\mathcal{E}^*$ is compact. Since $\mathcal{E}^* \subset \mathcal{A}$ and $\mathcal{A}$ is bounded, $\mathcal{E}^*$ is also bounded. To show that $\mathcal{E}^*$ is closed, choose a sequence $\{a^n\}$ with $a^n \to a^*$ and $a^n \in \mathcal{E}^*$ for all n. Choose any $i \in \mathcal{N}$ and $a_i' \in \mathcal{A}_i$. Since $a^n \in \mathcal{E}(f_{a^n})$, we have $u_i(\underline{a}_0, a^n) \geq u_i(\overline{a}_0, a_i', a_{-i}^n)$. Since $u_i(a_0, a)$ is continuous in a for $a_0 = \underline{a}_0, \overline{a}_0$, we obtain $u_i(\underline{a}_0, a^*) \geq u_i(\overline{a}_0, a_i', a_{-i}^*)$ by taking limits. This proves $a^* \in \mathcal{E}(f_{a^*})$ and thus $a^* \in \mathcal{E}^*$. ■

Extreme intervention rules are useful to characterize sustainable action profiles and intervention equilibria. However, they may not be desirable in practice. For example, when a user chooses an action different from the target action by mistake (i.e., trembling hands), an extreme intervention rule triggers the most severe punishment, resulting in a large performance loss. Thus, it is of interest to investigate intervention rules that use weaker punishments than extreme intervention rules do. To obtain concrete results, we assume that $\mathcal{A}_i = [\underline{a}_i, \overline{a}_i] \subset \mathbb{R}$ with $\underline{a}_i < \overline{a}_i$ for all $i \in \mathcal{N}_0$ in the remainder of this section. Below we define another class of intervention rules.

Definition 4: $f_{\tilde{a},c} : \mathcal{A} \to \mathcal{A}_0$ is a *(truncated) affine intervention rule with target action profile* $\tilde{a} \in \mathcal{A}$ *and intervention rate profile* $c \in \mathbb{R}^N$ if

$$f_{\tilde{a},c}(a) = [c \cdot (a - \tilde{a}) + \underline{a}_0]_{\underline{a}_0}^{\overline{a}_0},$$

where $[x]_\alpha^\beta = \min\{\max\{x, \alpha\}, \beta\}$.

The following proposition constructs an affine intervention rule to sustain an interior target action profile in the differentiable payoff case.

Proposition 4: Let $a^* \in \mathcal{A}$ be an action profile such that $a_i^* \in (\underline{a}_i, \overline{a}_i)$ for all $i \in \mathcal{N}$. Suppose that, for all $i \in \mathcal{N}$, u_i is twice continuously differentiable and $u_i(a_0, a^*)$ is strictly decreasing in a_0 on $[\underline{a}_0, \overline{a}_0]$. Let

$$c_i^* = -\frac{\partial u_i(\underline{a}_0, a^*)/\partial a_i}{\partial u_i(\underline{a}_0, a^*)/\partial a_0} \tag{3}$$

for all $i \in \mathcal{N}$.[2] Suppose that

$$\frac{\partial^2 u_i}{\partial a_i^2}(\underline{a}_0, a_i, a_{-i}^*) \leq 0 \quad \text{for all } a_i \in (\underline{a}_i, \overline{a}_i) \tag{4}$$

for all $i \in \mathcal{N}$ such that $c_i^* = 0$,

$$\frac{\partial^2 u_i}{\partial a_i^2}(\underline{a}_0, a_i, a_{-i}^*) \leq 0 \quad \text{for all } a_i \in (\underline{a}_i, a_i^*), \tag{5}$$

$$(c_i^*)^2 \frac{\partial^2 u_i}{\partial a_0^2} + 2c_i^* \frac{\partial^2 u_i}{\partial a_i \partial a_0} + \frac{\partial^2 u_i}{\partial a_i^2} \leq 0$$
$$\text{(left-hand side evaluated at } (c_i^*(a_i - a_i^*) + \underline{a}_0, a_i, a_{-i}^*))$$
$$\text{for all } a_i \in (a_i^*, \min\{\overline{a}_i, a_i^* + (\overline{a}_0 - \underline{a}_0)/c_i^*\}), \tag{6}$$

$$\frac{\partial u_i}{\partial a_i}(\overline{a}_0, a_i, a_{-i}^*) \leq 0 \quad \text{for all } a_i \in (a_i^* + (\overline{a}_0 - \underline{a}_0)/c_i^*, \overline{a}_i) \tag{7}$$

for all $i \in \mathcal{N}$ such that $c_i^* > 0$, and

$$\frac{\partial u_i}{\partial a_i}(\overline{a}_0, a_i, a_{-i}^*) \geq 0 \quad \text{for all } a_i \in (\underline{a}_i, a_i^* + (\overline{a}_0 - \underline{a}_0)/c_i^*),$$

$$(c_i^*)^2 \frac{\partial^2 u_i}{\partial a_0^2} + 2c_i^* \frac{\partial^2 u_i}{\partial a_i \partial a_0} + \frac{\partial^2 u_i}{\partial a_i^2} \leq 0$$
$$\text{(left-hand side evaluated at } (c_i^*(a_i - a_i^*) + \underline{a}_0, a_i, a_{-i}^*))$$
$$\text{for all } a_i \in (\max\{\overline{a}_i, a_i^* + (\overline{a}_0 - \underline{a}_0)/c_i^*\}, a_i^*), \tag{8}$$

$$\frac{\partial^2 u_i}{\partial a_i^2}(\underline{a}_0, a_i, a_{-i}^*) \leq 0 \quad \text{for all } a_i \in (a_i^*, \overline{a}_i)$$

for all $i \in \mathcal{N}$ such that $c_i^* < 0$.[3] Then f_{a^*,c^*} sustains a^*.

[2]We define $\partial u_i(\underline{a}_0, a^*)/\partial a_0$ as the right partial derivative of u_i with respect to a_0 at $(\underline{a}_0, a^*)$.

[3]We define $(\alpha, \beta) = \emptyset$ if $\alpha \geq \beta$.

Proof: See the Appendix. ■

Note that $\partial u_i(\underline{a}_0, a^*)/\partial a_0 < 0$ for all $i \in \mathcal{N}$ since $u_i(a_0, a^*)$ is strictly decreasing in a_0. Thus, c_i^*, defined in (3), has the same sign as $\partial u_i(\underline{a}_0, a^*)/\partial a_i$. With $\mathcal{A}_0 = [\underline{a}_0, \overline{a}_0]$, the action of the intervention device can be interpreted as the intervention level, and the users receive higher payoffs as the intervention level is smaller. The affine intervention rule f_{a^*,c^*}, constructed in Proposition 4, has the properties that the intervention device uses the minimum intervention level $\underline{a}_0$ when the users choose the target action profile a^*, i.e., $f_{a^*,c^*}(a^*) = \underline{a}_0$, and that the intervention level increases in the rate of $|c_i^*|$ as user i deviates to the direction in which its payoff increases at $(\underline{a}_0, a^*)$. The expression of c_i^* in (3) has an intuitive explanation. Since c_i^* is proportional to $\partial u_i(\underline{a}_0, a^*)/\partial a_i$ and inversely proportional to $-\partial u_i(\underline{a}_0, a^*)/\partial a_0$, a user faces a higher intervention rate as its incentive to deviate from $(\underline{a}_0, a^*)$ is stronger and as a change in the intervention level has a smaller impact on its payoff. The intervention level does not react to the action of user i when $c_i^* = 0$, because user i chooses a_i^* in its self-interest even when the intervention level is fixed at $\underline{a}_0$, provided that the other users choose a_{-i}^*. Finally, we note that if (f^*, a^*) is an intervention equilibrium and $f_{a^*,c}$ sustains a^* for some c, then $(f_{a^*,c}, a^*)$ is also an intervention equilibrium, since $f_{a^*,c}(a^*) = \underline{a}_0$.

IV. Illustration with Random Access Networks

In this section, we illustrate the analytical results of Section III with a random access network [8]. Time is divided into slots of equal length, and a user can transmit its packet or wait in each slot. Due to interference, a packet is successfully transmitted only if there is no other packet transmitted in the same slot. If more than one packet is transmitted simultaneously, a collision occurs. The intervention device can transmit its packets as the users do, and it interferes with all the users. All the users and the intervention device transmit their packets with fixed probabilities. The action of user i, a_i, is thus its transmission probability, and we have $\mathcal{A}_i = [0, 1]$ for all $i \in \mathcal{N}_0$. The average data rate for user $i \in \mathcal{N}$ when the intervention device and the users transmit according to the probabilities $(a_0, a) \in \mathcal{A}_0 \times \mathcal{A}$ is given by

$$r_i(a_0, a) = \gamma_i a_i \prod_{j \in \mathcal{N}_0 \setminus \{i\}} (1 - a_j),$$

where $\gamma_i > 0$ is the fixed peak data rate for user i. The benefit that user i obtains from its average data rate is represented by a utility function $U_i : \mathbb{R}_+ \to \mathbb{R}$, which is assumed to be continuous and strictly increasing. Hence, the payoff function of user i is given by $u_i(a_0, a) = U_i(r_i(a_0, a))$. The payoff function of the manager is assumed to be $u_0(a_0, a) = \sum_{i \in \mathcal{N}} u_i(a_0, a)$.

In this example, the minimum and maximum intervention levels are given by $\underline{a}_0 = 0$ and $\overline{a}_0 = 1$, respectively. Since $r_i(0, a) \geq 0$ and $r_i(1, a) = 0$ for all $a \in \mathcal{A}$, for all $i \in \mathcal{N}$, we have $\mathcal{E}^* = \mathcal{A}$. Then by Proposition 1(i), any action profile is sustainable, i.e., $\mathcal{E} = \mathcal{A}$.[4] Because the maximum intervention level yields zero rate to all the users regardless of the action profile, the most severe punishment is strong enough to prevent deviations from any target action profile.[5] By Proposition 2, (f_{a^*}, a^*) is an intervention equilibrium if and only if a^* maximizes $u_0(0, a)$ on $\mathcal{A}$. Since $\mathcal{A} = [0, 1]^N$ is compact and u_0 is continuous, a solution to $\max_{a \in \mathcal{A}} u_0(0, a)$ exists, which is consistent with Proposition 3. Also, we can apply Proposition 4 to show that any given action profile $a^* \in (0, 1)^N$ is sustained by f_{a^*,c^*} with $c_i^* = 1/a_i^*$ for all $i \in \mathcal{N}$. Suppose for the moment that the payoff of each user is given by its average data rate, i.e., $u_i(a_0, a) = r_i(a_0, a)$ for all $i \in \mathcal{N}$. Then for each $i \in \mathcal{N}$, we obtain $c_i^* = 1/a_i^* > 0$ by (3), and we can verify that the conditions (5)–(7) are satisfied. Thus, by Proposition 4, we can conclude that f_{a^*,c^*} sustains a^*. Note that the concept of an intervention rule sustaining an action profile is based on Nash equilibrium, which uses only the ordinal properties of payoff functions. Therefore, f_{a^*,c^*} continues to sustain a^* even when the payoff function of user i is given by $u_i(a_0, a) = U_i(r_i(a_0, a))$ for any strictly increasing function U_i, for all $i \in \mathcal{N}$.

The above argument points out an informational advantage that intervention may have over pricing. To highlight the informational advantage of intervention, suppose that the objective of the manager is to implement a target action profile $a^* \in (0, 1)^N$, determined independently of $(U_1, \ldots, U_N)$, while taking the minimum intervention level when the users choose a^*. Then the results in the previous paragraph show that the intervention rules f_{a^*} and f_{a^*,c^*} sustain the action profile a^* for *any* $(U_1, \ldots, U_N)$. Since intervention affects the payoffs of the users only through their rates, it suffices for the manager to have information about how the rates are determined, when it designs an intervention rule that sustains a given target action profile.[6] This property can be considered as the robustness of intervention with respect to the utilities of the users.

To draw a contrast, consider an alternative scenario where the manager provides incentives through pricing. In such a scenario, the action profile of the users determines their average data rates, i.e., $r_i(a) = \gamma_i a_i \prod_{j \in \mathcal{N} \setminus \{i\}} (1 - a_j)$ for all $i \in \mathcal{N}$, while a pricing scheme specifies the payments that the users make depending on their action profile. Let $f_i(a)$ be the payment of user i when the action profile a is chosen, and let $f(a) = (f_1(a), \ldots, f_N(a))$. We consider the payoff function of user i defined by

$$u_i(f(a), a) = U_i(r_i(a)) - f_i(a).$$

As can be seen from the above expression, pricing affects payoff by taking away utility units from the users, and thus

[4] A more interesting result where $\mathcal{E}$ is a strict subset of $\mathcal{A}$ can be obtained by restricting $\mathcal{A}_0$ to be $[0, \overline{a}_0]$ for some $0 < \overline{a}_0 < 1$. We omit the results of this exercise due to space limitation.

[5] In this example, an extreme intervention rule can prevent not only unilateral deviations from its target action profile but also joint deviations.

[6] Of course, if the target action profile depends on $(U_1, \ldots, U_N)$, the manager needs to know $(U_1, \ldots, U_N)$ to determine it.

the shapes and scales of the utility functions matter to the manager when designing a pricing scheme that implements a given target action profile. For example, consider a pricing scheme that charges each user an amount proportional to its average data rate, i.e., $f_i(a) = p_i r_i(a)$ for all $i \in \mathcal{N}$, where p_i is the price of unit data rate for user i. Then the pricing scheme implements an action profile a^* as Nash equilibrium when the manager sets $p_i = U_i'(r_i(a^*))$ for all $i \in \mathcal{N}$, assuming that U_i is differentiable and concave for all i. In order for the manager to achieve a desired outcome by using an incentive scheme, he needs to know how the incentive device used affects the payoff of the users. In many situations, this information is more easily obtained when the manager uses intervention rather than pricing, since intervention usually affects payoffs through measurable physical quantities arising from network usage.

V. Conclusion

In this paper, we have studied a class of incentive schemes based on intervention, which can be implemented by introducing an intervention device that can monitor the actions of the users and affect their network usage. By focusing on the case of perfect monitoring and assuming the existence of the most and least preferred intervention actions, we have characterized sustainable action profiles and intervention equilibria. Using an illustrative example based on random access networks, we have argued that intervention can be used to construct more robust and informationally efficient incentive schemes than pricing schemes. Our framework is general enough to capture not only efficiency but also fairness in the payoff of the manager, and we can analyze how different welfare criteria affect the design of intervention rules. In many practical scenarios, the intervention device can obtain only limited information about the actions of users, and studying the case of imperfect monitoring will provide us with richer insights about the design of incentive schemes based on intervention.

Appendix
Proof of Proposition 4

Proof: To prove that f_{a^*,c^*} sustains a^* is equivalent to show

$$a_i^* \in \arg\max_{a_i \in \mathcal{A}_i} u_i(f_{a^*,c^*}(a_i, a_{-i}^*), a_i, a_{-i}^*) \tag{9}$$

for all $i \in \mathcal{N}$. Note that $f_{a^*,c^*}(a_i, a_{-i}^*) = [c_i^*(a_i - a_i^*) + \underline{a}_0]_{\underline{a}_0}^{\overline{a}_0}$. We consider three cases depending on the sign of c_i^*.

Case 1: $c_i^* = 0$.

By (3), we have $\partial u_i(\underline{a}_0, a^*)/\partial a_i = 0$. Also, we have $f_{a^*,c^*}(a_i, a_{-i}^*) = \underline{a}_0$ for all $a_i \in \mathcal{A}_i$. Thus, the objective function in (9) reduces to $u_i(\underline{a}_0, a_i, a_{-i}^*)$. The condition (4) implies that $u_i(\underline{a}_0, a_i, a_{-i}^*)$ is a concave function with respect to a_i on $\mathcal{A}_i$. Also, the first-order optimality condition is satisfied at $a_i = a_i^*$ since $\partial u_i(\underline{a}_0, a^*)/\partial a_i = 0$. Therefore, a_i^* maximizes $u_i(\underline{a}_0, a_i, a_{-i}^*)$ on $\mathcal{A}_i$.

Case 2: $c_i^* > 0$.

Since $\partial u_i(\underline{a}_0, a^*)/\partial a_0 < 0$, we have $\partial u_i(\underline{a}_0, a^*)/\partial a_i > 0$ by (3). First, consider $a_i \in [\underline{a}_i, a_i^*]$. In this region, $f_{a^*,c^*}(a_i, a_{-i}^*) = \underline{a}_0$, and thus the objective function can be written as $u_i(\underline{a}_0, a_i, a_{-i}^*)$. Since the condition (5) implies that $u_i(\underline{a}_0, a_i, a_{-i}^*)$ is concave with respect to a_i on $[\underline{a}_i, a_i^*]$, $u_i(\underline{a}_0, a_i, a_{-i}^*)$ is strictly increasing in a_i on $[\underline{a}_i, a_i^*]$.

Second, consider $a_i \in [a_i^*, \min\{\overline{a}_i, a_i^* + (\overline{a}_0 - \underline{a}_0)/c_i^*\}]$. In this region, $f_{a^*,c^*}(a_i, a_{-i}^*) = c_i^*(a_i - a_i^*) + \underline{a}_0$, and thus the objective function can be written as $u_i(c_i^*(a_i - a_i^*) + \underline{a}_0, a_i, a_{-i}^*)$. The first derivative of $u_i(c_i^*(a_i - a_i^*) + \underline{a}_0, a_i, a_{-i}^*)$ with respect to a_i is given by

$$\left(c_i^* \frac{\partial u_i}{\partial a_0} + \frac{\partial u_i}{\partial a_i} \right) \bigg|_{(a_0, a_i, a_{-i}) = (c_i^*(a_i - a_i^*) + \underline{a}_0, a_i, a_{-i}^*)},$$

while the second derivative is given by the left-hand side of (6). The first derivative is zero at $a_i = a_i^*$ by (3), while the second derivative is non-positive by (6). Hence, the first derivative is non-positive on $(a_i^*, \min\{\overline{a}_i, a_i^* + (\overline{a}_0 - \underline{a}_0)/c_i^*\})$, and thus $u_i(c_i^*(a_i - a_i^*) + \underline{a}_0, a_i, a_{-i}^*)$ is non-increasing in a_i on $[a_i^*, \min\{\overline{a}_i, a_i^* + (\overline{a}_0 - \underline{a}_0)/c_i^*\}]$.

Lastly, consider $a_i \in [a_i^* + (\overline{a}_0 - \underline{a}_0)/c_i^*, \overline{a}_i]$. In this region, $f_{a^*,c^*}(a_i, a_{-i}^*) = \overline{a}_0$, and thus the objective function can be written as $u_i(\overline{a}_0, a_i, a_{-i}^*)$. Since the first derivative of $u_i(\overline{a}_0, a_i, a_{-i}^*)$ with respect to a_i is non-positive on $(a_i^* + (\overline{a}_0 - \underline{a}_0)/c_i^*, \overline{a}_i)$ by (7), $u_i(\overline{a}_0, a_i, a_{-i}^*)$ is non-increasing in a_i on $[a_i^* + (\overline{a}_0 - \underline{a}_0)/c_i^*, \overline{a}_i]$.

Case 3: $c_i^* < 0$.

In this case, $\partial u_i(\underline{a}_0, a^*)/\partial a_i < 0$ and

$$f_{a^*,c^*}(a_i, a_{-i}^*) = \begin{cases} \overline{a}_0, & \text{if } a_i \in [\underline{a}_i, a_i^* + (\overline{a}_0 - \underline{a}_0)/c_i^*], \\ c_i^*(a_i - a_i^*) + \underline{a}_0, & \\ & \text{if } a_i \in [\max\{\overline{a}_i, a_i^* + (\overline{a}_0 - \underline{a}_0)/c_i^*\}, a_i^*], \\ \underline{a}_0, & \text{if } a_i \in [a_i^*, \overline{a}_i]. \end{cases}$$

Following an analogous argument as in Case 2, we can show that the objective function is non-decreasing in a_i on $[\underline{a}_i, a_i^*]$ and strictly decreasing on $[a_i^*, \overline{a}_i]$, implying that $a_i = a_i^*$ maximizes the objective function on $\mathcal{A}_i$.

Note that if the inequalities in (4), (6), and (8) are strict, we have $a_i = a_i^*$ as a unique maximizer for all $i \in \mathcal{N}$. ■

References

[1] P. Dubey, "Inefficiency of Nash equilibria," *Math. Operations Research*, vol. 11, no. 1, pp. 1–8, Feb. 1986.

[2] J. K. MacKie-Mason and H. R. Varian, "Pricing congestible network resources," *IEEE J. Sel. Areas Commun.*, vol. 13, no. 7, pp. 1141–1149, Sep. 1995.

[3] M. M. Hallgren and A. K. McAdams, "The economic efficiency of Internet public goods," in *Internet Economics*, L. W. McKnight and J. P. Bailey, Eds. Cambridge, MA: MIT Press, 1997, pp. 455–478.

[4] G. Mailath and L. Samuelson, *Repeated Games and Reputations: Long-run Relationships*. Oxford, U.K.: Oxford Univ. Press, 2006.

[5] R. J. La and V. Anantharam, "Optimal routing control: repeated game approach," *IEEE Trans. Autom. Control*, vol.47, no.3, pp.437–450, Mar. 2002.

[6] J. Park and M. van der Schaar, "Stackelberg contention games in multiuser networks," *EURASIP J. Advances Signal Process.*, vol. 2009, Article ID 305978, 15 pages, 2009.

[7] D. Fudenberg and J. Tirole, *Game Theory*. Cambridge, MA: MIT Press, 1991.

[8] A. H. Mohsenian-Rad, J. Huang, M. Chiang, and V. W. S. Wong, "Utility-optimal random access without message passing," *IEEE Trans. Wireless Commun.*, vol. 8, no. 3, pp. 1073–1079, Mar. 2009.