Pricing and Distributed Power Control for Relay Networks†

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***Abstract*—In this paper, we consider a wireless amplify-and-forward relay network with one relay node and multiple source-destination pairs/users and propose a compensation framework such that the relay has incentives to forward the users' signals. Specifically, depending on the quality of the received signals, the relay sets the prices to maximize its revenue and correspondingly charges the users utilizing the relay for their transmissions. Given the specified price, the users competitively employ the relay node to forward their signals. We model each user as a strategic player, which aims at maximizing its own net utility through power allocation, and apply non-cooperative game theory to analyze the competition among the users. It is shown that, in the game played by the users, there always exists a unique Nash equilibrium point that can be achieved through distributed iterations. Then, subject to the availability of complete information about the users at the relay, we propose a low-complexity uniform pricing algorithm and an optimal differentiated pricing algorithm, in which the relay charges the users at a sub-optimal uniform price and at different prices, respectively.**

I. Introduction

In many wireless networks, the transmission between two distant users may have to be accomplished with the help of an intermediate node, i.e., relay, due to transmit power or other constraints [1]. Nevertheless, without a proper compensation framework, the relay may have no incentives to accommodate the users by forwarding their signals to the destinations at the expense of its own energy. Hence, pricing becomes an effective mechanism that reimburses the relay for using its resources by making payments[1], thereby providing the relay with the incentives for forwarding the users' signals [4]–[6].

It is worth noting that pricing mechanisms have been widely applied in the context of relay networks [4]–[6]. For instance, considering a multi-relay network, the authors in [4] cast the problem of distributed power control and relay selection into the Stackelberg formulation. In particular, the payment made by the user serves as a reimbursement that gives the relay an incentive to participate in the cooperation. Similar compensation frameworks enabling the relay to forward the users' signals are proposed in the literature as well, e.g., [5][6]. In [8], an auction-based spectrum sharing protocol is proposed such that the each user submits an optimal bid to the network manager to maximize the utility. The auction framework is also extended to a relay network setting [9].

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[1]The payments can be tokens, virtual money, etc., which can be used in the future by the relay to purchase resources from other nodes in the network.

In this paper, we focus on a relay network with one relay node and multiple source-destination pairs/users[2], and propose a pricing mechanism that gives the relay incentives to forward the users' signals. In the pricing mechanism, the price is determined by the relay to maximize its revenue, and each user is modeled as a strategic player in a non-cooperative game. Specifically, with the knowledge of its local channel state information (CSI), each user maximizes its utility by choosing its optimal power level, given the power allocation strategies of the other users. This process iterates until convergence. It is shown that, in the non-cooperative game, there always exists a unique Nash equilibrium point (NEP) that can be achieved through the iterative power allocation process. Next, with incomplete information about the users (i.e., the number of users and the sum signal to interference plus noise ratio (SINR) when all the users transmit with their maximum powers) at the relay, we propose a low-complexity uniform pricing algorithm, i.e., the relay charges all the users at the same price. Then, we extend the pricing algorithm to differentiated pricing by assuming that the relay has complete information about the users (i.e., channel coefficients, power constraints, etc.). Finally, extensive simulations are conducted to verify the performance of the proposed pricing algorithms.

The main contributions of this paper are threefold: (i) we focus on a relay network with multiple users modeled as strategic players competing for the network resource, i.e., relay, and study the NEP of the game; (ii) we propose a pricing mechanism that provides the relay with the incentives to forward the users' signals; (iii) we propose two pricing algorithms, i.e., uniform pricing with incomplete information and differentiated pricing with complete information.

The rest of this paper is organized as follows. Section II describes the network model and problem formulation. In Section III, a distributed power allocation algorithm along with two pricing algorithms are developed. Simulation results are shown in Section IV and concluding remarks are offered in Section V.

II. System Model

Consider a relay network consisting of one relay node and Q source-destination pairs, as illustrated in Fig. 1. Similar models are considered in the literature as well (e.g., [3]), and note that the following analysis can be extended to a multi-relay

[2]We interchangeably use "user" to represent the source-destination pair.

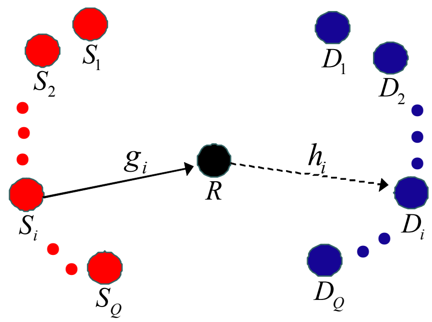

Fig. 1. Network model

network as long as different network clusters, each of which consists of one relay and multiple users, are transmitting over different channels [3].

A. Network Model

The source and destination are indexed by $\mathcal{S}_i$ and $\mathcal{D}_i$, respectively, for $i = 1, 2 \cdots Q$, and the relay node is represented by $\mathcal{R}$. We denote the coefficients for the $\mathcal{S}_i - \mathcal{R}$ and the $\mathcal{R} - \mathcal{D}_i$ channels by g_i and h_i, respectively. The transmit powers of $\mathcal{S}_i$ and $\mathcal{R}$ are p_i and $p_\mathcal{R}$, respectively. Local CSI, i.e., g_i and h_i, is available at user i, but neither g_j nor h_j is known to user i, if $j \neq i$, due to the distributed nature of the considered problem. Furthermore, we assume the zero-mean complex additive white Gaussian noise (AWGN) at each node to have a variance of N_0. Due to the half-duplex constraint, we consider orthogonal relaying transmissions, e.g., the source nodes and the relay node transmit in two non-overlapping time slots. The direct link between $\mathcal{S}_i$ and $\mathcal{D}_i$ is neglected due to, for instance, the shadowing effects [1]. We adopt the classical amplify-and-forward strategy as the relaying operation, which has been shown to be an appealing technique due to its low cost and easy implementation as compared to the decode-and-forward protocol [2]. Hence, the signals received at $\mathcal{R}$ and $\mathcal{D}_i$ can be written, respectively, as

$$y_\mathcal{R} = \sum_{j=1}^{Q} g_j \sqrt{p_j} x_j + n_\mathcal{R} \text{ and } y_i = \alpha h_i y_\mathcal{R} + n_i, \tag{1}$$

where x_i is the unit-variance transmit signal from $\mathcal{S}_i$ to $\mathcal{D}_i$, $\alpha = \sqrt{\frac{p_\mathcal{R}}{\sum_{j=1}^{Q} |g_j|^2 p_j + N_0}}$ is the amplification factor of $\mathcal{R}$, $n_\mathcal{R}$ and n_i are the statistically-independent AWGN terms at $\mathcal{R}$ and $\mathcal{D}_i$, respectively. Assuming that $\mathcal{D}_i$ is only interested in the signal x_i without applying interference cancelation [11], we can express the receive SINR at $\mathcal{D}_i$ as

$$\gamma_i = \frac{|g_i|^2 |h_i|^2 p_\mathcal{R} p_i}{|g_i|^2 N_0 p_i + (|h_i|^2 p_\mathcal{R} + N_0) \cdot \mathcal{I}_i}, \tag{2}$$

where $\mathcal{I}_i = \sum_{j=1, j\neq i}^{Q} |g_j|^2 p_j + N_0$. In general, the utility function is increasing and concave in the receive SINR [8]. Specifically, we adopt in the sequel the achievable rate[3] as the utility function of user i

$$R_i(p_i; p_{-i}) = \frac{1}{2} \log(1 + \gamma_i) \tag{3}$$

where $\frac{1}{2}$ is the spectral loss factor due to the half-duplex constraint [2], γ_i is given in (2), and $p_{-i} = (p_1 \cdots p_{i-1}, p_{i+1} \cdots p_Q)$ is the power allocation vector of all the users except for user i.

Before proceeding to the problem formulation, we briefly note that the relay amplifies and forwards the noise as well as the desired signal, whereas the source transmits noiseless signals in single-hop interference channels [11]. Hence, the analysis here can be regarded as a generalization of the existing results on Gaussian interference channels. In particular, if $|h_i|^2 \rightarrow \infty$, the dual-hop relay channel reduces to the conventional multi-access interference channel and the receive SINR of user i becomes $\gamma_i = \frac{|g_i|^2 p_i}{\sum_{j=1, j\neq i}^{Q} |g_j|^2 p_j + N_0}$.

B. Problem Formulation

It is known that the receive SINR, which is partially determined by the relay's power, measures the quality of the received signal and thus influences the utility of each user. Hence, it is reasonable to assume that the payment made to the relay is a function of the receive SINR. Mathematically, the payment that user i needs to make to the relay, which sets the price π_i, is defined as $\pi_i \gamma_i$. This payment rule charges each user in proportion to its receive SINR, which was similarly referred to as "SINR auction" in [8][9]. Other similar payment rules can be found in [7]. Given the payment rule, the net utility function of user i can therefore be expressed as

$$u_i(p_i; p_{-i}) = \frac{1}{2} \log(1 + \gamma_i) - \pi_i \gamma_i, \tag{4}$$

where the first term is the achievable rate of user i. From the relay's perspective, in order to maximize the revenue[4] collected from the users, the relay needs to set an optimal price vector $\mathbf{\Pi}^* = \{\pi_1^*, \pi_2^* \cdots \pi_Q^*\}$ such that

$$\mathbf{\Pi}^* = \arg\max_{\mathbf{\Pi} \succeq \mathbf{0}} \left(\sum_{i=1}^{Q} \pi_i \gamma_i (p_i; p_{-i}) \right). \tag{5}$$

III. Pricing and Distributed Power Control

In this section, we cast the user-level distributed power control problem into the framework of non-cooperative game theory, and jointly optimize the user's net utility and the relay's revenue.

A. Distributed Power Control

Given the price set by the relay, we mathematically characterize the competition among the strategic users using the following the non-cooperative game

$$\mathcal{G}_{\text{user}} = \{\Omega, \{\mathcal{P}_i\}_{i\in\Omega}, \{u_i(p_i; p_{-i})\}_{i\in\Omega}\} \tag{6}$$

[3]Note that the achievable rate is a widely-used utility function (see, e.g., [9][12]) and the analysis herein can be applied, after minor modifications, to other forms of utility functions as well.

[4]The relay incurs a fixed cost, e.g., power consumption, when forwarding the users' signals, and thus, revenue maximization is virtually equivalent to profit maximization [15].

where $\Omega \triangleq \{1, 2 \cdots Q\}$ is the set of active users (i.e., $\mathcal{S}_i - \mathcal{D}_i$ pair), $\mathcal{P}_i$ is the set of admissible power allocation strategies of user i defined as $\{p_i : 0 \leq p_i \leq p_i^{\max}\}$ and $u_i(p_i; p_{-i})$ is the net utility function of user i given in (4). The optimal power of user i in response to the power levels of all the other users is referred to as the *best response* function denoted by $\mathcal{B}_i(p_{-i})$. In the non-cooperative game, the NEP is a critical operating point at which the outcome of the game becomes stabilized [16], and it is achieved when user i, given p_{-i}, cannot increase its net utility $u_i(p_i; p_{-i})$ by unilaterally changing its own power p_i, for all $i \in \Omega$. Mathematically, the NEP, denoted by $\mathbf{p}^* = (p_1^*, p_2^* \cdots p_Q^*)$, of the game $\mathcal{G}_{\text{user}}$ in (6) is formally defined as [16]

$$u_i(p_i^*; p_{-i}^*) \geq u_i(p_i; p_{-i}^*), \quad \forall\, p_i \in \mathcal{P}_i,\ \forall\, i \in \Omega\,. \tag{7}$$

In particular, regarding the existence of NEP in the user game, we have the following theorem whose proof is omitted due to the space limitations and available in [14].

Theorem 1. Given any non-negative price vector $\mathbf{\Pi} \succeq \mathbf{0}$ set by the relay, there always exists at least one NEP in the non-cooperative game $\mathcal{G}_{\text{user}}$ played by the users. ∎

Next, we shall give the closed form of best response of user i, i.e., $\mathcal{B}_i(p_{-i})$, in the following. Note first that we can prove there exists a unique $\mathcal{B}_i(p_{-i})$, given any p_{-i} and π_i. To be more specific, depending on the price π_i set by the network, the unique $\mathcal{B}_i(p_{-i})$ can be easily derived and expressed in a compact form as

$$\mathcal{B}_i(p_{-i}) = \left[\frac{\delta_i(\pi_i)\left(|h_i|^2 p_{\mathcal{R}} + N_0\right) \cdot \mathcal{I}_i}{|g_i|^2 \cdot [|h_i|^2 p_{\mathcal{R}} - N_0 \cdot \delta_i(\pi_i)]}\right]_0^{p_i^{\max}} \tag{8}$$

where $\mathcal{I}_i = \sum_{j \neq i}^{Q} |g_j|^2 p_j + N_0$, $[\,\cdot\,]_a^b = \max\{\min\{\cdot, b\}, a\}$ and $\delta_i(\pi_i)$ is a non-increasing function of π_i defined as

$$\delta_i(\pi_i) = \begin{cases} 0, & \text{if } \frac{1}{2} < \pi_i, \\ \frac{1}{2\pi_i} - 1, & \text{if } \left(1 + \gamma_i(p_i^{\max}; \mathbf{0})\right)^{-1} < 2\pi_i \leq 1, \\ \gamma_i(p_i^{\max}; \mathbf{0}), & 0 \leq 2\pi_i \leq \left(1 + \gamma_i(p_i^{\max}; \mathbf{0})\right)^{-1}, \end{cases} \tag{9}$$

in which $\gamma_i(p_i^{\max}; \mathbf{0})$ is obtained by plugging $(p_i; p_{-i}) = (p_i^{\max}; \mathbf{0})$ into (2). In addition to the existence of NEP in the game $\mathcal{G}_{\text{user}}$, whether and how the non-cooperative game can eventually arrive at the NEP is another question we have yet to answer. To this end, we present an iterative algorithm that reaches the unique NEP of $\mathcal{G}_{\text{user}}$ and can be formally described as follows.

Algorithm I: Iterative Distributed Power Allocation

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Step 1: $n = 0$; **choose any feasible** $\mathbf{p}^0 = \left(p_1^0, p_2^0 \cdots p_Q^0\right)$
Step 2: $p_i^{(n+1)} = \mathcal{B}_i(p_{-i}^n)$, **for** $i = 1, 2 \cdots Q$
Step 3: $n = n + 1$; **go to Step 2 until convergence**

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To complete the algorithm description, we give Theorem 2 regarding the convergence of the proposed algorithm.

Theorem 2. Given any non-negative price vector $\pi \succeq \mathbf{0}$ and starting from any initial point $\mathbf{p}^0$, the iteration specified by $p_i^{(n+1)} = \mathcal{B}_i(p_{-i}^n)$, for $i \in \Omega$, always converges to the unique NEP of the game $\mathcal{G}_{\text{user}}$ as $n \to \infty$.

Proof: The proof is mainly based on the standard interference function [10] and can be found in [14]. ∎

Before concluding this section, we note that, as shown in (8), the local information required to compute $\mathcal{B}_i(p_{-i})$ at user i includes the local CSI (i.e., g_i and h_i), the relay's transmit power $p_{\mathcal{R}}$, the price π_i set by the network and the multi-user interference plus noise $\sum_{j=1, j\neq i}^{Q} |g_j|^2 p_j + N_0$. In particular, user i can obtain the local CSI through channel estimation (e.g., sending pilot signals) [9]. The relay's transmit power $p_{\mathcal{R}}$ and the price π_i are transmitted via control channels to user i prior to the users' transmissions. Regarding the multi-user interference, the relay node can broadcast to all the users its amplification factor α such that user i, for $i \in \Omega$, acquires the value of $\sum_{j=1, j\neq i}^{Q} |g_j|^2 p_j + N_0$ by computing $\frac{p_{\mathcal{R}}}{\alpha^2} - |g_i|^2 p_i$. It can therefore be seen that the proposed algorithm can be applied in a distributed manner. It should also be noted that Algorithm I is re-executed only when the price set by the relay is updated or the network condition changes, e.g., channel coefficients vary or additional users enter the system.

B. *Uniform Pricing With Incomplete Information*

In many wireless networks with limited information exchange among different nodes, the relay only has incomplete information about the users (e.g., the maximum power constraints of the users are private and thus unknown to the relay). Under such constraints, we propose a uniform pricing algorithm, i.e., the relay sets and broadcasts a uniform price, i.e., $\pi_1 = \pi_2 \cdots = \pi_Q = \pi$, to all the users.

Due to the lack of private information about the users, e.g., power strategy space, the relay cannot analytically compute the NEP of the user-level game $\mathcal{G}_{\text{user}}$ or directly set an optimal uniform price[5] such that $\pi^* = \arg\max_{\pi \geq 0}\left(\pi \sum_i^Q \gamma_i(p_i^*; p_{-i}^*)\right)$. As a consequence, we propose a low-complexity algorithm that can yield a close-to-optimal uniform price. Before stating the algorithm, we first define the lower and upper bounds on the optimal uniform price, i.e., $\pi_a = \frac{1}{2}\min_{i\in\Omega}\left\{1 + \gamma_i\left(\mathbf{p}^{\max}\right)\right\}^{-1}$ and $\pi_b = \frac{1}{2}\max_{i\in\Omega}\left\{1 + \gamma_i\left(\mathbf{p}^{\max}\right)\right\}^{-1}$, respectively, and summarize two instrumental properties of the revenue function $\rho(\pi) = \pi \sum_{i=1}^{Q} \gamma_i(\pi)$ in the following theorem whose proof can be found in [14].

Theorem 3. The revenue function has the following properties[6]:

1. $\rho(\pi) = \pi \sum_{i=1}^{Q} \gamma_i(\mathbf{p}^{\max})$ when $0 \leq \pi \leq \pi_a$;
2. There exists a certain value of price $\hat{\pi}$ satisfying

$$\begin{cases} \hat{\pi} < \pi_b, & \exists i, j \in \Omega \; s.t.\; \gamma_i\left(\mathbf{p}^{\max}\right) \neq \gamma_j\left(\mathbf{p}^{\max}\right) \\ \hat{\pi} = \pi_b, & \forall i, j \in \Omega \; s.t.\; \gamma_i\left(\mathbf{p}^{\max}\right) = \gamma_j\left(\mathbf{p}^{\max}\right) \end{cases}, \tag{10}$$

such that $\rho(\pi) = Q \cdot \left(\frac{1}{2} - \pi\right)$. ∎

[5] Note that, without knowing the power strategy of the users, the relay can only obtain the optimal uniform price through exhaustive search, which incurs a high implementation complexity.

[6] $\gamma_i(\mathbf{p}^{\max})$ is obtained by plugging $\mathbf{p}^{\max} = \left(p_1^{\max}, p_2^{\max} \cdots p_Q^{\max}\right)$ into (2).

Theorem 3 can be simply interpreted as follows: the optimal price of the relay lies in a certain interval that depends on the channel conditions and transmit power constraints. The non-trivial properties also form the basis of the proposed sub-optimal uniform pricing algorithm. Specifically, if we artificially increase π_a and decrease $\hat{\pi}$ simultaneously until they meet at $\overline{\pi}$ and assume that

$$\rho(\pi) = \begin{cases} \pi \cdot \sum_{i=1}^{Q} \gamma_i(\mathbf{p}^{\max}), & \text{if } 0 \leq \pi \leq \overline{\pi} \\ Q \cdot \left(\frac{1}{2} - \pi\right), & \text{if } \overline{\pi} < \pi \leq \frac{1}{2} \end{cases}, \tag{11}$$

we can easily obtain the "optimal" uniform price as

$$\pi^* \approx \overline{\pi} = \frac{Q}{2\left[\sum_{i=1}^{Q} \gamma_i(\mathbf{p}^{\max}) + Q\right]}. \tag{12}$$

Generally speaking, setting (12) as the price can only result in a sub-optimal revenue for the relay. Nevertheless, the high computational complexity incurred by the exhaustive search is avoided and only limited information is needed to calculate (12): the number of active users in the network, i.e., Q, and the value of $\sum_{i=1}^{Q} \gamma_i(\mathbf{p}^{\max})$. The relay can set a sufficiently low price π_l, given which the NEP is $\mathbf{p}^{\max}$, and find $\sum_{i=1}^{Q} \gamma_i(\mathbf{p}^{\max})$ by computing $\frac{\rho(\pi_l)}{\pi_l}$. The uniform price is determined in a similar way in the context of conventional cellular systems in [7]. Moreover, we note that, when there are a sufficient large number of users in the network or the users operate in low SINR regions, (12) is also a good approximation of the optimal uniform price. Specifically, when the number of users in the network is large, the sub-optimality of (12) can be explained as follows. It is natural that the level of interference observed by user i, i.e., $\sum_{j=1, j\neq i}^{Q} |g_j|^2 p_j$, increases when there are more active users. Hence, given a large value of Q, $\max_{i=1,2\cdots Q} \gamma_i(\mathbf{p}^{\max})$ becomes a small non-negative number due to the strong interference caused by the other users. Correspondingly, the difference between the lower bound and the upper bound on the optimal uniform price is not significant, i.e., $\hat{\pi} - \pi_a$ is a small number. Thus, the sub-optimal price (12), which lies between π_a and $\hat{\pi}$, is close to the optimal one. Note that the small non-negative number $\hat{\pi} - \pi_a$ is also a upper bound on the gap between (12) and the optimal uniform price. Similar statements can be made when the network operates in low SINR regions as well. As in the existing literature (e.g., [7]), it is challenging to determine a priori the exact gap between (12) and the optimal uniform price, and hence, we shall alternatively verify in numerical results that the loss of revenue is not significant in all the cases when the relay chooses (12), rather than the optimal one obtained through exhaustive search, as its uniform price.

C. Differentiated Pricing With Complete Information

It has been shown in [13] that the system performance can be improved if some users have complete information about the network. In the following analysis, we extend the uniform pricing to differentiated pricing by assuming that the relay knows the maximum power constraints of all the users[7], in addition to the channel coefficients. Under the differentiated pricing rule, we need to identify an optimal price vector $\mathbf{\Pi}^*$ set by the network such that $\mathbf{\Pi}^* = \arg\max_{\mathbf{\Pi}\succeq \mathbf{0}} \left(\sum_{i=1}^{Q} \pi_i \gamma_i(p_i^*; p_{-i}^*)\right)$. Differentiated pricing is also referred to as price discrimination in the economics literature [15]. Before developing the differentiated pricing algorithm, we first express the optimal value of π_i in terms of $\mathbf{p}^*$, for all $i \in \Omega$, in the following proposition, the proof of which is omitted here for brevity and can be found in [14].

Proposition 1. Assume that $\mathbf{\Pi}^* = \{\pi_1^*, \pi_2^* \cdots \pi_Q^*\}$ is the optimal price vector, which generates the maximum revenue for the network, and that $\tilde{\mathbf{p}}^* = \{\tilde{p}_1^*, \tilde{p}_2^* \cdots \tilde{p}_Q^*\}$ is the unique corresponding power allocation vector at the NEP of the user game $\mathcal{G}_{\text{user}}$. Then, $\mathbf{\Pi}^*$ can be expressed in terms of $\tilde{\mathbf{p}}^*$ as follows

$$\pi_i^* = \frac{1}{2(1+\gamma_i(\tilde{\mathbf{p}}^*))}, \quad \forall i \in \Omega, \tag{13}$$

where $\gamma_i(\tilde{\mathbf{p}}^*)$ is obtained by substituting $\tilde{\mathbf{p}}^*$ into (2). ■

Now, following Proposition 1, the problem of maximizing $\sum_{i=1}^{Q} \pi_i \gamma_i(p_i^*; p_{-i}^*)$ subject to $\mathbf{\Pi} \succeq \mathbf{0}$ can be reformulated as

$$\max \sum_{i=1}^{Q} \frac{\gamma_i(\tilde{\mathbf{p}}^*)}{2(1+\gamma_i(\tilde{\mathbf{p}}^*))} \Leftrightarrow \max \frac{\sum_{i=1}^{Q} \frac{|h_i|^2 p_{\mathcal{R}}}{|h_i|^2 p_{\mathcal{R}} + N_0} |g_i|^2 \tilde{p}_i^*}{2\left(\sum_{i=1}^{Q} |g_i|^2 \tilde{p}_i^* + N_0\right)}$$
$$s.t., \ \mathbf{0} \preceq \tilde{\mathbf{p}}^* \preceq \mathbf{p}^{\max}, \tag{14}$$

where the objective function is linear-fractional and hence quasi-concave in $\tilde{\mathbf{p}}^*$ [17]. Hence, the optimal value of $\tilde{\mathbf{p}}^*$ can be found by transforming (14) into a standard linear program [17], and the details of solving (14) are omitted due to the space limitations. Then, by uniqueness of the NEP of the game $\mathcal{G}_{\text{user}}$ given any price vectors stated in Theorem 2, it can be seen that $\tilde{\mathbf{p}}^*$ is the unique NEP of the game $\mathcal{G}_{\text{user}}$ if the relay sets $\mathbf{\Pi}^*$ according to (13) as its pricing vector. Therefore, we can solve (14) to find $\tilde{\mathbf{p}}^*$ and then $\mathbf{\Pi}^*$ can be determined using (13). Furthermore, from (14), we can see that the revenue that the relay can obtain by charging the users at the optimal differentiated prices is upper bounded by $\frac{1}{2}$.

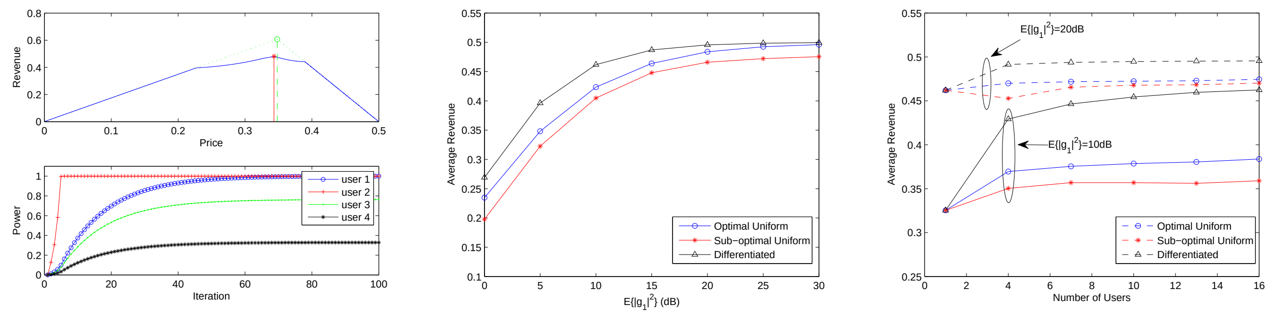
Next, we briefly discuss the complexity incurred by the relay to set the prices. With complete information about the users, i.e., channel coefficients and power strategy space, the relay can directly compute the optimal differentiated price vector $\mathbf{\Pi}^*$, by solving the linear-fractional optimization problem in (14), and thus, it only needs to broadcast once the optimal price vector to the users. However, in the case of uniform pricing, the relay needs to set a sufficiently low price before identifying the sub-optimal uniform price, since only incomplete information about the users is available by the relay.

IV. Numerical Results

For the convenience of illustration, g_i and h_i are modeled as independently Rayleigh distributed random variables, for $i \in \Omega$. The transmit power of the relay node and the maximum transmit power of each source node, as well as the variance of the Gaussian noise, are normalized to one. Moreover, we assume a homogeneous network topology[8], i.e., $\mathbb{E}\{|g_1|^2\} =$

[7]To implement the protocol, the user may be required to report its maximum transmit power level to the relay before entering the network.

[8]The results for general network topologies and channel conditions, which can be found in [14], are not shown in this paper due to space limitations



(a) Random channel gains

(b) Effects of channel gains

(c) Effects of the number of users

$\mathbb{E}\{|h_1|^2\} = \cdots = \mathbb{E}\{|g_Q|^2\} = \mathbb{E}\{|h_Q|^2\}$, where $\mathbb{E}\{\cdot\}$ is the expectation operator. First, given random channel gains in a four-user network, we illustrate in Fig. 2(a) the convergence of the proposed distributed power allocation algorithm and the sub-optimal uniform pricing algorithm. The upper plot shows that the sub-optimal uniform price (dashed line) is reasonably close to the optimal uniform price[9] (solid line), which validates the use of (12) as the uniform price by the relay.

1) Effects of Channel Gains: We consider a four-user network and examine the effects of channel gains on the performance in Fig. 2(b). As intuitively expected, the revenue of the relay increases as the channel condition becomes better. Fig. 2(b) also demonstrates that the revenue loss due to the sub-optimality of the uniform price is not significant compared to the optimal uniform price. Among all the three pricing schemes, differentiated pricing generates the maximum revenue for the relay at the expense of having more information about the users. Regarding the upper bound on the revenues, it can be observed that the maximum revenue is always less than $\frac{1}{2}$ regardless of the channel conditions,which verifies the analysis.

2) Effects of Number of Users: In Fig. 2(c), we fix the average channel gains and vary the number of active users. It shows that the average revenue of the network is increasing in the number of users and differentiated pricing achieves the maximum revenue. Fig. 2(c) also indicates that the sub-optimal revenue of the relay gained by setting (12) as the uniform price is close to the optimal uniform one obtained through exhaustive search.

V. Conclusion

In this paper, considering a wireless relay network with one relay node and multiple source-destination users, we proposed a pricing mechanism to provide the relay with incentives to forward the users' signals. We then modeled each user as a strategic player, which aims at maximizing its own benefit by choosing the optimal transmit power, and analyzed the competition among the users using the notion of non-cooperative game theory. It was proved that, in the non-cooperative game played by the users, there always exists a unique steady operating point, i.e., NEP, which can be achieved in a distributed manner. Next, we proposed a low-complexity algorithm, in which the relay charges the users at a sub-optimal uniform price. The analysis was then extended to differentiated pricing wherein the relay charges different users at different prices. Extensive simulations were finally conducted to verify the analysis from both the relay's as well as the user's perspective.

[9]The optimal uniform price in this paper is obtained numerically through exhaustive search.

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