

Distributed Interference Management in Large-Scale Small Cell Networks

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Abstract—We study the problem of interference management in large-scale small cell networks. Each user equipment (UE) needs to determine *in a distributed manner* when and at what power level it should transmit to its serving small cell base station (SBS) such that a given network performance criterion is maximized subject to minimum throughput requirements by the UEs. First, we propose a *distributed* algorithm for the UE-SBS pairs to find a *subset* of weakly interfering UE-SBS pairs, namely the maximal independent sets (MISs) of the interference graph in logarithmic time (with respect to the number of UEs). Then we propose a novel problem formulation which enables UE-SBS pairs to determine the optimal fractions of time occupied by each MIS in a *distributed* manner. We *analytically* bound the performance of our *distributed* policy in terms of the competitive ratio with respect to the optimal network performance, which is obtained in a centralized manner with NP (non-deterministic polynomial time) complexity. Remarkably, the competitive ratio is independent of the network size, which guarantees *scalability* in terms of performance for arbitrarily large networks. Through simulations, we show that our proposed policies achieve significant performance gains (up to 390%) over the existing policies.

I. INTRODUCTION

Dense deployment of low-cost heterogeneous small cells (e.g. picocells, femtocells) has become one of the most effective solutions to accommodate the exploding demand for wireless spectrum [1]. The success of it depends crucially on interference management by the small cells. Efficient interference management in *distributed* large-scale small cell networks is more challenging [1] due to the lack of central coordinators, compared to that in traditional cellular networks.

In this work, we propose a novel framework for designing interference management policies in the uplink of small cell networks, which specify when and at what power level each user equipment (UE) should transmit¹. Our proposed design framework and the resulting interference management policies fulfill *all* of the following three important requirements. First, effective policies should *deal with significant heterogeneity* of small cell networks, which is caused by the differences in cell sizes (e.g. pico/femtocells), numbers of UEs served, throughput requirements by the UEs and network performance criteria. Second, effective policies should *avoid strong interference* among neighboring cells, while exploiting the weak interference among non-neighboring cells by *spatial reuse*. The proposed policies achieve interference avoidance and spatial reuse by scheduling maximal independent sets (MISs)² of the

¹Although we focus on uplink transmissions in this paper, our framework can be easily applied to downlink transmissions.

²Consider the interference graph of the network, where each vertex is a UE-SBS pair and each edge indicates strong interference between the two vertices. An independent set (IS) is a set of vertices in which no pair is connected by an edge. An IS is a MIS if it is not a proper subset of another IS.

interference graph to transmit in each time slot. Third, the policies should be computed and implemented in a *distributed* manner, and should be *scalable*, i.e. achieve efficient network performance with low computational complexity. Next, we summarize our key contributions.

1. We propose a distributed method for the UEs to determine a subset of MISs such that each UE belongs to at least one MIS in the subset. Moreover, the subset can be generated in logarithmic time (logarithmic in the number of UEs in the network) for bounded-degree interference graphs³, which is significantly faster than the time (linear or quadratic in the number of UEs) required by existing works [2] [3].

2. Given the computed subsets of MISs, we propose a distributed algorithm in which each UE determines the optimal fractions of time occupied by the MISs with only local message exchange among neighbors in the interference graph.

3. Remarkably, we prove that the proposed distributed policy achieves a competitive ratio that is independent of the network size, with respect to the optimal network performance. Note that the optimal network performance can only be obtained in a centralized manner with global information (e.g. all the UEs' channel gains, maximum transmit power levels) and NP complexity, while our policy is computed in a distributed manner in polynomial time using only local information. Moreover, through simulations, we show significant (up to 390%) performance gains over existing policies.

The rest of the paper is organized as follows. In Section II we discuss the related works and their limitations. We describe the system model in Section III. Then we formulate the interference management problem in Section IV. We propose the design framework in Section V, and demonstrate the performance gain of our proposed policies in Section VI. Finally, we conclude the paper in Section VII.

II. RELATED WORKS

In this section, we will describe the related works and their limitations.

A. Distributed Interference Management Based on Power Control

Policies based on distributed power control, with representative references [4]–[11] have been used for interference management in both cellular and ad-hoc networks. In these policies, all the UEs in the network transmit at a *constant* power all the time (provided that the system parameters remain

³Bounded degree graphs are the graphs whose maximum degree can be bounded by a constant independent of the size of the graph, i.e. $\Delta = \mathcal{O}(1)$.

the same)⁴. The major limitation of policies based on power control is the difficulty in providing minimum throughput guarantees for each UE, especially in the presence of strong interference. Some works [4], [5], [7] use pricing to mitigate the strong interference. However, they [4], [5], [7] cannot strictly guarantee the UEs' minimum throughput requirements. Indeed, the low throughput experienced by some users, caused by strong interference, is the fundamental limitation of such power control approaches - even the optimal power control policy obtained by a central controller [12], [13] can be inefficient⁵. Since strong interference is very common in dense small cell deployments (e.g. in offices and apartments where SBSs are installed close to each other [15]), more efficient policies are required which can guarantee the individual UEs' throughput requirements. Also, there exist a different strand of work based on [16] which proposes a distributed algorithm to achieve the desired minimum throughput requirement for each UE. However, these works cannot optimize network performance criterion such as weighted sum throughput, minimum average throughput etc. and hence are suboptimal.

B. Distributed Spatial Reuse Based on Maximal Independent Sets

An efficient solution to mitigate strong interference is spatial reuse, in which only a subset of UEs (which do not significantly interfere with each other) transmit at the same time. Spatial Time reuse based Time Division Multiple Access (STDMA) has been widely used in existing works on broadcast scheduling in multi-hop networks [2], [3], [17]⁶. Specifically, these policies construct a cyclic schedule such that in each time slot an MIS of the interference graph is scheduled. The constructed schedule ensures that each UE is scheduled at least once in the cycle.

In terms of performance, STDMA policies [2], [3], [17] cannot guarantee the minimum throughput requirement of each UE, and usually adopt a fixed scheduling (i.e. follow a fixed order in which the MISs are scheduled), which may be very inefficient depending on the given network performance criteria. For example, the policies in [3] are inefficient in terms of fairness. In terms of complexity, for the distributed generation of the subsets of MISs, the STDMA policies in [2], [3], [17] require an ordering of all the UEs, and have a computational complexity (in terms of the number of steps executed by the algorithm) that scales as $\mathcal{O}(|V|)$ (in [3], [17]) or $\mathcal{O}(|V||E|)$ (in [2]), where $|V|$ and $|E|$ are the number of vertices/UEs and the number of edges in the interference graph, respectively. Hence, in large-scale dense deployments, the complexity grows superlinearly with the number of UEs, making the policies difficult to compute. By contrast, our

⁴Although some power control policies [4], [5], [7] go through a transient period of adjusting the power levels before the convergence to the optimal power levels, the users maintain constant power levels after the convergence.

⁵In the case of average sum throughput maximization given the minimum average throughput constraints of the UEs, the power control policies are inefficient if the feasible rate region is non-convex [14].

⁶These works [2], [3], [17] do not have the exactly same model as in our setting. However, these works can be adapted to our model. Hence, we also compare with these works to have a comprehensive literature review.

proposed distributed algorithm for generating subsets of MISs does not require the ordering of all the UEs, and has a complexity that scales as $\mathcal{O}(\log |V|)$, namely sublinearly with the number of the UEs, for bounded-degree graphs.⁷

Finally, the STDMA policies in [2], [3], [17] are designed for the MAC layer and assume that all the UEs are homogeneous at the physical layer. In practice, different UEs are heterogeneous due to their different distances from their SBSs, their different maximum transmit power levels, etc. This heterogeneity is important, and will be considered in our design framework.

C. Distributed Power Control and Spatial Reuse For Multi-Cell Networks

As we have discussed, the works in the above two categories either focus on distributed power control in the physical layer [4], [5], [7] or focus on distributed spatial reuse in the MAC layer [2], [3], [17]. Similar to our paper, some works (representative references [18]–[22]) adopted a cross-layer approach and proposed distributed joint power control and spatial reuse for multi-cell networks. However, although these works schedule a subset of UEs to transmit at the same time, the subset is not the MIS of the interference graph [20], [21]. For example, the policies in [20], [21] schedule one UE from each small cell at the same time, even if some UEs are from small cells very close to each other. In this case, the UEs will experience strong inter-cell interference. Hence, the works in [20], [21] cannot perfectly eliminate strong interference from neighboring cells and exploit weak interference from non-neighboring cells. Moreover, the works in [18]–[22] cannot provide minimum throughput guarantees for the UEs.

III. SYSTEM MODEL

A. Heterogeneous Network of Small Cells

We consider a heterogeneous network of K small cells operating in the same frequency band⁸ (see Fig. 1), which represents a common deployment scenario considered in practice [7]. Note that the small cells can be of different types (e.g. picocells, femtocells etc.) and thereby belong to different tiers in the heterogeneous network. Each small cell j has one SBS, (SBS- j), which serves a set of UEs under a closed access scenario [7]. Denote the set of UEs by $\mathcal{U} = \{1, \dots, N\}$. We write the association of UEs to SBSs as a mapping $T : \{1, \dots, N\} \rightarrow \{1, \dots, K\}$, where each UE- i is served by SBS- $T(i)$. We focus on the uplink transmissions; the extension to downlink transmissions is straightforward when each SBS serves one UE at a time (e.g. TDMA among UEs connected to the same SBS).

Each UE- i chooses its transmit power p_i from a compact set $\mathcal{P}_i \subseteq \mathbb{R}_+$. We assume that $0 \in \mathcal{P}_i$, $\forall i \in \{1, \dots, N\}$, namely any UE can choose not to transmit. The joint power profile of

⁷As will be shown in Theorem 5, for graphs which are not bounded degree graphs, even a centralized solution based on all the MISs cannot satisfy the minimum throughput requirements.

⁸Our solutions will be based on spatial time reuse assuming every UE uses the same frequency. Our solutions can be extended to spatial frequency reuse, where we let different MISs operate in non-overlapping frequency bands.

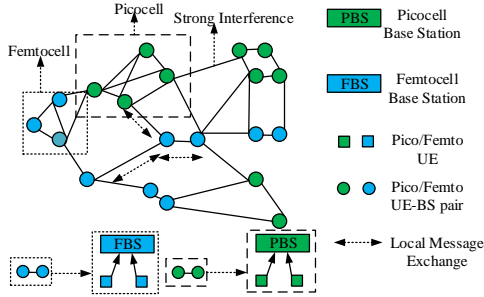


Figure 1. Illustration of a heterogeneous small cell network.

all the UEs is denoted by $\mathbf{p} = (p_1, \dots, p_N) \in \mathcal{P} \triangleq \prod_{i=1}^N \mathcal{P}_i$. Under the joint power profile \mathbf{p} , the signal to interference and noise ratio (SINR) of UE- i 's signal, experienced at its serving SBS- $j = T(i)$, can be calculated as $\gamma_i(\mathbf{p}) = \frac{g_{ij}p_i}{\sum_{k=1, k \neq i}^N g_{kj}p_k + \sigma_j^2}$,

where g_{ij} is the channel gain from UE- i to SBS- j , and σ_j^2 is the noise power at SBS j . The UEs do not cooperate to encode their signals to avoid interference, hence, each UE and its serving SBS, referred to as UE-SBS pair, treat the interference from other UEs as white noise. Each UE- i gets the following throughput [20], $r_i(\mathbf{p}) = \log_2(1 + \gamma_i(\mathbf{p}))^9$.

B. Interference Management Policies

The system is time slotted at $t = 0, 1, 2, \dots$, and the UEs are assumed to be synchronized as in [20]. At the beginning of each time slot t , each UE- i decides its transmit power p_i^t and obtains a throughput of $r_i(\mathbf{p}^t)$. Each UE i 's strategy, denoted by $\pi_i : \mathbb{Z}_+ = \{0, 1, \dots\} \rightarrow \mathcal{P}_i$, is a mapping from time t to a transmission power level $p_i \in \mathcal{P}_i$. The interference management policy is then the collection of all the UEs' strategies, denoted by $\pi = (\pi_1, \dots, \pi_N)$. The average throughput for UE i is given as $R_i(\pi) = \lim_{T \rightarrow \infty} \frac{1}{T+1} \sum_{t=0}^T r_i(\mathbf{p}^t)$, where $\mathbf{p}^t = (\pi_1(t), \dots, \pi_N(t))$ is the power profile at time t . We assume the channel gain to be fixed over the considered time horizon as in [20].

A policy based on power control discussed in Subsection ??, [4] [7] is defined as $\pi^{const}(t) = \mathbf{p}$ for all t , where $\mathbf{p} \in \mathcal{P}$. As we have discussed before, our proposed policy is based on MISs of the interference graph. The interference graph G has N vertices (See Fig. 1), each of which is one of the N UE-SBS pairs. There is an edge between two pairs/vertices if their cross interference is high (rules for deciding if interference is high will be discussed in Section V) and let there be M edges in the graph. Note that to refer to a vertex we will use UE-SBS pair/UE interchangeably. Given an interference graph, we write $\mathbf{I} = \{I_1, \dots, I_{N_{MIS}}\}$ as the set of all the MISs of the interference graph. Let \mathbf{p}^{I_j} be a power profile in which the UEs in the MIS I_j transmit at their maximum power levels and the other UEs do not transmit,

⁹We use the Shannon capacity here. However, our analysis is general and applies to the throughput models that consider the modulation scheme used.

namely $p_k = p_k^{max} \triangleq \max \mathcal{P}_k$ if $k \in I_j$ and $p_k = 0$ otherwise. Let $\mathcal{P}^{MIS} = \{\mathbf{p}^{I_1}, \dots, \mathbf{p}^{I_{N_{MIS}}}\}$ be the set of all such power profiles. Then π is a policy based on MIS if $\pi(t) \in \mathcal{P}^{MIS}$ for all t and the set of all such policies is $\Pi^{MIS} = \{\pi : \mathbb{Z}_+ \rightarrow \mathcal{P}^{MIS}\}$.

IV. PROBLEM FORMULATION

A. The Interference Management Policy Design Problem

We aim to optimize a chosen network performance criterion $W(R_1(\pi), \dots, R_N(\pi))$, defined as a function of the UEs' average throughput. We can choose any performance criterion that is concave in $R_1(\pi), \dots, R_N(\pi)$. For instance, W can be the weighted sum of all the UEs' throughput, i.e. $\sum_{i=1}^N w_i R_i(\pi)$ with $\sum_{i=1}^N w_i = 1$ and $w_i \geq 0$, minimum average throughput achieved by any user i.e. $\min_i R_i(\pi)$ etc. The policy design problem can be then formalized as follows:

Policy Design Problem (PDP)

$$\begin{aligned} \max_{\pi} \quad & W(R_1(\pi), \dots, R_N(\pi)) \\ \text{subject to} \quad & R_i(\pi) \geq R_i^{min}, \forall i \in \{1, \dots, N\} \end{aligned}$$

The above design problem is very challenging to solve even in a centralized manner (it is NP-hard [23] when the performance criterion is sum throughput, even when we restrict to policies based on power control π^{const}). Denote the optimal value of the PDP as \mathbf{W}_{opt} . Our goal is to propose distributed interference management policies, that achieve a constant competitive ratio with respect to \mathbf{W}_{opt} , with the competitive ratio independent of the network size. We achieve our goal by focusing on policies based on MISs Π^{MIS} , among other innovations that will be described in Section V.

V. DESIGN FRAMEWORK FOR DISTRIBUTED INTERFERENCE MANAGEMENT

A. Proposed Design Framework

Our proposed design framework (See Fig. 2) consists of four steps. In Step 1, each UE-SBS pair identifies UE-SBS pairs which interfere with it strongly. In Step 2, the UE-SBS pairs determine a subset of MISs in a distributed fashion, such that each UE-SBS pair belongs to at least one MIS in the subset. In Step 3, each UE-SBS pair determines the optimal fraction of time allocated to each MIS found in Step 2 in a distributed fashion (The optimal fraction depends on the performance criterion.). In Step 4, UE-SBSs determine the cycle length and the number of slots taken by each MIS, based on the optimal fractions of time computed in Step 3. Next, we describe the four steps in detail.

Step 1. Identification of the interfering neighbors: Each UE-SBS pair obtains a *local* view (i.e. its neighbors) of the interference graph using distributed methods that either use local measurements of Received Signal Strength (RSS) [24] or use locations of the UE-SBSs in the geographical proximity to identify another pair [25] as interfering or not. If a UE-SBS pair is identified by another pair as interfering, then the two are connected by an edge in the interference graph. Each UE-SBS pair is informed by another pair if it identifies it as interfering through the back-haul network/X2 interface, which is used for inter-cell interference coordination (ICIC) [26].

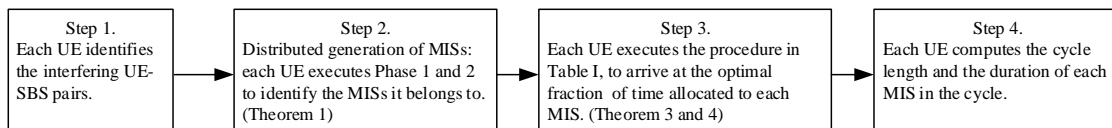


Figure 2. Steps in the Design Framework.

Step 2. Distributed generation of MISs that span all the UEs: In Step 2, the UE-SBS pairs generate a subset of MISs in a distributed fashion. It is important that the generated subset spans all the UEs, namely every UE is contained in at least one MIS in the subset. Otherwise, some UEs will never be scheduled. The distributed algorithm is comprised of two phases: first, distributed coloring of the interference graph based on [27], and second, extension of the color classes to the MISs. We assume that all the UEs are synchronized and carry out their computation simultaneously (i.e. not sequentially). We now explain the algorithm in detail. The pseudo-code can be found in Tables II and III given at the end.

Phase 1. Distributed coloring of the interference graph: The goal of this phase is to let each UE- i choose a color from $C_i^0 = \{1, \dots, H\} \cap \{1, \dots, d_i + 1\}$, such that no neighbors choose the same color. Here H^{10} is the maximum number of colors given to all SBSs at the installation and d_i is the degree (number of neighbors in the interference graph) of i^{th} pair.

The distributed coloring works as follows. At the beginning of each time slot t , each UE- i chooses a color from the set of remaining colors C_i^t randomly, and informs its neighbors of its tentative choice. This information can be transmitted using the back-haul network/X2 interface. If the tentative choice of the UE does not conflict with any of its neighbor, then it fixes and confirms the color choice, informs the neighbors of the confirmation and will not contend for colors any further in Phase 1. The neighbors delete the color chosen by i from their lists $C_j^{t+1}, \forall j \in \mathcal{N}(i)$, where $\mathcal{N}(i)$ is the set of i 's neighbors. On the other hand, if there is a conflict then the UE does not choose that color and repeats the same procedure in the next time slot. There are $\lceil c_1 \log_{\frac{4}{3}} N \rceil + 1$ time slots in Phase 1, known to the SBSs at installation where c_1 is the parameter given by the protocol. Phase 1 is successful if all the UEs acquire a color which implies that the set of color classes (i.e. the set of UEs with the same color) obtained span all the UEs.

Phase 2. Extending color classes to the MISs: Each color class obtained at the end of Phase 1 is an independent set (IS) of the graph. In Phase 2, we extend each of these ISs to MISs and generate additional MISs in this step. Note that after Phase 1, each UE has chosen one color and deleted some colors from its list. But there may still be remaining colors in its list that are not acquired by any of its neighbors. The goal of Phase 2 is to utilize these remaining colors. At each time slot in

Phase 2, UE- i chooses each color from the remaining colors in its list independently with probability c . Each UE- i then sends this set of its tentative choices to its neighboring UEs, and receives its neighbors' choices. For any tentative choice of color, if there is a conflict with at least one neighbor, then that color is not fixed; otherwise, it is fixed. At the end of each time slot, each UE deletes its set of fixed colors from its list, and transmits this set of fixed colors to its neighbors, who will delete these fixed colors from their lists as well. Hence, a UE deletes a particular color, if and only if this UE itself or some of its neighbors have chosen this color. Based on this key observation, we can see that if a color is not in any UE's list, the set of UEs who have chosen that color is a MIS. If all the UEs have an empty list, then for any color in the set $\{1, \dots, H\}$, the set of UEs with this color is a MIS. There are $\lceil c_2 \log_x N \rceil + 1$ time slots in Phase 2, known to SBSs at installation, where $x = \frac{1}{1 - (c)^H (1-c)^{H^2}}$, and c_2 is the parameter given by the protocol. We say that Phase 2 is successful, if it finds H MISs, equivalently, if all the UEs have an empty list after $\lceil c_2 \log_x N \rceil + 1$ time slots.

Example: Fig. 3 shows the interference graph of a network of 5 UE-SBS pairs. At the start, each UE-SBS pair has a list of 3 colors, {Red, Yellow, Green} to choose from. At the end of Phase 1, which runs for $P1 = \lceil c_1 \log_{\frac{4}{3}} 5 \rceil$ time slots, UEs 2,3 acquire Yellow, UEs 4,5 acquire Red and UE 1 acquires Green. UEs 2,3 delete all the colors from their list of remaining colors, as Yellow is acquired by them and Red, Green by their neighbors. UE 1 also deletes all the colors, since its neighbors use Yellow and Red, but UE 4(UE 5) has Yellow(Green) color in its list. At the end of Phase 1, Red color class is MIS, while Yellow and Green are not. At the end of Phase 2, which runs for $P2 = \lceil c_2 \log_x 5 \rceil + 1$ time slots, UE 4 (UE 5) acquires the remaining color Yellow (Green). At the end of Phase 2, the color classes Green and Yellow are MISs. Next theorem shows that the Phase 1 and 2 are successful with a high probability.

Theorem 1. For any interference graph with the maximum degree $\Delta \leq H - 1$, the phase 1 and 2 of the proposed algorithm output a set of H MISs spanning all the UEs in $(\lceil c_1 \log_{\frac{4}{3}} N \rceil + \lceil c_2 \log_x N \rceil + 2)$ time slots with a probability no smaller than $(1 - \frac{1}{N^{c_1-1}})(1 - \frac{1}{N^{c_2-1}})$, where c_1, c_2 are design parameters that trade-off run time and the success probability.

Proof 1: The success probability of Phase 1 is high, $(1 - \frac{1}{N^{c_1-1}})$ (lower bound), (see [27] for detail), here we analyze Phase 2.

We first show that, if the list of remaining colors given as, C_i^n is empty at $n \geq \lceil c_1 \log_{\frac{4}{3}} N \rceil + \lceil c_2 \log_x N \rceil + 2$ and if this holds $\forall i \in \{1, \dots, N\}$ then the Phase 2 has converged to a set of H MISs which span all the UEs. Let us assume

¹⁰The maximum number of colors H should be set to be larger than the maximum number of UE-SBS pairs interfering strongly with any UE-SBS pair. The SBSs can determine H according to the deployment scenario. H in general will also include the number of UEs that use the same SBS who interfere with each other along with the other neighboring UEs. For example, H can be 10-15 in an office and can be 3-5 in a residential area.

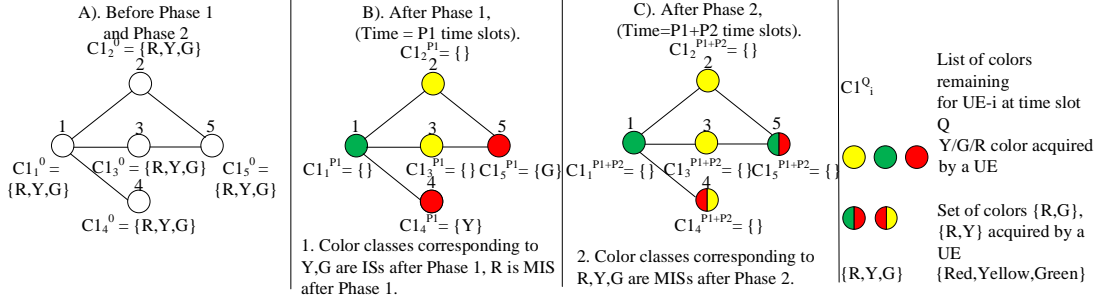


Figure 3. Illustration of the distributed generation of MISs in Step 2.

otherwise, i.e. $C1_i^n$ is empty $\forall i \in \{1, \dots, N\}$ however, the set corresponding to some color $h \in \{1, \dots, H\}$, I'_h is not a MIS. I'_h has to be an IS. Assume otherwise, i.e. I'_h is not an IS, which implies that there must exist a pair of UEs, i and j , which are neighbors and are a part of I'_h . If this is true then both then both acquired the color h either in the same time slot or in different time slots, in Phase 1 or 2. In case the color is acquired in different time slots, then after the first time slot when either of the UEs in the pair acquires the color it will transmit the final color choice, h to the neighbors (see Table II and III) who in turn delete that color. However, if the color is deleted by the neighbor then it cannot acquire it in the future thus, ruling out the case that the colors were acquired in two different time slots. If the color was acquired by the UEs in the same time slot, then it implies that despite the conflict in tentative choice the UEs acquire the color which is not possible (see Table II and III). This shows that I'_h is an IS.

Since I'_h is not maximal then \exists at least one UE- $j \notin I'_k$ which can be added to this set without violating independence. From the assumption, we have $C1_j^n = \phi$ which implies that the color h was deleted at some stage from the original list of all the colors either in Phase 1 or 2. The deletion of h was a result of that color being acquired finally by at least one of the neighbors $k \in \mathcal{N}(j)$ since $j \notin I'_k$. In that case, j cannot acquire h as it will violate the independence property.

Next, we show that indeed the list of all colors available $C1_i^n$ is empty at the end of Phase 2 with a high probability. Let U^n correspond to the number of UEs which have a non-empty list at the beginning of time slot n and, let $Tn(U^n)$ correspond to the total time needed before all the UEs have an empty list. The probability that a UE at time slot n with a non-empty list will have an empty list in next time slot is always greater than $c^H(1-c)^{H^2}$. This can be explained as, if the UE chooses all the colors in the list assuming (worst case H number of colors remain) and all the neighbors (worst case H neighbors) do not choose any color, then all the colors in the UE's list will be deleted. From this, we get $E(U^{n+1}) \leq (1-c^H(1-c)^{H^2})U^n = \frac{1}{x}U^n$ and $Tn(U^n) = 1 + Tn(U^{n+1})$. Assuming that the Phase 2 will start with N UEs whose list are non-empty (worst case) and from [28] we get $P(Tn(N) \geq \lceil c_2 \log_x N \rceil) \leq \frac{1}{N^{c_2-1}}$. This gives the lower bound on success

probability of Phase 2 and thereby the result in the Theorem. (Q.E.D)

Theorem 1 characterizes the performance of our proposed algorithm, in terms of the run time of the algorithm and the lower bound of the success probability. When the parameters c_1 and c_2 are chosen to be large, the lower bound of the success probability increases at the expense of a longer run time. When the maximum degree of the interference graph is larger, we need to set a higher H , which results in a longer run time. This is reasonable, because it is harder to find coloring and MISs when the number of interfering neighbors is higher. Finally, we can see that the lower bound of the success probability is very high even under smaller c_1 and c_2 , especially if the number of UEs is large. Note that the exact success probability should depend on the probability c , while the lower bound in Theorem 1 does not.

Step 3. Distributed computation of the optimal fractions of time for each MIS: Let the set of MISs generated in Step 2 be $\{I'_1, \dots, I'_H\}$. Here, the UE-SBS pairs compute the fractions of time allocated to each MIS in a distributed manner.

Note that when an MIS is scheduled, the UEs in this MIS transmit at their maximum power levels, and the other UEs do not transmit. Define R_i^k as the *instantaneous* throughput obtained by UE i in the MIS I'_k , which can be calculated

as: $\log_2(1 + \frac{g_{iT(i)}P_i^{I'_k}}{\sum_{r=1, r \neq i}^N g_{rT(i)}P_r^{I'_k} + \sigma_T^2(i)})$, where $p_i^{I'_k} = p_i^{max}$ if

$i \in I'_k$ and $p_i^{I'_k} = 0$ otherwise. To determine R_i^k , the UE needs to know the accumulative interference it experiences when transmitting in I'_k . This can be done by having an initial cycle of transmissions by the MISs, i.e. UEs in MISs, in the order of the indices (i.e. the order of their colors), in which the SBSs of the UEs can measure the received SINR.

From now on, we assume that the network performance criterion $W(\mathbf{y})$ is concave in \mathbf{y} and is separable, namely $W(y_1, \dots, y_N) = \sum_{i=1}^N W_i(y_i)$. Examples of separable criteria include weighted sum throughput and proportional fairness. Note that our framework can be extended to deal with minimum average throughput, although it is not separable (see the discussion in Appendix at the end). Then the problem of computing the optimal fractions of time for the MISs becomes:

Coupled Problem (CP)

$$\begin{aligned} & \max_{\alpha} \sum_{i=1}^N W_i \left(\sum_{k=1}^H \alpha_k R_i^k \right) \\ & \text{subject to } \sum_{i=1}^N \alpha_k R_i^k \geq R_i^{\min}, \forall i \in \{1, \dots, N\} \\ & \sum_{k=1}^H \alpha_k = 1, \alpha_k \geq 0, \forall k \in \{1, \dots, H\} \end{aligned}$$

Since each UE i knows only its own utility function W_i and its minimum throughput requirement R_i^{\min} , it cannot solve the above problem by itself directly. To be able to solve the problem, each UE- i computes a local estimate of the fractions of time allocated to all the MISs (including those that do not include UE- i). Denote UE- i 's local estimate of the fraction of time allocated to MIS I_k by β_i^k . We impose an additional constraint that all the UEs' local estimates are the same, such that they reach to consensus. Such a constraint is still global, because any two UEs, even when they are not neighbors and far away from each other, need to have the same local estimate. Hence, global message exchange among any pair of UEs is needed to solve CP, which is not allowed.

Now we reformulate the CP into a decoupled problem (DP) that involves only local coupling among the neighbors and that can be solved by Alternating Direction Method of Multipliers (ADMM) [29]. If UE i and l are connected by an edge (i, l) then for each set I_k define $\theta_{(i,l)i}^k = \beta_i^k$ and $\theta_{(i,l)l}^k = -\beta_l^k$. Note that these auxiliary variables are introduced to formulate the problem into the ADMM framework [29]. Define a polyhedron for each i , $\mathcal{T}_i = \{\beta_i | \text{s.t. } \mathbf{1}^t \beta_i = 1, \beta_i \geq \mathbf{0}, \mathbf{R}_i \beta_i \geq R_i^{\min}\}$, here $\beta_i = (\beta_i^1, \dots, \beta_i^H)$ and $\mathbf{R}_i = (R_i^1, \dots, R_i^H)$ and $()^t$ corresponds to the transpose. Let $\beta = (\beta_1, \dots, \beta_N) \in \mathcal{T}$, where $\mathcal{T} = \prod_{i=1}^N \mathcal{T}_i$ and \prod corresponds to the Cartesian product of the sets. Also, let $\beta^k = (\beta_1^k, \dots, \beta_N^k), \forall k \in \{1, \dots, H\}$. Define another polyhedron $\Theta_{(i,l)}^k = \{(\theta_{(i,l)i}^k, \theta_{(i,l)l}^k) : \theta_{(i,l)i}^k + \theta_{(i,l)l}^k = 0, -1 \leq \theta_{(i,l)s}^k \leq 1, \forall s \in \{i, l\}\}$, $\Theta^k = \prod_{(i,l) \in E} \Theta_{(i,l)}^k$ here $E = (e_1, \dots, e_M)$ is the set of all the M edges in the interference graph. A vector $\theta^k \in \Theta^k$ is written as $\theta^k = (\theta_{e_1, z(e_1)}^k, \theta_{e_1, t(e_1)}^k, \dots, \theta_{e_M, z(e_M)}^k, \theta_{e_M, t(e_M)}^k)$, here $z(e_i), t(e_i)$ correspond to the vertices in the edge, e_i . Similarly define, $\theta = (\theta^1, \dots, \theta^H) \in \Theta$, where $\Theta = \prod_{k=1}^H \Theta^k$.

Decoupled Problem (DP)

$$\begin{aligned} & \min_{\beta \in \mathcal{T}, \theta \in \Theta} - \sum_{i=1}^N W_i(\mathbf{R}_i^t \beta_i) \\ & \text{subject to } \mathbf{D}^k \beta^k - \theta^k = \mathbf{0}, \forall k \in \{1, \dots, H\} \end{aligned}$$

Here, $\mathbf{D}^k \in \mathbb{R}^{2M \times N}$, is a matrix in which each row has exactly one non-zero element which is 1 or -1 . Each element of the matrix, D_{vj}^k is evaluated as follows, the index v can be uniquely expressed in terms of quotient q and the remainder w as $v = 2q + w$, and if $j \neq z(e_{q+1}), j \neq t(e_{q+1})$ then $D_{vj}^k = 0$. If $w = 1, j = z(e_{q+1})$, then $D_{vj}^k = 1$ else if $w = 0, j = z(e_{q+1})$ then $D_{vj}^k = 0$. Also, if $w = 0, j = t(e_{q+1})$, then $D_{vj}^k = -1$ else if $w = 1, j = t(e_{q+1})$ then $D_{vj}^k = 0$.

Theorem 2: For any connected interference graph, the coupled problem is equivalent to the decoupled problem.

Proof 2: The two problems which are introduced to transit from CP to DP are,

Global Primal Problem (GPP)

$$\begin{aligned} & \max \sum_{k=1}^H W_i \left(\sum_{i=1}^N \beta_i^k R_i^k \right) \\ & \text{subject to } \sum_{k=1}^H \beta_i^k R_i^k \geq R_i^{\min}, \forall i \in \{1, \dots, N\} \\ & \sum_{k=1}^H \beta_i^k = 1, \forall i \in \{1, \dots, N\} \\ & \beta_i^k = \beta_l^k, \forall i \neq l, \forall k \in \{1, \dots, H\}, \\ & \beta_i^k \geq 0, \forall i \in \{1, \dots, N\}, \forall k \in \{1, \dots, H\} \end{aligned}$$

The second problem, Local Primal Problem (LPP) is the same as GPP except we choose a subset of the constraints from the above problem. Basically, instead of an equality constraint between the UE's estimate and every other UE in the network, we only keep the equality constraints between the UE and its neighbors, i.e. $\beta_i^k = \beta_l^k, \forall k \in \{1, \dots, H\}, \forall l \in \mathcal{N}(i)$. This is formally stated below:

Local Primal Problem (LPP)

$$\begin{aligned} & \max \sum_{k=1}^H W_i \left(\sum_{i=1}^N \beta_i^k R_i^k \right) \\ & \text{subject to } \sum_{k=1}^H \beta_i^k R_i^k \geq R_i^{\min}, \forall i \in \{1, \dots, N\} \\ & \sum_{k=1}^H \beta_i^k = 1, \forall i \in \{1, \dots, N\} \\ & \beta_i^k = \beta_l^k, \forall l \notin \mathcal{N}(i), \forall k \in \{1, \dots, H\}, \\ & \beta_i^k \geq 0, \forall i \in \{1, \dots, N\}, \forall k \in \{1, \dots, H\} \end{aligned}$$

To show that problems CP and GPP are equivalent, we need to show that from $\beta^* = (\beta_1^*, \dots, \beta_N^*)$, an optimal argument of GPP, we can obtain an optimal argument of CP, i.e. α^* and vice versa. Since β^* is the optimal value (assuming feasibility) we know that $\beta_i^* = \beta_j^*$ (component-wise) holds $\forall i, j \in \{1, \dots, N\}$.

a). Let $\alpha' = \beta_i^*$. α' satisfies the constraints in CP. The objective of CP at α' attains the optimal value of GPP. We need to establish that α' is indeed the optimal argument of CP. Assume that α' is not the optimal value, then there exists another α^* which is indeed the optimal. Next, using α^* , we can obtain another β' as follows, $\beta_1' = \alpha^*$ and $\beta_i' = \beta_1', \forall i \in \{1, \dots, N\}$. The objective of GPP at β' should be higher than β^* which contradicts β^* being the optimal argument. Note that if either of CP or GPP is infeasible then the other problem can be shown to be infeasible as well. On the same lines we can show that from an α^* we can obtain β^* as well.

b). Let α^* be the optimal solution to CP, and define β'' a solution to GPP as follows. Let $\beta_1'' = \alpha^*$ and $\beta_i'' = \beta_j'', \forall j \neq i$ and since α^* satisfies the constraints of CP, i.e. it is feasible, implies that β'' as well satisfies constraints of GPP. We want to show that β'' is the optimal value as well, assume that it is not and there exists an argument β^* for which the objective takes a higher value. If this is the case then, from β^* we can construct a α' as in part a). which, if β^* takes a higher value than β'' , takes a higher value than α^* thus, contradicting optimality.

To show that GPP and LPP are equivalent, we use the following fact, since LPP consists of a subset of the constraints then the solution of LPP is an upper bound of the

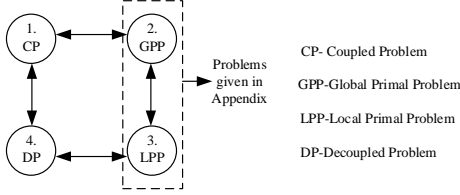


Figure 4. Problems used to transit from the Coupled Problem (CP) to Decoupled Problem (DP).

solution to GPP. We need to show that the gap between the solution of LPP and GPP is always 0. Note that for an optimal solution of LPP, $\gamma^* = (\gamma_1^*, \dots, \gamma_N^*)$ we know that $\gamma_i^* = \gamma_j^* \forall j \in \mathcal{N}(i)$ (component-wise). If we can show that $\gamma_i^* = \gamma_j^* \forall j \in \{1, \dots, N\}$ then LPP and GPP will be equivalent, since it will also satisfy all the constraints of GPP. Assume that this does not hold then $\exists i, j$ such that $\gamma_i^* \neq \gamma_j^*$. Since, the interference graph is connected \exists a path $i \rightarrow j = \{i_1, \dots, i_s\}$ which implies, $\gamma_i^* = \gamma_{i_1}^* \dots = \gamma_j^*$. This leads to a contradiction, thereby establishing the claim.

Lastly, to show that DP is equivalent LPP. Given γ^* , define $\kappa = \gamma^*$ and a $\theta = (\theta^1, \dots, \theta^H)$ to satisfy $D^k \kappa^k - \theta^k = \mathbf{0}, \forall k \in \{1, \dots, H\}$, where $\kappa^k = (\gamma_1^{*,k}, \dots, \gamma_N^{*,k})$. It can be shown using the same approach as we did for GPP and CP that (κ, θ) is indeed optimal argument for DP. Assume that (κ, θ) is not the optimal solution then we know that there exists (κ^*, θ^*) for which the objective in DP takes a higher value. If this is the case, let us define $\gamma' = \kappa^*$, here γ' satisfies the constraints in LPP. Also, since the objective in DP at (κ^*, θ^*) takes a higher value than that at (κ, θ) , this yields that the objective in LPP at γ' should take a higher value than that at γ^* , which contradicts optimality of γ^* . On the same lines, it can be easily shown that from (κ^*, θ^*) we can construct the optimal solution γ^* of the LPP. This, will establish equivalence between LPP and DP. Hence, all the four problems are equivalent. This is shown in Fig. 4.

(Q.E.D)

The above theorem is important, because it shows that the CP, which cannot be solved without global information and global message exchange, is now transformed into an equivalent problem, DP which can be solved by ADMM with local message exchange. We denote the optimal solution to the DP by $\mathbf{W}_{\text{distributed}}^G$ and corresponding optimal argument, i.e. fractions of time allocated to the MISs as $\gamma^* = (\gamma^{*,1}, \dots, \gamma^{*,H})$. We solve the DP by ADMM briefly described next (more detail in Table I at the end), and prove the rate of convergence, namely how fast the error from the optimal solution $\mathbf{W}_{\text{distributed}}^G$ decreases as the number of iterations increase.

We associate with each constraint $D^k \beta^k - \theta^k = \mathbf{0}$ a price vector which we denote as follows, $\lambda^k = (\lambda_{e_1 z(e_1)}^k, \lambda_{e_1 t(e_1)}^k, \dots, \lambda_{e_M z(e_M)}^k, \lambda_{e_M, t(e_M)}^k)$. We can write the augmented Lagrangian for DP as follows, $L_y(\beta, \theta, \lambda) = -\sum_{i=1}^N \mathbf{W}_i(\mathbf{R}_i' \beta_i) + \sum_{k=1}^H (\lambda^k)' (D^k \beta^k - \theta^k) + \frac{\mu}{2} \sum_{k=1}^H \|(D^k \beta^k - \theta^k)\|^2$, here $\lambda = (\lambda^1, \dots, \lambda^H)$. The ADMM procedure relies on computing the optimal vector

$\beta_i(t) = (\beta_i^1(t), \dots, \beta_i^H(t))$, by each UE- i in the current time slot t given the price variables and auxiliary variables at time $t-1$, i.e. $\lambda_{ei}^k(t-1)$ and $\theta_{ei}^k(t-1) \forall k \in \{1, \dots, H\}, \forall e \in \tilde{E}(i)$, here $\tilde{E}(i)$ is the set of edges with UE- i as a vertex. Then, the price variable $\lambda_{ei}^k(t-1)$ and auxiliary variable $\theta_{ei}^k(t-1)$ is updated parallelly by each UE- i based on the $\beta_i^k(t)$ and the neighbor j 's $\beta_j^k(t)$, here $e = (i, j), \forall e \in \tilde{E}(i), \forall k \in \{1, \dots, H\}$. This iteration of updating $\beta_i(t)$ and price, auxiliary variables is repeated P times. DP is feasible if the set of vectors satisfying the constraints in DP is non-empty.

Theorem 3: If DP is feasible, then the ADMM algorithm in Table I converges to the optimal value $\mathbf{W}_{\text{distributed}}^G$ with a rate of convergence $\mathcal{O}(\frac{1}{P})$.

Proof 3: We only need to show that the assumptions in [29], namely the feasibility of DP, along with compactness of \mathcal{T}, Θ to ensure convergence at rate $\mathcal{O}(\frac{1}{P})$. Both \mathcal{T}, Θ are closed and bounded polyhedron implying that they are compact. (Q.E.D)

Step 4. Determining the cycle length and transmission times: MISs are scheduled in cycles in the order of their indices, with the numbers of slots allocated in a cycle proportional to the optimal fractions of time, $\gamma^* = (\gamma^{*,1}, \dots, \gamma^{*,H})$.

B. Performance Guarantees for Large Networks and Properties of Interference Graphs

In this subsection, we provide performance guarantees for our proposed framework described in Subsection V-A. Specifically, we prove that the network performance, $\mathbf{W}_{\text{distributed}}^G$, achieved by the proposed distributed algorithm has a constant competitive ratio with respect to the optimal value, \mathbf{W}_{opt} , of the PDP. Moreover, we prove that the competitive ratio does not depend on the network size. Our result is strong, because the PDP is NP-hard and requires global information to solve, while our proposed framework requires the UEs to have only local information based message exchange, and converges in polynomial time.

Before characterizing the competitive ratio analytically, we define some auxiliary variables. Define the upper and lower bounds on the UEs' maximum transmit power levels and throughput requirements as, $0 < p_{lb}^{max} \leq p_i^{max} \leq p_{ub}^{max}, \forall i \in \{1, \dots, N\}$ and, $0 < R_{lb}^{min} \leq R_i^{min} \leq R_{ub}^{min}, \forall i \in \{1, \dots, N\}$ respectively. Let D_{ij} is the distance between UE i and SBS j . Define upper and lower bounds on the distance between any UE and its serving SBS and the noise power at the SBSs as, $0 < D^{lb} \leq D_{iT(i)} \leq D^{ub}, \forall i \in \{1, \dots, N\}$ and, $\sigma_{lb}^2 \leq \sigma_j^2 \leq \sigma_{ub}^2, \forall j \in \{1, \dots, K\}$ respectively. We assume that the channel gain is $g_{ij} = \frac{1}{(D_{ij})^{np}}$, np is the path loss exponent.

Definition (Weak Non-neighboring Interference): The interference graph G exhibits ζ Weak Non-neighboring Interference (ζ -WNI) if for each UE i the maximum interference from its non-neighbors is bounded, i.e. $\sum_{j \notin \mathcal{N}(i), j \neq i} g_{jT(i)} p_j^{max} \leq (2^\zeta - 1) \sigma_{ub}^2, \forall i \in \{1, \dots, N\}$.

Define $\Delta^{max} = \frac{\log_2(1 + \frac{p_{lb}^{max}}{(D^{ub})^{np} 2^\zeta \sigma_{ub}^2})}{R_{ub}^{min}} - 1$. Then we state the theorem for network performance criterion, sum throughput.

Theorem 4: For any interference graph, if the maximum degree $\Delta \leq \Delta^{max}$ and it exhibits ζ -WNI then, our proposed framework of interference management described in Subsection V-A achieves a performance $\mathbf{W}_{distributed}^G \geq \Gamma \cdot \mathbf{W}_{opt}$, where $\Gamma = \frac{R_{ub}^{min}}{\log_2(1 + \frac{p_{ub}^{max}}{(D^{lb})^{np}\sigma^2})}$ is the constant competitive ratio, which is independent of the network size.

Proof 4: Here, we need to show three things,

i). if $\Delta \leq \Delta^{max}$ then the distributed policy yields a feasible solution,

ii). the size of any MIS is $\geq \frac{N}{\Delta+1}$, thereby using this to show that the distributed policy, if feasible will yield a network performance of at least $\frac{N}{\Delta+1} \log_2(1 + \frac{p_{ib}^{max}}{(D^{ub})^{np}2\zeta\sigma^2})$ and

iii). the upper bound on the network performance, sum throughput here is $N \log_2(1 + \frac{p_{ub}^{max}}{(D^{lb})^{np}\sigma^2})$.

i). In the Phase 1 of the algorithm the maximum number of colors used is $\Delta + 1$, since each UE selects colors from subset of $\{1, \dots, H\} \cap \{1, \dots, d_i + 1\}$. The first $\Delta + 1$ output MISs, $\{I'_1, \dots, I'_{\Delta+1}\}$ span all the UEs in the network. If the fraction of time assigned to each of these $\Delta + 1$ MISs is, $\alpha'_k = \frac{R_{ub}^{min}}{\log_2(1 + \frac{p_{ib}^{max}}{(D^{ub})^{np}2\zeta\sigma^2})}$, $\forall k \in \{1, \dots, \Delta + 1\}$ then such

an assignment satisfies the constraint that sum of fractions assigned to all the colors cannot be more than 1, i.e. since $\Delta \leq \Delta^{max} \implies (\Delta + 1) \frac{R_{ub}^{min}}{\log_2(1 + \frac{p_{ib}^{max}}{(D^{ub})^{np}2\zeta\sigma^2})} \leq 1$.

Using the fact that network exhibits ζ -WNI we can write the minimum instantaneous throughput that can be obtained by UE- i as, $\log_2(1 + \frac{p_i^{max}}{(D_{iT(i)})^{np}2\zeta\sigma_{ub}^2})$, and minimum instantaneous throughput of any UE as, $\log_2(1 + \frac{p_{ib}^{max}}{(D^{ub})^{np}2\zeta\sigma_{ub}^2})$.

Thus, given the fractions assigned to the MISs, $\alpha_k = \frac{R_{ub}^{min}}{\log_2(1 + \frac{p_{ib}^{max}}{(D^{ub})^{np}2\zeta\sigma_{ub}^2})}$, $\forall k \in \{1, \dots, \Delta + 1\}$, which span all the UEs. each UE i 's throughput requirement is satisfied, $\frac{R_{ub}^{min}}{\log_2(1 + \frac{p_{ib}^{max}}{(D^{ub})^{np}2\zeta\sigma_{ub}^2})} \log_2(1 + \frac{p_i^{max}}{(D_{iT(i)})^{np}2\zeta\sigma_{ub}^2}) \geq R_{ub}^{min}$.

ii). Assume that \exists an MIS whose size is $S < \frac{N}{\Delta+1}$. Each UE in the MIS can exclude a maximum of Δ UEs from being included in the MIS. This implies that $S(\Delta + 1)$, represents the total number of UEs excluded and the UEs in the MIS which put together should exceed N . Since this is not the case here, the contradiction implies that $S \geq \frac{N}{\Delta+1}$. This combined with minimum instantaneous throughput of any UE, we get the lower bound $\frac{N}{\Delta+1} \log_2(1 + \frac{p_{ib}^{max}}{(D^{ub})^{np}2\zeta\sigma_{ub}^2})$, for our policy.

iii). The upper bound on the optimal network performance is obtained by summing maximum instantaneous throughput of any UE $\log_2(1 + \frac{p_{ub}^{max}}{(D^{lb})^{np}\sigma_{lb}^2})$ for all UEs, $N \log_2(1 + \frac{p_{ub}^{max}}{(D^{lb})^{np}\sigma_{lb}^2})$. Computing the ratio of the lower bound of proposed scheme $\frac{N}{\Delta+1} \log_2(1 + \frac{p_{ib}^{max}}{(D^{ub})^{np}2\zeta\sigma_{ub}^2})$ and $N \log_2(1 + \frac{p_{ub}^{max}}{(D^{lb})^{np}\sigma_{lb}^2})$, we get $\frac{\log_2(1 + \frac{p_{ib}^{max}}{(D^{ub})^{np}2\zeta\sigma_{ub}^2})}{(\Delta+1) \log_2(1 + \frac{p_{ub}^{max}}{(D^{lb})^{np}\sigma_{lb}^2})}$ which

is no less than, $\Gamma = \frac{R_{ub}^{min}}{\log_2(1 + \frac{p_{ub}^{max}}{(D^{lb})^{np}\sigma_{lb}^2})}$ since $\Delta \leq \Delta^{max}$. (Q.E.D)

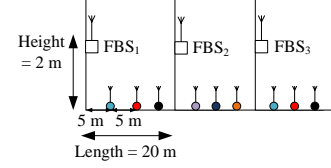


Figure 5. Illustration of the setup used in Section VI.

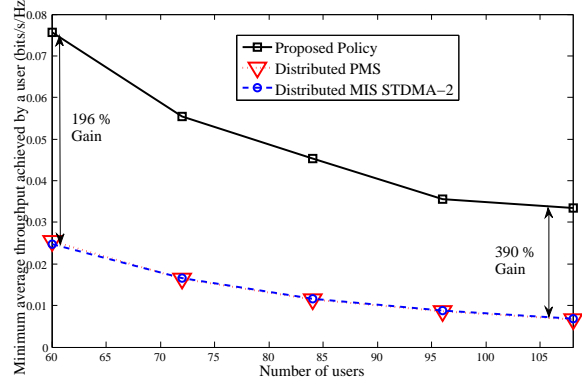


Figure 6. Comparing minimum average throughput achieved by proposed policy with state of the art for large networks.

Note that the analytical expression of competitive ratio, Γ does not depend on the size of the network. The bound on maximum number of neighbors Δ^{max} can be interpreted as a sufficient condition to guarantee the minimum throughput requirement of each UE. Our results are derived under the conditions that the interference graph has a maximum degree bounded by Δ^{max} , and that the interference from non-neighbors is bounded (i.e. ζ -WNI). Note that these do not restrict the size of the network, we will explain this with an example next. Also, see that the competitive ratio, Γ holds as long as conditions in the theorem are satisfied and it does not explicitly depend on the rule used for constructing the interference graph.

Example: Consider a layout of SBSs in a $K \times K$ square grid, i.e. K^2 SBSs with a distance of 5m between the nearest SBSs. Assume that each UE is located vertically below its SBS at a distance of 1 m. Fix the parameters $p_i^{max} = 100$ mW, $\sigma_i^2 = 3$ mW, $R_i^{min} = 0.1$ bits/s/Hz, $\forall i \in \{1, \dots, K^2\}$, $np = 4$. We construct the interference graph based on the distance rule [30], namely there is an edge between two pairs if the distance between their SBSs exceeds 6m, which gives us the maximum degree $\Delta = 4$. We can also verify that the interference graphs under any number K^2 of SBSs exhibit ζ -WNI with $\zeta = 0.15$ and $\Delta < \Delta^{max}$, where $\Delta^{max} = 48$. Given $\Delta = 4$ and $\zeta = 0.15$, from Theorem 4, we get the performance guarantee of 0.17 for any network size K^2 . Note that the number 0.17 is a performance guarantee, and that the actual performance is much higher compared to the performance guarantee as well as those achieved by state-of-the-art policies (see Section VI).

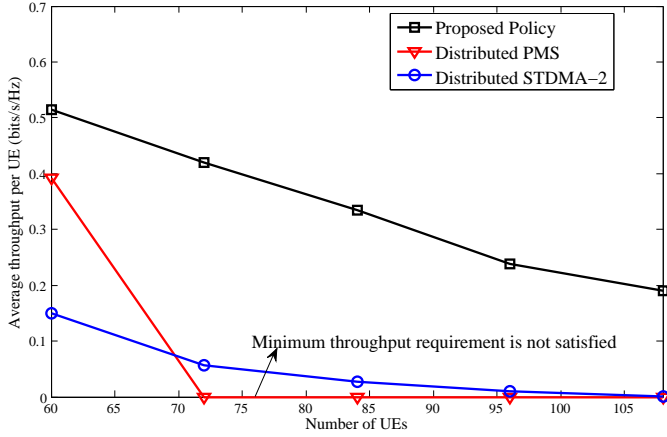


Figure 7. Comparing average throughput per UE achieved by proposed policy with state of the art for large networks.

VI. ILLUSTRATIVE RESULTS

We evaluate our proposed policy for a large network for performance criterion, minimum average throughput achieved by any UE and average throughput per UE. We compare our policy with the optimal centralized constant power control policy [23], distributed MIS STDMA-2 [2] and distributed Power Matched Scheduling (PMS) [20]. We do not compare with distributed constant power control policies [4] [7], because their performance is upper bounded by the optimal centralized power control [23]. Also, we do not compare with distributed MIS STDMA-1 as the transmission slots assignment in it is unfair in comparison to MIS STDMA-2, hence it is bound to have a lower performance.

Consider the uplink of a femtocell network in a building with 12 rooms adjacent to each other. Fig. 5 illustrates 3 of the 12 rooms with 3 UEs in each room. Each room has a length of 20 meters. In each room, there are P uniformly spaced UEs who transmit to their corresponding SBS which is installed on the left wall of the room at a height of 2m. Based on the path loss model in [31], the channel gain from each SBS i to a UE j is $\frac{1}{(D_{ij})^2 \Delta^{n_{ij}}}$, where $\Delta = 10^{0.25}$ is the coefficient representing the loss from the wall, and n_{ij} is the number of walls between UE i and SBS j . Each UE has a maximum transmit power level of 50 mW and a minimum throughput requirement of $R_i^{min} = 0.025$ bits/s/Hz and the noise power at each receiver is 10^{-11} mW. Here, we consider that the UEs use a distance based threshold rule as in Section V-B with $D^{th} = 30$ m. This results in interference graphs which connects all the UE-SBS pairs within the room and in the adjacent rooms. We vary the number P of UEs in each room from 5 to 9 and compare the performance in Fig. 6. Note that the optimal centralized constant power policy cannot satisfy the feasibility conditions for any number of UEs in each room. Therefore, only the performance of distributed MIS STDMA-2 and distributed PMS is shown in Fig. 6 and Fig. 7. We can see the performance gain of our proposed policy can be up to 390%. Note that since the number of UEs is large a comparison with the PDP (which is NP-hard) is not possible.

Table I
ADMM UPDATE METHOD FROM STEP 3 IN SUBSECTION V-A, FOR UE i

<p>Initialization: $\beta_i(0) = \beta_i^{init}, \beta_i^{init} \in \mathcal{T}_i$ and it can be chosen arbitrarily and $\theta_{ei}^k(0) = \theta^{init}, \forall k \in \{1, \dots, H\}, \forall e \in \tilde{E}(i), \theta^{init} \in \Theta_{ei}^k$ and $\tilde{E}(i)$ is the set of edges which have i as an end point and set $\lambda_{ei}^k(0) = 0, \forall e \in \tilde{E}(i), \forall k \in \{1, \dots, H\}$</p> <p>For $t=0$ to $P-1$</p> <p>$\beta_i(t+1) = \arg \min_{\beta_i \in \mathcal{T}_i} -(\mathbf{W}_i(\mathbf{R}'_i \beta_i) - \sum_{k=1}^H \sum_{e \in \tilde{E}(i)} (\lambda_{ei}^k(t) D_{ei}^k \beta_i^k + \frac{y}{2} (D_{ei}^k \beta_i^k - \theta_{ei}^k(t))^2)$</p> <p>$\beta_i(t+1)$ is transmitted to all the interfering neighbors, $\mathcal{N}(i)$. $\lambda_{ei}^k(t)$ is transmitted to the neighbor with the edge $e, \forall k \in \{1, \dots, H\}$ and $\forall e \in \tilde{E}(i)$</p> <p>Update $\forall k \in \{1, \dots, H\}$ and $\forall e \in \tilde{E}(i)$</p> <p>$\lambda_{ei}^k(t+1) = \frac{1}{2}(\lambda_{ei}^k(t) + \lambda_{ej}^k(t)) - \frac{y}{2} (D_{ei}^k \beta_i^k(t+1) + D_{ej}^k \beta_j^k(t+1))$, here j is the other end point of e.</p> <p>$\theta_{ei}^k(t+1) = \frac{1}{y}(\lambda_{ei}^k(t+1) - \lambda_{e,i}^k(t)) + D_{ei}^k \beta_i^k(t+1)$</p> <p>end</p>

VII. CONCLUSION

We propose a framework for *distributed* interference management in large-scale small cell networks, in which each UE has only local knowledge about the network and communicates only with its interfering neighbors. First, we propose a novel distributed algorithm for the UEs to generate a set of MISs that span all the UEs. The distributed algorithm for generating MISs requires $\mathcal{O}(\log N)$ steps (which is much faster than state-of-the-art) before it converges to the set of MISs with a high probability. Second, we reformulate the problem of determining the optimal fractions of time allocated to the MISs in a novel manner such that the optimal solution can be determined by a distributed algorithm based on ADMM. Remarkably, we prove that the proposed policy can achieve a constant competitive ratio with respect to the policy design problem which is NP-hard. Our simulations show that our policy can achieve large performance gains (up to 390%).

APPENDIX

Discussion on minimum average throughput: We now discuss as to how the proposed framework can be extended to incorporate inseparable function like minimum average throughput. The coupled problem with minimum average throughput objective is restated below:

Coupled Problem (CP)

$$\begin{aligned} \max_{\alpha} \quad & \min_{i \in \{1, \dots, N\}} W_i \left(\sum_{k=1}^H \alpha_k R_i^k \right) \\ \text{subject to} \quad & \sum_{k=1}^H \alpha_k R_i^k \geq R_i^{min}, \forall i \in \{1, \dots, N\} \\ & \sum_{k=1}^H \alpha_k = 1, \alpha_k \geq 0, \forall k \in \{1, \dots, H\} \end{aligned}$$

Transforming the above problem into an equivalent problem with auxiliary variable t is given as

Table II
GENERATING MISS IN A DISTRIBUTED MANNER, ALGORITHM FOR UE i

<p>Initialization: $\text{Tx}_{\text{tent}}^i = \phi, \text{Tx}_{\text{final}}^i = \phi$, tentative and final choice of UE i, $\text{Rx}_{\text{tent}}^{\mathcal{N}(i)} = \phi, \text{Rx}_{\text{final}}^{\mathcal{N}(i)} = \phi$ tentative and final choice made by the neighbors, $C_i^0 = \{1, \dots, H\} \cap \{1, \dots, d_i + 1\}$ the current list of subset of available colors, $C_i = \phi$, list of colors used by i, $F_i^{\text{colored}} = \phi$ indicator if i has acquired a color, $C1_i^0 = \{1, \dots, H\}$, the current list of all available colors</p> <p>for $n = 0$ to $\lceil c_1 \log_{\frac{4}{3}} N \rceil$</p> <p>$\text{Tx}_{\text{tent}}^i = \phi, \text{Tx}_{\text{final}}^i = \phi$</p> <p>if($F_i^{\text{colored}} = \phi$)</p> <p>$\text{Tx}_{\text{tent}}^i = \text{rand}\{C_i^n\}$, rand represents randomly select a color and inform the neighbors about it.</p> <p>$\text{Rx}_{\text{tent}}^{\mathcal{N}(i)} = \{\text{Tx}_{\text{tent}}^k, \forall k \in \mathcal{N}(i)\}$</p> <p>If($\text{Tx}_{\text{tent}}^i \neq \text{Rx}_{\text{tent}}^{\mathcal{N}(i)}(j), \forall j \in \mathcal{N}(i)$), UE-$i$ checks if there is a conflict with any of the neighbor's choice</p> <p>$\text{Tx}_{\text{final}}^i = \text{Tx}_{\text{tent}}^i, C_i = \{\text{Tx}_{\text{final}}^i\}$, if no conflict then UE-i transmits its final color choice to the neighbors,</p> <p>else</p> <p>$\text{Tx}_{\text{final}}^i = \phi$</p> <p>end</p> <p>end</p> <p>$\text{Rx}_{\text{final}}^{\mathcal{N}(i)} = \{\text{Tx}_{\text{final}}^k, \forall k \in \mathcal{N}(i)\}$</p> <p>$C_i^{n+1} = C_i^n \cap \{\text{Rx}_{\text{final}}^{\mathcal{N}(i)} \cup \text{Tx}_{\text{final}}^i\}^c$</p> <p>$C1_i^{n+1} = C1_i^n \cap \{\text{Rx}_{\text{final}}^{\mathcal{N}(i)} \cup \text{Tx}_{\text{final}}^i\}^c$</p> <p>if($\text{Tx}_{\text{final}}^i \neq \phi$)</p> <p>$F_i^{\text{colored}} = 1$</p> <p>end</p> <p>end</p>

Table III
PHASE 2 OF THE DISTRIBUTED MIS GENERATION

<p>Phase 2-Initialization: $\text{Tx}_{\text{tent},i}^{\text{set}} = \phi, \text{Tx}_{\text{final},i}^{\text{set}} = \phi$, the set of tentative and final colors chosen by i, $\text{Rx}_{\text{tent},i}^{\text{set}} = \phi, \text{Rx}_{\text{final},i}^{\text{set}} = \phi$, the set of tentative and final colors chosen by the neighbors, $x = \frac{1}{1-(c)^H(1-c)^{H^2}}$</p> <p>for $n = \lceil c_1 \log_{\frac{4}{3}} N \rceil + 1$ to $\lceil c_1 \log_{\frac{4}{3}} N \rceil + \lceil c_2 \log_x N \rceil + 1$</p> <p>$\text{Tx}_{\text{tent},i}^{\text{set}} = \phi, \text{Tx}_{\text{final},i}^{\text{set}} = \phi$,</p> <p>for $m = 1$ to $C1_i^n$</p> <p>with probability c, $\text{Tx}_{\text{tent},i}^{\text{set}}(m) = C1_i^n(m)$, randomly selecting and informing the neighbors about tentative choice</p> <p>with probability $1 - c$, $\text{Tx}_{\text{tent},i}^{\text{set}}(m) = \phi$</p> <p>end</p> <p>$\text{Rx}_{\text{tent},i}^{\text{set}} = \cup_{k \in \mathcal{N}(i)} \text{Tx}_{\text{tent},k}^{\text{set}}$, set of tentative color choices of the neighbors of i</p> <p>for $r = 1$ to $\text{Tx}_{\text{tent},i}^{\text{set}}$</p> <p>If($\text{Tx}_{\text{tent},i}^{\text{set}}(r) \neq \text{Rx}_{\text{tent},i}^{\text{set}}(j), \forall j \in \mathcal{N}(i)$)</p> <p>$\text{Tx}_{\text{final},i}^{\text{set}}(r) = \text{Tx}_{\text{tent},i}^{\text{set}}(r)$</p> <p>else</p> <p>$\text{Tx}_{\text{final},i}^{\text{set}}(r) = \phi$</p> <p>end</p> <p>$C_i = C_i \cup \text{Tx}_{\text{final},i}^{\text{set}}$</p> <p>$\text{Rx}_{\text{final},i}^{\text{set}} = \cup_{k \in \mathcal{N}(i)} \text{Tx}_{\text{final},k}^{\text{set}}$, set of final color choices of the neighbors of i</p> <p>$C1_i^{n+1} = C1_i^n \cap \{\text{Rx}_{\text{final},i}^{\text{set}} \cup \text{Tx}_{\text{final},i}^{\text{set}}\}^c$</p> <p>end</p>
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$$\begin{aligned} & \max_{\alpha, t} \quad t \\ & \text{subject to} \quad W_i \left(\sum_{k=1}^H \alpha_k R_i^k \right) \geq t, \forall i \in \{1, \dots, N\} \\ & \quad \sum_{k=1}^H \alpha_k R_i^k \geq R_i^{\min}, \forall i \in \{1, \dots, N\} \\ & \quad \sum_{k=1}^H \alpha_k = 1, \alpha_k \geq 0, \forall k \in \{1, \dots, H\} \end{aligned}$$

To decouple the above problem, we introduce local variables for each UE i given as, $\{\beta_i^1, \dots, \beta_i^{H+1}\}$. Now we state a problem which we claim is equivalent to CP, (the proof to this claim is very similar to the proof of Theorem 2 and we will highlight this fact in the proof clearly).

P1

$$\begin{aligned} & \max_{\beta} \quad \sum_{i=1}^N \beta_i^{H+1} \\ & \text{subject to} \quad W_i \left(\sum_{k=1}^H \beta_i^k R_i^k \right) \geq \beta_i^{H+1}, \forall i \in \{1, \dots, N\} \\ & \quad \sum_{k=1}^H \beta_i^k R_i^k \geq R_i^{\min}, \forall i \in \{1, \dots, N\} \\ & \quad \sum_{k=1}^H \beta_i^k = 1, \beta_i^k \geq 0, \forall k \in \{1, \dots, H\}, \forall i \in \{1, \dots, N\} \\ & \quad \beta_i^k = \beta_j^k, \forall j \in \mathcal{N}(i), \forall k \in \{1, \dots, H+1\} \end{aligned}$$

Here, $\beta = (\beta_1, \dots, \beta_N)$, with $\beta_i = (\beta_i^1, \dots, \beta_i^{H+1}), \forall i \in \{1, \dots, N\}$. Now, given the two problems CP and the problem P1 are equivalent, we focus on solving P1. P1 can be changed to a problem similar to DP. To do that we introduce some additional variables similar to the ones introduced for DP. If UE i and l are connected by an edge (i, l) then for each set I'_k define $\theta_{(i,l)i}^k = \beta_i^k$ and $\theta_{(i,l)l}^k = -\beta_l^k$, note that these auxiliary variables are introduced to formulate the problem into the ADMM framework [29]. Define a polyhedron for each i , $\mathcal{T}'_i = \{(\beta_1)_i \mid \text{s.t. } \mathbf{1}^t(\beta''_i) = 1, (\beta_1)_i \geq \mathbf{0}, \mathbf{R}'_i(\beta''_i) \geq R_i^{\min}, W_i(\mathbf{R}'_i(\beta''_i)) - \beta_i^{H+1} \geq 0\}$, here $\beta''_i = (\beta_i^1, \dots, \beta_i^H)$ and $\mathbf{R}_i = (R_i^1, \dots, R_i^H)$ and $(\cdot)^t$ corresponds to the transpose. Let $\beta = (\beta_1, \dots, \beta_N) \in \mathcal{T}'$, where $\mathcal{T}' = \prod_{i=1}^N \mathcal{T}'_i$ and \prod corresponds to the Cartesian product of the sets. Also, let $\beta^k = (\beta_1^k, \dots, \beta_N^k), \forall k \in \{1, \dots, H\}$. Define another polyhedron $\Theta_{(i,l)}^k = \{(\theta_{(i,l)i}^k, \theta_{(i,l)l}^k) : \theta_{(i,l)i}^k + \theta_{(i,l)l}^k = 0, -1 \leq \theta_{(i,l)s}^k \leq 1, \forall s \in \{i, l\}\}$, $\Theta^k = \prod_{(i,l) \in E} \Theta_{(i,l)}^k$ here $E = (e_1, \dots, e_M)$ is the set of all the M edges in the interference graph. A vector $\theta^k \in \Theta^k$ is written as $\theta^k = (\theta_{e_1, z(e_1)}^k, \theta_{e_1, t(e_1)}^k, \dots, \theta_{e_M, z(e_M)}^k, \theta_{e_M, t(e_M)}^k)$, here $z(e_i), t(e_i)$ correspond to the vertices in the edge, e_i . Similarly define, $\theta = (\theta^1, \dots, \theta^{H+1}) \in \Theta'$, where $\Theta' = \prod_{k=1}^{H+1} \Theta^k$. The reformulated problem is stated as follows:

$$\text{DPI} \quad \min_{\beta \in \mathcal{T}', \theta \in \Theta'} - \sum_{i=1}^N \mathbf{W}_i(\mathbf{R}_i' \beta_i)$$

$$\text{subject to } \mathbf{D}^k \beta^k - \theta^k = \mathbf{0}, \forall k \in \{1, \dots, H+1\}$$

Then, DPI can be solved using the ADMM procedure similar to the one described for DP.

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