Learning to Compete for Resources in Wireless Stochastic Games

Fangwen Fu and Mihaela van der Schaar, Senior Member, IEEE

4 Abstract—In this paper, we model the various users in a wireless 5 network (e.g., cognitive radio network) as a collection of selfish 6 autonomous agents that strategically interact to acquire dynami-7 cally available spectrum opportunities. Our main focus is on devel-8 oping solutions for wireless users to successfully compete with each 9 other for the limited and time-varying spectrum opportunities, 10 given experienced dynamics in the wireless network. To analyze 11 the interactions among users given the environment disturbance, 12 we propose a stochastic game framework for modeling how the 13 competition among users for spectrum opportunities evolves over 14 time. At each stage of the stochastic game, a central spectrum 15 moderator (CSM) auctions the available resources, and the users 16 strategically bid for the required resources. The joint bid actions 17 affect the resource allocation and, hence, the rewards and future 18 strategies of all users. Based on the observed resource allocations 19 and corresponding rewards, we propose a best-response learning 20 algorithm that can be deployed by wireless users to improve their 21 bidding policy at each stage. The simulation results show that 22 by deploying the proposed best-response learning algorithm, the 23 wireless users can significantly improve their own bidding strate-24 gies and, hence, their performance in terms of both the application 25 quality and the incurred cost for the used resources.

26 *Index Terms*—Delay-sensitive transmission, interactive learn-27 ing, multiuser resource management, reinforcement learning, 28 stochastic games, wireless networks.

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I. INTRODUCTION

30 **D** YNAMIC resource management in heterogeneous wire-31 **D** less networks is a challenging problem [3]. The wireless 32 stations and radio systems that must coexist in such a network 33 differ in their individual utility functions, transmission actions, 34 resource demands, and capabilities. Thus, various levels of 35 strategic¹ interaction and adaptation are necessary to cope 36 with the widely varying dynamics. In this paper, we focus on 37 synthesizing new, dynamic, and informationally decentralized 38 resource-management mechanisms to achieve high utility in 39 competitive and heterogeneous wireless networks (including 40 cognitive radio networks [1]–[3]). Specifically, our focus is 41 on designing associated communication algorithms that enable

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The authors are with the Department of Electrical Engineering, University of California at Los Angeles, Los Angeles, CA 90095 USA (e-mail: fwfu@ee.ucla.edu; mihaela@ee.ucla.edu).

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¹By strategic users, we mean users that are not price takers and do not have an *a priori* consensus on resource allocation.

self-interested autonomous wireless stations to strategically 42 compete for the available spectrum resources in either ISM 43 bands [1] or bands shared with licensed users, according to 44 *a priori* mandated or negotiated rules. 45

This paper is primarily concerned with the tensions and 46 relationships among autonomous adaptation by secondary 47 (unlicensed) users (SUs), the competition among these users, 48 the interaction of these users with spectrum moderators hav- 49 ing their own goals, e.g., making money, imposing fairness 50 rules, ensuring compliance with the Federal Communications 51 Commission (FCC) [1], and local regulations with respect to 52 primary (licensed) users (PUs), etc. Unlike previous works on 53 resource management [6], [21], [26], our main focus is on 54 discussing how users can adapt, predict, learn, and determine 55 how they compete for the time-varying resources, as well as 56 how they select the associated transmission strategies, given the 57 experienced "dynamics."

In wireless networks, these dynamics can be categorized into 59 two types: One is the disturbance due to the "environment," 60 and the other is the impact caused by competing users. The 61 disturbance due to the environment results from variations 62 (uncertainties) of the wireless channels or source (e.g., mul- 63 timedia) characteristics. For example, the stochastic behavior 64 of the PUs, the time-varying channel conditions experienced 65 by the SUs, and the time-varying source traffic that needs to 66 be transmitted by the SUs can be considered as environmental 67 disturbances. These types of dynamics are generally modeled 68 as stationary processes. For instance, the use of each channel 69 by the PUs can be modeled as a two-state Markov chain 70 with ON-state (the channel is used by PUs) and OFF-state (the 71 channel is available for the SUs) [7]. The channel conditions 72 can be modeled using a finite-state Markov model [24]. The 73 packet arrival of the source traffic can be modeled as a Poisson 74 process² [11]. 75

Conventionally, wireless stations have only considered these 76 environment disturbances when adapting their cross-layer 77 strategies [12] for delay-sensitive transmission. The other type 78 of dynamics—the impact from competing users, which is due to 79 the noncollaborative, autonomous, and strategic SUs in the net- 80 work transmitting their traffic—is less well studied to wireless 81 communication networks. 82

The goal of this paper is to provide solutions and associ- 83 ated metrics that can be used by an autonomous SU to ana- 84 lyze and predict the outcome of various dynamic interactions 85 among competing SUs in dynamic multiuser communication 86

²Other packet arrival models can also been used in our proposed framework.

87 systems and, based on this forecast, adapt and optimize its 88 transmission strategy. In our considered wireless networks, 89 the SUs are modeled as rational and strategic. We model the 90 spectrum management as a stochastic game [22] in which the 91 SUs simultaneously and repeatedly make their own resource 92 bids. The competition for dynamic resources is assisted by a 93 central coordinator (similar to that in existing wireless LAN 94 (WLAN) standards such as 802.11e HCF [13]). We refer to this

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95 coordinator as the central spectrum moderator (CSM). The role 96 of the CSM is to allocate resources to the SUs based on the 97 predetermined utility maximization rule.³

In this paper, to explicitly consider the strategic behavior of 98 99 autonomous SUs and the informationally decentralized nature 100 of the competition for wireless resources, we assume that the 101 CSM deploys an auction mechanism for dynamically allocat-102 ing resources. Auction theory has extensively been studied 103 in economics [19], and it has also been recently applied to 104 network resource allocation [4]–[6]. Note that the role of the 105 CSM⁴ in our resource management game for our considered 106 wireless networks will be kept to a minimum. Unlike alternative 107 existing solutions [21], the CSM will not require knowledge 108 of the private information of the users and will not perform 109 complex computations for deciding the resource allocation. Its 110 only role will be the implementation of the spectrum etiquette 111 rules as in [8] and ensuring that the available spectrum holes 112 are auctioned among users. To capture the network dynamics, 113 we allow the CSM to repeatedly auction the available spectrum 114 opportunities based on the PUs' behaviors. Meanwhile, each 115 SU is allowed to strategically adapt its bidding strategy based 116 on information about the available spectrum opportunities, its 117 source and channel characteristics, and the impact of the other 118 SU bidding actions.

119 Using this stochastic wireless allocation framework, we de-120 velop a learning methodology for SUs to improve their policies 121 for playing the auction game, i.e., the policies for generating 122 the bids to compete for available resources. Specifically, during 123 repeated multiuser interaction, the SUs can observe partial his-124 toric information of the outcome of the auction game, through 125 which the SUs can estimate the impact on their future rewards 126 and then adopt their best response to effectively compete for 127 channel opportunities. The estimation of the impact on the 128 expected future reward can be performed using different types 129 of interactive learning [18]. In this paper, we focus on reinforce-130 ment learning [17], [27] because this allows the SUs to improve 131 their bidding strategy based only on the knowledge of their own 132 past received payoffs without knowing the bids or payoffs of 133 the other SUs. Our proposed best-response learning algorithm 134 is inspired from the Q-learning for the single agent interact-135 ing with the environment. Unlike Q-learning, the proposed 136 best-response learning explicitly considers the interactions and 137 coupling among SUs in the wireless network. By deploying 138 the best-response learning algorithm, the SUs can strategically

predict the impact of current actions on future performance and 139 then optimally make their resource bids. 140

This paper is organized as follows. In Section II, we intro- 141 duce a stochastic game formulation for multiuser interaction 142 in wireless networks. In Section III, we show how a one- 143 stage auction mechanism can be used to divide the spectrum 144 allocation among strategic SUs. In Section IV, we present 145 the state definition, state transition model, and stage reward 146 function for the SUs in the stochastic game. In Section V, 147 we discuss the bidding strategies of the SUs for playing the 148 stochastic game. In Section VI, we propose a best-response 149 learning approach for the SUs to predict their future rewards 150 based on the observed historic information. In Section VII, 151 we present the simulation results, followed by conclusions and 152 future research in Section VIII.

II. STOCHASTIC GAME FORMULATION FOR 154 DYNAMIC MULTIUSER INTERACTION 155

We consider a spectrum consisting of N channels, each 156 indexed by $j \in \{1, \ldots, N\}$. The N wireless channels are orig- 157 inally licensed to a primary network (PN) whose users (i.e., 158 PUs) exclusively access the channels. In the secondary network 159 (SN), the $M(M \ge N)$ autonomous SUs, each indexed by 160 $i \in \{1, \ldots, M\}$ and transmitting delay-sensitive data, compete 161 for the spectrum opportunities released by the PUs in these 162 N channels. Although the available transmission opportunities 163 (TxOps) for SUs depend on the access patterns of PUs and the 164 detection systems [2], we do not discuss the detection methods 165 in this paper but rather rely on the existing literature for this 166 purpose [3]. Instead, we assume that the available TxOps in 167 each channel change over time due to the PUs joining or leaving 168 the network and can be modeled as a two-state Markov chain, 169 as in [7] and [10]. Our goal is to develop a general framework 170 for multiuser interaction in the SN, where users can compete 171 for dynamically available TxOps. Moreover, we also aim to 172 provide solutions for SUs to improve their strategies for playing 173 the repeated resource-management game by considering their 174 past interactions with other SUs. 175

The communications of the PUs are assumed to follow a 176 synchronous slot structure. The time slot has length of ΔT 177 seconds. We assume that during each time slot, each channel 178 is either exclusively occupied by PUs or that there is no PU 179 accessing the channel [7], [10]. Hence, during each time slot, 180 the channel is in one of the following two states: ON-state 181 (this channel is currently used by the PUs) or OFF-state (this 182 channel is not used by the PUs, and hence, the SUs can use this 183 channel). Note that if this is an unlicensed band, the channel 184 will always be in the off mode and can be utilized by the 185 SUs at all times. The TxOp of channel j at time slot $t \in \mathbb{N}$ 186 is denoted by $y_i^t \in \{0, 1\}$, where y_i^t is 0 if the channel is 187 in the ON-state and 1 if it is in the OFF-state. In this paper, 188 the TxOp y_j^t of channel j is modeled by a two-state Markov 189 chain with transition probability $p_j^{FN} = p(y_j^{t+1} = 0 | y_j^t = 1)$ 190 and $p_j^{NF} = p(y_j^{t+1} = 1 | y_j^t = 0)$. The TxOp profile of the 191 N channels is represented by $y^t = [y_1^t, \dots, y_N^t]$.

As in [13], we assume that a polling-based medium-access 193 protocol is deployed in the SN, which is arbitrated by a CSM. 194

³Other fairness rules can also be deployed in the CSM such as air-time fairness, utility-based fairness, etc. [12].

⁴It should be noted that this approach can also allow for multiple CSMs to manage the spectrum by fairly dividing their responsibilities, e.g., based on their geolocation or frequency band in which they are operating, or by competing against each other for the number of SUs that will associate with them.



Fig. 1. Conceptual overview of the multi-SU interaction in the SN.

195 The polling policy is only changed at the start of every time 196 slot. For simplicity, we assume that each SU can access a 197 single channel, and that each channel can be accessed by 198 a single SU within the time slot. The SUs can switch the 199 channels only when crossing time slots. Note that this simple 200 medium-access model used for illustration in this paper can 201 easily be extended to more sophisticated models [10], where 202 each SU can simultaneously access multiple channels or the 203 channels are being shared by multiple SUs, etc. When using 204 this time-division channel access, we assume that the wireless 205 users deploy constant transmission power and experience no 206 interference. Furthermore, we assume that the wireless users 207 move slowly, and thus, their experienced channel conditions 208 slowly change.

209 During each time slot, an SU needs to first determine how to 210 compete with the other SUs for the time-varying TxOps. This 211 represents its external actions, since they determine the inter-212 action between this SU and the other SUs, and the amount of 213 resources allocated to that SU. The external actions at time slot t214 are denoted by $a_i^t \in A_i$, where A_i is the set of possible external 215 actions available to SU *i*. Based on the allocated resources, 216 the SU determines how to transmit its traffic (application layer 217 data) by selecting the various strategies at different layers of 218 the OSI stack (e.g., through cross-layer adaptation [12]). These

218 the OSI stack (e.g., through cross-layer adaptation [12]). These 219 actions are referred to as internal actions, since they only 220 determine the SU's utility at the current time. The internal 221 actions at time slot t are denoted by $b_i^t \in B_i$, where B_i is the set 222 of possible internal actions available to SU i. In this paper, we 223 propose an auction mechanism deployed in the CSM. Hence, 224 the external action a_i^t of SU i is the bid it submits to CSM. The 225 auction mechanism will be detailed in Section III. The environ-226 ment experienced by an SU i can be characterized by its current 227 "state" $s_i^t \in S_i$, which will be discussed in Section IV. At each 228 time slot t, SU i generates the external action a_i^t to compete 229 for the TxOps y^t . The competition result is ϑ_i^t , based on which 230 SU i performs its internal action b_i^t and obtains the reward r_i^t at 231 this time slot. After packet transmission, SU i transits to the 232 next state $s_i^{t+1} \in S_i$. The conceptual overview of the multi-233 SU interactions in the repeated auctions is illustrated in Fig. 1. The repeated competition among the SUs can be modeled as 234 a stochastic game [16], [22]. The time slot corresponds to the 235 term "stage," which is commonly used in stochastic games. In 236 the remainder of this paper, we interchangeably use the terms 237 "time slot" and "stage." 238

We define the stochastic game for SN resource allocation as 239 $\langle \langle S_i, A_i, B_i, O_i, q_i, r_i \rangle_{i=1}^M, \mathcal{Y} \rangle$, where each SU *i* is associated 240 with a tuple $\langle S_i, A_i, B_i, O_i, q_i, r_i \rangle$. Specifically, we have the 241 following. 242

- 1) \mathcal{Y} is a finite set of possible TxOps available for SUs. 243 In this paper, $\mathcal{Y} = \{0, 1\}^N$, and $\mathbf{y}^t \in \mathcal{Y}$ is the avail- 244 able TxOps at stage t, which is common information 245 for SUs. 246
- 2) S_i is a finite local state space of SU *i*. We let S := 247 $\prod_{k=1}^{N} S_k$ be the global state space of all SUs and 248 $S_{-i} := \prod_{k \neq i} S_k$ be the global state space of SUs other 249 than *i*. At stage *t*, the global state is denoted by $s^t = 250$ $(s_1^t, \ldots, s_M^t) = (s_i^t, s_{-i}^t)$, where -i represents all the 251 SUs other than *i*. 252
- 3) A_i is a finite set of external actions performed by SU *i* 253 to compete for the available TxOps. The external action 254 vector at stage *t* for all SUs will be $a^t = (a_1^t, \dots, a_M^t)$. 255
- 4) B_i is a finite set of internal actions performed by SU *i* to 256 determine the packet transmission. 257
- 5) O_i is a finite set of possible output from multi-SU com- 258 petition. In this paper, the output $\vartheta_i^t \in O_i$ is the auction 259 result computed by the CSM for SU *i* at stage *t*. We will 260 give the specific form of the output in Section III. 261
- 6) q_i is the state transition probability for SU *i*. Thus, 262 $q_i(s_i^{t+1}, y^{t+1}|s_i^t, y^t, \vartheta_i^t, b_i^t)$ is the probability that the state 263 of SU *i* transits from s_i^t to s_i^{t+1} and TxOp transits from 264 y^t to y^{t+1} if the competition output is ϑ_i^t and the internal 265 action is b_i^t . The reason that the transition probability 266 includes the common TxOp y^t is because the channel 267 condition transition of SU *i* depends on the available 268 TxOp. 269
- 7) r_i is the stage reward (immediate reward) received by SU 270 *i*, where $r_i : (S_i, O_i, B_i) \mapsto \mathbb{R}$. It should be noted that 271

272	the reward function r_i depends on the competition output
273	and, hence, indirectly depends on the other SUs' external

275 To design a stochastic game for the SN with strategic SUs, 276 we have to consider the following: 1) What auction mech-277 anism can be deployed to resolve the competition among 278 SUs; 2) how the dynamic environment experienced by each 279 SU can be modeled; and 3) how the SUs can forecast the 280 impact of their bids made at the current time on their future 281 performance?

282 III. AUCTION MECHANISM—ONE STAGE 283 RESOURCE ALLOCATION

In this paper, we assume that the CSM is aware of the 284 285 TxOp y^t and allocates (through polling the SUs) those channels 286 with $y_i^t = 1$ to the SUs. To efficiently allocate the available 287 resources (opportunities), the CSM needs to collect information 288 about the SUs [21]. However, as mentioned in Section I, in a 289 wireless network, the information is decentralized, and thus, 290 the information exchange between the SUs and the CSM needs 291 to be kept limited due to the incurred communication cost. 292 On the other hand, the SUs competing with each other are 293 selfish and strategic, and hence, the information they hold is 294 private, and they may not desire to reveal this information to 295 the CSM. Therefore, one of our key interests in this paper is 296 to determine what information should be exchanged between 297 the SUs and the CSM and how this information should be 298 exchanged. In the following, we present an auction mechanism 299 for dynamically coordinating the interactions among SUs and 300 discuss the computational complexity in the CSM and the 301 communication cost between SUs and CSM.

First, the CSM announces the auction by broadcasting the 302 303 TxOp y^t . The SUs receive the announcement and determine the 304 external action (i.e., the bid vector) $a_i^t = [a_{i1}^t, \dots, a_{iN}^t] \in \mathbb{R}^N$ 305 based on the announced information and their own private 306 information about the environment they experience, which is 307 discussed in detail in Section IV. Subsequently, each SU sub-308 mits the bid vector to the CSM. After receiving the bid vectors 309 from the SUs, the CSM computes the channel allocation $z_i^t =$ 310 $[z_{i1}^t, \ldots, z_{iN}^t] \in \{0, 1\}^N$ for each SU i based on the submitted 311 bids. To compel the SUs to truthfully declare their bids [23], 312 the CSM also computes the payment $\tau_i^t \in \mathbb{R}_-$ that the SUs have 313 to pay for the use of resources during the current stage of the 314 game. The negative value of the payment means the absolute 315 value that SU i has to pay the CSM for the used resources. 316 Hence, the competition output ϑ_i^t in this auction mechanism 317 includes the channel allocation z_i^t and the payment τ_i^t , i.e., 318 $\vartheta_i^t = (z_i^t, \tau_i^t)$. The competition output is then transmitted back 319 to the SUs. The computation of the channel allocation z_i^t and 320 payment τ_i^t is described as follows.

After each SU submits the bid vector, the CSM performs After each SU submits the bid vector, the CSM performs putation. Note that most existing multiuser wireless resource allocation solutions can be modeled as such repeated auctions for resources. If the resources are priced or the users may lie about their resource needs, taxes associated with the resource



Fig. 2. Information exchange between CSM and SU i.

usage will need to be imposed [14]. Otherwise, these taxes can 327 be considered to be zero throughout the paper. 328

We denote the channel allocation matrix $Z^t = [z_{ij}^t]_{M \times N}$ 329 with z_{ij}^t being 1 if channel j is assigned to SU i, and 0 330 otherwise. The feasible set of channel assignments is denoted 331 as $Z^t = \{Z^t | \sum_{i=1}^M z_{ij}^t = y_j^t, \forall j, \sum_{j=1}^N z_{ij}^t \leq 1, \forall i, z_{ij}^t \in 332$ $\{0,1\}\}$. The channel allocation matrix without the pres- 333 ence of SU i is denoted $Z_{-i}^t = [z_{kj}^t]_{(M-1)\times N}$, and the 334 corresponding feasible set is $Z_{-i}^t = \{Z_{-i}^t | \sum_{k=1,k\neq i}^M z_{kj}^t = 335$ $y_j^t \quad \forall j, \sum_{j=1}^N z_{kj}^t \leq 1 \quad \forall k \neq i, z_{kj}^t \in \{0,1\}\}$, where -i = 336 $\{1, \ldots, i-1, i+1, \ldots, M\}$. During the first phase, the CSM 337 allocates the channels to SUs based on its adopted fairness rule, 338 e.g., maximizing the total "social welfare,"⁵ as

$$Z^{t,\text{opt}} = \arg \max_{Z^t \hat{I} \mathcal{Z}^t} \sum_{i=1}^M \sum_{j=1}^N z_{ij}^t a_{ij}^t.$$
 (1)

If the resources are priced, we will consider in this paper, 340 for illustration, a second-price auction mechanism [19], [23] for 341 determining the tax that needs to be paid by SU *i* based on the 342 above optimal channel assignment $Z^{t,\text{opt}} = [z_{ij}^{t,\text{opt}}]_{M \times N}$. This 343 tax is equal to 344

$$\tau_i^t = \sum_{k=1, k \neq i}^M \sum_{j=1}^N z_{kj}^{t, \text{opt}} a_{kj}^t - \max_{Z_{-i}^t \in \mathcal{Z}_{-i}^t} \sum_{k=1, k \neq i}^M \sum_{j=1}^N z_{kj}^t a_{kj}^t.$$
(2)

Note that when N = 1, the generalized auction mechanism 345 presented above becomes the well-known second-price auction 346 [19]. Although the optimization problems in (1) and (2) are 347 discrete optimizations, they can efficiently be solved using 348 linear programming. As argued in [20], the linear optimization 349 problem can be solved in polynomial time, and hence, the CSM 350 only requires limited computational complexity. 351

The information exchange between the CSM and the SUs 352 is illustrated in Fig. 2. From Fig. 2, we note that, at each 353 stage, the CSM first broadcasts the available TxOps to all the 354 SUs for the auction, and then each SU submits its own bid 355 vector over all the available TxOps. After receiving the bids, the 356 CSM computes the auction results and sends back to the users 357 the channel allocations and the corresponding payments. The 358 signaling required for the auction is most often implemented 359 at the application layer. In the worst case, the amount of 360

⁵Note that other fairness solutions than maximizing the social welfare could be adopted, and this will not influence our proposed solution.

361 data communicated between the CSM to the SUs is equal to 362 (M+1)N + nN bits, where *n* is the amount of bits repre-363 senting the payment for each SU. The amount of data commu-364 nicated by each SU to the CSM is n'N bits, where n' is the 365 amount of bits representing the bid submitted to the CSM on 366 each channel.

367 Compared with traditional one-stage resource allocation 368 methods, our proposed auction mechanism has the following 369 advantages.

1) Unlike traditional centralized resource allocation meth-370 ods [30], our proposed auction mechanism is not required 371 to know the SUs' utility functions or preferences, which 372 is often the private information of the users and is not 373 common knowledge. In fact, our auction mechanism only 374 375 requires the SUs to submit their bid vectors for the available TxOps. The bid vector computation is performed 376 by the SUs, but not the CSM, based on their utili-377 ties, preferences, action sets, experienced environment 378 379 characteristics, etc.

380 2) Unlike traditional decentralized resource allocation methods [28] where multiple iterations are required before 381 convergence, our proposed auction mechanism only re-382 quires the SUs to submit the bid vectors once. Hence, 383 our proposed auction mechanism is suitable for online 384 resource management. Moreover, we do not assume as in 385 [29] that users are price takers and that there is consensus 386 about what is a fair distribution of the resources. Instead, 387 in the proposed framework, users are strategic and are 388 able to determine their own bid vectors for resources 389 based on their knowledge, utilities, preferences, etc. 390

391 IV. USER MODELING IN THE STOCHASTIC 392 GAME FRAMEWORK

393 A. Definition of SU States

As discussed in Section I, each SU needs to cope with two 95 types of "uncertainties," i.e., disturbances from the environment 96 and interactions with other SUs. The environment is charac-97 terized by packet arrivals from the source (i.e., source/traffic 98 characterization) connected with the transmitter and the chan-99 nel conditions. In this section, we will illustrate how these 400 disturbances can be modeled. However, note that other models 401 of the environment existing in the literature can be adopted. The 402 use of a specific model will only affect the performance of the 403 proposed solution and not the general framework for multiuser 404 interaction proposed in this paper.

For illustration, we assume that each SU i maintains a buffer 406 with limited size X_i , which can be interpreted as a time window 407 that specifies which packets are considered for transmission at each time based on their delay deadlines. Expired packets are 408 dropped from the buffer. This model has extensively been used 409 for delay-sensitive data transmission, e.g., leaky bucket model 410 for video transmission [25]. The number of packets in the buffer 411 at time slot t is denoted as $x_i^t (0 \le x_i^t \le X_i)$. We assume that 412 the packets arrive from the source at the beginning of each time 413 slot, i.e., x_i^t is only updated at the beginning of a time slot. The 414 number of packets arriving into the buffer during one time slot 415 is a random variable independent of the time t and denoted as 416 χ_i . χ_i follows the Poisson distribution with the average arrival 417 rate $\overline{\chi}_i$ packets per second [11]. However, note that the Poisson 418 process is simply used for illustration purposes, and other traffic 419 models (e.g., renewal process, etc.) can also be used in our 420 framework. The average number of packets arriving during one 421 time slot is equal to $\overline{\chi}_i \Delta T$ [11]. 422

The condition of channel j experienced by SU i is rep- 423 resented by the signal-to-noise ratio (SNR) and denoted as 424 ρ_{ij}^t (in decibels). When $y_j^t = 1$, we assume that the channel 425 condition of each channel can be represented by a set of discrete 426 SNR values, i.e., $\rho_{ij}^t \in {\sigma_{ij}^1, \ldots, \sigma_{ij}^K}$. Note that the number of 427 discrete SNR values K can be determined by SU i by trading 428 off the complexity (a larger K leads to a larger state space) and 429 the resulting impact on performance. When $y_j^t = 0$, we set ρ_{ij}^t 430 equal to $-\infty$, which means that the channel is unavailable to 431 SUs at that time. As shown in [24], when $y_j^t = 1$, the channel 432 condition (in terms of SNR) can also be modeled as a finite-state 433 Markov chain, where the transition from channel condition σ_{ij}^l 434 at time t to channel condition σ_{ij}^k at time t+1 takes place with 435 probability $p_{ij}^{l \to k}$. These transition probabilities can easily be 436 estimated by SU *i* by repeatedly interacting with the channel. 437 We denote by $p_{ij}^{-\infty \to k}$ the probability that the channel condi- 438 tion is σ_{ij}^k at time t + 1, knowing that $y_j^t = 0$ and $y_j^{t+1} = 1.439$ The probability that the channel condition transition to $-\infty$, 440 knowing that $y_j^{t+1} = 0$, is 1 no matter in what condition the 441 channel j is at time t. Then, the combination (y_i^t, ρ_{ij}^t) is still a 442 Markov chain with state transition probability as in (3), shown 443 at the bottom of the page. 444

To model the dynamics experienced by SU *i* at time *t* in 445 the SN, we define a "state" $s_i^t = (v_i^t, \rho_i^t) \in S_i$, where $\rho_i^t = 446$ $(\rho_{i1}^t, \dots, \rho_{iN}^t)$. The state encapsulates the current buffer state 447 as well as the state of each channel. S_i is the set of possible 448 states.⁶ The total number of possible states for SU *i* is equal to 449 $|S_i| = (X_i + 1) \times (K + 1)^N$. We will show later in this paper 450 that the state information is sufficient for SU *i* to compete for 451 resources (make bid vector) at the current time. 452

⁶We assume that the channel state and the transmission buffer independently evolve as time goes by.

$$p\left(y_{j}^{t+1},\rho_{ij}^{t+1}|y_{j}^{t},\rho_{ij}^{t}\right) = \begin{cases} \left(1-p_{j}^{FN}\right)p_{ij}^{l\to k}, & \text{if } y_{j}^{t}=1, \quad \rho_{ij}^{t}=\sigma_{ij}^{l}, \quad y_{j}^{t+1}=1, \quad \rho_{ij}^{t+1}=\sigma_{ij}^{k} \\ p_{j}^{NF}p_{ij}^{-\infty\to k}, & \text{if } y_{j}^{t}=0, \quad y_{j}^{t+1}=1, \quad \rho_{ij}^{t+1}=\sigma_{ij}^{k} \\ p_{j}^{FN}, & \text{if } y_{j}^{t}=1, \quad \rho_{ij}^{t}=\sigma_{ij}^{l}, \quad y_{j}^{t}=0 \\ 1-p_{j}^{NF} & \text{o. w.} \end{cases}$$
(3)

453 B. State Transition and Stage Reward

454 We will now discuss the state transition process. Remember 455 that the state of SU *i* includes the buffer state v_i^t and the 456 channel state ρ_i^t . In this paper, we assume that the channel 457 state transition is independent of the buffer state transition. 458 In the above, we describe the transition of the channel state 459 ρ_i^t and the TxOp y^t . The buffer state transition is determined 460 by the number of packets arriving and the channel allocation 461 z_i^t as well as the internal action b_i^t during that time slot. 462 The number of packets transmitted at stage *t* is denoted by 463 $\mathcal{N}_i(s_i^t, z_i^t, b_i^t)$. Given the channel allocation, SU *i* can adapt 464 its own internal action to maximize the number of transmitted 465 packets, i.e.,

$$n_i\left(s_i^t, z_i^t\right) = \max_{b_i^t \in B_i} \mathcal{N}_i\left(s_i^t, z_i^t, b_i^t\right).$$
(4)

466 The optimization can be performed by a cross-layer adaptation 467 algorithm as in [5], [12], and [21]. Since our focus is on the 468 multi-SU interaction, we assume that the internal action will 469 always be performed to maximize the number of transmitted 470 packets. We simply use $n_i(s_i^t, z_i^t)$ to represent the number 471 of transmitted packets and omit the internal actions in the 472 following notations.

473 The evolution of the buffer state is captured by 474 $v_i^{t+1} = \min\{(v_i^t - n(s_i^t, z_i^t))^+ + \chi_i^t, X_i\}$. We define $h = v_i^{t+1} -$ 475 $(v_i^t - n(s_i^t, z_i^t))^+$. Based on the packet arrival model, the buffer 476 state transition probability is computed as in (5), shown at the 477 bottom of the page. The state transition combined with TxOps, 478 given the current resource allocation z_i^t , can be computed as

$$q_{i}\left(s_{i}^{t+1}, \boldsymbol{y}^{t+1} | s_{i}^{t}, \boldsymbol{y}^{t}, \boldsymbol{z}_{i}^{t}\right) = \underbrace{p_{i}^{\text{buf}}\left(v_{i}^{t+1} | v_{i}^{t}, \boldsymbol{z}_{i}^{t}\right)}_{\text{buffer state transition}} \underbrace{\prod_{j=1}^{N} p\left(y_{j}^{t+1}, \rho_{ij}^{t+1} | y_{j}^{t}, \rho_{ij}^{t}\right)}_{\text{channel state transition}}$$
(6)

479 where the first term represents the buffer state transition, which 480 is independent of the second term of the channel state transition. 481 Based on the channel allocation z_i^t , the SU transmits 482 the available packets in the buffer. In the next time slot, 483 new packets arrive into the buffer. Newly incoming packets 484 may lead to packets already existing in the buffer being 485 dropped whenever the buffer is full or their delay dead-486 line has passed. Clearly, the performance of the application 487 (e.g., video quality) improves when fewer packets are lost. 488 Hence, we can interpret a negative value of the number of 489 lost packets as the stage gain, which is denoted by g_i^t , i.e., 490 $g_i^t(s_i^t, z_i^t) = -((v_i^t - n_i(s_i^t, z_i^t))^+ + \chi_i^t - X_i)^+$. The reward at 491 time t for SU i is expressed using the quasi-linear form 492 $r_i(s_i^t, \vartheta_i^t) = g_i^t + \tau_i^t$. Note that the gain g_i^t and payment τ_i^t depend on the states and bids of all the competing SUs in the 493 SN. Hence, the reward is also rewritten as $r_i(s^t, y^t, a^t)$. 494

497

A. Best-Response Bidding Policy

In the SN, we assume that the stochastic game is played 498 by all the SUs for an infinite number of stages. This 499 assumption is reasonable for applications having a long 500 duration, such as video streaming. In our network setting, we 501 define a history of the stochastic game up to time t as $h^t = 502$ $\{m{s}^0,m{y}^0,m{a}^0,m{z}^0,m{ au}^0,\dots,m{s}^{t-1},m{y}^{t-1},m{a}^{t-1},m{z}^{t-1},m{ au}^{t-1},m{s}^t,m{y}^t\}\in$ 503 \mathcal{H}^t , which summarizes all previous states, available TxOps, 504 and the actions taken by the SUs as well as the outcomes at 505 each stage of the auction game, and \mathcal{H}^t is the set of all possible 506 histories up to time t. However, during the stochastic game, 507 each SU *i* cannot observe the entire history but rather part of 508 the history h^t . The observation of SU *i* is denoted as $o_i^t \in \mathcal{O}_i^t$ 509 and $o_i^t \subset h^t$. Note that the current state s_i^t can always be 510 observed, i.e., $s_i^t \in o_i^t$. In this paper, we focus on the external 511 action selection for the SUs. The external action selection 512 for SU i to play the stochastic game is also referred to as a 513 bidding policy $\pi_i^t : \mathcal{O}_i^t \mapsto A_i$ for SU *i* at time *t* and defined 514 as a mapping from the observations up to the time t into the 515 specific action, i.e., $a_i^t = \pi_i^t(o_i^t)$. Furthermore, a policy profile 516 π_i for SU *i* aggregates the bidding policies about how to play 517 the game over the entire course of the stochastic game, i.e., 518 $\pi_i = (\pi_i^0, \ldots, \pi_i^t, \ldots)$. The policy profile for all the SUs at 519 time slot t is denoted as $\boldsymbol{\pi}^t = (\pi_1^t, \dots, \pi_M^t) = (\pi_i^t, \boldsymbol{\pi}_{-i}^t).$ 520

The policy π_i is said to be Markov if the bidding policy 521 π_i^t for $\forall t$ is, given the current state s_i^t and current avail- 522 able TxOp y^t , independent of the states, TxOps, and actions 523 prior to the time t, i.e., $\pi_i^t(\boldsymbol{o}_i^t) = \pi_i^t(\boldsymbol{s}_i^t, \boldsymbol{y}^t)$. The policy π_i 524 is said to be stationary if the bidding policy $\pi_i^t = \pi_i$ for 525 $\forall t$. The reward $r_i(s^k, y^k, a^k)$ of the stage k is discounted 526 by the factor $(\alpha_i)^{k-t}$ at time t. The factor $\alpha_i (0 \le \alpha_i < 1)$ 527 is the discounted factor determined by a specific application 528 (for instance, for video streaming applications, this factor can 529 be set based on the tolerable delay). The total discounted sum 530 of rewards $Q_i^t(s^t, y^t, \pi)$ for SU *i* can be calculated at time 531 t starting from the state profile s^t , assuming that all SUs 532 deploy stationary and Markov policies $\boldsymbol{\pi} = (\pi_i, \boldsymbol{\pi}_{-i})$, as in (7), 533 shown at the bottom of the next page. The total discounted 534 sum of rewards in (7) consists of two parts: 1) the current 535 stage reward and 2) the expected future reward discounted by 536 α_i . Note that SU *i* cannot independently determine the above 537 value without explicitly knowing the policies and states of other 538 SUs. The SU maximizes the total discounted sum of future 539 rewards to select the bidding policy, which explicitly considers 540

$$p_{i}^{\text{buf}}\left(v_{i}^{t+1}|v_{i}^{t},\boldsymbol{z}_{i}^{t}\right) = \begin{cases} \frac{(\mu_{i}\Delta T)^{h}e^{-\mu_{i}\Delta T}}{h!}, & \text{if } 0 \leq h < X_{i} - \left(v_{i}^{t} - n\left(s_{i}^{t},\boldsymbol{z}_{i}^{t}\right)\right)^{+}\\ \sum_{k=h}^{\infty} \frac{(\mu_{i}\Delta T)^{k}e^{-\mu_{i}\Delta T}}{k!}, & \text{if } h = X_{i} - \left(v_{i}^{t} - n\left(s_{i}^{t},\boldsymbol{z}_{i}^{t}\right)\right)^{+} \end{cases}$$
(5)

541 the impact of the current bid vector on the expected future 542 rewards. We define the *best response* β_i for SU *i* to other SUs' 543 policies π_{-i} as

$$\beta_i(\boldsymbol{\pi}_{-i}) = \operatorname*{arg\,max}_{\boldsymbol{\pi}_i} Q_i^t \left(\boldsymbol{s}^t, \boldsymbol{y}^t, (\boldsymbol{\pi}_i, \boldsymbol{\pi}_{-i}) \right). \tag{8}$$

The central issue in our stochastic game is how the best-544 545 response policies can be determined by the SUs. In the repeated 546 auction mechanism discussed in Section III, the procedure that 547 each SU i follows to compete for the channel opportunities is 548 illustrated in Fig. 3. In this procedure, the bidding strategy π_i^t is 549 continuously improved by the "bidding strategy improvement" 550 module. In Section V-B, we discuss the challenges involved in 551 building such a module, and in Section VI, we develop a best-552 response learning algorithm that can be used to improve the 553 bidding strategy.

554 B. Challenges for Selecting the Best-Response 555 Bidding Policy

556 Recall that during each time slot, the CSM announces an 557 auction based on the available TxOps, and then SUs bid for 558 the resources. To enable the successful deployment of this 559 resource auction mechanism, we can prove (similar to our 560 prior work in [21]) that SUs have no incentive to misrepresent 561 their information, i.e., they adhere to the "truth telling" policy. 562 We assume that at each time slot t, SU i has preference u_{ij}^t 563 over the channel j, which captures the benefit derived when 564 using that channel. The preference u_{ij}^t is interpreted as the 565 benefit obtained by SU i when using channel j compared to the 566 benefit when this channel is not used. Note that this benefit also 567 includes the expected future rewards. The optimal bid $a_{ij}^{t,\text{opt}}$ 568 that SU i can take on channel j at time t is the bid maximizing 569 the net benefit $u_{ij}^t + \tau_i^t$. In the auction discussed in Section III, 570 the optimal bid that SU *i* can make is $a_{ij}^{t,\text{opt}} = u_{ij}^t$, i.e., the 571 optimal bid for SU i is to announce its true preference to the 572 CSM [21]. The proof is omitted here due to space limitations, 573 since it is similar to that in [21]. The payment made by SU i is

computed by the CSM based on the inconvenience incurred by 574 other SUs due to SU *i* during that time slot [23]. 575

Next, we define the preference u_{ij}^t in the context of the 576 stochastic game model. Using the channel j, SU i obtains 577 the immediate gain $g_i^t(s_i^t, \boldsymbol{y}^t, \boldsymbol{e}_j)$ by transmitting the pack- 578 ets in its buffer, where e_j indicates that channel j is al- 579 located to SU *i* during the current time slot. SU *i* then 580 moves into the next state s_i^{t+1} from which it may ob-581 tain the future reward $Q_i^{t+1}(s^{t+1}, y^{t+1}, \pi)$. On the other 582 hand, if no channel is assigned to SU i, it receives the 583 immediate gain $g_i^t(s_i^t, \boldsymbol{y}^t, \boldsymbol{0})$ and then moves into the next 584 state s_i^{t+1} , from which it may obtain the future reward 585 $Q_{i}^{t+1}(s^{t+1}, y^{t+1}, \pi)$. We define a feasible set of channel as- 586 signments to SU i's opponents (given SU i's channel allocation 587 \mathcal{Z}_{i}^{t} as $\mathcal{Z}_{-i}^{t}(\boldsymbol{z}_{i}^{t})$, with $\mathcal{Z}_{-i}^{t}(\boldsymbol{z}_{i}^{t}) = \{Z_{-i}^{t}|\sum_{k=1,k\neq i}^{M} z_{kj}^{t} = y_{j}^{t} - 588$ $z_i^t \forall j, \sum_{j=1}^N z_{kj}^t \le 1 \ \forall k \neq i, \quad z_{kj}^t \in \{0,1\}\}.$ 589 The preference over the current state can then be computed as 590

$$u_{ij}^{t}(\boldsymbol{s}^{t}, \boldsymbol{y}^{t}) = \left[g_{i}^{t}(\boldsymbol{s}_{i}^{t}, \boldsymbol{y}^{t}, \boldsymbol{e}_{j}) + \alpha_{i} \sum_{\boldsymbol{y}^{t+1} \in \{0,1\}^{N}} \\ \times \left[q_{i}(\boldsymbol{s}_{i}^{t+1}, \boldsymbol{y}^{t+1} | \boldsymbol{s}_{i}^{t}, \boldsymbol{y}^{t}, \boldsymbol{e}_{j}) \sum_{Z_{-i}^{t} \in \mathcal{Z}_{-i}^{t}(\boldsymbol{e}_{j})} \\ \times \left[\prod_{k=1}^{M} q_{k}(\boldsymbol{s}_{k}^{t+1}, \boldsymbol{y}^{t+1} | \boldsymbol{s}_{k}^{t}, \boldsymbol{y}^{t}, \boldsymbol{z}_{k}^{t}) Q_{i}^{t+1}(\boldsymbol{s}^{t+1}, \boldsymbol{y}^{t+1}, \boldsymbol{\pi}) \right] \right] \right] \\ - \left[g_{i}^{t}\left(\boldsymbol{s}_{i}^{t}, \boldsymbol{y}^{t}, \boldsymbol{0} \right) + \alpha_{i} \sum_{\boldsymbol{y}^{t+1} \in \{0,1\}^{N}} \\ \times \left[q_{i}(\boldsymbol{s}_{i}^{t+1}, \boldsymbol{y}^{t+1} | \boldsymbol{s}_{i}^{t}, \boldsymbol{y}^{t}, \boldsymbol{0}) \sum_{Z_{-i}^{t} \in \mathcal{Z}_{-i}^{t}(\boldsymbol{0})} \\ \times \left[\prod_{k=1}^{M} q_{k}(\boldsymbol{s}_{k}^{t+1}, \boldsymbol{y}^{t+1} | \boldsymbol{s}_{k}^{t}, \boldsymbol{y}^{t}, \boldsymbol{z}_{k}^{t}) Q_{i}^{t+1}(\boldsymbol{s}^{t+1}, \boldsymbol{y}^{t+1}, \boldsymbol{\pi}) \right] \right] \right].$$
(9)

$$Q_{i}^{t}(\boldsymbol{s}^{t},\boldsymbol{y}^{t},\boldsymbol{\pi}) = \sum_{k=t}^{\infty} (\alpha_{i})^{k-t} r_{i}\left(\boldsymbol{s}^{k},\boldsymbol{y}^{k},\boldsymbol{\pi}(\boldsymbol{s}^{k},\boldsymbol{y}^{k})\right) = \underbrace{r_{i}\left(\boldsymbol{s}^{t},\boldsymbol{y}^{t},\boldsymbol{\pi}(\boldsymbol{s}^{t},\boldsymbol{y}^{t})\right)}_{\text{stage reward at time t}} + \alpha_{i} \underbrace{\sum_{\boldsymbol{y}^{t+1} \in S}_{\boldsymbol{y}^{t+1} \in \{0,1\}^{N}} \left\{ \prod_{k=1}^{M} q_{k}\left(\boldsymbol{s}_{k}^{t+1},\boldsymbol{y}^{t+1}|\boldsymbol{s}_{k}^{t},\boldsymbol{y}^{t},\boldsymbol{z}_{k}^{t}\left(\boldsymbol{\pi}(\boldsymbol{s}^{t},\boldsymbol{y}^{t})\right) \times Q_{i}^{t+1}(\boldsymbol{s}^{t+1},\boldsymbol{y}^{t+1},\boldsymbol{\pi}) \right\}}_{\text{expected future reward}} = \left\{ \underbrace{g_{i}^{t}\left(\boldsymbol{s}_{i}^{t},\boldsymbol{y}^{t},\boldsymbol{z}_{i}^{t}\left(\boldsymbol{\pi}(\boldsymbol{s}^{t},\boldsymbol{y}^{t})\right)\right) + \tau_{i}^{t}\left(\boldsymbol{\pi}(\boldsymbol{s}^{t},\boldsymbol{y}^{t})\right)}_{\text{stage reward at time t}} + \alpha_{i} \underbrace{\sum_{\boldsymbol{y}^{t+1} \in \{0,1\}^{N}}^{\boldsymbol{s}^{t+1} \in S} \left\{ \prod_{k=1}^{M} q_{k}\left(\boldsymbol{s}_{k}^{t+1},\boldsymbol{y}^{t+1}|\boldsymbol{s}_{k}^{t},\boldsymbol{y}^{t},\boldsymbol{z}_{k}^{t}\left(\boldsymbol{\pi}(\boldsymbol{s}^{t},\boldsymbol{y}^{t})\right) \times Q_{i}^{t+1}(\boldsymbol{s}^{t+1},\boldsymbol{y}^{t+1},\boldsymbol{\pi}) \right\}}_{(7)} \right\}$$

expected future reward



Fig. 3. Procedure for SU i to play the auction game at time slot t.

591 From this equation, it is clear that the true value u_{ij}^t depends 592 not only on its own current state s_i^t but also on the other SUs' 593 states s_{-i}^t , the channel allocations $\mathcal{Z}_{-i}^t(e_j)$ to the other users 594 when channel j is assigned to SU i, $\mathcal{Z}_{-i}^t(\mathbf{0})$ when SU i is 595 not assigned to any channel, and the state transition models 596 $q_k(s_k^{t+1}, \mathbf{y}^{t+1} | s_k^t, \mathbf{y}^t, \mathbf{z}_k^t) \forall k$. However, the other SUs' states, 597 the channel allocations, and the state transition models of other 598 SUs are not known to SU i, and it is, thus, impossible for each 599 SU to determine its preference $u_{ij}^t(\mathbf{s}^t, \mathbf{y}^t)$.

600 Without knowing the other SUs' states and state transition 601 models, SU *i* cannot derive its optimal bidding strategy 602 $a_{ij}^{t,\text{opt}} = u_{ij}^t(s^t, y^t)$. However, if SU *i* chooses the bid 603 vector by only maximizing the immediate reward $g_i^t + \tau_i^t$, 604 i.e., the total discounted sum of reward degenerates in 605 $Q_i^t(s^t, y^t, \pi) = g_i^t(s_i^t, y^t, z_i^t(\pi(s^t, y^t))) + \tau_i^t(\pi(s^t, y^t))$ by 606 setting $\alpha_i = 0$. Then, the preference over channel *j* becomes 607 $u_{ij}^t(s^t, y^t) = g_i^t(s_i^t, y^t, e_j) - g_i^t(s_i^t, y^t, 0)$. Now, since u_{ij}^t 608 only depends on the state s_i^t , SU *i* can compute both the 609 optimal bid vector and the optimal bidding policy. We refer to 610 this optimal bidding policy as the "myopic" policy since it only 611 takes the immediate reward into consideration and ignores the 612 future impact. The myopic policy is referred to as π_i^{myopic} . To solve the difficult problem of optimal bidding policy selection 613 when $\alpha_i \neq 0$, an SU needs to forecast the impact of its current 614 bidding actions on the expected future rewards discounted by 615 α_i . The forecast can be performed using learning from its past 616 experiences. 617

A. How to Evaluate Learning Algorithms? 620

Section V-B shows that an SU needs to know the other SUs' 621 states and state transition models to derive its own optimal 622 bidding policy. This coupling among SUs is due to the shared 623 nature of wireless resources. However, an SU cannot exactly 624 know the other SUs' models and private information in wireless 625 networks. Thus, to improve the bidding policy, an SU can only 626 predict the impacts of dynamics (uncertainties) caused by the 627 competing SUs based on its observations from past auctions. 628 In this paper, we propose a learning algorithm for predicting 629 these impacts. We define a learning algorithm \mathcal{L}_i for SU *i* as 630 a function taking the observation o_i^t as input and having the 631 bidding policy π_i^t as output. 633 Before developing a learning algorithm, we first discuss how 634 to evaluate the performance of a learning algorithm in terms 635 of its impact on the SU's reward. Unlike existing multiagent 636 learning research, which is aimed at achieving converge to an 637 equilibrium point for the interacting agents, we develop learn-638 ing algorithms based on the performance of the bidding strategy 639 on the SU's reward. We denote a bidding policy generated by 640 the learning algorithm \mathcal{L}_i as $\pi_i^{\mathcal{L}_i}$. An SU will learn to improve 641 its bidding policy and its rewards from participating in the 642 auction game. The performance of the bidding strategy π_i is 643 defined as the time average reward that SU *i* obtains in a time 644 window with length *T* when it adopts π_i , i.e.,

$$\mathcal{V}^{\pi_i}(T) = \frac{1}{T} \sum_{k=1}^T r_i^k.$$
 (10)

645 Using this definition, the performance of two learning al-646 gorithms can easily be compared. For instance, given two 647 algorithms \mathcal{L}'_i and \mathcal{L}''_i , if $\mathcal{V}^{\pi_i^{\mathcal{L}'_i}} > \mathcal{V}^{\pi_i^{\mathcal{L}''_i}}$, then we say that the 648 learning algorithm \mathcal{L}'_i is better than \mathcal{L}''_i .

649 B. What Information to Learn From?

650 First, let us consider what information the SU can 651 observe while playing the stochastic game in our SN. As 652 shown in Fig. 1, at the beginning of time slot t, the SUs 653 submit the bids $a_i^t \forall i$. Then, the CSM returns the channel 654 allocations $z_i^t \forall i$ and $\tau_i^t \forall i$. If SU i is not allowed to 655 observe the bids, the channel allocations, and payments 656 for other SUs, then the observation of SU i becomes $o_i^t =$ 657 $\{s_i^0, y^0, a_i^0, z_i^0, \tau_i^0, \ldots, s_i^{t-1}, y^{t-1}, a_i^{t-1}, z_i^{t-1}, \tau_i^{t-1}, s_i^t, y^t\}$. If 658 the information is exchanged among SUs or broad-659 casted and overheard by all SUs, the observed infor-660 mation by SU i becomes $o_i^t = \{s_i^0, y^0, a^0, z^0, \tau^0, \ldots, 661 s_i^{t-1}, y^{t-1}, a^{t-1}, z_i^{t-1}, \tau^{t-1}, s_i^t, y^t\}$. Now, the problem that 662 needs to be solved by SU i is how it can improve its own policy 663 for playing the game by learning from the observation o_i^t . In 664 this paper, we assume that SU i observes the information $o_i^t =$ 665 $\{s_i^0, y^0, a_i^0, z_i^0, \tau_i^0, \ldots, s_i^{t-1}, y^{t-1}, a_i^{t-1}, z_i^{t-1}, \tau_i^{t-1}, s_i^t, y^t\}$.

666 C. What to Learn?

667 In Section VI-A, we introduce learning as a tool to predict the 668 impacts of dynamics and, hence, improve the bidding policy. 669 However, a key question is what needs to be learned. Recall that 670 the optimal bidding policy for SU *i* is to generate a bid vector 671 that represents its preferences for using different channels. 672 From (9), we can see that SU *i* needs to learn the following: 673 1) the state space of other SUs, i.e., S_{-i} ; 2) the current state of 674 other SUs, i.e., s_{-i}^t ; 3) the transition probability of other SUs, 675 i.e., $\prod_{k \neq i} q_k(s_k^{t+1}, \boldsymbol{y}^{t+1} | s_k^t, \boldsymbol{y}^t, \boldsymbol{z}_k^t)$; 4) the resource allocations 676 $\mathcal{Z}_{-i}^t(e_j) \forall j$ and $\mathcal{Z}_{-i}^t(\mathbf{0})$; and 5) the discounted sum of rewards 677 $Q_i^{t+1}(\boldsymbol{s}^{t+1}, \boldsymbol{y}^{t+1}, \boldsymbol{\pi})$.

678 However, SU *i* can only observe the information $o_i^t = \{s_i^0, 679 \ y^0, a_i^0, z_i^0, \tau_i^0, \dots, s_i^{t-1}, y^{t-1}, a_i^{t-1}, z_i^{t-1}, \tau_i^{t-1}, s_i^t, y^t\}$ from 680 which SU *i* cannot accurately infer the other SUs' state space 681 and transition probability. Moreover, capturing the exact in-

formation about other SUs requires heavy computational and 682 storage complexity. Instead, we allow SU *i* to classify the space 683 S_{-i} into H_i classes, each of which is represented by a representing 684 tative state $\tilde{s}_{-i,h}$, $h \in \{1, \ldots, H_i\}$. We discuss how the space 685 S_{-i} is decomposed in Section VI-D. By dividing the state space 686 S_{-i} , the transition probability $\prod_{k \neq i} q_k(s_k^{t+1}, \tilde{y}^{t+1} | s_k^t, y^t, z_k^t)$ 687 is approximated by $q_{-i}(\tilde{s}_{-i}^{t+1}, y^{t+1} | \tilde{s}_{-i}^t, y^t, z_i^t)$, where \tilde{s}_{-i}^t and 688 \tilde{s}_{-i}^{t+1} are the representative states of the classes to which s_{-i}^t and 689 s^{t+1} belong. This approximation is performed by aggregating 690 all the other SUs' states into one representative state and assum- 691 ing that the transition depends on the resource allocation z_{i}^{t} . 692 The transition probability approximation is also discussed in 693 Section VI-D. The discounted sum of rewards $Q_i^{t+1}(s^{t+1}, 694, y^{t+1}, \pi)$ is approximated by $V_i^{t+1}((s_i^{t+1}, \tilde{s}_{-i}^{t+1}), y^{t+1})$. 695 Note that the classification on the state space S_{-i} and the 696 approximation of the transition probability and discounted sum 697 of rewards affect the learning performance. Hence, a user can 698 tradeoff an increased complexity for an increased performance. 699 After the classification, the preference computation can be 700 approximated as 701

$$\begin{split} t_{ij}^{t} \left(\left(s_{i}^{t}, \tilde{s}_{-i}^{t} \right), \boldsymbol{y}^{t} \right) \\ = & \left[g_{i}^{t} q\left(s_{i}^{t}, \boldsymbol{y}^{t}, \boldsymbol{e}_{j} \right) + \alpha_{i} \sum_{\substack{\left(s_{i}^{t+1}, \tilde{s}_{-i}^{t+1} \right) \in \left(S_{i}, \tilde{S}_{-i} \right) \\ \boldsymbol{y}^{t+1} \in \left\{ 0, 1 \right\}^{N}} \right.} \\ & \times \left[q_{i} \left(s_{i}^{t+1}, \boldsymbol{y}^{t+1} | s_{i}^{t}, \boldsymbol{y}^{t}, \boldsymbol{e}_{j} \right) \times q_{-i} \left(\tilde{s}_{-i}^{t+1}, \boldsymbol{y}^{t+1} | \tilde{s}_{-i}^{t}, \boldsymbol{y}^{t}, \boldsymbol{e}_{j} \right) \right. \\ & \left. \times V_{i}^{t+1} \left(\left(s_{i}^{t+1}, \tilde{s}_{-i}^{t+1} \right), \boldsymbol{y}^{t+1} \right) \right] \right] \\ & \left. - \left[g_{i}^{t} \left(s_{i}^{t}, \boldsymbol{y}^{t}, \boldsymbol{0} \right) + \alpha_{i} \sum_{\substack{\left(s_{i}^{t+1}, s_{-i}^{t+1} \right) \in \left(S_{i}, \tilde{S}_{-i} \right) \\ \boldsymbol{y}^{t+1} \in \left\{ 0, 1 \right\}^{N}} \right. \\ & \times \left[q_{i} \left(\tilde{s}_{i}^{t+1}, \boldsymbol{y}^{t+1} | s_{i}^{t}, \boldsymbol{y}^{t}, \boldsymbol{0} \right) \times q_{-i} \left(\tilde{s}_{-i}^{t+1}, \boldsymbol{y}^{t+1} | \tilde{s}_{-i}^{t}, \boldsymbol{y}^{t}, \boldsymbol{0} \right) \right. \\ & \left. \times V_{i}^{t+1} \left(\left(s_{i}^{t+1}, \tilde{s}_{-i}^{t+1} \right), \boldsymbol{y}^{t+1} \right) \right] \right] . \end{split}$$

 \mathbf{A}

In this setting, to find the approximated preference and, 702 thus, the approximated optimal bidding policy, we need 703 to learn the following from past observations: 1) how 704 the space \tilde{S}_{-i} is classified; 2) the transition probability 705 $q_{-i}(\tilde{s}_{-i}^{t+1}, \boldsymbol{y}^{t+1} | \tilde{s}_{-i}^{t}, \boldsymbol{y}^{t}, \boldsymbol{z}_{i}^{t})$; and 3) the approximated future 706 rewards $V_{i}^{t+1}((s_{i}^{t+1}, \tilde{s}_{-i}^{t+1}), \boldsymbol{y}^{t+1})$. 707

D. How to Learn?

In this section, we develop a learning algorithm to estimate 709 the terms listed in Section VI-C. 710

708

1) Decomposition of the Space S_{-i} : As discussed 711 in Section VI-B, only $o_i^t = \{s_i^0, y^0, a_i^0, z_i^0, \tau_i^0, \dots, s_i^{t-1}, 712 y^{t-1}, a_i^{t-1}, z_i^{t-1}, \tau_i^{t}, y^t\}$ are observed. From the auction 713 mechanism presented in Section III, we know that the value of 714 715 the tax τ_i^t is computed based on the inconvenience that SU *i* 716 causes to the other SUs. In other words, a higher value of $|\tau_i^t|$ 717 indicates that the network is more congested.⁷ Based on the 718 bid vector \boldsymbol{b}_i^t , the channel allocation \boldsymbol{z}_i^t , and the tax τ_i^t , SU *i* 719 can infer network congestion and thus, indirectly, the resource 720 requirements of the competing SUs. Instead of knowing the 721 exact state space of other SUs, SU *i* can classify the space S_{-i} 722 as follows.

723 We assume that the maximum absolute tax is Γ . We split the 724 range $[0, \Gamma]$ into $[\Gamma_0, \Gamma_1), [\Gamma_1, \Gamma_2), \dots, [\Gamma_{H_i-1}, \Gamma_{H_i}]$ with 0 =725 $\Gamma_0 \leq \Gamma_1 \leq \dots \leq \Gamma_{H_i} = \Gamma$. Here, we assume that the values 726 of $\{\Gamma_1, \dots, \Gamma_{H_i-1}\}$ are equally located in the range of $[0, \Gamma]$. 727 (Note that more sophisticated selection for these values can be 728 deployed, and this forms an interesting area of future research.) 729 We need to consider three cases to determine the representa-730 tive state \tilde{s}_{-i}^t at time t.

1) If the resource allocation $z_i^t \neq 0$, then the representative state of the other SUs is chosen as

$$\tilde{s}_{-i}^t = h, \quad \text{if} \quad \left|\tau_i^t\right| \in [\Gamma_{h-1}, \Gamma_h).$$
(12)

2) If the resource allocation $z_i^t = 0$ but $y^t \neq 0$, the tax is 733 734 0. In this case, we cannot use the tax to predict network congestion. However, we can infer that the congestion 735 is more severe than the minimum bid for those avail-736 able channels, i.e., $\min_{j \in \{l: y_i^t \neq 0\}} \{a_{ij}^t\}$. This is because, 737 in this current stage of the auction game, only SU i' 738 with $a_{i'j}^t \ge a_{ij}^t$ can obtain channel j, which indicates 739 that $|\tau_i^t| \geq \min_{j \in \{l: y_i^t \neq 0\}} \{a_{ij}^t\}$ if SU i is allocated any 740 channel. Then, the representative state of the other SUs 741 is chosen as 742

$$\tilde{s}_{-i}^{t} = h, \quad \text{if} \quad \min_{j \in \{l: y_{l}^{t} \neq 0\}} \{a_{ij}^{t}\} \in [\Gamma_{h-1}, \Gamma_{h}). \quad (13)$$

743 3) If the resource allocation $z_i^t = 0$ and $y^t = 0$, there is 744 no interaction among the SUs in this time slot. Hence, 745 $\tilde{s}_{-i}^t = \tilde{s}_{-i}^{t-1}$.

⁷When the CSM deploys a mechanism without tax for resource management, the space classification for other SUs can also be done based on the announced information and corresponding resource allocation. 2) Estimating the Transition Probability: To estimate the 746 transition probability, SU *i* maintains a table *F* with size $H_i \times 747$ $H_i \times (N + 1)$. Each entry $f_{h',h'',j}$ in the table *F* represents the 748 number of transitions from state $\tilde{s}_{-i}^t = h''$ to state $\tilde{s}_{-i}^{t+1} = h' 749$ when the resource allocation $z_i^t = e_j$ (or **0** if j = 0). It is 750 clear that H_i will significantly influence the complexity and 751 memory requirements, etc., of SU *i*. The update of *F* is simply 752 based on the observation o_i^t and the state classification in the 753 above section. Then, we use the frequency to approximate the 754 transition probability [15], i.e., 755

$$q_{-i}\left(\tilde{s}_{-i}^{t+1} = h'|\tilde{s}_{-i}^{t} = h'', \boldsymbol{e}_{j}\right) = \frac{f_{h',h'',j}}{\sum_{h'} f_{h',h'',j}}.$$
 (14)

3) Learning the Future Reward: By classifying the state 756 space S_{-i} and estimating the transition probability, SU *i* 757 can now forecast the value of the average future reward 758 $V_i^{t+1}((s_i^{t+1}, \tilde{s}_{-i}^{t+1}), y^{t+1})$ using learning. Equation (7) can be 759 approximated by (15), shown at the bottom of the page. 760

Similar to the Q-learning established in [17], we also use 761 the received rewards to update the estimation of future rewards. 762 However, the main difference between our proposed algorithm 763 and Q-learning is that our solution explicitly considers the 764 impacts of other SUs' bidding actions through the state clas- 765 sifications and transition probability approximation. 766

We use a 3-D table to store the value $V_i((s_i, \tilde{s}_{-i}), \boldsymbol{y})$ with 767 $s_i \in S_i, \tilde{s}_{-i} \in \tilde{\boldsymbol{S}}_{-i}$. The total number of entries in V_i is $|S_i| \times$ 768 $H_i \times 2^N$. SU *i* updates the value of $V_i((s_i, \tilde{s}_{-i}), \boldsymbol{y})$ at time 769 *t* according to the rules in (16), shown at the bottom of the 770 page, where $\gamma_i^t \in [0, 1)$ is a learning rate factor satisfying 771 $\sum_{t=1}^{\infty} \gamma_i^t = \infty$, and $\sum_{t=1}^{\infty} (\gamma_i^t)^2 < \infty$ [17]. In summary, the 772 learning procedure that is developed for an SU is shown in 773 Table I. 774

E. Complexity of Learning

775

In Section III, we have discussed the computation complexity 776 incurred by the CSM and the communication cost between 777 the CSM and the SUs. In this section, we further quantify 778 the complexity of learning in terms of the computational and 779 storage burden. We use a floating-point operation ("flop") as a 780 measure of complexity, which will provide us an estimation of 781

$$Q_{i}^{t}\left(\left(s_{i}^{t},\tilde{s}_{-i}^{t}\right),\boldsymbol{y}^{t},\boldsymbol{\pi}\right) \doteq \left\{g_{i}^{t}\left(s_{i}^{t},\boldsymbol{y}^{t},\boldsymbol{z}_{i}^{t}\left(\boldsymbol{\pi}(\boldsymbol{s}^{t},\boldsymbol{y}^{t})\right)\right) + \tau_{i}^{t}\left(\boldsymbol{\pi}(\boldsymbol{s}^{t},\boldsymbol{y}^{t})\right) + \alpha_{i}\sum_{\substack{\left(s_{i}^{t+1},\tilde{s}_{-i}^{t+1}\right)\in\left(S_{i},\tilde{S}_{-i}\right)\\\boldsymbol{y}^{t+1}\in\left\{0,1\right\}^{N}}} \times \left\{q_{i}\left(s_{i}^{t+1},\boldsymbol{y}^{t+1}|s_{i}^{t},\boldsymbol{y}^{t},\boldsymbol{z}_{i}^{t}\left(\boldsymbol{\pi}(\boldsymbol{s}^{t},\boldsymbol{y}^{t})\right)\right)q_{-i}\left(\tilde{s}_{-i}^{t+1},\boldsymbol{y}^{t+1}|\tilde{s}_{-i}^{t},\boldsymbol{y}^{t},\boldsymbol{z}_{i}^{t}\left(\boldsymbol{\pi}(\boldsymbol{s}^{t},\boldsymbol{y}^{t})\right)\right)V_{i}^{t+1}\left(\left(s_{i}^{t+1},\tilde{s}_{-i}^{t+1}\right),\boldsymbol{y}^{t+1}\right)\right\}\right\}$$
(15)

$$V_{i}^{t}((s_{i},\tilde{s}_{-i}),\boldsymbol{y}) = \begin{cases} (1-\gamma_{i}^{t})V_{i}^{t-1}((s_{i},\tilde{s}_{-i}),\boldsymbol{y}) + \gamma_{i}^{t}Q_{i}^{t}((s_{i},\tilde{s}_{-i}),\boldsymbol{y},\boldsymbol{\pi}), & \text{if } (s_{i}^{t},\tilde{s}_{-i}^{t}) = (s_{i},\tilde{s}_{-i}), & \boldsymbol{y}^{t} = \boldsymbol{y} \\ V_{i}^{t-1}((s_{i},\tilde{s}_{-i}),\boldsymbol{y}), & \text{otherwise} \end{cases}$$
(16)



TABLE I LEARNING PROCEDURE

Fig. 4. Bidding strategies based on the required information.

782 the computational complexity required to perform the learning 783 algorithm. In addition, based on this, we can determine how 784 complexity grows with the increasing number of SUs [20]. At 785 each stage, SU performs the classification of other the SUs' 786 states, which, in the worst case, requires a number of "flops" of 787 approximately N. The number of "flops" to estimate the transi-788 tion probability of other SUs' states as in (14) is approximately 789 $(H_i + 1)$. The number of "flops" to learn the future reward 790 is approximately $(2|S_i|H_i + 6)$. Therefore, the total number 791 of "flops" incurred by the SU is $N + H_i + 2|S_i|H_i + 7$, from 792 which we can note that the complexity of learning for each SU 793 is proportional to the number of possible states of that SU and 794 the number of classes in which the other SUs' state space is 795 decomposed.

To perform the learning algorithm, the SU needs to store two 797 tables (i.e., transition probability table and state value table), 798 which, in total, have $(H_i^2(N+1) + 2^N |S_i|H_i)$ entries. We 799 also note that the storage complexity is also proportional to the 800 number of possible states of that SU and the number of classes 801 in which the other SUs' state space is decomposed.

802 VII. SIMULATION RESULTS

In this section, we aim at quantifying the performance of we are proposed stochastic interaction and learning framework. We sos assume that the SUs compete for available spectrum opportunities to transmit delay-sensitive multimedia data. First, we compare the performance of various bidding strategies. Next, we we quantify the performance of our proposed learning algorithm so9 in various network environments. We will only present here sto several illustrative examples. However, the same observations stor channels.

A. Various Bidding Strategies for Dynamic Multiuser Interaction

In this section, we highlight the merits of the stochastic 814 game framework proposed in Section II by comparing the 815 performance of different SUs, which deploy different bidding 816 strategies. The SUs are required to submit the bid vector on 817 the available channels. The SUs can deploy different bidding 818 strategies to generate their bid vector. 819

- 1) Fixed bidding strategy π_i^{fixed} : This strategy generates a 820 constant bid vector during each stage of the auction game, 821 irrespective of the state that SU *i* is currently in and of the 822 states other SUs are in. In other words, π_i^{fixed} does not 823 consider any of the dynamics defined in Section IV. 824
- 2) Source-aware bidding strategy π_i^{source} : This strategy gen- 825 erates various bid vectors by considering the dynamics in 826 source characteristics (based on the current buffer state) 827 but not the channel dynamics. 828
- 3) Myopic bidding strategy π_i^{myopic} : This strategy takes 829 into account the disturbance due to the environment as 830 well as the impact caused by other SUs, as discussed in 831 Section V-B. However, it does not consider the impact on 832 future rewards.
- 4) Bidding strategy based on best-response learning $\pi_i^{\mathcal{L}_i}$: 834 This strategy is produced using the learning algorithm 835 proposed in Section VI. $\pi_i^{\mathcal{L}_i}$ considers the two types of 836 dynamics defined in Section IV and the interaction impact 837 on future reward. 838

In terms of required information, the above bidding strategies 839 are illustrated in Fig. 4. For instance, the fixed bidding strategy 840 π_i^{fixed} does not require information about SU *i*'s state or other 841 SUs' states. The source-aware bidding strategy π_i^{buff} considers 842

812 813

1.7837

2.2967

PERFORMANCE OF SU I AND 2 WITH VARIOUS BIDDING STRATEGIES IN THE I WO SU NETWORKS								
		SU 1			SU 2			
	Bidding Strategies	Packet loss rate (%)	Average tax	Average cost	Packet loss rate (10%)	Average tax	Average cost	
Scenario 1	$\pi_1^{fixed}, \pi_2^{fixed}$	32.53	0.4875	2.8966	31.05	0.5095	2.6104	
Scenario 2	$\pi_1^{fixed}, \pi_2^{myopic}$	34.36	0.1222	2.6337	14.39	0.5495	1.5105	
Scenario 3	$\pi_1^{source}, \pi_2^{myopic}$	29.83	0.3147	2.4915	18.11	0.6048	1.6116	

0.4669

0.6923

1.9767

1.7428

19.55

27.29

0.3763

0.4197

21.55

15.14

 $1.\pi^{myop}$

 TABLE
 II

 Performance of SU 1 and 2 With Various Bidding Strategies in the Two SU Networks



Fig. 5. Accumulated packet loss and cost of SU 1 in the five scenarios. (a) Accumulated packet loss over the time slot. (b) Accumulated cost over the time slot.

843 the source characteristics based on the current buffer state. 844 However, the myopic bidding strategy π_i^{myopic} requires full 845 information about SU *i*'s state. The bidding strategy based on 846 best-response learning $\pi_i^{\mathcal{L}_i}$ also requires information about the 847 states of other SUs.

Scenario 4

Scenario 5

In this simulation, we consider the SN as an extension of WLANs with spectral agile capability [9]. In the following, we first simulate the case that two SUs compete for the channel opportunities and then extend to the case with multiple \$22 (five) SUs.

1) Competition Among Two SUs for Channel Opportunities: 854 We first consider a simple illustrative network with two SUs 855 competing for available TxOps. The packet arrivals of the SUs 856 are modeled using a Poisson process with the same average 857 arrival rate of 1 Mb/s. For simplicity of illustration, the channel 858 condition of SU 1 (SU 2) on each channel only takes three val-859 ues (K = 3), which are 18, 23, and 26 dB. The transition prob-860 abilities are $p_{ij}^{0\to 1} = p_{ij}^{0\to 2} = 0.4$, $p_{ij}^{0\to 3} = 0.2$, $p_{1j}^{l\to 1} = p_{1j}^{l\to 2} =$ 861 0.4, and $p_{1j}^{l\to 3} = 0.2 \forall i, j, l$. The transition probability of the 862 availability of channels to SUs is $p_j^{NF} = p_j^{FN} = 0.5$. For sim-863 plicity of illustration, the environment parameters experienced 864 by the two SUs are the same. The length of the time slot ΔT 865 is 10^{-2} s.

In this simulation, we consider five scenarios. In scenario 1, 867 both SU 1 and SU 2 deploy the fixed bidding strategy π_1^{fixed} . 868 In scenarios 2–5, SU 1 deploys the fixed bidding strategy 869 π_1^{fixed} , source-aware bidding strategy π_1^{source} , myopic bidding strategy π_1^{myopic} , and best-response learning-based bidding 870 strategy $\pi_1^{\mathcal{L}_1}$, respectively, and SU 2 always deploys the myopic 871 bidding strategy π_2^{myopic} . The discounted factor for the best- 872 response learning algorithm is set to 0.8. As discussed in 873 Section IV-B, the stage reward is defined as $r_i^t = (g_i^t + \tau_i^t)$, 874 with $(-g_i^t - \tau_i^t)$ being the number of packet lost plus the tax 875 charged by the CSM (note that $\tau_i^t \leq 0$). This can be interpreted 876 as the cost incurred at each stage. Similar to (10), we use the 877 average cost over the time window T = 1000 to evaluate the 878 performance of the bidding strategies. Hence, the lower 879 the average cost, the better the performance of the bidding 880 strategy. The packet loss rate, average tax, and cost per time slot 881 are presented in Table II. The accumulated packet loss and cost 882 of SU 1 for the five scenarios are plotted in Fig. 5(a) and (b), 883 respectively.

From this simulation, comparing scenario 2 with scenario 1, 885 we observe that when SU 2 deploys the myopic strategy against 886 SU 1, which adopted the fixed bidding strategy, SU 2 reduces 887 its average cost by around 42% and the average packet loss 888 rate by around 16.6%. This significant improvement is because 889 SU 2 can more accurately value the channel opportunities by 890 modeling and considering its experienced dynamics, i.e., source 891 characteristics, channel conditions, and availability. 892

In scenario 3, SU 1 improves its bidding strategy (i.e., 893 it deploys now a source-aware bidding strategy) by partially 894 considering its experienced environment, i.e., SU 1 generates 895 its bid vector by only considering the source dynamics though 896

24.20

1.2993

13

PERFORMANCE OF SU 1–5 WITH VARIOUS BIDDING STRATEGIES IN THE FIVE SU NETWORKS										
	SU	U 1	SU	J 2	S	U 3	S	U 4	S	U 5
	Packet		Packet		Packet		Packet		Packet	
	Loss	Average								
	Rate	cost								
	(%)		(%)		(%)		(%)		(%)	
1	21.14	1.2002	19.99	1.1666	22.05	1.2123	21.37	1.1949	24.17	1.3101

25.72

1.3338

26.02

1.3568

 TABLE III

 Performance of SU 1–5 With Various Bidding Strategies in the Five SU Networks

897 its current buffer state. Compared with scenario 2, if SU 1 898 considers more information about its own state, it can further 899 reduce its packet loss rate by an average of 4.5% and an 900 average cost by around 5.4%. This observation verifies that the 901 information about the SU's state improves the bidding strategy. 902 In scenario 4, SU 1 deploys a myopic bidding strategy, which 903 is more advanced than the source-aware bidding strategy since 904 it considers both types of dynamics defined in Section IV 905 (including the dynamics regarding the source characteristics, 906 channel conditions, and channel availability, and the interaction 907 with other SUs in the auction mechanism). The significant 908 improvement in terms of packet loss rate (13% reduced) and 909 average cost (25% reduced), compared with scenario 2, indi-910 cates that the myopic bidding strategy provides the optimal bid 911 vector when only current benefits are considered, as shown in 912 Section V-B.

25.03

1.2992

913 In scenario 5, SU 1 further improves the bidding strat-914 egy using the best-response learning algorithm developed in 915 Section VI. Using learning, SU 1 reduces the packet loss rate to 916 15.14% and the average cost to 1.7428 (11.8% lower compared 917 with scenario 4). This significant improvement is due to the 918 ability of the SU to learning and forecast the future impact of 919 its current actions.

It is also worth noting that the reduction of packet loss rate 921 of SU 1 in scenarios 2–5 comes from two parts: One is the 922 advanced bidding strategies, which allows the SU to take into 923 consideration more information about its own states and the 924 other SUs' states and, based on this better forecast, the impact 925 of various actions; the other one is the increase in the amount 926 of resources consumed by SU 1, which corresponds to a higher 927 tax charged by the CSM, as shown in Table II.

We further note that the bidding strategy deployed by SU 1 928 929 will affect the performance of SU 2. For example, comparing 930 scenario 2 with scenario 4, the fixed bidding strategy of SU 1 931 in scenario 2 leads to a lower average cost (15% reduced) for 932 SU 2. This is because SU 1 uses a fixed bidding strategy, which 933 does not account for the dynamic changes in its environment, 934 while SU 2 minimizes its current cost (the number of packets 935 lost plus the tax) based on its current state. However, when 936 comparing scenario 5 with scenario 4, SU 1 using learning 937 not only improves its prediction of the current environment 938 dynamics but also better predicts the impact on the future cost 939 based on the observations. The improvement leads to higher 940 resource allocation (hence, incurring higher tax, see in Table II) 941 for SU 1, thereby resulting in worse performance for SU 2 (i.e., 942 the average cost is increased by 22.2%).

943 2) Multiple SUs Competition for Channel Opportunities:
944 In this simulation, we consider five SUs competing for the
945 available TxOps in the WLAN-like SN. The packet arrivals of

all the five SUs are modeled using a Poisson process with the 946 same average arrival rate of 1 Mb/s. The number of channels 947 is 3, and the channel condition of all the five SUs on each 948 channel takes only three values (K = 3), which are 18, 23, 949 and 26 dB. The transition probabilities are $p_{ij}^{0\to1} = p_{ij}^{0\to2} = 0.4$, 950 $p_{ij}^{0\to3} = 0.2$, $p_{1j}^{L\to1} = p_{1j}^{L\to2} = 0.4$, and $p_{1j}^{L\to3} = 0.2 \forall i, j, l$. The 951 parameters of the model of the availability of the channels to 952 the SUs are $p_j^{NF} = 0.7$ and $p_j^{FN} = 0.3$. The length of the time 953 slot ΔT is also 10^{-2} s. Similar parameters are used for the five 954 SUs to clearly illustrate the performance differences obtained 955 based on the different strategies.

9.56

1.0988

In this simulation, we consider only two scenarios. In sce- 957 nario 1, all SUs deploy a myopic bidding strategy π_i^{myopic} , i = 958 1, 2, ..., 5, whereas in scenario 2, SU 5 deploys the multiuser 959 learning-based bidding strategy $\pi_5^{\mathcal{L}_5}$ with the discount factor 960 of 0.5, and the other SUs deploy the myopic bidding strategy 961 π_i^{myopic} , i = 1, ..., 4. The packet loss rate and cost per time slot 962 incurred by the SUs are presented in Table III. The accumulated 963 packet loss and cost of SU 5 for the five scenarios are plotted in 964 Fig. 6(a) and (b), respectively.

Similar to the two-SU network, SU 5 significantly reduces 966 the packet loss rate by 14.6% and average cost by 16.1% 967 by adopting the best-response learning-based bidding strategy. 968 Fig. 6(a) and (b) further verifies the improvement of the per- 969 formance for SU 1. However, the other SUs' performances are 970 decreased as they now need to compete against a learning SU 971 (i.e., SU 5), which is able to make better bids for the available 972 resources. 973

B. Multiuser Learning and Delay Impact in a974Wireless Test Bed975

To validate the performance of multiuser learning and the 976 impact of various delays in a realistic network setting, we 977 considered two SUs competing for the available TxOps in our 978 802.11a-enabled wireless test bed [31]. The channel condition 979 experienced by the SUs varied between 10 and 30 dB, and 980 we represented this variation using ten states (K = 10). The 981 parameters of the TxOp model are $p_j^{NF} = 0.6$ and $p_j^{FN} = 0.4$. 982 The length of the time slot ΔT is also 10^{-2} s. The SUs stream 983 the delay-sensitive video traffic (e.g., the Mobile sequence en- 984 coded using an H.264 video encoder) to their own destinations 985 with an average data rate of 1.5 Mb/s. We compare three 986 scenarios. In scenario 1, both SUs deploy a myopic bidding 987 strategy π_i^{myopic} , i = 1, 2. In scenario 2, SU 1 deploys the 988 learning-based bidding strategy $\pi_1^{\mathcal{L}_1}$ with a discount factor of 989 0.5, and SU 2 deploys a myopic strategy π_2^{myopic} . In scenario 3, 990 both SUs deploy the learning-based bidding strategy $\pi_i^{\mathcal{L}_i}$, i = 991 1, 2. In the mentioned three scenarios, video applications are 992



Fig. 6. Accumulated packet loss and cost of SU 5 in the two scenarios. (a) Accumulated packet loss over the time slot. (b) Accumulated cost over the time slot. TABLE IV

	Bidding	SL	J1	SU	J 2
	strategies	PSNR (dB)	Average cost	PSNR (dB)	Average cost
Scenario 1	$\pi_1^{myopic}, \pi_2^{myopic}$	30.8	5.8951	30.7	5.8845
Scenario 2	$\pi_1^{\mathcal{L}_1}, \pi_2^{myopic}$	33.0	5.3449	29.9	6.2236
Scenario 3	$\pi_1^{\mathcal{L}_1},\pi_2^{\mathcal{L}_2}$	31.8	5.6493	31.9	5.6536
Scenario 4	$\pi_1^{\mathcal{L}_1}, \pi_2^{myopic}$	31.2	7.5439	29.2	6.6748

993 considered to tolerate a delay⁸ of 533 ms, which is used in some 994 real-time video streaming applications. In scenario 4, SU 1 995 deploys the learning-based bidding strategy $\pi_1^{\mathcal{L}_1}$ with a discount 996 factor of 0.5, and SU 2 deploys a myopic strategy π_2^{myopic} . 997 However, in this scenario, SU 1 streams a video sequence that 998 can only tolerate a delay of 266 ms, which is typical for video 999 conferencing applications.

1000 Table IV shows the average video quality in terms of peak 1001 SNR (PSNR)⁹ and incurred cost for both SUs under various 1002 scenarios. Comparing scenario 2 with scenario 1, we observe 1003 that the SU using the learning-based bidding strategy improves 1004 the received video quality by 2.2 dB and reduces the incurred 1005 cost by 9.3%. However, as the performance of SU 1 improves, 1006 this also results in worse performance for SU 2. This observa-1007 tion is similar to the results in Section VII-A1 and has the same 1008 explanation.

1009 In scenario 3, both SUs deploy the learning-based bidding 1010 strategies and are able to better predict the impact of their 1011 current bidding actions on the future cost based on their ob-1012 servations. Thus, compared with scenario 1, the performance of 1013 both SUs has improved: SU 1 (SU 2) increases by 1 dB (1.2 dB) 1014 in terms of PSNR and reduces its cost by 4.3% (4.0%). Com-1015 pared to scenario 2, if SU 2 also deploys the learning-based 1016 approach, then SU 2 also observes its estimated future reward 1017 and will increase its bid, thereby reducing the performance of SU 1. From Table IV, we note that the PSNR of SU 1 is 1018 decreased by 1.2 dB, whereas the PSNR of SU 2 is increased 1019 by 2 dB. We also observe that the cost of SU 1 is increased by 1020 around 5.6%, whereas the cost of SU is decreased by 9.1%.

In scenario 4, since SU 1 streams a video application with a 1022 lower delay deadline, it has to bid more to ensure that packets 1023 with stringent delay deadline are transmitted to the destination, 1024 and hence, SU 1 incurs a higher transmission cost (41% 1025 increased) compared with scenario 2. Although SU 1 bids 1026 more for the limited available resources, the video quality of 1027 SU 1 is reduced by 1.8 dB due to its stringent delay deadline. 1028 Interestingly, the stringent delay deadline of the SU 1's 1029 application also increases the transmission cost of SU 2 and also 1030 reduces its video quality. This is because the higher bid of SU 1 1031 on limited resources automatically increases the bid of SU 2. 1032

C. Learning With Imperfect Information

In this section, we consider that SU 1 deploys the learning- 1034 based bidding strategy and SU 2 deploys the myopic strategy. 1035 The environment parameters are the same as in Section VII-B. 1036 To quantify the impact of imperfect information about the 1037 environment on SUs' performance, we assume that SU 1 has the 1038 transition probability of TxOps $(p_j^{NF} = 0.55 \text{ and } p_j^{FN} = 0.45)$, 1039 which is slightly different from the true one (i.e., $p_j^{NF} = 0.6$ 1040 and $p_j^{FN} = 0.4$). Table V shows the PSNRs and corresponding 1041 cost of both SUs when SU 1 has perfect or imperfect informa- 1042 tion about the TxOps.

1033

From Table V, we observe that an inaccurate model of TxOps 1044 reduces the performance of SU 1 (i.e., the PSNR decreases by 1045

⁸During the simulations, for simplicity, we assume that the packets within one Group of Picture (GOP) have the same delay deadline.

⁹PSNR is a widely adopted metric to objectively measure the video quality. A PSNR difference of 1 dB is significant and can be seen by an untrained human observer.

	Bidding	S	U 1	SU 2	
	strategies	PSNR (dB)	Average cost	PSNR (dB)	Average cost
Scenario 1 (SU 1 has perfect information)	$\pi_1^{\mathcal{L}_1}, \pi_2^{myopic}$	33.0	5.3449	30.7	6.2236
Scenario 2 (SU 1 has imperfect information)	$\pi_1^{\mathcal{L}_1}, \pi_2^{myopic}$	32.7	5.5685	30.5	6.4385

 $\begin{tabular}{ll} TABLE & V \\ Performance Comparison Between the Scenarios Whether SU 1 Has Perfect Information or Not \\ \end{tabular}$

TABLE VI
CHANNEL AVAILABILITY PROBABILITY

		Channel 1			Channel 2	
	NF	<i>"FN</i>	Number of	p_2^{NF}	p_2^{FN}	Number of
	p_1	p_1	opportunities			opportunities
Scenario 1	0.8	0.2	3502	0.8	0.2	3498
Scenario 2	0.5	0.5	2490	0.5	0.5	2462
Scenario 3	0.4	0.6	1960	0.4	0.6	1968

TABLE VII Average Packet Loss Rate and Cost for the SUs Under Various Resource Constraints

		SU	1	🔺 SU 2	
		Packet	Average	Packet	Average
		loss rate	cost	loss rate	cost
Saamania 1	$\pi_1^{myopic}, \pi_2^{myopic}$	3.08	0.2678	2.90	0.2844
Scenario I	$\pi_1^{\mathcal{L}_1}, \pi_2^{myopic}$	2.69	0.3092	4.17	0.4110
Soonaria 2	$\pi_1^{myopic}, \pi_2^{myopic}$	21.36	1.8954	23.85	1.7471
Scenario 2	$\pi_1^{\mathcal{L}_1}, \pi_2^{myopic}$	14.54	1.6764	30.67	2.1744
Scenario 3	$\pi_1^{myopic}, \pi_2^{myopic}$	45.01	3.6283	45.42	3.8289
	$\pi_1^{\mathcal{L}_1}, \pi_2^{myopic}$	35.21	3.2590	56.44	4.5162

1046 0.3 dB and increases the cost by 4.2%). We further note that this 1047 will also affect the performance of SU 2. In this simulation, the 1048 PSNR of SU 2 is reduced by 0.2 dB, and the cost is increased 1049 by 3.5%. This performance loss can be explained as follows. 1050 Since SU 1 has an inaccurate model about the available TxOps, 1051 it may generate a suboptimal bid vector at each stage, which 1052 will accordingly result in a suboptimal allocation (TxOps and 1053 payment) among the SUs. This suboptimal allocation will also 1054 lead to the performance loss of other SUs. Hence, it is essential 1055 for the users to learn and accurately predict their environment.

1056 D. Impact of Various Dynamics on Learning

1057 In Section VII-A, we demonstrated that the best-response 1058 learning algorithm improves the bidding strategy, thereby lead-1059 ing to a reduced packet loss rate and average cost. In this 1060 simulation, we further investigate how various dynamics impact 1061 the learning algorithm proposed in Section VI-D. Specifically, 1062 we compare the learning performance under different channel 1063 dynamics, i.e., various available spectrum opportunities for the 1064 SUs, as discussed in Section II. The source characteristics and 1065 channel conditions experienced by the SUs are kept the same as 1066 in Section VII-A1. We consider three types of channel dynam-1067 ics corresponding to scenarios 1-3. The transition probabilities 1068 of TxOps for all three scenarios are listed in Table VI. In each 1069 scenario, we compare two cases. In the first one, both SUs 1070 deploy myopic bidding strategies, and in the second one, SU 1071 1 deploys the best-response learning-based bidding strategy, 1072 while SU 2 still uses the myopic bidding strategy.

1073 Table VII shows the average packet loss rate and cost ex-1074 perienced by the SUs under various channel dynamics. Interestingly, we observe from these results that even though 1075 the learning algorithm reduces the packet loss rate, it does 1076 not reduce the cost associated with SU 1 when the channel 1077 resources are abundant as in scenario 1. As the resources 1078 become increasingly scarce, the learning algorithm helps SU 1 1079 to simultaneously reduce the packet loss rate and cost, e.g., 1080 in scenarios 2 and 3. This observation can be explained as 1081 follows. When the resources are abundant, the cost (including 1082 the packet loss and tax) is small, i.e., the "value" of the chan- 1083 nel is limited, and hence, the learning-based bidding strategy 1084 does not significantly benefit. On the other hand, when the 1085 resources are scarce, the bid vectors of the SUs in the current 1086 time slot will significantly affect the transition of their states 1087 through the channel allocation compared with the case when 1088 the resources are abundant. For example, if an SU makes low 1089 bids as compared to other SUs, it might have no resources 1090 (channels) allocated to it when resources are scarce (i.e., the SN 1091 is congested). In this case, the learning-based bidding strategy 1092 will carefully plan the bid by considering the future impact, and 1093 thus, it is able to successfully improve the performance of SU 1 1094 in terms of reducing the average cost. 1095

VIII. CONCLUSION AND FUTURE RESEARCH 1096

In this paper, we have modeled the dynamic resource allo- 1097 cation problem as a "stochastic game" played among strategic 1098 SUs. At each stage of the game, the CSM deploys a general- 1099 ized second-price auction mechanism to allocate the available 1100 spectrum resource. The SUs are allowed to simultaneously 1101 and independently make bid decisions on that resource by 1102

We note that the constraint of the perfect information about 1109 1110 the available wireless resources can be relaxed for the case 1111 when the CSM and wireless users do not have perfect infor-1112 mation about the available resources. In this case, the wire-1113 less users can estimate and build a belief about the available 1114 resource. Hence, the stochastic game model can be extended 1115 to partially observably stochastic games [32]. This is one of 1116 our interesting future research topics. We also note that we 1117 can allow the wireless users to adapt their transmission power, 1118 which will lead to different interference levels to other users. 1119 In this case, the wireless users compete with each other for 1120 lower interference levels incurred by other users [6] instead 1121 of competing for the transmission time. This can also be for-1122 mulated as a stochastic game, and similar learning algorithms 1123 can be developed. This forms another interesting topic of our 1124 future research. Our future work also includes analyzing the 1125 performance of SNs, where multiple SUs are deploying various 1126 learning strategies and protocols.

- 1127 REFERENCES
- 1128 [1] Fed. Commun. Comm., Spectrum Policy Task Force, Nov. 2002.
 1129 Rep. ET Docket No. 02-135.
- 1130 [2] S. Haykin, "Cognitive radio: Brain-empowered wireless communications," *IEEE J. Sel. Areas Commun.*, vol. 23, no. 2, pp. 201–220, Feb. 2005.
- [3] I. F. Akyildiz, W. Y. Lee, M. C. Vuran, and S. Mohanty, "NeXt generation/dynamic spectrum access/cognitive radio wireless networks: A survey," *Comput. Netw.*, vol. 50, no. 13, pp. 2127–2159, Sep. 2006.
- [4] C. Kloeck, H. Jaekel, and F. Jondral, "Auction sequence as a new resource allocation mechanism," in *Proc. VTC*, Dallas, TX, Sep. 2005, pp. 240–244.
- [5] F. Fu, A. R. Fattahi, and M. van der Schaar, "Game-theoretic paradigm for resource management in spectrum agile wireless networks," in *Proc. IEEE ICME*, 2006, pp. 873–876.
- 1142 [6] J. Huang, R. Berry, and M. L. Honig, "Auction-based spectrum sharing,"
 1143 ACM Mobile Netw. Appl. J. (MONET), vol. 11, no. 3, pp. 405–418,
 1144 Jun. 2006.
- 1145 [7] Y. Xing, R. Chandramouli, and C. M. Cordeiro, "Price dynamics in competitive agile spectrum access markets," *IEEE J. Sel. Areas Commun.*, vol. 25, no. 3, pp. 613–621, Apr. 2007.
- 1148 [8] L. Berlemann, S. Mangold, G. R. Hiertz, and B. H. Walke, "Policy defined spectrum sharing and medium access for cognitive radios," *J. Commun.*, vol. 1, no. 1, pp. 1–12, Apr. 2006.
- [9] C. T. Chou, S. Shankar N, H. Kim, and K. Shin, "What and how much to gain by spectrum agility?" *IEEE J. Sel. Areas Commun.*, vol. 25, no. 3, pp. 576–588, Apr. 2007.
- 1154 [10] S. Shankar, C. T. Chou, K. Challapali, and S. Mangold, "Spectrum agile radio: Capacity and QoS implications of dynamic spectrum assignment," in *Proc. Global Telecommun. Conf.*, Nov. 2005, pp. 2510–2516.
- 1157 [11] D. Bertsekas and R. Gallager, *Data Networks*. Upper Saddle River, NJ:
 1158 Prentice–Hall, 1987.
- 1159 [12] M. van der Schaar and S. Shankar, "Cross-layer wireless multimedia transmission: Challenges, principles, and new paradigms," *IEEE Wireless Commun.*, vol. 12, no. 4, pp. 50–58, Aug. 2005.
- 1162 [13] Wireless Medium Access Control (MAC) and Physical Layer (PHY) Specifications: Medium Access Control (MAC) Enhancements for Quality of
- 1164 Service (QoS), Draft Supplement, IEEE Std. 802.11e/D5.0, Jun. 2003. 1165 [14] R. W. Lucky, "Tragedy of the commons," *IEEE Spectr.*, vol. 43, no. 1,
- 1165 [14] R. W. Lucky, "Tragedy of the commons," *IEEE Spectr.*, vol. 43, no. 1, p. 88, Jan. 2006.
- 1167 [15] R. G. Gallager, *Discrete Stochastic Processes*. Norwell, MA: Kluwer, 1168 1996.

- [16] L. S. Shapley, "Stochastic games," Proc. Nat. Acad. Sci. U.S.A., vol. 39, 1169 no. 10, pp. 1095–1100, Oct. 1953. 1170
- [17] C. Watkins and P. Dayan, "Q-learning, technical note," *Mach. Learn.*, 1171 vol. 8, no. 3/4, pp. 279–292, May 1992.
- [18] M. Bowling and M. Veloso, "Rational and convergent learning in stochastic games," in *Proc. 17th IJCAI*, Aug. 2001, pp. 1021–1026. 1174
- [19] P. Klemperer, "Auction theory: A guide to the literature," J. Econ. Surv., 1175 vol. 13, no. 3, pp. 227–286, Jul. 1999.
- [20] S. P. Boyd and L. Vandenberghe, *Convex Optimization*. Cambridge, 1177 U.K.: Cambridge Univ. Press, 2004. 1178
- [21] F. Fu and M. van der Schaar, "Noncollaborative resource management for 1179 wireless multimedia applications using mechanism design," *IEEE Trans.* 1180 *Multimedia*, vol. 9, no. 4, pp. 851–868, Jun. 2007.
- [22] D. Fudenberg and D. K. Levine, *The Theory of Learning in Games*. 1182 Cambridge, MA: MIT Press, 1999. 1183
- [23] M. Jackson, "Mechanism theory," in Encyclopedia of Life Support 1184 Systems. Oxford, U.K.: EOLSS, 2003. 1185
- [24] Q. Zhang and S. A. Kassam, "Finite-state Markov model for Rayleigh 1186 fading channels," *IEEE Trans. Commun.*, vol. 47, no. 11, pp. 1688–1692, 1187 Nov. 1999.
- [25] A. Ortega, "Variable bit-rate video coding," in *Compressed Video Over* 1189 *Networks*, M.-T. Sun and A. R. Reibman, Eds. New York: Marcel 1190 Dekker, 2000, pp. 343–382. 1191
- [26] S. Lal and E. S. Sousa, "Distributed resource allocation for DS-CDMA- 1192 based multimedia ad hoc wireless LANs," *IEEE J. Sel. Areas Commun.*, 1193 vol. 17, no. 5, pp. 947–967, May 1999. 1194
- [27] R. S. Sutton and A. G. Barto, *Reinforcement Learning: An Introduction*. 1195 Cambridge, MA: MIT Press, 1998. 1196
- [28] M. Chiang, S. H. Low, A. R. Calderbank, and J. C. Doyle, "Layering as 1197 optimization decomposition: A mathematical theory of network architectures," *Proc. IEEE*, vol. 95, no. 1, pp. 255–312, Jan. 2007. 1199
- [29] F. Kelly, A. Maulloo, and D. Tan, "Rate control for communication net- 1200 works: Shadow prices, proportional fairness and stability," *J. Oper. Res.* 1201 Soc., vol. 49, no. 3, pp. 237–252, Mar. 1998.
- [30] X. Zhu, P. Agrawal, J. P. Singh, T. Alpcan, and B. Girod, "Rate allocation 1203 for multi-user video streaming over heterogeneous access networks," in 1204 *Proc. ACM MM*, Sep. 2007, pp. 37–46. 1205
- [31] D. Krishnaswamy and J. Vicente, "Scalable adaptive wireless networks 1206 for multimedia in the proactive enterprise," *Intel Technol. J.*, vol. 8, no. 4, 1207 pp. 291–301, Nov. 2004. [Online]. Available: http://developer.intel.com/ 1208 technology/itj/2004/volume08issue04/art04_scalingwireless/p01_abstract. 1209 htm 1210
- [32] D. S. Bernstein, E. A. Hansen, S. Zilberstein, and C. Amato, "Dynamic 1211 programming for partially observable stochastic games," in *Proc. AAAI* 1212 *Spring Symp. Bridging Multi-Agent Multi-Robot. Res. Gap*, Stanford, CA, 1213 Mar. 2004. 1214

Fangwen Fu received the B.S. and M.S. degrees from Tsinghua University, 1215 AQ3 Beijing, China, in 2002 and 2005, respectively. He is currently working toward 1216 the Ph.D. degree with the Department of Electrical Engineering, University of 1217 California at Los Angeles. 1218

During the summer of 2006, he was an Intern with the IBM T. J. Watson 1219 Research Center, Yorktown Heights, NY. His research interests include wireless 1220 multimedia streaming, resource management for networks and systems, applied 1221 game theory, and video processing and analysis. 1222

Mihaela van der Schaar (SM'04) received the Ph.D. degree from the 1223 AQ4 Eindhoven University of Technology, Eindhoven, The Netherlands, in 2001. 1224

She is currently an Associate Professor with the Department of Electrical 1225 Engineering, University of California, Los Angeles. She is the holder of 1226 30 granted U.S. patents and three ISO awards. Her research interests are in 1227 multimedia communications, networking, processing, and systems. 1228

Dr. van der Schaar received the National Science Foundation Career Award 1229 in 2004, the Best Paper Award from IEEE TRANSACTIONS ON CIRCUITS AND 1230 SYSTEMS FOR VIDEO TECHNOLOGY in 2005, the Okawa Foundation Award 1231 in 2006, the IBM Faculty Award in 2005, 2007, and 2008, and the Most Cited 1232 Paper Award from *EURASIP: Image Communications* in 2006. 1233