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Towards Efficient, Stable, and Fair Random Access Networks: A Conjectural Equilibrium Approach

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***Abstract*— For wireless LANs, such as IEEE 802.11 networks, the channel utilization *efficiency*, the system *stability*, and the *fairness* of bandwidth allocation are three important criteria for designing medium access control (MAC) protocols. This paper aims to design a simple access mechanism optimized for all the aforementioned issues from a *game theoretic* perspective. In particular, this paper enables nodes to form simple *internal belief functions* on how their competitors would react to their transmission actions. The steady-state outcome of this multi-user interaction can be characterized as a *conjectural equilibrium* (CE). We propose a distributed algorithm, *Conjecture-based Random Access* (CBRA), which enables nodes to independently update their transmission probabilities based on their internal beliefs and local observations. For CBRA, we first derive the sufficient conditions that guarantee its local stability and global convergence. We analytically show that all the achievable operating points in the throughput region are essentially stable CE corresponding to different belief initializations. Moreover, we show that CBRA approximately achieves the weighted fairness for the nodes carrying different traffic classes. Numerical simulations verify that the system performance significantly outperforms existing protocols, such as the 802.11 DCF and the priority-based fair medium access control (P-MAC) protocol, in terms of throughput, fairness, convergence, and stability.**

I. Introduction

There are three key issues to be addressed when designing the medium access control (MAC) protocols in wireless channels. First, how can we schedule the transmissions of different devices in order to operate the network close to its capacity and maximize its efficiency? Second, how can we stabilize the entire network to sustain the desired operating points to combat the randomness in traffic loads, channel conditions, etc? Finally, how should we design the protocols such that the devices can access the wireless medium in a fair manner? More importantly, from a practical point of view, it is desirable to design distributed protocols that can improve the system performance (in terms of efficiency, stability, and fairness), while minimizing the required information exchange among different devices. In addition, the complexity of the adopted protocol should be reasonable and limited to allow for lowcost implementations. The goal of this paper is to show how a simple game theoretic concept, i.e. conjectural equilibrium, can inspire the design of a constructional framework aimed at simultaneously achieving efficiency, stability, and fairness in the wireless random access networks with no need for any real-time information exchange among the wireless devices. The main contributions of this paper are as follows.

First, to cultivate cooperation and sustain efficiency in random access networks, we enable autonomous nodes to form independent local beliefs about how their probabilities of experiencing an idle channel vary as a linear function of their own actions. The steady state of such a play among belief-forming devices can be characterized as a conjectural equilibria (CE). We design a simple distributed algorithm, called *Conjecture-based Random Access* (CBRA), in which all the nodes' beliefs and actions will be dynamically revised by observing the outcomes of past mutual interaction over time. We investigate the stability of different operating points and derive sufficient conditions that guarantee their global convergence and prove that all the operating points in the throughput region are stable CE. Second, we investigate the relationship between the parameter initialization of beliefs and the efficiency of the achieved CE if deploying CBRA. Importantly, it is shown that the CBRA algorithm can operate stably and arbitrarily close to the Pareto boundary of the throughput region while approximately maintaining the weighted fairness across the entire network. Therefore, our investigation provides useful insights that help define convergent dynamic adaptation schemes that are apt to drive distributed random access networks towards efficient, stable, and fair configurations.

In recent years, game theory has been extensively applied to study random access control, see, e.g. [5]- [11]. First of all, the existence, stability, and convergence of noncooperative Nash equilibria have been studied in both Aloha and CSMA/CA networks [5]- [8]. The existence of and convergence to the Nash equilibrium in random access games have also been studied in other scenarios, where individual nodes have various utility functions. Several recent works also investigate how to design new distributed algorithms that provably converge to the Pareto boundary of the network throughput region [8]- [11]. Furthermore, it is shown in [11] that network utility maximization in random access networks can be achieved without real-time message passing among nodes. The key idea is to estimate the other nodes' transmission probabilities from local observations, which in fact increases the internal computational overhead of individual nodes. To address the fairness issue, the P-MAC protocol is proposed in [14] to maximize the utilization subject to the weighted fairness among multiple nodes in a single cell. The performance improvement is achieved by estimating the number of contending stations and setting the transmission probability to an optimal value based on the estimation.

The paper is organized as follows. Section II presents the system model and introduces the concept of CE. Section III develops the CBRA algorithm and establishes its stability, efficiency, and fairness. We provide numerical simulations in Section IV. Conclusions are drawn in Section V. Due to space limitations, the formal proofs are omitted; for these proofs, the reader is referred to [17].

II. System Model and Conjectural Equilibrium

A. System Model of Random Access Networks

We model the interaction among multiple autonomous wireless nodes in random access networks as a random access game. Consider a set $\mathcal{K} = \{1, 2, \ldots, K\}$ of wireless nodes and each node represents a transmitter-receiver pair (link). We define Tx_k as the transmitter node of link k and Rx_k as the receiver node of link k. We first assume a single-cell wireless network, where every node can hear every other node in the network. The system operates in discrete time with evenly spaced time slots. We assume that all nodes always have a data packet to transmit at each time slot and packet loss occurs only due to collision. The action of a node in this game is to select its transmission probability and a node k will independently attempt transmission of a packet with transmit probability p_k. The action set available to node k is $P_k = [0, 1]$ for all $k \in \mathcal{K}$[1]. Once the nodes decide their transmission probabilities based on which they transmit their packets, an action profile is determined. We denote the action profile in the random access game as a vector $\mathbf{p} = (p_1, \ldots, p_K)$ in $P = P_1 \times \cdots \times P_K$. Then the throughput of node k is given by[2]

$$u_k(\mathbf{p}) = p_k \prod_{i \in \mathcal{K} \setminus \{k\}} (1 - p_i). \tag{1}$$

To capture the performance tradeoff in the network, the throughput (payoff) region is defined as

$$\mathscr{T} = \{(u_1(\mathbf{p}), \ldots, u_K(\mathbf{p})) | \; \exists \; \mathbf{p} \in P\}.$$

The random access game can be formally defined by the tuple $\Gamma = \langle \mathcal{K}, (P_k), (u_k) \rangle$ [1]. Denote the transmission probability for all nodes but k by $\mathbf{p}_{-k} = (p_1, \ldots, p_{k-1}, p_{k+1}, \ldots, p_K)$. From (1), we can see that node k's throughput depends not only on its own transmission probability p_k, but also the other nodes' transmission probabilities $\mathbf{p}_{-k}$.

In the random access game, one of the most investigated problems is whether or not a Nash equilibrium exists. The definition of Nash equilibrium is given as follows [1].

Definition 1: A profile $\mathbf{p}$ of actions is a *Nash equilibrium* of Γ if $u_k(p_k, \mathbf{p}_{-k}) \geq u_k(p'_k, \mathbf{p}_{-k})$ for all $p'_k \in P_k$ and $k \in \mathcal{K}$.

The Nash equilibrium of the investigated random access game has been addressed in the similar context of CSMA/CA networks where selfish nodes deliberately control their random deferment by altering their contention window sizes [8]. Specifically, the transmission probability p_k in our model can be related to the contention window CW_k in the CSMA/CA protocol, where $p_k = \frac{2}{1+CW_k}$. It has been shown in [8] that at the Nash equilibrium, at least one selfish node will set $CW_k = 1$ (i.e. always transmit). If more than one selfish node sets its contention window to 1, it will cause zero throughput for all the nodes in the system. This result is known as *the tragedy of the commons*. Novel mechanisms are required to encourage cooperation among the selfish devices.

B. Conjectural Equilibrium

Players in games operating at equilibrium can be viewed as decision makers behaving optimally with respect to their *beliefs* about the strategies of other players.

To rigorously define CE, we need to include two new elements $\mathcal{S}$ and s and, based on this, reformulate the random access game $\Gamma' = \Big(\mathcal{K}, (P_k), (u_k), (\mathcal{S}_k), (s_k)\Big)$ [12]. $\mathcal{S} = \times_{k \in \mathcal{K}} \mathcal{S}_k$ is the *state space*, where $\mathcal{S}_k$ is the part of the state relevant to the node k. Specifically, the state in the random access game is defined as the contention probability that nodes experience. The utility function u_k is a map from the nodes' state space to real numbers, $u_k : \mathcal{S}_k \times P_k \rightarrow \mathcal{R}$. The *state determination function* $s = \times_{k \in \mathcal{K}} s_k$ maps joint action to state with each component $s_k : P \rightarrow \mathcal{S}_k$. Each node cannot directly observe the actions (transmission probabilities) chosen by the others, and each node has some belief about the state that would result from performing its available actions. The *belief function* $\tilde{s}_k$ is defined to be $\tilde{s}_k : P_k \rightarrow \mathcal{S}_k$ such that $\tilde{s}_k(p_k)$ represents the state that node k believes it would result in if it selects action p_k . Notice that the beliefs are not expressed in terms of other nodes' actions and preferences, and the multi-user coupling in these beliefs is captured directly by individual nodes forming conjectures of the effects of their own actions. Moreover, each node chooses the action $p_k \in P_k$ if it believes that this action will maximize its utility.

Definition 2: In the game Γ' defined above, a configuration of belief functions $(\tilde{s}^*_1, \ldots, \tilde{s}^*_K)$ and a joint action $p^* = (p^*_1, \ldots, p^*_K)$ constitute a *conjectural equilibrium*, if for each $k \in \mathcal{K}$, $\tilde{s}^*_k(p^*_k) = s_k(p^*_1, \ldots, p^*_K)$ and $p^*_k = \arg\max_{p_k \in P_k} u_k(\tilde{s}^*_k(p_k), p_k)$.

From the above definition, we can see that, at CE, all nodes' expectations based on their beliefs are realized and each node behaves optimally according to its expectation. The key challenges are how to configure the belief functions such that cooperative behavior is encouraged.

III. Conjecture-based Random Access

In this section, we enable each node to configure its belief about its expected contention as a linear function of its own transmission probability. We show that all the operating points in the throughput region $\mathscr{T}$ are essentially CE. Furthermore, we propose a distributed algorithm for these nodes to dynamically achieve the CE and provide the sufficient conditions that guarantee its stability and convergence. Finally, we prove the stability of the throughput region $\mathscr{T}$.

[1]The action set can be alternatively defined to be $P_k = [P^{\min}_k, P^{\max}_k]$ and the analysis in this paper still applies.

[2]The throughput can be alternatively expressed as $u_k(\mathbf{p}) = r_k p_k \prod_{i \in \mathcal{K} \setminus \{k\}} (1 - p_i)$ in which r_k denotes the fixed peak data rate for node k. The constant term r_k is dropped here for simplicity. The throughput formula that considers unequal intervals by incorporating the PHY and MAC layers' parameters will be presented in Section III.C.

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A. Individual Behavior

Both the state space and belief functions need to be defined in order to investigate the existence of CE. In the random access game, we define the state $s_k = \prod_{i \in \mathcal{K} \setminus \{k\}} (1 - p_i)$ to be the contention measure signal representing the probability that all nodes except node k do not transmit. This is because besides its own transmission probability, its throughput only depends on the probability that the remaining nodes do not transmit. State s_k indicates the aggregate effects of the other nodes' joint actions on node k's payoff. In practice, it is hard for wireless nodes to estimate the transmission probabilities of their opponents [11]. Therefore, we assume that s_k is the only information that node k has about the contention level of the entire network, because it is a metric that node k can easily compute based on local observations. Notice that the action available to node k is to choose the transmission probability $p_k \in P_k$. By the definition of belief function, we need to express the expected contention measure $\tilde{s}_k$ as a function of its own transmission probability p_k. The simplest approach is to deploy the following linear belief models

$$\tilde{s}_k(p_k) = \bar{s}_k - a_k(p_k - \bar{p}_k), \tag{2}$$

for $k \in \mathcal{K}$. The values of $\bar{s}_k$ and $\bar{p}_k$ are specific states and actions, called *reference points* [2] and a_k is a positive scalar. In other words, node k assumes that other nodes will observe its deviation from its reference point $\bar{p}_k$ and the aggregate contention probability deviates from the referent point $\bar{s}_k$ by a quantity proportional to the deviation of $p_k - \bar{p}_k$. How to configure $\bar{s}_k, \bar{p}_k$, and a_k will be addressed in the rest of this paper. As we will show later in Section III.C, building and optimizing over such simple beliefs is sufficient for the network to achieve almost any operating point in the throughput region as a stable CE.

The goal of node k is to maximize its expected throughput $p_k \cdot \tilde{s}_k(p_k)$ taking into account the conjectures that it has made about the other nodes. Therefore, the optimization a node needs to solve becomes:

$$\max_{p_k \in P_k} p_k \Big[\bar{s}_k - a_k(p_k - \bar{p}_k) \Big]. \tag{3}$$

For $a_k > 0$, node k believes that increasing its transmission probability will increase its experienced contention probability. The optimal solution of (3) is given by

$$p_k^* = \min\Big\{ \frac{\bar{s}_k}{2a_k} + \frac{\bar{p}_k}{2}, 1 \Big\}. \tag{4}$$

The following theorem indicates that forming simple linear beliefs in (2) can cause all the operating points in the achievable throughput region to be CE.

Theorem 1: All the operating points in the throughput region $\mathscr{T}$ are essentially conjectural equilibria.

Theorem 1 establishes the existence of CE, i.e. for a particular $\mathbf{p}^* \in P$, how to choose the parameters $\{\bar{s}_k, \bar{p}_k, a_k\}_{k=1}^K$ such that $\mathbf{p}^*$ is a CE. However, it neither tells us how these CE can be achieved and sustained in the dynamic setting nor clarifies how different belief configurations can result in various CE.

TABLE I

BASIC CONJECTURE-BASED RANDOM ACCESS

Initialize: $t = 0$, $p_k^0 \in [0,1]$, $a_k > 0$, $\forall k \in \mathcal{K}$.
Procedure
Locally at each node k, iterate through t:
Set $t \leftarrow t + 1$.
for all $k \in \mathcal{K}$
At stage t, $p_k^t \leftarrow \min\{\frac{p_k^{t-1}}{2} + \frac{\prod_{i \in \mathcal{K} \setminus \{k\}} (1 - p_i^{t-1})}{2a_k}, 1\}$.
(or equivalently, $CW_k^t = 2/p_k^t - 1$)
end for
end procedure

In the dynamic scenarios, nodes autonomously modify their conjectures based on their new observations. Specifically, we first allow the nodes to revise their reference points based on their past local observations. Let $s_k^t, p_k^t, \tilde{s}_k^t, \bar{s}_k^t, \bar{p}_k^t$ be user k's state, transmission probability, belief function, and reference points at stage t[3], in which $s_k^t = \prod_{i \in \mathcal{K} \setminus \{k\}} (1 - p_i^t)$. We propose a simple rule for individual nodes to update their reference points. At stage t, node k set its $\bar{s}_k^t$ and $\bar{p}_k^t$ to be s_k^{t-1} and p_k^{t-1}. In other words, node k's conjectured utility function at stage t is

$$u_k^t(\tilde{s}_k^t(p_k), p_k) = p_k \Big[\prod_{i \in \mathcal{K} \setminus \{k\}} (1 - p_i^{t-1}) - a_k(p_k - p_k^{t-1}) \Big]. \tag{5}$$

The remainder of this paper will investigate the dynamic properties and the performance trade-off at the resulting CE. For fixed $\{a_k\}_{k=1}^K$, Sections III.B will embed the individual optimization scheme in a dynamic process in which all the nodes update their transmission probabilities over time. Section III.C further allows individual nodes adaptively update their parameters $\{a_k\}_{k=1}^K$ such that desired efficiency can be attained. For given $\{a_k\}_{k=1}^K$, Section III.D will derive the resulting CE $\mathbf{p}^*$ and show that it achieves weighted fairness.

B. Basic CBRA

For fixed $\{a_k\}_{k=1}^K$, we propose a basic Conjecture-Based Random Access (CBRA) Algorithm that adopts the best response update mechanism. In particular, each node adjusts its transmission probability using the best response that maximizes its conjectured utility function (5). Therefore, at stage t, node k chooses a transmission probability

$$\begin{aligned} p_k^t &= \arg\max_{p_k \in P_k} u_k^t(\tilde{s}_k^t(p_k), p_k) \\ &= \min\Big\{ \frac{p_k^{t-1}}{2} + \frac{\prod_{i \in \mathcal{K} \setminus \{k\}} (1 - p_i^{t-1})}{2a_k}, 1 \Big\}. \end{aligned} \tag{6}$$

Each node takes the best response based on its internal belief of how its own action will impact the other nodes' behavior. The detailed description of this CBRA Algorithm is summarized in Table I. Next, we are interested in the stability and convergence of this algorithm.

[3] A stage contains multiple time slots. The superscript t in this paper represents the numbering of the stages unless specified.

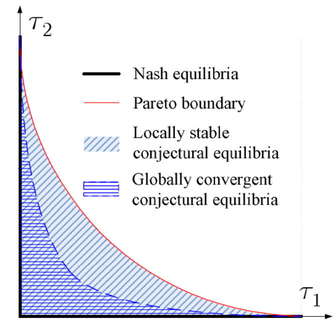

Fig. 1. Comparison of different solution concepts.

TABLE II
ADAPTIVE CBRA

Initialize: stepsize δ and γ, p_k^0, and $a_k > \lvert\mathcal{K}\rvert$, $\forall k \in \mathcal{K}$.
Procedure
outer loop: For each node k,
if efficiency increases, $a_k \leftarrow a_k(1-\delta)$;
else $a_k \leftarrow a_k + \frac{a_k}{a_1}\gamma$;
inner loop:
Locally at each node k, use CBRA to update p_k^t, until it converges.
end inner loop
end outer loop
end procedure

1) Local Stability: Although Theorem 1 indicates all the points in $\mathscr{T}$ are CE, they may not be stable. An unstable CE is not desirable, because small perturbations might cause the operating point to move away from the original CE. The next theorem describes a subset in P that consists of stable CE.

Theorem 2: For any $\mathbf{p}^* = (p_1^*, \ldots, p_K^*) \in P$, if $\sum_{k=1}^K p_k^* < 1$, $\mathbf{p}^*$ is a stable CE.

2) Global Convergence: Note that Theorem 2 only investigates the stability for different fixed points, i.e. CBRA converges to these points when initial values are close enough to them. In addition to local stability, we are also interested in characterizing the global convergence of CBRA when using various a_k to initialize the belief function $\tilde{s}_k$.

Theorem 3: Regardless of any initial value chosen for $\{p_k^0\}_{k=1}^K$, if the parameters $\{a_k\}_{k=1}^K$ in the belief functions $\{\tilde{s}_k\}_{k=1}^K$ satisfy $a_k > K-1, \forall k \in \mathcal{K}$, CBRA converges to a unique CE.

C. Adaptive CBRA

The results in the previous subsections describe the values of $\{p_k\}_{k=1}^K$ and $\{a_k\}_{k=1}^K$ for which local stability and global convergence can be guaranteed in CBRA. This subsection directly investigates the stability of achievable operating points in the throughput region $\mathscr{T}$.

Theorem 4: Regardless of the number of nodes in the network, CBRA can operate on any feasible point within the Pareto boundary of $\mathscr{T}$ as a stable CE.

Fig. 1 compares the throughput performance among various game-theoretic solution concepts, including Nash equilibria, Pareto frontier, locally stable conjectural equilibria, and globally convergent conjectural equilibria, in random access games. As proven in Theorem 4, Fig. 1 shows that, the entire space spanning between the Nash equilibria and Pareto frontier essentially consists of stable conjectural equilibria. In addition, as discussed in , the set of globally convergent CE is a subset of the stable CE set.

In practice, it is more important to construct algorithmic mechanisms to attain the desirable CE that operate stably and closely to the Pareto boundary. To this end, we develop an adaptive CBRA algorithm and summarize it in Table II. Specifically, this algorithm has an inner loop and an outer loop. The inner loop adopts the basic CBRA to achieve convergence for fixed $\{a_k\}_{k=1}^K$. This algorithm initializes $a_k > \lvert\mathcal{K}\rvert$ such that it initially globally converges. In practice, individual nodes do not need to know or estimate $\lvert\mathcal{K}\rvert$ and they can initialize a_k to be sufficiently large. After converging to a stable CE, the outer loop adaptively adjusts $\{a_k\}_{k=1}^K$. The outer loop updates $\{a_k\}_{k=1}^K$ in the *Proportional Increase Multiplicative Decrease* (PIMD) manner due to two reasons. First, reducing $\{a_k\}_{k=1}^K$ individually increases $\{p_k^t\}_{k=1}^K$ and $\sum_{k=1}^K p_k$ and hence, move the operating point towards the Pareto boundary. Second, PIMD can maintain weighted fairness among different nodes. Both reasons will be analytically explained in the Section III.D. During the outer loop iteration, individual nodes can measure the system efficiency by monitoring their common observation of the aggregate ideal throughput $\sum_{k\in\mathcal{K}} u_k^t$ or the protocol-specific aggregate throughput

$$\mathcal{T} = \frac{P_s L_d}{(1-P_{tr})T_{slot} + P_s T_s + P_{tr} T_c - P_s T_c}, \qquad (7)$$

where $P_s = \sum_{k=1}^K p_k \cdot \prod_{m\neq k}(1-p_m)$ is the probability that a transmission occurring on the channel is successful, $P_{tr} = 1 - \prod_{k=1}^K (1-p_k)$ is the probability that at least one transmission attempt happens, L_d is the average data frame size, T_s is the average time of a successful transmission, and T_c is the average duration of a collision. Note that L_d, T_s, and T_c are determined by the protocol's PHY and MAC specifications.

D. Fairness

We can show that the CBRA algorithm approximately achieves the weighted fairness for users with different qualify-of-service requirements. Consider a network with $N > 1$ different classes of nodes. Let ϕ_n denote the parameter that class-n nodes choose for their conjectured utility functions (i.e. the parameter a_k if node k belongs to class-n) and $\mathcal{F}_n$ denote the set of nodes that set their algorithm parameters to be ϕ_n, $1 \le n \le N$. At equilibrium, the transmission probabilities of the same class of nodes are equal, denoted as $\tilde{p}_n$. Before we proceed, we first define the weighted fairness for the random access game [14]. Denote T_k^t and T_{-k}^t as the events that node k transmits data at time slot t and any node in $\mathcal{K}\backslash\{k\}$ transmits data at time slot t, respectively. Let $\mathbf{1}_a$ denote an indicator function of event a taking place. For each traffic class n, we associate with a positive weight χ_n. Then the weighted fairness intended for the random access game satisfy

$$\forall i, j \in \{1, 2, \cdots, N\}, \forall s \in \mathcal{F}_i, \forall s' \in \mathcal{F}_j,$$
$$\frac{\mathrm{E}\{\mathbf{1}_{\{T_{-s}=0\}}\mathbf{1}_{\{T_s=1\}}\}}{\chi_i} = \frac{\mathrm{E}\{\mathbf{1}_{\{T_{-s'}=0\}}\mathbf{1}_{\{T_{s'}=1\}}\}}{\chi_j}. \quad (8)$$

which means that the probability of an successful transmission attempt for traffic class n is proportional to its weight χ_n. By simple manipulation, we have the equivalent form for equation (8) [14]:

$$\forall i, j \in \{1, \cdots, N\}, \frac{p_i^{WF}}{(1-p_i^{WF})\chi_i} = \frac{p_j^{WF}}{(1-p_j^{WF})\chi_j}. \quad (9)$$

The following theorem indicates the quantitative relationship between the chosen algorithm parameters $\{\phi_n\}_{n=1}^N$, the sizes of different classes $\{\mathcal{F}_n\}_{n=1}^N$, and the resulting steady-state transmission probabilities $\{\tilde{p}_n\}_{n=1}^N$. More importantly, it also shows that if the network size is large, the conjecture-based algorithms approximately achieve weighted fairness.

Theorem 5: Suppose that $\phi_n \geq 2$, $\forall 1 \leq n \leq N$. The achieved steady-state transmission probabilities $\{\tilde{p}_n\}_{n=1}^N$ are given by

$$\tilde{p}_n = \frac{1}{2}\left(1 - \sqrt{1 - \frac{4\varrho}{\phi_n}}\right), \quad (10)$$

where ϱ satisfies

$$\varrho = \frac{1}{2^K}\prod_{n=1}^{N}\left[\left(1 + \sqrt{1 - \frac{4\varrho}{\phi_n}}\right)^{|\mathcal{F}_n|}\right]. \quad (11)$$

Remark 1: There are several intuitions and observations that we can obtain from Theorem 5. First, according to (10) and (11), $\{\tilde{p}_n\}_{n=1}^N$ increases as long as any one parameter in $\{\phi_n\}_{n=1}^N$ decreases. Therefore, the multiplicative decreasing update in the adaptive CBRA moves the operating points towards Pareto boundary. A quantitative approximation between the steady-state transmission probability $\tilde{p}_n$ and the algorithm parameter ϕ_n of each traffic class can be derived if a large number of nodes coexist. Since $\varrho \to 0$ when $|\mathcal{F}_n|$ is large, using the Taylor expansion, $\tilde{p}_n$ can be approximated as ϱ/ϕ_n, i.e. the steady-state transmission probability $\tilde{p}_n$ decays as the inverse first power of parameter ϕ_n that indicates the "aggressiveness" of traffic class n. Finally, we observe that, if $|\mathcal{F}_n|$ is large, $\tilde{p}_n \to 0$ and $1 - \tilde{p}_n \approx 1$. Therefore,

$$\forall i, j, \phi_i\tilde{p}_i(1-\tilde{p}_i) = \phi_j\tilde{p}_j(1-\tilde{p}_j) \Rightarrow \frac{\phi_i\tilde{p}_i}{1-\tilde{p}_i} \approx \frac{\phi_j\tilde{p}_j}{1-\tilde{p}_j}.$$

It indicates that CBRA approximately achieves weighted fairness given in (9) with weight $\chi_n = 1/\phi_n$. Moreover, it is worth mentioning that the weighted fairness is purely an implicit by-product of the CBRA algorithm and it can be sustained with stability. Therefore, adaptive CBRA uses PIMD to maintain the weighted fairness.

IV. Simulations

This section numerically compares the performance of the existing 802.11 DCF protocol, the P-MAC protocol, and the CBRA algorithm. In the simulation, we assume that each wireless device operates at the IEEE 802.11a PHY mode-8 [15] and the RTS/CTS mechanism is disabled. The contention window sizes in DCF are $CW_{min} = 16$ and $CW_{max} = 1024$. The parameters in P-MAC are set according to [14]. Our comparison results are summarized as follows.

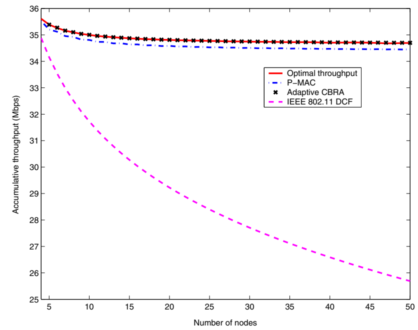

Fig. 2. Throughput Comparison.

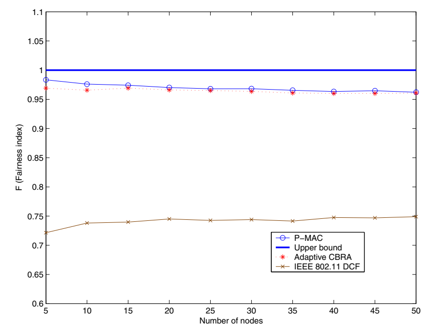

Fig. 3. Fairness Comparison.

A. Throughput

We vary the total number of nodes K in a single cell from 4 to 50, in which $\lceil K/2 \rceil$ nodes carry class-1 traffic and the remaining nodes carry class-2 traffic. The positive weights of class-1 and class-2 are $\chi_1 = 1$ and $\chi_2 = 0.5$. The initial parameters in adaptive CBRA are chosen to be $\phi_1 = 3K/\chi_1$ and $\phi_2 = 3K/\chi_2$. The saturation throughput is calculated based on the analysis in [16]. As shown in Fig. 2, both adaptive CBRA and P-MAC significantly outperform the IEEE 802.11 DCF. The IEEE 802.11 DCF achieves the lowest throughput, because the lack of adaptation mechanism of the contention window size causes more frequent packet collisions as the number of nodes increases. Surprisingly, the performance of the conjecture equilibrium attained by adaptive CBRA achieves the optimal throughput limit. It also outperforms P-MAC, because P-MAC uses approximation to derive closed-form expressions for the transmission probabilities of different traffic class.

B. Fairness

We evaluate the short-term fairness of different protocols using the quantitative fairness index introduced in [14]

$$\mathbf{F} = \frac{\mu(\mathcal{T}_k/\chi_n)}{\mu(\mathcal{T}_k/\chi_n) + \sigma(\mathcal{T}_k/\chi_n)}, k \in \mathcal{F}_n \quad (12)$$

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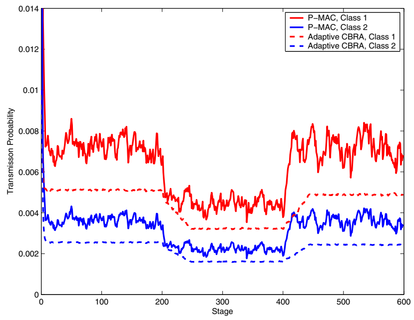

Fig. 4. Dynamics of Transmission Probabilities.

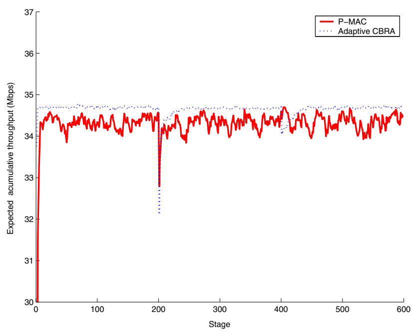

Fig. 5. Dynamics of Accumulative Throughput.

in which $\mathcal{T}_k$ denotes the throughput of node k that belongs to traffic class n, and μ and σ are, respectively, the mean and the standard deviation of $\mathcal{T}_n/\chi_n$ over all the active data traffic flows. We simulate a transmission duration of 3 minutes. The stage duration in adaptive CBRA is set as 50 successful transmissions. As shown in Fig. 3, we can see that adaptive CBRA and P-MAC are comparable in their fairness performance and the achieved fairness index is always above 0.95 regardless of the network configuration. On the other hand, the fairness performance of 802.11 DCF is much poorer than the previous two algorithms because the DCF protocol provides no fairness guarantee.

C. Convergence and Stability

In order to compare the convergence and the stability of different protocols for time-varying traffic, we simulate a network in which the number of active nodes fluctuates over time. At the beginning, $|\mathcal{F}_1| = |\mathcal{F}_2| = 25$. At stage 200, 15 class-1 and 15 class-2 nodes join the network. These nodes leave the network at the 400th stage. The algorithm parameter a_k is updated every 5 stages. Fig. 4 and Fig. 5 show the variation of the transmission probabilities for both traffic classes and the expected accumulative throughput over time. We can see that P-MAC does not converge due to the lack of feedback control, which agrees with the observation about the instability of P-MAC reported in [7]. In addition, the optimal transmission probabilities computed by P-MAC and adaptive CBRA are different under the same network parameters because of the approximation used in P-MAC. As shown in Fig. 4, nodes deploying P-MAC transmit with a higher probabilities than adaptive CBRA, which create a more congested environments. As a result, the accumulative throughput achieved by P-MAC is slightly lower than the optimal throughput. In contrast, as shown in Fig. 4 and Fig. 5, during stage [200,250] and [400,440], adaptive CBRA enables the nodes adaptively tune their parameters a_k to maximize the network throughput while maintaining the weighted fairness as well as the system stability.

V. Conclusion

This paper proposes a distributed algorithm that enables autonomous nodes to simultaneously achieve throughput efficiency, system stability, and weighted fairness in random access networks. Specifically, we allow individual nodes to form internal belief functions about the impact of its various actions can alter the interaction outcome and its throughput. The proposed CBRA algorithm achieve significant performance improvement against existing protocols in terms of not only fairness and throughput but also convergence and stability. Future investigation will focus on studying how losses incurred by channel fading impacts the performance of CBRA.

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