

# Queuing-Based Dynamic Channel Selection for Heterogeneous Multimedia Applications over Cognitive Radio Networks

Hsien-Po Shiang<sup>\*</sup> and Mihaela van der Schaar

Department of Electrical Engineering (EE), University of California Los Angeles (UCLA) Los Angeles, CA

{hpshiang, mihaela} @ee.ucla.edu

## ABSTRACT

In this paper, we propose a dynamic channel selection solution for autonomous wireless users transmitting delay-sensitive multimedia applications over cognitive radio networks. Unlike prior works that seldom consider the requirement of the application layer, our solution explicitly considers various rate requirements and delay deadlines of heterogeneous multimedia users. Note that the users usually possess private utility functions, application requirements, and distinct channel conditions in different frequency channels. To efficiently manage available spectrum resources in a decentralized manner, information exchange among users is necessary. Hence, we propose a novel priority virtual queue interface that determines the required information exchanges and evaluates the expected delays experienced by various priority traffics. Such expected delays are important for multimedia users due to their delay-sensitivity nature. Based on the exchanged information, the interface evaluates the expected delays using priority queuing analysis that considers the wireless environment, traffic characteristics, and the competing users' behaviors in the same frequency channel. We propose a Dynamic Strategy Learning (DSL) algorithm deployed at each user that exploits the expected delay and dynamically adapts the channel selection strategies to maximize the user's utility function. We simulate multiple video users sharing the cognitive radio network and show that our proposed solution significantly reduces the packet loss rate and outperforms the conventional single-channel dynamic resource allocation by almost 2dB in terms of video quality.

**Index Terms:** cognitive radio networks, resource management for heterogeneous users, delay-sensitive multimedia applications, queuing analysis.

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## I. INTRODUCTION

The demand for wireless spectrum has increased rapidly in recent years due to the emergence of a variety of applications, such as wireless Internet browsing, file downloading, streaming, etc. In the foreseeable future, the requirements for wireless spectrum will increase even more with the introduction of multimedia applications such as YouTube, peer to peer multimedia networks, and distributed gaming. However, scanning through the radio spectrum reveals its inefficient occupancy [2] in most frequency channels. Hence, the Federal Communications Commission (FCC) suggested in 2002 [1] improvements on spectrum usage to efficiently allocate frequency channels to license-exempt users without impacting the primary licensees. This forms cognitive radio networks that 1) enhance the spectrum usage of the traditional licensing system, and 2) release more spectrum resources for the unlicensed allocations in order to fulfill the required demand.

The emergence of cognitive radio networks have spurred both innovative research and ongoing standards [3][4][6][7]. Cognitive radio networks have the capability of achieving large spectrum efficiencies by enabling *interactive* wireless users to *sense* and *learn* the surrounding environment and correspondingly *adapt* their transmission strategies. Three main challenges arise in this context. The first problem is how to sense the spectrum and model the behavior of the primary licensees. The second problem is how to manage the available spectrum resources and share the resource to the license-exempt users to satisfy their transmission requirements while not interfering with the primary licensees. The third problem is how to maintain seamless communication during the transition (hand-off) of selected frequency channels. In this paper, we focus on the second challenge and rely on the existing literature for the remaining two challenges [23][26].

Prior research such as [3][6] focus on centralized solutions for the resource management problem in cognitive radio networks. However, due to the informationally-decentralized nature of wireless networks, the complexity of the optimal centralized solutions for spectrum allocation is prohibitive [8] for delay-sensitive multimedia applications. Moreover, the centralized solution requires the propagation of private information back and forth to a common coordinator, thereby incurring delay that may be unacceptable for delay-sensitive applications. Hence, it is important to implement decentralized solutions for dynamic channel selection by relying on the wireless multimedia users' capabilities to sense and adapt their frequency channel selections. Moreover, unlike

most of the existing research on resource management in the cognitive radio networks [10][22] that ignores the multimedia traffic characteristics in the application layer and assumes that all competing users in the networks are of the same type (applications, radio capabilities), we consider *heterogeneous* users in this paper, meaning that the users can have 1) different types of utility functions and delay deadlines, 2) different traffic priorities and rates, and 3) experience distinct channel conditions in different frequency channels. For example, the multimedia users can differ in their preferences of utility functions, priorities of accessing the frequency channels, traffic rate requirements, capabilities of transmitting data in different frequency channels. Note that in the informationally-decentralized wireless network, these utility functions, traffic characteristics, and the channel conditions are usually considered as private information of the users. Hence, the main challenge here is how to coordinate the spectrum sharing among heterogeneous multimedia users in a decentralized manner.

To do this, information exchange across the multimedia users is essential. Since the decisions of a user will impact and be impacted by the other users selecting the same frequency channel, without explicit information exchange, the heterogeneous users will consume additional resources and respond slower to the time-varying environment [27]. The key questions are what information exchanges are required, and how autonomous users adapt their channel selections based on the limited information exchange to efficiently maximize their private utilities. In this paper, we propose a novel priority virtual queue interface to abstract multimedia users' interactions and determine the required information exchange according to the priority queuing analysis. Note that such information exchanges can rely on a dedicated control channel for all users, or can use a group-based scheme without a common control channel [19].

In this paper, we model the traffic of the users (including the licensed users and the license-exempt users) and the channel conditions (e.g. Signal-to-Noise Ratio, Bit-Error-Rate) by stationary stochastic models similar to [22]. Our approach endows the primary licensees with the priority to preempt the transmissions of the license-exempt users in the same frequency channel. Based on the priority queuing analysis, each wireless user can evaluate its utility impact based on the behaviors of the users deploying the same frequency channel (including the primary licensees, to which the highest priority is assigned). The behavior of a user is represented by its probability profile for selecting different frequency channels, which is referred as the channel selection strategy in this paper. Based on the expected utility evaluation, we propose a Dynamic Strategy Learning (DSL)

algorithm for an autonomous multimedia user to adapt its channel selection strategy.

In summary, our paper addresses the following important issues:

**a) Separation of the utility evaluation and channel selection using the priority virtual queue interface.**

We propose a novel priority virtual queue interface for each autonomous user to exchange information and maximize its private utility in cognitive radio networks. Through the interface, the user can model the strategies of the other users with higher priorities and evaluates the expected utility of selecting a certain frequency channel. Importantly, the interface provides a simple model that facilitates the user's learning of what is the best channel selection strategy.

**b) Priority virtual queuing analysis for heterogeneous multimedia users.**

Unlike prior works on cognitive radio networking, which seldom consider multimedia traffic characteristics and delay deadlines in the application layer, our priority virtual queue framework enables the autonomous multimedia users to consider 1) priorities of accessing the frequency channels, 2) different traffic loads and channel conditions in different frequency channels, and 3) heterogeneous preferences for various types of utility functions based on the deployed applications. Note that the priority queuing model allows the primary licensees to actively share the occupied channels instead of excluding all the other wireless users. However, by assigning highest preemptive priorities to the licensees, the unlicensed users do not impact the licensees.

**c) DSL algorithm for dynamic channel selections by wireless stations.**

Based on the expected utility evaluation from the interface, we propose a decentralized learning algorithm that dynamically adapts the channel selection strategies to maximize the private utility functions of users. Note that a frequency channel can be shared by several users. A wireless user can also select multiple frequency channels for transmission. Our learning algorithm addresses how multimedia users distribute traffic to multiple available frequency channels to maximize their own utility functions.

The rest of this paper is organized as follows. Section II provides the specification of cognitive radio networks and models the dynamic resource management problem as a multi-agent interaction problem. In Section III, we give an overview of our dynamic resource management for the heterogeneous multimedia users, including the priority virtual queue interface and the dynamic channel selection. In Section IV, we provide the queuing analysis for the priority virtual queue interface and determine the required information exchange. In Section V,

we focus on the dynamic channel selection and propose the DSL algorithm to adapt the channel selection strategy for the multimedia users. Simulation results are given in Section VI. Section VII concludes the paper.

## II. MODELING THE COGNITIVE RADIO NETWORKS AS MULTI-AGENT INTERACTIONS

### A. Agents in a cognitive radio network

In this paper, we assume that the following agents interact in the cognitive radio network:

- **Primary Users** are the incumbent devices possessing transmission licenses for specific frequency bands (channels). We assume that there are  $M$  channels in the cognitive radio network, and that there are several primary users in each frequency channel. These primary users can only occupy their assigned frequency channels. Since the primary users are licensed users, they will be provided with an interference-free environment [4][23].
- **Secondary Users** are the autonomous wireless stations that perform channel sensing and share the available spectrum holes [3]. We assume that there are  $N$  secondary users in the system. These secondary users are able to transmit their traffic using various frequency channels. If multiple users select the same frequency channel, they will time share the chosen frequency channel. Moreover, these secondary users are license-exempt, and hence, they cannot interfere with the primary users.

In this paper, we consider the users sharing a single-hop wireless ad-hoc network. Figure 1 provides an illustration of the considered network model. We assume the secondary users as transmitter-receiver pairs with information exchange among these pairs. In order to maintain stationary property, we assume that these network agents are static (i.e. we do not consider mobility effects). Next, we model the interaction among secondary users accessing the same frequency channel.

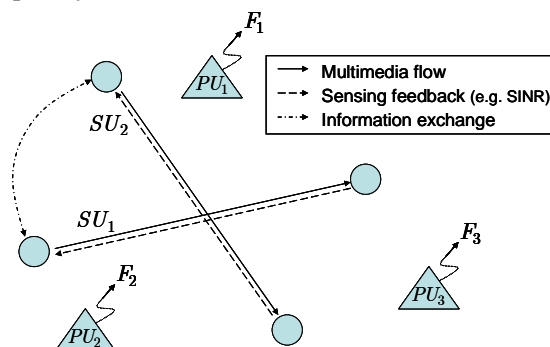


Fig. 1 An illustration of the considered network model.

### B. Modeling of the dynamic resource management problem as a multi-agent interaction

- **Users:** As indicated above, there are two sets of users – aggregate primary users in each channel  $\text{PU} = \{\text{PU}_1, \dots, \text{PU}_M\}$ <sup>1</sup> and the secondary users  $\text{SU} = \{\text{SU}_1, \dots, \text{SU}_N\}$ . The priorities of users in cognitive radio networks are pre-assigned depending on their Quality of Service (QoS) requirements and their right to access the frequency channels.
- **Resources:** The resources are the frequency channels  $\mathbf{F} = \{F_1, \dots, F_M\}$ . Multiple users can time share the same frequency channel. Note that even if the same time sharing fraction is assigned to the users choosing the same frequency channel, their experienced channel conditions may differ.
- **Actions:** The considered actions of the secondary users are the selection of the frequency channel for each packet transmission. We denote the actions of a secondary user  $\text{SU}_i$  using  $\mathbf{a}_i = [a_{i1}, a_{i2}, \dots, a_{iM}] \in \mathcal{A}^M$ , where  $a_{ij} \in \mathcal{A}$  ( $\mathcal{A} = \{0,1\}$ ).  $a_{ij} = 1$  indicates that  $\text{SU}_i$  chooses the frequency channel  $F_j$ . Otherwise,  $a_{ij} = 0$ . Let  $\mathbf{a}_{-i}$  denote the actions of the other secondary users except  $\text{SU}_i$ . Let  $\mathbf{A} = [\mathbf{a}_1^T, \dots, \mathbf{a}_N^T] \in \mathcal{A}^{M \times N}$  denote the total action profile across all secondary users.
- **Strategies:** A strategy of a secondary user  $\text{SU}_i$  is a vector of probabilities  $\mathbf{s}_i = [s_{i1}, s_{i2}, \dots, s_{iM}] \in \mathcal{S}^M$ , where  $s_{ij} \in \mathcal{S}$  ( $\mathcal{S} \in [0,1]$ ) represents the probability of the secondary user  $\text{SU}_i$  to take the action  $a_{ij}$  (i.e. to choose the frequency channel  $F_j$ ). Hence, the summation over all the frequency channels is  $\sum_{j=1}^M s_{ij} = 1$ . Note that  $s_{ij}$  can also be viewed as the fraction of data from  $\text{SU}_i$  transmitted on frequency channel  $F_j$ , and hence, multiple frequency channels are selected for a secondary users with  $s_{ij} > 0$ . Let  $\mathbf{S} = [\mathbf{s}_1^T, \dots, \mathbf{s}_N^T] \in \mathcal{S}^{M \times N}$  denote the total strategy profile across all secondary users.
- **Utility functions:** Each secondary user has its own utility function. Based on the adopted actions of the secondary users, we denote the utility function of  $\text{SU}_i$  as  $u_i$ . Conventionally, the utility function of a specific user is often modeled solely based on its own action, i.e.  $u_i(\mathbf{a}_i)$  without modeling the other

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<sup>1</sup> From the secondary users' point of view, there is no need to differentiate different primary users in one frequency channel. Hence, we reduce the primary users in one frequency channel into one aggregate primary user. A secondary user needs to back-off and wait for transmission or select another frequency channel, once any of the primary users starts to transmit in the same frequency channel.

secondary users [8][28]. However, the utility function for multimedia users relates to the effective delay and throughput that a secondary user can derive from the selected frequency channel, which is coupled with the actions of other secondary users. Hence, the utility function  $u_i$  is also influenced by the action of other secondary users that select the same frequency channel. In other words, the utility function can be regarded as  $u_i(\mathbf{a}_i, \mathbf{a}_{-i})$ . We will discuss this utility function in detail in Section III.C.

- **Expected utility function with dynamic adaptation:** In an informationally-decentralized cognitive wireless network that consists of heterogeneous secondary users, the secondary user  $SU_i$  may not know the exact actions of other secondary users  $\mathbf{a}_{-i}$ . Moreover, even if all the actions are known, it is unrealistic to assume that the exact action information can be collected timely to compute and maximize the actual utility function  $u_i(\mathbf{a}_i, \mathbf{a}_{-i})$ . Hence, a more practical solution is to dynamically model the other secondary users' behavior by updating their probabilistic strategy profile of actions  $\mathbf{s}_{-i}$  based on the observed information, and then compute the optimal channel selection strategy  $\mathbf{s}_i$  that maximizes the expected utility function of  $SU_i$ , i.e.

$$U_i(\mathbf{s}_i, \mathbf{s}_{-i}) = E_{(\mathbf{s}_i, \mathbf{s}_{-i})}[u_i(\mathbf{a}_i, \mathbf{a}_{-i})], \quad (1)$$

where  $E_{(\mathbf{s}_i, \mathbf{s}_{-i})}[u_i(\mathbf{a}_i, \mathbf{a}_{-i})]$  is the expected utility function, given a fixed strategy profile  $\mathbf{S} = (\mathbf{s}_i, \mathbf{s}_{-i})$ . In the next section, we discuss how secondary users perform dynamic resource management that maximizes the expected utility function  $U_i(\mathbf{s}_i, \mathbf{s}_{-i})$  by modeling the strategy (behavior)  $\mathbf{s}_{-i}$  of the other users in cognitive radio networks.

### III. DYNAMIC RESOURCE MANAGEMENT FOR HETEROGENEOUS SECONDARY USERS USING PRIORITY QUEUING

In this section, we provide our dynamic resource management solution using the multi-agent interaction settings in the previous section. We first emphasize the heterogeneity of the secondary users in cognitive radio networks and then introduce our solution with the priority queuing interface and adaptive channel selection strategies.

### A. Prioritization of the users

We assume that there are  $K$  priority classes of users in the system. The highest priority class  $C_1$  is always reserved for the primary users **PU** in each frequency channel. The heterogeneous secondary users **SU** can be categorized into the rest of  $K - 1$  priority classes ( $C_2, \dots, C_K$ ) to access the frequency channels<sup>2</sup>. We assume that the users in higher priority classes can preempt the transmission of the lower priority classes to ensure an interference-free environment for the primary users [14]. The priority of a user affects its ability of accessing the channel. Primary users in the highest priority class  $C_1$  can always access their corresponding channels at any time. Secondary users, on the other hand, need to sense the channel and wait for transmission opportunities for transmission (when there is no higher priority users using the channel) based on their priorities. We assume that there are  $N_k$  users in each of the class  $C_k$ . Hence,  $N_1 = M$  (number of aggregate primary users) and  $\sum_{k=2}^K N_k = N$  (number of secondary users).

Various multiple access control schemes can be adopted for the secondary users to share the spectrum resource. For simplicity, in this paper, we consider a MAC protocol similar to IEEE 802.11e HCF [12]<sup>3</sup> to assign transmission opportunities (i.e. TXOP) and ensure that a secondary users in the lower priority class will stop accessing the channel and wait in the queue or change its action (channel selection) if a higher priority user is using the frequency channel. Note that for secondary users, they not only can have different priorities to access the frequency channels, but they can also have different channel conditions and possess their own preferences for a certain type of utility function, which is discussed in the following subsections.

### B. Channel conditions of the heterogeneous secondary users

For a certain frequency channel  $F_j$ , the secondary users can experience various channel conditions for the same frequency channel. We denote  $T_{ij}$  and  $p_{ij}$  as the resulting physical transmission rate and packet error rate for the secondary user  $SU_i$  transmitting through a certain frequency channel  $F_j$ . Let  $R_{ij} = [T_{ij}, p_{ij}] \in \mathcal{R}$

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<sup>2</sup> The prioritization of the secondary users can be determined based on their applications, prices paid for spectrum access, or other mechanism design based rules. In this paper, we will assume that the prioritization was already performed.

<sup>3</sup> Either the polling-based HCCA or contention-based EDCA protocols can be applied, as long as the priority property of the users is provided. However, a more sophisticated MAC protocols can also be considered to deal with the spectrum heterogeneity (such as HD-MAC in [19]). Different MAC protocols will have different overheads including the time of waiting for the MAC acknowledgement, contention period, etc. that affect the service time distribution of the M/G/1 queuing model.



be the channel conditions of the channel  $F_j$  for the secondary user  $SU_i$ . We denote the channel condition matrix as  $\mathbf{R} = [R_{ij}] \in \mathcal{R}^{M \times N}$ . The expected physical transmission rate and packet error rate can be approximated as sigmoid functions of measured Signal-to-Interference-Noise-Ratio (SINR) and the adopted modulation and coding scheme as in [17]. Note that the expected  $T_{ij}$  and  $p_{ij}$  of the same frequency channel can be different for various secondary users.

### C. Goals of the heterogeneous secondary users

In general, the utility function  $u_i$  is a non-decreasing function of the available transmission rates. Several types of objectives for the secondary users can be considered in practice, such as minimizing the end-to-end delay, loss probability, or maximizing the received quality, etc. For simplicity, we assume only two types of utility functions<sup>4</sup> in this paper.

- **The delay-based utility** for *delay-sensitive* multimedia applications.

Let  $D_i(\mathbf{a}_i, \mathbf{a}_{-i})$  represent the end-to-end packet delay (transmission delay plus the queuing delay) for the secondary user  $SU_i$ . Let  $d_i$  represent the delay deadline of the application of secondary user  $SU_i$ . We consider this type of utility function as (as in [20]):

$$u_i^{(1)}(\mathbf{a}_i, \mathbf{a}_{-i}) = \text{Prob}(D_i(\mathbf{a}_i, \mathbf{a}_{-i}) \leq d_i), \quad (2)$$

which depends on the end-to-end delay  $D_i(\mathbf{a}_i, \mathbf{a}_{-i})$  and the delay deadline  $d_i$  imposed by the application.

- **The throughput-based utility** for *delay-insensitive* applications.

Let  $T_i^{eff}$  represent the effective available throughput for the secondary user  $SU_i$ . The second type of utility function is assumed to be directly related to the throughput (as in [18]). In this paper, we define it as:

$$u_i^{(2)}(\mathbf{a}_i, \mathbf{a}_{-i}) = \begin{cases} \frac{T_i^{eff}(\mathbf{a}_i, \mathbf{a}_{-i})}{T_i^{\max}}, & \text{if } T_i^{eff}(\mathbf{a}_i, \mathbf{a}_{-i}) \leq T_i^{\max} \\ 1 & , \text{ if } T_i^{eff}(\mathbf{a}_i, \mathbf{a}_{-i}) > T_i^{\max} \end{cases}, \quad (3)$$

where  $T_i^{\max}$  is the physical throughput required by the secondary user  $SU_i$ .

We assume that a secondary user can possess multiple applications that can be either delay-sensitive

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<sup>4</sup> This model can be easily extended to more types of utility functions. Moreover, our utility function can also be easily modified to a quality-type utility function using different priorities. For simplicity, we do not consider the quality impact of different multimedia packets in our utility function.

multimedia traffic or delay-insensitive data traffic. Hence, we define the utility function of a secondary user as a multi-criterion objective function (as in [6][21]) of these two types of utility functions. Different secondary users can have different preferences  $\theta_i^5$  ( $0 \leq \theta_i \leq 1$ ). Specifically, the goal of a secondary user  $SU_i$  is to maximize the following utility function:

$$u_i(\mathbf{a}_i, \mathbf{a}_{-i}) = \theta_i \cdot u_i^{(1)}(\mathbf{a}_i, \mathbf{a}_{-i}) + (1 - \theta_i) \cdot u_i^{(2)}(\mathbf{a}_i, \mathbf{a}_{-i}). \quad (4)$$

Note that, in this setting,  $0 \leq u_i(\mathbf{a}_i, \mathbf{a}_{-i}) \leq 1$ .

#### D. Example of three priority classes with different utility functions

Let  $\mathbf{A}_k$  be the action set of the secondary users in the classes  $C_2, \dots, C_k$ , i.e.  $\mathbf{A}_k = \{\mathbf{a}_i \mid SU_i \in C_l, l = 2, \dots, k\}$ . Note that  $\mathbf{A}_{k-1} \subseteq \mathbf{A}_k \subseteq \mathbf{A}$ . Due to the priority queuing structure, the actions of the secondary users with lower priority will not affect the users in the higher priority class [11]. Hence, the decentralized optimizations are performed starting from the higher priority classes to the lower priority classes. In other words, the decentralized optimization of a secondary user in a lower priority class also needs to consider the actions of the users in higher priority classes. For example, three classes can be assumed ( $K = 3$ ) – the first priority class is composed by the primary users whose actions are fixed (no channel selection capability). The second priority class  $C_2$  is composed by the secondary users transmitting delay-sensitive multimedia applications, and the third priority class  $C_3$  is composed by the secondary users transmitting regular data traffic, which requires throughput maximization. The objective function for each of the secondary users in priority class  $C_2$  is ( $\theta_i = 1$ , for  $SU_i \in C_2$ ):

$$\begin{aligned} & \text{maximize } U_i^{(1)}(\mathbf{s}_i, \mathbf{s}_{-i}) \\ \Rightarrow & \text{maximize } E_{(\mathbf{s}_i, \mathbf{s}_{-i})}[\text{Prob}(D_i(\mathbf{A}_2) \leq d_i)] \end{aligned} \quad (5)$$

Then, the objective function for the secondary users in the class  $C_3$  is ( $\theta_i = 0$ , for  $SU_i \in C_3$ ):

$$\begin{aligned} & \text{maximize } U_i^{(2)}(\mathbf{s}_i, \mathbf{s}_{-i}) \\ \Rightarrow & \text{maximize } E_{(\mathbf{s}_i, \mathbf{s}_{-i})}[T_i^{eff}(\mathbf{A})] \end{aligned} \quad (6)$$

with the constraint that  $\mathbf{A}_2 \subseteq \mathbf{A}$  are predetermined by (5). The effective transmission rate of each secondary

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<sup>5</sup> In this paper, we assume that the preferences  $\theta_i$  are predetermined by the secondary users. The preferences  $\theta_i$  of the multi-criterion optimization can be determined based on the applications. See e.g. [15].

user can be expressed as:

$$E_{(s_i, s_{-i})}[T_i^{eff}(\mathbf{A})] = \sum_{j=1}^M s_{ij} T_{ij} (1 - p_{ij}). \quad (7)$$

From the above three classes example, note that delay analysis is essential for the heterogeneous secondary users with delay-sensitive applications in a cognitive radio network.

To maximize the expected utility function as stated in equation (1), a secondary user needs to consider the impact of the other secondary users. In order to efficiently regulate the information exchange among heterogeneous users and efficiently provide expected utility evaluation, a coordination interface must be developed. Based on this interface, the secondary users can interact with each other in a decentralized manner. In the next subsection, we propose a novel dynamic resource management with such an interface for a secondary user  $SU_i$  to adapt its frequency selection strategy  $s_i$ .

#### *E. Dynamic resource management with priority virtual queue interface*

The resource management for delay-sensitive multimedia applications over cognitive radio networks needs to consider the heterogeneous wireless users having various utility functions, priorities of accessing the channel, traffic rates, and channel conditions. Specifically, the main challenge is how to coordinate the spectrum sharing among competing users and select the frequency channel to maximize the utility functions in a decentralized manner. For this, we propose a novel priority virtual queue interface. Unlike prior research assuming that secondary users apply 2-state ‘‘spectrum holes’’ (on-off model [22]) for spectrum access [4] in our priority virtual queue interface, we allow secondary users to obtain transmission opportunities once the primary user in a specific channel stops transmitting. The primary users have the highest priority, thereby being able to preempt the transmission of the secondary users’ transmission.

The priority virtual queue interface has two main tasks – 1) determines the required information exchange and 2) evaluates the utility impact from the wireless environment as well as the competing users’ behaviors in the same frequency channel. In the priority virtual queue interface of a user, the virtual queues are preemptive priority queues [14] for each of the frequency channels. They are emulated by each multimedia user to estimate the delay of selecting a specific frequency channel for transmission. Figure 2 illustrates the architecture of the proposed dynamic resource management with priority virtual queue interface that exchanges information and

emulates the expected delay. Note that these virtual queues are in fact distributed (physically located) at the secondary users.

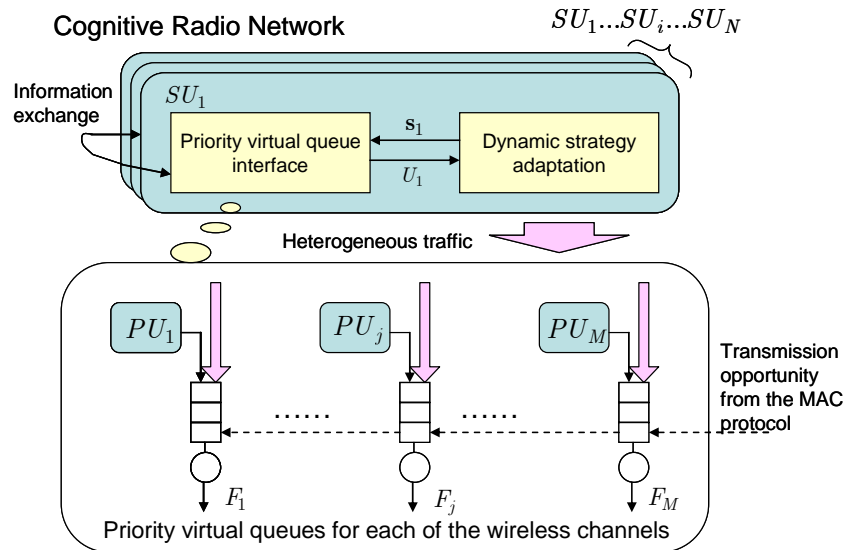


Fig. 2 The architecture of the proposed dynamic resource management with priority virtual queue interface.

The implementation of the dynamic resource management with priority virtual queue interface of the secondary users is presented below:

1. **Information exchange collection:** The secondary user  $SU_i$  collect the required information from other secondary users through the priority virtual queue interface. The required information exchange will be discussed in Section IV.D based on the queuing analysis.
2. **Priority queuing analysis:** The interface estimates  $s_{-i}$  and performs priority queuing analysis based on the observed information to evaluate the expected utility  $U_i(s_i, s_{-i})$ . The priority queuing analysis will be discussed in details in Section IV.
3. **Dynamic strategy adaptation:** Based on the expected utility  $U_i(s_i, s_{-i})$ , the secondary user adapts its channel selection strategy  $s_i$ . We propose a dynamic strategy learning algorithm, which will be discussed in detail in Section V.
4. **Assign actions for each packet based on the strategy:** Based on current channel selection strategy  $s_i$ ,  $SU_i$  can assign to each packet an action (select frequency channel according to the probability profile). As the channel selection strategy adapts to the network changes, the behavior of a secondary user selecting the frequency channels for its packets will also change.
5. **Wait for the transmission opportunity and transmit the packets:** The packets wait in queues to be

transmitted. Based on the priorities of the users, the higher priority secondary users will have a better chance to access the channel and transmit their packets.

Note that the primary users will transmit whenever needed in their corresponding frequency channels.

Next, we present the priority queuing analysis for delay-sensitive multimedia users to evaluate  $U_i(\mathbf{s}_i, \mathbf{s}_{-i})$ .

#### IV. PRIORITY QUEUING ANALYSIS FOR DELAY-SENSITIVE MULTIMEDIA USERS

In this section, we discuss the priority queuing analysis for delay-sensitive multimedia applications. It is important to note that the packets of the competing wireless users are physically waiting at different locations. Figure 3 gives an example of the physical queues for the case of  $M$  frequency channels and  $N$  secondary users. Each secondary user maintains  $M$  physical queues for the various frequency channels, which allows users to avoid the well-known head-of-line blocking effect [24]. The channel selection decisions are based on the queuing analysis, which will be discussed in detail in Section V. In this section, we focus on the priority queuing analysis from the perspective of each secondary user to evaluate  $U_i(\mathbf{s}_i, \mathbf{s}_{-i})$ .

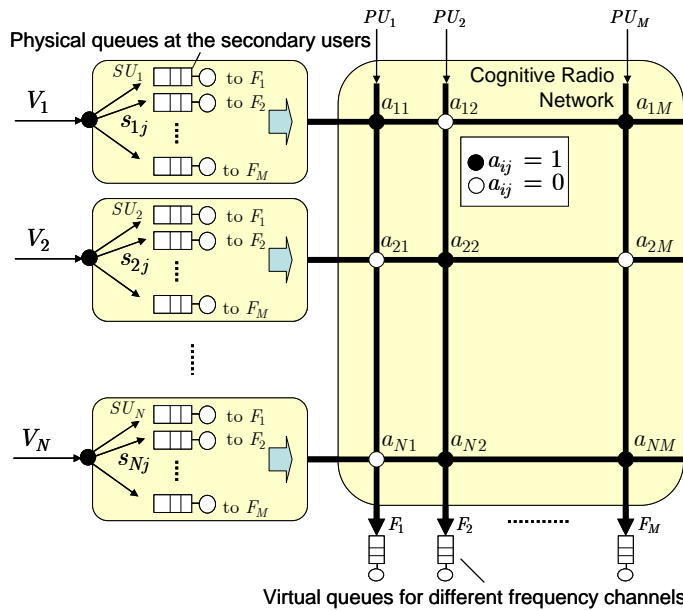


Fig. 3 Actions of the secondary users  $a_{ij}$  and their physical queues for each frequency channel

##### A. Traffic models of the users

###### • Traffic model for primary users

We assume that the stationary statistics of the traffic patterns of primary users can be modeled by all secondary users. The packet arrival process of a primary user is modeled as a Poisson process with average

packet arrival rate  $\lambda_j^{PU}$  for the primary user  $PU_j$  using the frequency channel  $F_j$ . Note that the aggregation of Poisson processes of primary users in the same frequency channel is still Poisson. We denote the  $m^{\text{th}}$  moments of the service time distribution of the primary user  $PU_j$  in frequency channel  $F_j$  as  $E[(X_j^{PU})^m]$ . We adopt an M/G/1 model for the traffic descriptions. Note that this traffic model description is more general than a Markov on-off model [22], which is a sub-set of our queuing model with an exponential idle period and an exponential busy period.

- **Traffic model for secondary users**

We assume that the average rate requirement for the secondary user  $SU_i$  is  $B_i$  (bit/s). Let  $\lambda_{ij}$  denote the average packet arrival rate of the secondary user  $SU_i$  using the frequency channel  $F_j$ . Since the strategy  $s_{ij}$  represents the probability of the secondary user  $SU_i$  taking action  $a_{ij}$  (transmitting using the frequency channel  $F_j$ ), we have

$$\lambda_{ij} = s_{ij} \frac{B_i}{L_i}, \quad (8)$$

where  $L_i$  denotes the average packet length of the secondary user  $SU_i$ . If a certain secondary user  $SU_i$  can never use the frequency channel  $F_j$ , we fix its strategy to  $s_{ij} = 0$ , and hence,  $\lambda_{ij} = 0$ . For simplicity, we also model the packet arrival process of the secondary users using a Poisson process. Note that the average arrival rate is the only sufficient statistics required to describe a Poisson process.

Since packet errors are unavoidable in a wireless channel, we assume that packets will be retransmitted, if they are not correctly received. This can be regarded as a protection scheme similar to the Automatic Repeat Request protocol in IEEE 802.11 networks [12]. Hence, the service time of the users can be modeled as a geometric distribution [13]. Let  $E[X_{ij}]$  and  $E[X_{ij}^2]$  denote the first two moments of the service time of the secondary user  $SU_i$  using the frequency channel  $F_j$ . We have:

$$E[X_{ij}] = \frac{L_i + L_o}{T_{ij}(1 - p_{ij})}, \quad (9)$$

$$E[X_{ij}^2] = \frac{(L_i + L_o)^2(1 + p_{ij})}{T_{ij}^2(1 - p_{ij})^2}, \quad (10)$$

where  $L_i$  is the average packet length of the secondary user  $SU_i$  and  $L_o$  represents the effective control overhead including the time for protocol acknowledgement<sup>6</sup>, information exchange, and channel sensing delay, etc. (see [12] for details). Let us denote  $\mathbf{X}_i = [E[X_{ij}] | j = 1, \dots, M]$  and  $\mathbf{X}_i^2 = [E[X_{ij}^2] | j = 1, \dots, M]$ . To describe the traffic model, we define the traffic specification<sup>7</sup> for the secondary user  $SU_i$  as  $\mathbf{TS}_i = [C_k, B_i, L_i, \mathbf{X}_i, \mathbf{X}_i^2]$ , if  $SU_i \in C_k$ . This information needs to be exchanged among the secondary users, which will be discussed in detail in Section IV.D.

### B. Priority virtual queuing analysis

In order to evaluate the expected utility  $U_i(s_i, s_{-i})$  for delay-sensitive multimedia applications, we need to calculate the distribution of the end-to-end delay  $D_i(\mathbf{a}_i, \mathbf{a}_{-i})$  for the secondary user  $SU_i$  to transmit its packets. The expected end-to-end delay<sup>8</sup>  $E[D_i]$  of the secondary user  $SU_i$  can be expressed as:

$$E[D_i(\mathbf{a}_i, \mathbf{a}_{-i})] = \sum_{j=1}^M s_{ij} \cdot E[D_{ij}(R_{ij}(\mathbf{A}))], \quad (11)$$

where  $E[D_{ij}(R_{ij}(\mathbf{A}))]$  is the average end-to-end delay if the secondary user  $SU_i$  chooses the frequency channel  $F_j$ . Note that  $s_{ij}$  is the strategy of the action  $a_{ij}$  in  $\mathbf{A}$ .

Using the queuing model in Figure 3, each arriving packet of  $SU_i$  will select a physical queue to join (action  $a_{ij}$ ) according to the strategy  $s_{ij}$ . Note that there are  $N$  physical queues from  $N$  secondary users for a frequency channel  $F_j$ . Only one of them can transmit its packets at any time. Hence, we form a “virtual queue” for the same frequency channel as illustrated in Figure 3. In a virtual queue, the packets of the different secondary users wait to be transmitted. Importantly, the total sojourn time (queue waiting time plus the transmission service time) of this virtual queue now becomes the actual service time at each of the physical queues. The concept is similar to the “service on vacation” [11] in queuing theory, and the waiting time of the virtual queue can be regarded as the “vacation time”.

<sup>6</sup> Here we only consider retransmission due to channel errors. We consider the protocol overhead in the MAC layer including possible contention period, time for acknowledgement, etc. in the effective control overhead.

<sup>7</sup> The traffic specification is similar to the TSPEC in current IEEE 802.11e [12] for multimedia transmission.

<sup>8</sup> In order to simplify the notation, we use simple expectation notation  $E[\cdot]$  instead of the expectation over the action strategies  $E_{(s_i, s_{-i})}[\cdot]$  hereafter in this paper.

Since the number of the secondary users in a regular cognitive radio network is usually large, we can approximate the virtual queue using prioritized M/G/1 queuing model (i.e. when  $N \rightarrow \infty$ , the input traffic of the virtual queue can be modeled as a Poisson process). The average arrival rate of the virtual queue of the frequency channel  $F_j$  is  $\sum_{i=1}^N \lambda_{ij}$ . Let us denote the first two moments of the service time for the virtual queue of the frequency channel  $F_j$  as  $E[\tilde{X}_j]$  and  $E[\tilde{X}_j^2]$ . For a packet in the virtual queue of frequency channel  $F_j$ , we determine the probability of the packet coming from the secondary user  $SU_i$  as:

$$f_{ij} = \frac{\lambda_{ij}}{\sum_{k=1}^N \lambda_{kj}}. \quad (12)$$

Hence,

$$E[\tilde{X}_j] = \sum_{i=1}^N f_{ij} \times E[X_{ij}], \quad E[\tilde{X}_j^2] = \sum_{i=1}^N f_{ij} \times E[X_{ij}^2]. \quad (13)$$

Since there are  $K$  priority classes among users ( $K > 2$ ,  $\mathbf{PU} \in C_1$ ,  $\mathbf{SU} \in \{C_2, \dots, C_K\}$ ), we assume that  $\mu_{jk}$  represents the normalized traffic loading of all the class  $C_k$  secondary users using the frequency channel  $F_j$ . By the definition of the normalized traffic loading [11], we have:

$$\mu_{jk} = \sum_{\forall SU_i \in C_k} \lambda_{ij} \times E[\tilde{X}_j], \text{ and } \mu_{jk}^2 = \sum_{\forall SU_i \in C_k} \lambda_{ij} \times E[\tilde{X}_j^2]. \quad (14)$$

Assume that  $E[\tilde{D}_{jk}]$  and  $E[\tilde{W}_{jk}]$  represent the average virtual queuing delay and average virtual queue waiting time experienced by the secondary users in class  $C_k$  in the virtual queue of the frequency channel  $F_j$ . By applying the Mean Value Analysis (MVA) [14], we have:

$$E[\tilde{D}_{jk}] = E[\tilde{W}_{jk}] + E[\tilde{X}_j] = \frac{\rho_j^2 + \sum_{l=2}^k \mu_{jl}^2}{2(1 - \rho_j - \sum_{l=2}^{k-1} \mu_{jl})(1 - \rho_j - \sum_{l=2}^k \mu_{jl})} + E[\tilde{X}_j], \quad (15)$$

where  $\rho_j$  represents the normalized loading of the primary user  $PU_j$  for the frequency channel  $F_j$ , and

$$\rho_j = \lambda_j^{PU} E[X_j^{PU}], \quad \rho_j^2 = \lambda_j^{PU} E[(X_j^{PU})^2]. \quad (16)$$

Recall that the average input rate of the primary user  $PU_j$  is  $\lambda_j^{PU}$ , and the first two moments of the service time is  $E[X_j^{PU}]$  and  $E[(X_j^{PU})^2]$ .



Since the average virtual queuing delay  $E[\tilde{D}_{jk}]$  is the average service time of the physical queue, the average end-to-end delay of the secondary user  $SU_i$  sending packets through frequency channel  $F_j$  is approximately:

$$E[D_{ij}] = \frac{E[\tilde{D}_{jk}]}{1 - \lambda_{ij}E[\tilde{D}_{jk}]}, \text{ for } \lambda_{ij}E[\tilde{D}_{jk}] < 1, SU_i \in C_k. \quad (17)$$

Strategies  $(s_i, s_{-i})$  such that  $\lambda_{ij}E[\tilde{D}_{jk}] \geq 1$  will result in an unbounded delay  $E[D_{ij}]$ , which is undesirable for delay-sensitive applications. The advantage of this approximation is that once the average delay of the virtual queue  $E[\tilde{D}_{jk}]$  is known by the secondary user  $SU_i$ , the secondary user can immediately calculate the expected end-to-end delay  $E[D_{ij}]$  of a packet transmitting using the frequency channel  $F_j$ . Note that in equation (17), we assume that once a packet selects a physical queue, it cannot switch to another queue (change position to the other queues). However, by considering current physical queue size  $q_{ij}$  for user  $SU_i$  using the frequency channel  $F_j$ , a packet can change its channel selection after it is put in the physical queue. The switching probability from a longer queue  $q_{ia}$  to a shorter queue  $q_{ib}$  in a time interval  $t$  can be defined as  $1 - \exp(-t \times (q_{ia} - q_{ib}))$ . To evaluate such expected end-to-end delay  $E[D_{ij}]$ , a more sophisticated queuing model with jockey impatient customers [30] needs to be considered.

Let  $P_{ij}(s_i, s_{-i})$  represent the probability of packet loss for the secondary user  $SU_i$  sending packets through frequency channel  $F_j$ . By applying G/G/1 approximation based on the work of [16], we have:

$$P_{ij}(s_i, s_{-i}) = \begin{cases} \lambda_{ij}E[\tilde{D}_{jk}] \exp\left(-\frac{\lambda_{ij}E[\tilde{D}_{jk}] \times d_i}{E[D_{ij}]}\right), & \text{for } \lambda_{ij}E[\tilde{D}_{jk}] < 1, SU_i \in C_k \\ 1, & \text{for } \lambda_{ij}E[\tilde{D}_{jk}] \geq 1 \end{cases}. \quad (18)$$

For a delay-sensitive secondary user  $SU_i$ , the objective function in (5) becomes:

$$\begin{aligned} & \underset{s_i}{\text{maximize}} \quad U_i^{(1)}(s_i, s_{-i}) \\ \Rightarrow & \underset{s_i}{\text{maximize}} \quad \sum_{j=1}^M s_{ij}(1 - P_{ij}(s_i, s_{-i})) \\ \Rightarrow & \underset{s_i}{\text{minimize}} \quad \sum_{j=1}^M s_{ij} \lambda_{ij} E[\tilde{D}_{jk}] \exp\left(-\frac{\lambda_{ij}E[\tilde{D}_{jk}] \times d_i}{E[D_{ij}]}\right), \text{ for } SU_i \in C_k \end{aligned} \quad (19)$$

### C. The overhead of required information exchange and the aggregate virtual queue effects

In the previous subsection, we calculate  $P_{ij}(s_i, s_{-i})$ , the packet loss probability for a packet of the secondary user  $SU_i$  transmitting using the frequency channel  $F_j$ . In a general case, we can calculate the expected utility function of equation (4) as:

$$\begin{aligned} E[u_i(\mathbf{a}_i, \mathbf{a}_{-i})] &= \theta_i \cdot U_i^{(1)} + (1 - \theta_i) \cdot U_i^{(2)} \\ &= \theta_i \sum_{j=1}^M s_{ij} \cdot (1 - P_{ij}(s_i, s_{-i})) + (1 - \theta_i) \sum_{j=1}^M s_{ij} T_{ij} (1 - p_{ij}) / T_i^{\max}, \\ &= \sum_{j=1}^M s_{ij} \cdot E[V_{ij}(\mathbf{a}_i, \mathbf{a}_{-i})] \end{aligned} \quad (20)$$

where  $E[V_{ij}(\mathbf{a}_i, \mathbf{a}_{-i})] = \theta_i(1 - P_{ij}(s_i, s_{-i})) + (1 - \theta_i)T_{ij}(1 - p_{ij})/T_i^{\max}$ .  $E[V_{ij}(\mathbf{a}_i, \mathbf{a}_{-i})]$  represents the aggregate virtual queue effect for the secondary user  $SU_i$  of class  $C_k$  transmitting using the frequency channel  $F_j$ .

Note that  $E[V_{ij}(\mathbf{a}_i, \mathbf{a}_{-i})] \leq 1$ .

The aggregate virtual queue effect  $E[V_{ij}(\mathbf{a}_i, \mathbf{a}_{-i})]$  can be regarded as a metric of the dynamic wireless environment and the competing wireless users' behaviors [4][5], which reflects the impact of the time-varying environment and the impact of the other users (including the primary user and the other secondary users) on the secondary user  $SU_i$  in the specific frequency channels  $F_j$ . To evaluate  $E[V_{ij}(\mathbf{a}_i, \mathbf{a}_{-i})]$ , modeling other secondary users is necessary<sup>9</sup>. Our priority virtual queue interface requires the following information to compute  $\mu_{jl}$  and  $\mu_{jl}^2$  in (15):

1. Priority: the secondary users' priorities.
2. Normalized loading: the secondary users' normalized loading parameters  $\lambda_{ij} \times E[\tilde{X}_j]$ , which not only include the information of  $s_i$ , but also reflects the input traffic loading and the expected transmission time using a specific frequency channel.
3. Variance statistics: the secondary users' variance statistics with the normalized parameter  $\lambda_{ij} \times E[\tilde{X}_j^2]$ .

To determine the above information, two kinds of information need to be exchanged:

- Information exchange of other secondary users' traffic specification  $TS_{-i}$  (see Section IV.A).

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<sup>9</sup> Although we apply M/G/1 priority queuing analysis, more sophisticated queuing models can be applied for evaluating the aggregate virtual queue effects, if using different traffic model description.

- Information exchange of the action of the other secondary users  $\mathbf{a}_{-i}$  (to model the strategies  $\mathbf{s}_{-i}$ ).

Since the traffic specification  $\mathbf{TS}_i$  only varies when the frequency channels change dramatically (we do not consider mobility effects and this information exchange is assumed to be truthfully revealed), the traffic specification can be exchanged only when a secondary user joins the network to reduce the overhead. On the other hand, the action information can be observed (sensed) more frequently (once per packet/service interval [12]). Note that since the users in the higher priority classes will not be affected by the users in the lower priority classes, they do not need the information from the users in a lower priority class. Hence, higher priority secondary users will have small information exchange overhead and computational complexity. In conclusion, the information overheads for higher importance secondary users are limited.

Based on the action information observation, the interface updates the strategies  $(s_i, \mathbf{s}_{-i})$  and compute all the required information to evaluate the aggregate virtual queue effect  $E[V_{ij}(\mathbf{a}_i, \mathbf{a}_{-i})]$ . Next, we discuss how to make use of  $E[V_{ij}(\mathbf{a}_i, \mathbf{a}_{-i})]$  to determine the frequency channel selection.

## V. DYNAMIC CHANNEL SELECTION WITH STRATEGY LEARNING

From Section III, we know that the goals of the secondary users are to maximize their utility functions. We define the *best response strategy* for the decentralized optimization by considering the strategy that yields the highest utility  $U_i$  of the secondary user  $SU_i$ . To simplify the description, we now consider all the secondary users in one class<sup>10</sup>. The decentralized optimization is:

$$\mathbf{s}_i^* = \arg \max_{\mathbf{s}_i \in \mathcal{S}^M} E_{(\mathbf{s}_i, \mathbf{s}_{-i})} [u_i(\mathbf{a}_i, \mathbf{a}_{-i})]. \quad (21)$$

From equation (20), the decentralized optimization problem in equation (21) can be written as:

$$\mathbf{s}_i^* = \arg \max_{\mathbf{s}_i \in \mathcal{S}^M} \sum_{j=1}^M s_{ij} \cdot E[V_{ij}(\mathbf{a}_i, \mathbf{a}_{-i})]. \quad (22)$$

Based on the strategy  $\mathbf{s}_i^*$ , a secondary user can choose its action (frequency channel), and then the secondary user models  $\mathbf{s}_{-i}$  based on the action information exchange revealed by the other secondary users (i.e.  $\mathbf{a}_{-i}$ ) in

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<sup>10</sup> For multiple priority classes' case, the same algorithm can be applied consecutively from higher priority classes to lower priority classes without losing generality.

order to evaluate a new  $E[V_{ij}(\mathbf{a}_i, \mathbf{a}_{-i})]$ . The concept is similar to the fictitious play [25] in multi-agent learning in game theory. The difference is that a user not only models the strategies of the other users, but also explicitly calculates the aggregate virtual queue effect  $E[V_{ij}(\mathbf{a}_i, \mathbf{a}_{-i})]$  that directly impacts the utility function. Based on the priority queuing analysis in Section IV, the aggregate virtual queue effect  $E[V_{ij}(\mathbf{a}_i, \mathbf{a}_{-i})]$  can be evaluated using equation (20) by each of the secondary users. The iterative learning algorithm based on  $E[V_{ij}(\mathbf{a}_i, \mathbf{a}_{-i})]$  can be written as:

$$\begin{aligned} \mathbf{s}_i^*(n) &= \arg \max_{\mathbf{s}_i \in \mathcal{S}^M} U_i(\mathbf{s}_i, \mathbf{s}_{-i}(n-1)) \\ &= \arg \max_{\mathbf{s}_i \in \mathcal{S}^M} \sum_{j=1}^M s_{ij} \cdot E[V_{ij}(\mathbf{a}_i(n-1), \mathbf{a}_{-i}(n-1))] \end{aligned} \quad (23)$$

where the initial stage is  $\mathbf{s}_i(0)$ . We show the system diagram of a secondary user in Figure 4. The optimal strategy  $\mathbf{s}_i^*$  can be determined by the secondary user  $SU_i$  for a given  $E[V_{ij}(\mathbf{a}_i, \mathbf{a}_{-i})]$  from the interface. Then, based on the best response strategy  $\mathbf{s}_i^*(n)$ , a packet of the secondary user  $SU_i$  selects an action  $\mathbf{a}_i(n)$ .

Let the frequency channel with the largest  $E[V_{ij}(\mathbf{A}(n-1))]$  be  $F^*(n)$ , i.e.  $F^*(n) = \arg \max_{F_j \in \mathbf{F}_i} \{E[V_{ij}(\mathbf{A}(n-1))]\}$ . Recall that  $\mathbf{A}(n-1) = [\mathbf{a}_i(n-1), \mathbf{a}_{-i}(n-1)]$ . The solution of (23) is:

$$\mathbf{s}_i^*(n) = \begin{cases} s_{ij} = 1, & \text{if } F_j = F^*(n) \\ s_{ij} = 0, & \text{otherwise} \end{cases} \quad (24)$$

For a specific frequency channel  $F_j$ , even though the corresponding primary user's traffic is stationary, it is not guaranteed that the secondary users' strategies will converge to a steady state, since the secondary users mutually impact each other. Hence, our solution adopts a multi-agent learning which resembles the gradient play [25] in game theory. Our approach does not employ a best response strategy, but rather adjusts a strategy in the direction of the perceived "better" response. In addition, due to the cost of frequency hopping and the hardware limitations, only a limited set of selectable frequency channels can be selected by a secondary user for transmission. Hence, we assume that the selectable frequency channels for the secondary user  $SU_i$  are in a set  $\mathbf{F}_i \subseteq \mathbf{F}$ . Let us denote  $\mathbf{H}_i = \{F_j \mid s_{ij} > 0\} \subseteq \mathbf{F}_i$  as the set of frequency channels with  $s_{ij} > 0$ . The maximum number of selected frequency channel is  $H_i$ , i.e.  $|\mathbf{F}_i| \leq H_i$ .

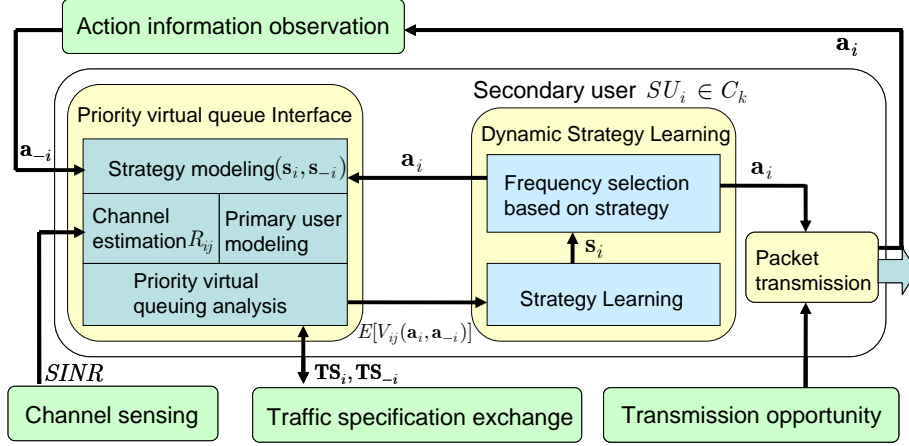


Fig 4. The block diagram of the priority virtual queue interface and dynamic strategy learning of a secondary user.

Note that changing the selected frequency channels requires channel sensing, control signaling, and also additional incurred delays, etc. for the spectrum handoff [23]. In Appendix, we discuss the convergence properties of the proposed algorithm considering the cost of changing the frequency selection strategy. We refer to this cost for the secondary user  $SU_i$  as  $\chi_i(s_i(n), s_i(n-1))$ , which is a function of the difference between the selected strategy and the previous strategy (see Appendix for more detail). The utility function of  $SU_i$  now becomes  $U_i(s_i(n), s_{-i}(n-1)) = \sum_{j=1}^M s_{ij}(n) \times E[V_{ij}(\mathbf{A}(n-1))] - \chi_i(s_i(n), s_i(n-1))$ .

The steps in our DSL algorithm are summarized below:

### Step 1. Model the strategy matrix from the action information exchange:

The priority virtual queue interface collects the action information from the other users and accordingly updates the strategy matrix.

### Step 2. Calculate virtual queue effects:

Given the strategy matrix of the previous stage,  $\mathbf{S}(n-1) = [s_i(n-1), s_{-i}(n-1)]$  and the channel loading specification, we calculate the aggregate virtual queue effects  $E[V_{ij}(\mathbf{A}(n-1))]$  based on equation (18) and (20).

### Step 3. Determine the set of selected frequency channels:

Determine the set  $\mathbf{H}_i$  of selected frequency channels from  $\mathbf{F}_i$ :

$$\mathbf{H}_i(n) = \arg \max_{F_j \in \mathbf{F}_i}^{(H_i)} \{E[V_{ij}(\mathbf{A}(n-1))]\}, \quad (25)$$

where we denote the operation  $\max^{(N)}(X)$  as the largest  $N$  choices from a set  $X$ . Recall that the frequency

channel with the largest  $E[V_{ij}(\mathbf{A}(n-1))]$  be  $F^*(n)$ .

**Step 4. Determine the channel selection strategies:**

Based on  $\mathbf{H}_i(n)$ , we determine the strategy  $\tilde{s}_{ij}(n)$  using the following policy:

$$\tilde{s}_{ij}(n) = \begin{cases} \max(0, s_{ij}(n-1) - \sigma) & , \text{ if } F_j \in \mathbf{H}_i(n), F_j \neq F^*(n) \\ 1 - \sum_{F_j \neq F^*(n)} \max(0, s_{ij}(n-1) - \sigma) & , \text{ if } F_j \in \mathbf{H}_i(n), F_j = F^*(n), \\ 0 & , \text{ if } F_j \notin \mathbf{H}_i(n) \end{cases} \quad (26)$$

where  $\sigma$  is a constant step size of changing the strategies such that the policy favors a frequency channel leading to a larger  $V_{ij}(\mathbf{S}(n-1))$ . Specifically, the policy concentrates the traffic distribution to the frequency channel  $F^*(n)$  from the other frequency channels in  $\mathbf{H}_i$ , while learning from the previous strategy  $s_{ij}(n-1)$ .

**Step 5. Update the new strategy:**

Update the new strategy  $s_{ij}(n)$  if the strategy  $\tilde{s}_{ij}(n)$  leads to an improved utility.

$$s_{ij}(n) = \begin{cases} \tilde{s}_{ij}(n), & \text{if } U_i(\tilde{\mathbf{s}}_i(n), \mathbf{s}_{-i}(n-1)) > U_i(\mathbf{s}_i(n-1), \mathbf{s}_{-i}(n-1)) \\ s_{ij}(n-1), & \text{otherwise} \end{cases} \quad (27)$$

**Step 6. Determine a frequency channel for packet transmissions based on the strategy.**

The proposed dynamic channel selection algorithm has the following advantages:

1. Decentralized decision making allows heterogeneous secondary users (in terms of their priorities, utilities, source traffic and channel conditions) to optimize their own utility functions based on the information exchanges.
2. Virtual queuing analysis provides the expected utility impacted by other users using the same frequency channel and hence, simplifies the required information exchange.
3. The iterative algorithm provides real-time adaptation to the changing network conditions and source traffic variations of the primary users or other secondary users.

## VI. SIMULATION RESULTS

First, we simulate a simple network with two secondary users and three frequency channels (i.e.  $N = 2$ ,  $M = 3$ ) in order to show the results of our solution using a simple example such that the behavior of the

proposed cognitive radio model can be clearly understood. We assume that each secondary user can choose all the frequency channels, i.e.  $H_i = 3$ . The two secondary users are in the same priority class. The simulation parameters of the secondary users are presented in Table I including the channel conditions  $R_{ij} = [T_{ij}, p_{ij}]$ , and initial strategies  $s_i(0)$ , etc. The normalized traffic statistics of the primary users are in Table II. Given these statistics, Figure 5 provides the analytical experienced delays  $E[D_{ij}]$  (using equation (17)) that are bounded by the delay deadlines for the two secondary users using different strategy pairs  $(s_{1j}, s_{2j})$  in the three frequency channels. Importantly, a strategy pair  $(s_{1j}, s_{2j})$  that results in an unbounded  $E[D_{ij}]$  will make the utility function drop abruptly for delay-sensitive applications (see equation (2)), which is undesirable for these secondary users. Hence, equation (17) provides the analytical operation points for the strategy pairs. In the following subsection, each secondary user applies the proposed DSL algorithm from a uniform traffic distribution over the three channels to find the channel selection strategies.

**TABLE I**  
SIMULATION PARAMETERS OF THE SECONDARY USERS

Secondary users	Physical transmission rate $T_{ij}$ (Mbps)			Physical packet error rate $p_{ij}$			Initial strategy $s_{ij}(0)$			Satisfaction rate $T_i^{\max} = 3B_i$ (Mbps)	Rate requirement $B_i$ (Mbps)	Packet length $L_i$ (bytes)	Delay deadline $d_i$ (sec)
	$F_1$	$F_2$	$F_3$	$F_1$	$F_2$	$F_3$	$F_1$	$F_2$	$F_3$				
$SU_1$	1.90	1.21	1.78	0.09	0.16	0.12	1/3	1/3	1/3	2.77	0.92	1000	0.5
$SU_2$	0.46	0.97	1.52	0.01	0.09	0.15	1/3	1/3	1/3	2.21	0.74	1000	0.5

**TABLE II**  
SIMULATION PARAMETERS OF THE PRIMARY USERS

Primary users	Normalized loading $\rho_j$	Second moment normalized loading $\rho_j^2$
$PU_1$	0.2	$1 \times 10^{-4}$
$PU_2$	0.1	$1 \times 10^{-4}$
$PU_3$	0.3	$1 \times 10^{-4}$

#### A. Impact of the delay sensitivity preference of the applications

In this simulation, we show that the delay sensitivity preferences of the secondary users affect the stability of utility and also the resulting channel selection strategies. Figure 6 gives the strategies and the resulting utilities of the two secondary users with two different  $\theta_i$  (applications that care less about delay with  $\theta_i = 0.2$ ,  $i = 1, 2$  in Figure 6(a) and applications that care more about delay with  $\theta_i = 0.8$ ,  $i = 1, 2$  in Figure 6(b)).

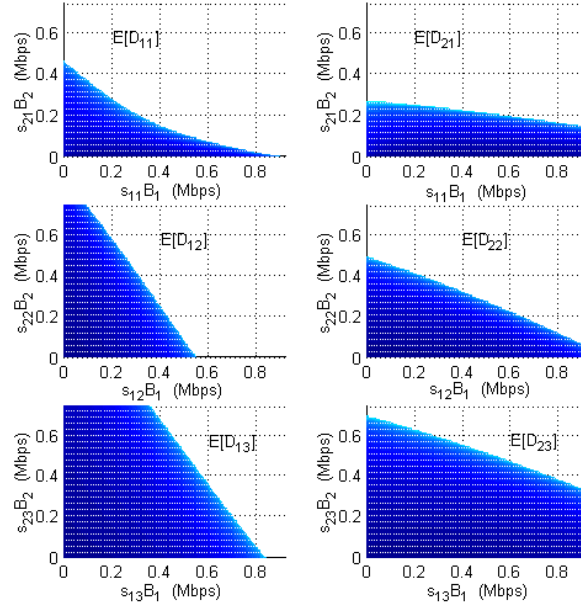


Fig. 5 Analytical expected delay of the secondary users with various strategies in different frequency channels, shadow part represents a bounded delay below the delay deadline (stable region).

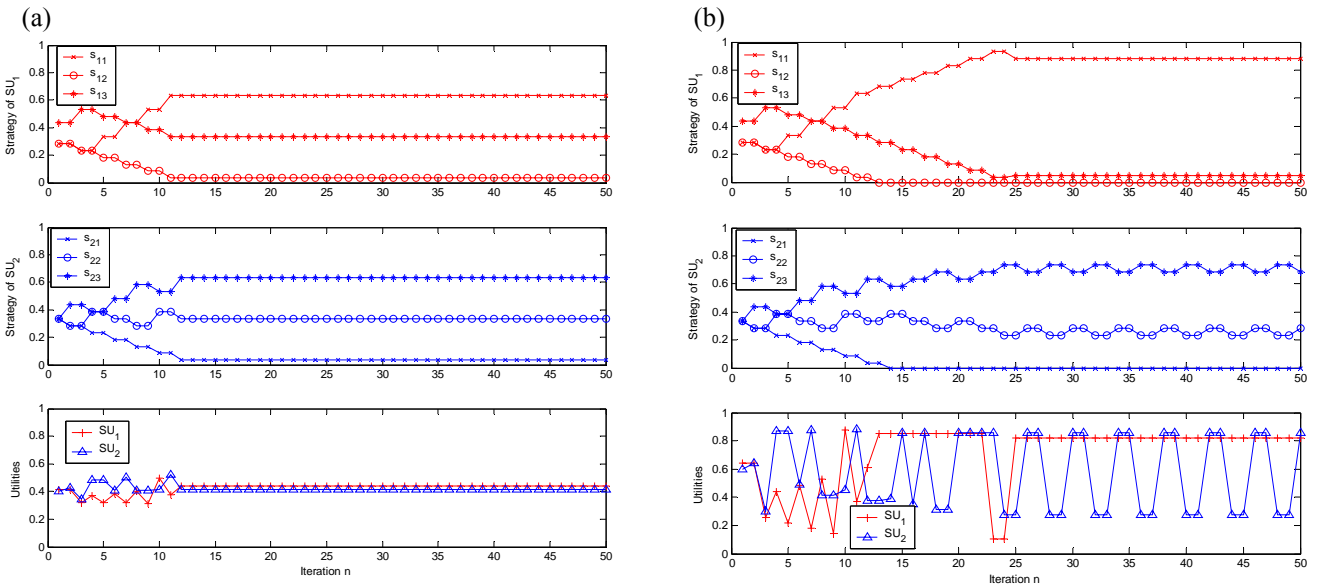


Fig. 6 (a) Simulation results of the DSL algorithm – strategies of the secondary users and the utility functions of less delay-sensitive applications ( $\theta_i = 0.2, \sigma = 0.05, \chi_{ij} = 0$ ).

(b) Simulation results of the DSL algorithm – strategies of the secondary users and the utility functions of delay-sensitive applications ( $\theta_i = 0.8, \sigma = 0.05, \chi_{ij} = 0$ ).

The delay-sensitive applications in Figure 6(b) do not achieve a steady state, since the small changes in the channel selection strategies can push the experienced delay over the delay deadline and hence, impact the utility function dramatically. Moreover, compared with the resulting strategies of the applications in Figure 6(a), Figure 6(b) shows that the delay-sensitive applications prefer a channel without other secondary users to transmit the



data –  $SU_1$  transmits most of its data through channel  $F_1$ , while  $SU_2$  transmits through  $F_2$  and  $F_3$  (i.e.  $s_{11} \cong 1$ ,  $s_{21} \cong 0$ ). This is because for a secondary user with delay sensitive applications, the utility function is more sensitive to the traffic in a frequency channel. The data traffic from other secondary users can increase the uncertainty of the channel, which makes such channel undesirable for the delay sensitive applications. Moreover, the resulting utility is more unstable for the applications with a larger  $\theta_i$ . The resulting strategy  $(s_{11}, 0)$ ,  $(0, s_{22})$ , and  $(0, s_{23})$  of Figure 6(b) are closer to the region with unbounded delay for  $E[D_{11}]$ ,  $E[D_{22}]$ , and  $E[D_{23}]$ , respectively (see Figure 5).

### B. Impact of the primary users in different channels

Next, we simulate the impact of the highest priority users – the primary users in Figure 7. We change the normalized traffic loading of  $PU_1$  in the frequency channel  $F_1$  from 0 to 1 and fix the normalized loading of the other two primary users as in Table II. Due to the priority queuing, we know that once  $\rho_1$  reaches 1, frequency channel  $F_1$  is not accessible for the secondary users. For different normalized loading of  $PU_1$ , Figure 7 shows the resulting strategies and the utilities of the two secondary users after convergence. Both  $s_{11}$  and the utility value  $U_1$  decreases when the available resource from  $F_1$  decreases ( $\rho_1 > 0.6$ ). Interestingly, even though  $SU_2$  does not utilize channel  $F_1$  ( $s_{21} \cong 0$ ) and the resulting strategies do not change with  $\rho_1$ ,  $U_2$  also decreases. This is because more traffic from  $SU_1$  will now be distributed to  $F_2$  and  $F_3$ . This simple example illustrates that the traffic of a higher priority class user can still affect the utilities of the secondary users in lower priority classes even when these secondary users avoid selecting the same channels as the higher priority class user.

### C. Comparison with other cognitive radio resource management solutions

In this subsection, we simulate a larger number of secondary users and a larger number of frequency channels. First, we look at the case with 6 secondary users with video streaming applications (“Coastguard”, frame rate of 30Hz, CIF format, delay deadline 500ms) sharing 10 frequency channels ( $N = 6, M = 10, \theta_i = 1$ ). We compare our DSL algorithm with other two resource allocation algorithms – the “Static Assignment” [10] and the “Dynamic Least Interference” [9]. In the “Static Assignment” algorithm, a secondary user will statically select a

frequency channel with the best effective transmission rate without interacting with other secondary users. This work has the drawback that it is merely a decentralized scheme without any information exchange. In the ‘‘Dynamic Least Interference’’ algorithm, a secondary user will dynamically select a single frequency channel that has the least interference from the other users (both secondary users and primary users), which is also similar to the rule  $D$  in [29]. This work has the drawback of considering only the interference and the resulting throughput in the physical layer. We simulate 100 different frequency channel conditions as well as the traffic loadings and then compute the average the video PSNR and the standard deviation of the PSNR over the one hundred cases in Table III for the 6 video applications. Unlike the ‘‘Dynamic Least Interference’’ that only considers the interference and the resulting throughput in the physical layer, our proposed multi-agent learning algorithm tracks the strategies of the other users through information exchange and adequately adapts the channel selection to maximize the multimedia utility in the application layer. The results show that our DSL algorithm outperforms the other two algorithms for delay-sensitive multimedia applications in terms of packet loss rate (PLR) and video quality (PSNR).

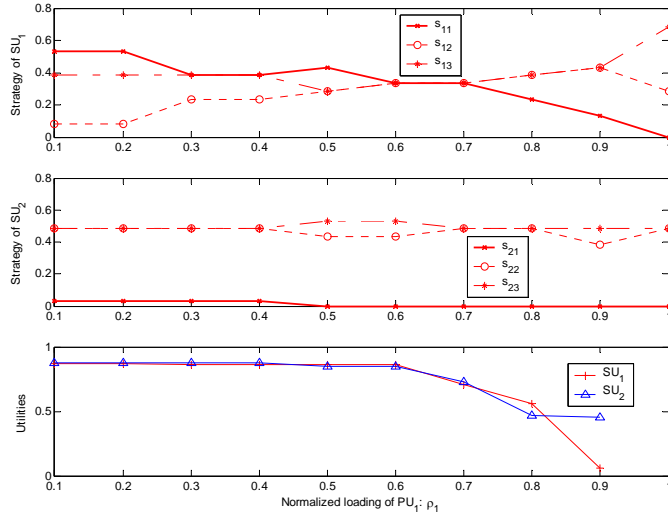


Fig. 7 Steady state strategies of the secondary users and the utility functions vs. the normalized loading of  $PU_1$  for delay-sensitive applications ( $\theta_i = 0.8$ ,  $\sigma = 0.05$ ,  $\chi_{ij} = 0.02$ ).

Next, we simulate the case with 20 secondary users with video streaming applications ( $\theta_i = 1$ ) mixed with  $r$  secondary users with delay insensitive ( $\theta_i = 0$ ) applications. These secondary users are in the same priority class and share 10 frequency channels. The average  $T_{ij}$  of the frequency channels is now set to 3 Mbps, instead of 1.25 Mbps and 1 Mbps in the previous simulation. Table IV shows the average packet loss rate and the

average PSNR over the 20 video streams (instead of over 100 different channel conditions in the previous simulation) with different  $r$  for the three solutions. Larger  $r$  reduces the available resources that can be shared by the video streams, and hence, decreases the received video quality. The results show that the video streaming of the “Static Assignment” is impacted severely by the different channel conditions to the secondary users. The standard deviations of the “Static Assignment” are larger than the results of the “Dynamic Least Interference” and our DSL algorithm. The results again show that our DSL algorithm outperforms the other two algorithms for multimedia applications in terms of packet loss rate and video quality.

**TABLE III**

COMPARISONS OF THE CHANNEL SELECTION ALGORITHMS FOR DELAY-SENSITIVE APPLICATIONS WITH  $N = 6, M = 10$ .

Medium bandwidth case: (average $T_{ij} = 1.25$ Mbps)	“Static Assignment – Largest Bandwidth”			“Dynamic Least Interference”			“Dynamic Learning Algorithm”		
	PLR	Average Y-PSNR (dB)	Y-PSNR Standard Deviation	PLR	Average Y-PSNR (dB)	Y-PSNR Standard Deviation	PLR	Average Y-PSNR (dB)	Y-PSNR Standard Deviation
$SU_1$	15.24 %	32.93	3.92	17.44 %	32.55	3.49	7.61 %	34.17	1.52
$SU_2$	25.38 %	31.48	4.31	19.80 %	32.20	3.45	8.74 %	33.97	1.82
$SU_3$	21.34 %	32.03	4.24	15.45 %	32.86	3.50	11.85 %	33.44	2.28
$SU_4$	20.38 %	32.17	4.35	12.98 %	33.26	3.40	8.22 %	34.06	1.77
$SU_5$	27.17 %	31.21	4.29	20.56 %	32.09	3.55	12.61 %	33.32	2.21
$SU_6$	19.26 %	32.32	4.33	12.86 %	33.27	3.61	9.38 %	33.86	2.27
Low bandwidth case: (average $T_{ij} = 1$ Mbps)	“Static Assignment – Largest-Bandwidth”			“Dynamic Least Interference”			“Dynamic Learning Algorithm”		
	PLR	Average Y-PSNR (dB)	Y-PSNR Standard Deviation	PLR	Average Y-PSNR (dB)	Y-PSNR Standard Deviation	PLR	Average Y-PSNR (dB)	Y-PSNR Standard Deviation
$SU_1$	42.01 %	29.48	4.94	38.16 %	29.89	4.32	18.30 %	32.42	1.97
$SU_2$	38.21 %	29.90	4.89	34.07 %	30.35	4.29	17.02 %	32.62	2.42
$SU_3$	39.97 %	29.69	5.02	33.85 %	30.37	4.41	18.76 %	32.36	2.26
$SU_4$	32.30 %	30.59	4.98	29.74 %	30.87	4.37	16.12 %	32.75	2.31
$SU_5$	42.19 %	29.48	4.98	38.34 %	29.87	4.41	18.45 %	32.40	2.33
$SU_6$	37.07 %	30.01	5.04	31.52 %	30.65	4.46	19.40 %	32.26	2.67

**TABLE IV**

COMPARISONS OF THE CHANNEL SELECTION ALGORITHMS FOR DELAY-SENSITIVE APPLICATIONS WITH  $N = 20 + r, M = 10$ ,  
WHERE  $r$  IS THE SECONDARY USERS WITH DELAY INSENSITIVE  $\theta_k = 0$  APPLICATIONS.

Average $T_{ij} = 3$ Mbps	“Static Assignment – Largest Bandwidth”			“Dynamic Least Interference”			“Dynamic Learning Algorithm”		
	PLR	Average Y-PSNR (dB)	Y-PSNR Standard Deviation	PLR	Average Y-PSNR (dB)	Y-PSNR Standard Deviation	PLR	Average Y-PSNR (dB)	Y-PSNR Standard Deviation
$r = 2$	20.00%	28.49	14.24	12.64%	33.76	2.59	0.06%	35.60	0.0013
$r = 5$	35.00%	23.15	16.98	15.81%	33.30	2.83	2.86%	35.23	1.64
$r = 10$	50.00%	17.81	17.80	24.34%	32.32	3.36	8.12%	34.50	2.55

## VII. CONCLUSIONS

In this paper, we propose a priority virtual queue interface for heterogeneous multimedia users in cognitive

radio networks, based on which they can exchange information and time share the various frequency channels in a decentralized fashion. Based on the information exchange, the secondary users are able to evaluate the expected utility impact from the dynamic wireless environment as well as the competing wireless users' behaviors and learn how to efficiently adapt their channel selection strategies. We focus on delay-sensitive multimedia applications, and propose a dynamic learning algorithm based on the priority queuing analysis. Importantly, unlike conventional channel allocation schemes that select the least interfered channel merely based on the channel estimation, the proposed multi-agent learning algorithm allows the secondary users to track the actions of the other users and adequately adapt their own strategies and actions to the changing multi-user environment. The results show that our proposed solution outperforms the fixed channel allocation and the dynamic channel allocation that selects the least interfered channel, in terms of video quality. Without primary users using the highest priority class, the proposed approach can also be used to support QoS for general multi-radio wireless networks. This situation also emerges in wireless systems such as those discussed in [23], where the secondary users are competing in the unlicensed band (i.e. ISM band) and there is no primary user. The proposed DSL algorithm can be implemented by the secondary users to switch channels, suspend/resume channel operation, and add/remove channels, etc., while complying with emerging MAC solutions for cognitive radio networks [3].

## APPENDIX: CONVERGENCE OF THE DECENTRALIZED APPROACH

If we consider the additional cost (penalty)  $\chi_i(\mathbf{s}_i, \mathbf{s}_i(n-1))$  when the channel selection strategies are not the same, equation (23) can be rewritten as:

$$\begin{aligned} \mathbf{s}_i^*(n) &= \arg \max_{\mathbf{s}_i \in \mathcal{S}^M} U_i(\mathbf{s}_i, \mathbf{s}_{-i}(n-1)) \\ &= \arg \max_{\mathbf{s}_i \in \mathcal{S}^M} \left( \sum_{j=1}^M s_{ij} \cdot E[V_{ij}(\mathbf{A}(n-1))] - \chi_i(\mathbf{s}_i, \mathbf{s}_i(n-1)) \right), \end{aligned} \quad (28)$$

For example, the penalty function can be

$$\chi_i(\mathbf{s}_i, \mathbf{s}_i(n-1)) = \begin{cases} \chi_i^+, & \text{if } s_{ij}(n-1) = 0, s_{ij} > 0 \\ \chi_i^-, & \text{if } s_{ij}(n-1) > 0, s_{ij} = 0, \\ 0, & \text{otherwise} \end{cases} \quad (29)$$

where  $\chi_i^+$  and  $\chi_i^-$  represent the cost of selecting a new channel and the cost of hopping away from a used

frequency channel.

From equation (28), the secondary user  $SU_i$  will keep updating its channel for transmission, unless the utility difference of selecting a new strategy  $s_i^*(n)$  becomes small. Hence, in the proposed DSL algorithm in Section V, assume the difference between the estimated strategy  $\tilde{s}_{ij}(n)$  and the previous strategy  $s_{ij}(n-1)$  is  $e_{ij}(n)$  for  $SU_i$  using the frequency channel  $F_j$ , i.e.  $e_{ij}(n) = \tilde{s}_{ij}(n) - s_{ij}(n-1)$ . Let  $U_i^{diff}(n) = \sum_{j=1}^M e_{ij}(n) \times E[V_{ij}(\mathbf{A}(n-1))]$  be the utility difference between the estimated strategy and the previous strategy.

**Claim 1:** If  $e_{ij}(n)$  satisfies the following condition:

$$U_i^{diff}(n) \leq \chi_i(\tilde{\mathbf{s}}_i(n), \mathbf{s}_i(n-1)), \quad (30)$$

for all the secondary users, the channel selection strategies converge to a steady state.

**Proof:** Equation (30) can be derived as:

$$\begin{aligned} \chi_i(\tilde{\mathbf{s}}_i(n), \mathbf{s}_i(n-1)) &\geq \sum_{j=1}^M (\tilde{s}_{ij}(n) - s_{ij}(n-1)) \times E[V_{ij}(\mathbf{A}(n-1))] \\ \Rightarrow \sum_{j=1}^M s_{ij}(n-1) \times E[V_{ij}(\mathbf{A}(n-1))] &\geq \sum_{j=1}^M \tilde{s}_{ij}(n) \times E[V_{ij}(\mathbf{A}(n-1))] - \chi_i(\tilde{\mathbf{s}}_i(n), \mathbf{s}_i(n-1)) \\ \Rightarrow U_i(\mathbf{s}_i(n-1), \mathbf{s}_{-i}(n-1)) &\geq U_i(\tilde{\mathbf{s}}_i(n), \mathbf{s}_{-i}(n-1)) \end{aligned}$$

From Step 5 of the DSL algorithm in Section V, the strategies will remain unchanged and converge to a steady state. ■

**Claim 2:** If the penalty function  $\chi_i(\mathbf{s}_i, \mathbf{s}_i(n-1))$  is a convex function of  $\mathbf{s}_i$ , when the DSL algorithm converges to a steady state, the channel selection strategy  $\mathbf{s}_i^*$  is the best response strategy that maximizes  $U_i$ .

**Proof:** As long as the penalty function  $\chi_i(\mathbf{s}_i, \mathbf{s}_i(n-1))$  is a convex function of  $\mathbf{s}_i$ , the utility function  $U_i(\mathbf{s}_i, \mathbf{s}_{-i}(n-1))$  is a concave function, since for each iteration, the  $E[V_{ij}(\mathbf{A}(n-1))]$  in equation (28) does not change with  $\mathbf{s}_i$ . Hence, when the DSL algorithm converges to a steady state, the local optimum in equation (28) converges to the global optimum. ■

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