A New Perspective on Multi-user Power Control Games in Interference Channels

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ABSTRACT

This paper considers the problem of how to allocate power among competing users sharing a frequency-selective interference channel. We model the interaction between these selfish users as a non-cooperative game. As opposed to the existing iterative water-filling algorithm, which studies the myopic behavior of users, this paper studies how a foresighted user, who knows the channel state information and response strategies of its competing users, should behave. To characterize this multi-user interaction, the Stackelberg equilibrium is introduced, and the existence of this equilibrium for the investigated non-cooperative game is shown. We analyze such interactions in more detail using a simple two-user example, where we define the strategic behavior of a foresighted user as a bi-level programming problem, and derive the necessary optimality conditions. It is analytically shown that a foresighted user can improve its performance, if it has the required information about its competitors. Due to the computationally prohibitive nature of the optimal solution, a practical low-complexity approach is proposed based on the intuition gained from the derived necessary conditions. Numerical simulations verify the performance improvements. Possible ways of acquiring the required information and of extending the analysis to multiple users are also discussed.

Index Terms— interference channel, power control, non-cooperative game theory, Stackelberg equilibrium

I. INTRODUCTION

The multi-user power control problem in frequency-selective interference channels was investigated from the game theoretic point of view [1]-[7]. In these multi-user power control games, users are modeled as players with individual goals and strategies. They are competing and cooperating with each other until they agree on an acceptable resource allocation outcome. Existing research can be categorized into two types, *non-cooperative* games and *cooperative* games.

First, the formulation of the multi-user environment as a non-cooperative game has appeared in several recent works [1][2]. An iterative water-filling (IW) algorithm has been proposed to mitigate the mutual interference and optimize the performance without the need for a central controller [1]. At every decision stage, selfish users deploying this algorithm try to maximize their achievable rates by water-filling across the whole frequency band until a Nash equilibrium is reached. Alternatively, self-enforcing protocols are studied in the non-cooperative scenario [2], in which incentive

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compatible allocations are guaranteed and no individual has the incentive to deviate. By imposing punishment in the case of misbehavior and enforcing users to cooperate, efficient, fair, and incentive compatible spectrum sharing is shown to be possible.

Second, there also have been a number of related works studying dynamic spectrum management in the setting of cooperative games [3]-[7]. Two optimal but centralized DSM algorithms, the Optimal Spectrum Balancing (OSB) algorithm [4] and the Iterative Spectrum Balancing (ISB) algorithm [5][6], were proposed to address the problem of maximization of a certain user's achievable rate while satisfying the minimum rate requirements of all the other competing users. OSB has an exponential complexity in the number of users and ISB only has a quadratic complexity in the number of users because it implements the optimization in an iterative fashion. Recently, an autonomous spectrum balancing (ASB) technique is proposed to achieve near-optimal performance autonomously without real-time explicit information exchanges [7]. These works focus on cooperative games, because it is well-known that the IW algorithm may lead to Pareto-inefficient solutions [8], i.e. selfishness is detrimental in the interference channel.

In short, previous research mainly concentrates on studying the existence and performance of Nash equilibrium in non-cooperative games and developing efficient algorithms to approach the Pareto boundary in cooperative games. However, an important intrinsic dimension of this information-decentralized multi-user interaction still remains unexplored. Prior research does not consider the users' availability of information about other users and their ability in improving their performance by having this information. Hence, determining what is the best response strategy of a selfish user if it gets the information about competing users in the non-cooperative game still needs to be determined. Moreover, it still needs to be established if such strategy can lead to a better performance than adopting the IW algorithm. Intuitively, a "clever" user with more information in this non-cooperative game should be able to gain more benefits [9]. It is important to look at these scenarios in order to assess the significance of information availability from the users' viewpoint in non-cooperative games and show why selfish users have incentives to learn their environment and adapt their rational response strategies [10].

Throughout this paper, we differentiate two types of selfish users based on their response strategies:

1) *Myopic users*: Users that always act to maximize their immediate achievable rates. They are myopic in the sense that, at each decision stage, they treat other users' actions as fixed, ignore the impact of its competitors' reactions over its own performance, and determine their responses to gain the maximal immediate increases in their payoffs.

2) *Foresighted users*: Users that behave by taking into account the long-term impacts of their actions. They avoid shortsighted actions, anticipate how the others will react, and maximize their performance by considering the reactions of the others. It should be highlighted that such users require additional information to assist their decision making.

As opposed to previous approaches considering myopic users [1], we discuss in this paper how foresighted users should behave in non-cooperative power control games. We explicitly show that a strategic user can benefit if it takes its competitors' information and strategies into account. The concept of Stackelberg equilibrium is introduced in order to characterize the strategic behavior of a user by considering the response of its competing users. "Strategic behavior" in this paper refers to the action that a foresighted user takes in order to improve its own performance. Using a simple two-user case, we formulate this behavior to be a bi-level programming problem and derive the necessary optimality conditions. Based on the intuition gained from the optimality conditions, we provide a low-complexity solution of the original intractable non-convex optimization problem. Furthermore, noticing that a large amount of information is required to achieve the Stackelberg equilibrium, we propose that users can estimate and learn the required information by repeatedly interacting with the environment.

We also note that there are already some papers applying Stackelberg equilibrium to allocate the resources in networking [11]. However, the problems and the proposed solutions in these papers are completely different from this paper. The focus here is to study the strategic behavior of selfish users, which has not been yet investigated in multi-user interference channels.

The rest of the paper is organized as follows. Section II presents the non-cooperative game model and introduces the concept of Stackelberg equilibrium. In Section III, using a simple two-user example, we define the strategic behavior of a foresighted user to be a bi-level programming problem, and derive the necessary optimality conditions. Section IV discusses the complexity of the optimal solution and proposes a practical sub-optimal approach. Simulations show that a strategic user can achieve substantial performance improvement over the myopic case. In the same section, how the required information can be obtained by the strategic users and the formulation of problem in more general multi-user cases are also discussed. Conclusions are drawn in Section V.

II. System Model

In this section, following the notations in [1], we describe the mathematical model of the frequency-selective interference channel and formulate the non-cooperative multi-user power control game. We introduce the concept of Stackelberg Equilibrium and prove the existence of the Stackelberg equilibrium in the power control game.

A. *System Description*

Fig. 1 illustrates a frequency-selective Gaussian interference channel model. There are K transmitters and K receivers in the system. Each transmitter and receiver pair can be viewed as a player (or user). The transfer function of the channel from transmitter i to receiver j is denoted as $H_{ij}(f)$, where $0 \le f \le F_s$. The noise power spectral density (PSD) that receiver k experiences is denoted as $\sigma_k(f)$. Denote player k's transmit PSD as $P_k(f)$. For user k, the transmit PSD is subject to its power constraint:

$$\int_0^{F_s} P_k(f)\,df \le \mathbf{P_k}\,. \tag{1}$$

For a fixed $P_k(f)$, if treating interference as noise, user k can achieve the following data rate:

$$R_k = \int_0^{F_s} \log_2\left(1 + \frac{P_k(f)\left|H_{kk}(f)\right|^2}{\sigma_k(f) + \sum_{j\neq k} P_j(f)\left|H_{jk}(f)\right|^2}\right) df\,. \tag{2}$$

To fully capture the performance tradeoff in the system, the concept of a rate region is defined as

$$\mathcal{R} = \left\{(R_1,\cdots,R_K) : \exists\left(P_1(f),\cdots,P_K(f)\right)\ satisfying\ (1)\ and\ (2)\right\}. \tag{3}$$

Because of the non-convexity in the capacity expression as a function of power allocations, the computational complexity of optimal solutions (e.g., doing exhaustive search) in finding the rate region is prohibitively high. Existing works [4]-[7] aim to approach the Pareto boundary of this rate region and provide near-optimal performance. Moreover, it should be noted that cooperation among users is indispensable for the multi-user system to operate at the Pareto boundary. On the other hand, the interference channel can also be modeled as a non-cooperative game among multiple competing users. Instead of solving the optimization problem globally, the IW algorithm models the users as myopic decision makers [1]. This means that they optimize their transmit PSD by water-filling and compete to increase their transmission data rates with the sole objective of maximizing their own performance regardless of the coupling among users. Under a wide range of realistic conditions [1][14], the existence and uniqueness of the competitive optimal point (Nash equilibrium) is demonstrated and can be obtained by the IW algorithm, which significantly outperforms the static spectrum management algorithms.

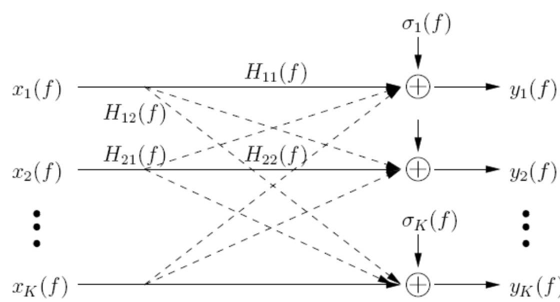

Fig. 1. Gaussian interference channel model.

Throughout this paper, we still concentrate on the non-cooperative game setting. In the IW algorithm, users are assumed to be myopic, i.e., they update actions shortsightedly without considering the long-term impacts of taking these actions. We argue that the myopic behavior can be further improved because it neglects the coupling nature of players' actions and payoffs. In contrast with previous approaches, we study the problem of how a strategic user should behave rather than taking myopic actions. This investigation provides us some insights in the following problems: what is the benefit that a foresighted user can achieve compared with the myopic case and thus, why a strategic user has the incentive to sense its environment and learn the anticipatory behavior of its competitors?

Fig. 2 shows a simple example, a Stackelberg stage game [12], of the foresighted behavior. Note that in this game, the row player has a strictly dominant strategy [13], *Down*. Therefore, two players will end up with a (*Down*, *Left*) play if the row player is myopic. However, if the row player is aware of the column player's coupled reaction, they will end up with a (*Up*, *Right*) play, which leads to an increased payoff for both players. It is also worth noticing that additional information is needed to attain this performance improvement. The row player needs to know the payoff and the response strategy of the column player. In order to formulate how a strategic user can take such foresighted actions, we introduce the concept of Stackelberg equilibrium. The following subsection will formally define the Stackelberg equilibrium and show its existence in our power control game.

	Left	*Right*
Up	1, 0	3, 2
Down	2, 1	4, 0

Fig. 2. Stackelberg stage game: the row player's payoff is given first in each cell, with the column player's payoff following.

B. *Stackelberg Equilibrium*

Game theory formally studies the interaction of rational players. Let $\mathcal{G} = \left[\mathcal{K}, \{\mathcal{A}_k\}, U_k\right]$ represent a game where $\mathcal{K} = \{1, \cdots, K\}$ is the set of players, $\mathcal{A}_k$ is the set of actions available to user k, and U_k is the user k's payoff [13]. In the power control game, the payoffs for the players are the respective achievable data rates and their strategies are to determine their own transmit PSD. In other words, U_k is user k's achievable rate R_k, and $\mathcal{A}_k$ is the set of transmit PSDs satisfying the constraint in (1). Recall that the Nash equilibrium is defined to be any $\left(a_1^*, \cdots, a_K^*\right)$ satisfying

$$U_k\left(a_k^*, a_{-k}^*\right) \geq U_k\left(a_k, a_{-k}^*\right) \; for \; all \; a_k \in \mathcal{A}_k \; and \; k = 1, \cdots, K \,, \tag{4}$$

where $a_{-k}^* = \left(a_1^*, \cdots, a_{k-1}^*, a_{k+1}^*, \cdots, a_K^*\right)$ [13].

We also define the action a_k^* to be a best response (BR) to actions a_{-k} if

$$U_k\left(a_k^*, a_{-k}\right) \geq U_k\left(a_k, a_{-k}\right), \; \forall \; a_k \in \mathcal{A}_k \,. \tag{5}$$

The set of user k's best response to a_{-k} is denoted as $BR_k\left(a_{-k}\right)$.

The Nash equilibrium is the best response of a user only in a competitive optimality sense [1]. The Stackelberg equilibrium is a best response when a hierarchy of actions exists between users [13]. Only one player is the *leader* and the other ones are *followers*. The leader begins the game by announcing its action. Then, the followers act according to the leader's action. Stackelberg equilibrium prescribes an optimal strategy for the leader if its followers always react by playing their Nash equilibrium strategies in the smaller sub-game. For example, in a two player game, where user 1 is the leader and user 2 is the follower, an action a_1^* is the Stackelberg equilibrium strategy for user 1 if

$$U_1\left(a_1^*, BR_2\left(a_1^*\right)\right) \geq U_1\left(a_1, BR_2\left(a_1\right)\right), \forall a_1 \in \mathcal{A}_1 \,. \tag{6}$$

For example, in Fig. 2, *Up* is the Stackelberg equilibrium strategy for the row player.

Next, we also define Stackelberg equilibrium in the general case. Let $NE\left(a_k\right)$ be the Nash equilibrium strategy of the remaining players if player k chooses to play a_k, i.e.

$$NE(a_k) = a_{-k},\ \forall a_i = BR_i(a_{-i}), a_i \in \mathcal{A}_i, i \neq k . \tag{7}$$

The strategy profile $\left(a_k^*, NE\left(a_k^*\right)\right)$ is a Stackelberg equilibrium with user k leading iff

$$U_k\left(a_k^*, NE\left(a_k^*\right)\right) \geq U_k\left(a_k, NE(a_k)\right), \forall a_k \in \mathcal{A}_k . \tag{8}$$

Note that there might exist multiple Nash equilibria in the followers' sub-game, in which case the definition of Stackelberg equilibrium becomes more complicated. Interested readers can refer to [11][15] for more details. In this paper, we will not discuss this case, because for most of the channel realizations, only a single Nash equilibrium exists in the sub-game [14].

In fact, the requirement of hierarchic actions in the original definition can also be removed in our problem if we consider the repeated interaction among all the users. Note that temporarily we assume only a single foresighted user exists in this game and all its competing myopic users will adopt the IW algorithm. Extension to multiple users will be discussed in Section IV. The foresighted user can always regard itself as the leader and perform the Stackelberg equilibrium strategy. Regardless of the transmit PSD which the foresighted user chooses, the other users will water-fill to gain an immediate increase in transmission rates until the system converges to an equilibrium. Therefore, the initial action order does not impact the outcome of this game. The following theorem formally establishes the existence of Stackelberg equilibrium in the considered power control game.

Theorem 1**:** Under a wide range of realistic channels [14], the Stackelberg equilibrium always exists in the multi-user power control game.

Proof **:** Suppose user k is the only foresighted user in this game. First, R_k is bounded because

$$0 \leq R_k \leq \int_0^{F_s} \log_2\left(1 + \frac{P_k^*(f)\left|H_{kk}(f)\right|^2}{\sigma_k(f)}\right) df , \tag{9}$$

where $P_k^*(f) = \left(\lambda - \frac{\sigma_k(f)}{\left|H_{kk}(f)\right|^2}\right)^+$ is the water-filling solution, $(x)^+ = \max(0, x)$, and λ is a constant satisfying the constraint in (1) with equality.

Second, it has been shown that in realistic channel settings, e.g., arbitrary symmetric interference environment and diagonally dominant asymmetric channel with any number of users, the existence and uniqueness of Nash equilibrium are always guaranteed [14]. In the interference channel consisting of the $K-1$ followers, whatever form of $P_k^*(f) \in \mathcal{A}_k$ user k chooses, they will regard user k's transmit PSD as part of the background noise PSD, i.e. $\tilde{\sigma}_j(f) = \sigma_j(f) + \left|H_{jk}(f)\right|^2 P_k(f),\ j \neq k$. Since the channel gains still satisfy the requirements in [14], the convergence to a unique Nash equilibrium always holds, i.e. a single $NE(a_k)$ exists for $\forall a_k = P_k^*(f) \in \mathcal{A}_k$.

To summarize, since R_k is bounded, and for $\forall a_k \in \mathcal{A}_k$, the remaining players' action will always lead to a Nash equilibrium, we have

$$0 \le U_k\left(a_k^*, NE\left(a_k^*\right)\right) \le \int_0^{F_s} \log_2\left(1 + \frac{P_k^*(f)\left|H_{kk}(f)\right|^2}{\sigma_k(f)}\right) df,\ \forall a_k = P_k^*(f) \in \mathcal{A}_k . \tag{10}$$

Therefore, there exist $a_k^* \in \mathcal{A}_k$ such that $U_k\left(a_k^*, NE\left(a_k^*\right)\right) = \sup_{a_k \in \mathcal{A}_k}\left\{U_k\left(a_k, NE\left(a_k\right)\right)\right\}$. We can conclude that a Stackelberg equilibrium always exists for this power control game. ■

III. PROBLEM FORMULATION

In this section, we study how to achieve the Stackelberg equilibrium in the two-user power control game. We formulate the strategic behavior as a bi-level programming problem, and derive the necessary optimality conditions. Since finding the optimal solution is computationally prohibitive, intuition gained from this optimality conditions is used to develop sub-optimal approach to solve of the original intractable optimization problem [16]. We start from the simplest two-user version because it is illustrative for understanding the interactions emerging among competing users. The extension of multiple-user case is hard to handle directly [1][14]. We briefly discuss it in Section V.

A. *A Bi-level Programming Formulation*

The Stackelberg equilibrium applied to the two-user power control game in the frequency-selective interference channel model can be represented by a bi-level mathematical problem [15], in which the foresighted user always act as the leader and the other user behaves as the follower. The leader chooses an appropriate transmit PSD to maximize its own benefits by considering the response of its follower, who always reacts to the given transmit PSD of the leader by water-filling over the entire frequency band. Thus, the Stackelberg equilibrium can be formulated as

$$\begin{aligned}
&\max_{P_1(f),P_2(f)} \int_0^{F_s} \ln\left(1 + \frac{P_1(f)}{N_1(f) + \alpha_2(f) P_2(f)}\right) df && (a)\\
&s.t.\ \int_0^{F_s} P_1(f)\, df \le \mathrm{P}_1 && (b)\\
&\quad P_1(f) \ge 0 && (c)\\
&\quad P_2(f) = \arg\max_{P_2'(f)} \int_0^{F_s} \ln\left(1 + \frac{P_2'(f)}{N_2(f) + \alpha_1(f) P_1(f)}\right) df && (d)\\
&\quad s.t.\ P_2'(f) \ge 0 && (e)\\
&\quad\quad \int_0^{F_s} P_2'(f)\, df \le \mathrm{P}_2 && (f)
\end{aligned}, \tag{11}$$

where $N_1(f) = \frac{\sigma_1(f)}{\left|H_{11}(f)\right|^2}, \alpha_1(f) = \frac{\left|H_{12}(f)\right|^2}{\left|H_{22}(f)\right|^2}, N_2(f) = \frac{\sigma_2(f)}{\left|H_{22}(f)\right|^2}, \alpha_2(f) = \frac{\left|H_{21}(f)\right|^2}{\left|H_{11}(f)\right|^2}$. The sub-problem in (11.a)-(11.c) is called the *upper-level problem* and (11.d)-(11.f) corresponds to the *lower-level problem*.

The bi-level programming formulation is different from the existing IW approach. By letting $P_1(f)$ and $P_2(f)$ to be the individual transmit PSD of the IW algorithm, we can see that the Nash equilibrium actually gives the lower bound of the problem in (11). Furthermore, by taking the opponent's reaction into account, the user can improve the myopic IW approach and improve its performance. Recall that some additional information is needed in the Stackelberg stage game in Fig. 2. Similarly, the information, which include the other user's channel condition, $N_2(f)$ and $\alpha_2(f)$, and response strategy, the IW strategy, is also indispensable to achieve the Stackelberg equilibrium.

Bi-level programming problems belong to *the mathematical programs with optimization problems in the constraints*, and they are intrinsically hard to solve [15]. In this paper, we first focus on study the necessary optimality conditions, and then develop a sub-optimal solution using intuition gained from the optimality conditions.

Noting that the lower-level problem is a standard convex programming problem, Karush-Kuhn-Tucker (KKT) conditions below are necessary and sufficient for the lower-level problem to achieve the optimum:

$$\begin{aligned}
&\lambda_2(f) \geq 0 \\
&P_2(f) \geq 0 \\
&\lambda_2(f) P_2(f) = 0 \\
&\int_0^{F_s} P_2(f)\, df \leq \mathrm{P}_2 \\
&K_2 \geq 0 \\
&K_2 \left(\mathrm{P}_2 - \int_0^{F_s} P_2(f)\, df \right) = 0 \\
&P_2(f) = \frac{1}{K_2 - \lambda_2(f)} - N_2(f) - \alpha_1(f) P_1(f)
\end{aligned} \quad . \tag{12}$$

The KKT conditions in (12) can be further simplified. Because the myopic user will always transmit at its maximal power to maximize its achievable rate, we always have $\int_0^{F_s} P_2(f)\, df = \mathrm{P}_2$ and $K_2 > 0$. By replacing the lower-level problem with the KKT conditions, it leads to the single-level reformulation of the problem in (11):

$$\begin{aligned}
&\max_{P_1(f), P_2(f), \lambda_2(f), K_2} \int_0^{F_s} \ln\left(1 + \frac{P_1(f)}{N_1(f) + \alpha_2(f) P_2(f)} \right) df \\
&s.t. \;\; \int_0^{F_s} P_1(f)\, df \leq \mathrm{P}_1 \\
&\quad P_1(f) \geq 0 \\
&\quad \lambda_2(f) \geq 0 \\
&\quad P_2(f) \geq 0 \\
&\quad \lambda_2(f) P_2(f) = 0 \\
&\quad \int_0^{F_s} P_2(f)\, df = \mathrm{P}_2 \\
&\quad K_2 > 0 \\
&\quad P_2(f) = \frac{1}{K_2 - \lambda_2(f)} - N_2(f) - \alpha_1(f) P_1(f)
\end{aligned} \quad . \tag{13}$$

This single-level reformulation is equivalent to the original problem in (11) and will be investigated in the following. However, the above mathematical problem is not easy to solve because of the non-convexities that occur in the

Lagrangian constraints of the lower-level problem. Therefore, in the following sub-section, we first study the necessary optimality conditions.

B. *Necessary Conditions of Optimality*

Although the formulation in (13) is a non-convex and hard to solve [16], the KKT conditions are still necessary for the optimal solution [17]. The Lagrangian function of (13) can be written as a function of $P_1(f), P_2(f), \lambda_2(f), K_2$:

$$\begin{aligned}
&\mathcal{L}\left(P_1(f), P_2(f), \lambda_2(f), K_2, \boldsymbol{\mu}(f), K_1', K_2', K_3'\right) = \\
&\int_0^{F_s} \ln\left(1 + \frac{P_1(f)}{N_1(f) + \alpha_2(f) P_2(f)}\right) df - K_1'\left(\int_0^{F_s} P_1(f)\, df - \mathrm{P}_1\right) \\
&+\int_0^{F_s} \mu_1(f) P_1(f) df + \int_0^{F_s} \mu_2(f)\lambda_2(f)\, df + \int_0^{F_s} \mu_3(f) P_2(f)\, df + \int_0^{F_s} \mu_4(f)\lambda_2(f) P_2(f)\, df \\
&+K_2'\left(\int_0^{F_s} P_2(f)\, df - \mathrm{P}_2\right) + K_3' K_2 + \int_0^{F_s} \mu_5(f)\left[P_2(f) - \frac{1}{K_2 - \lambda_2(f)} + N_2(f) + \alpha_1(f) P_1(f)\right] df
\end{aligned} \tag{14}$$

where $\mu_1(f), \mu_2(f), \mu_3(f), \mu_4(f), \mu_5(f), K_1', K_2'$, and K_3' are Lagrangian multipliers. Table I gives the relationship between the constraints and the dual variables.

Constraints of the primal problem	Dual variables
$\int_0^{F_s} P_1(f)\, df \le \mathrm{P}_1$	$K_1' \ge 0$
$P_1(f) \ge 0$	$\mu_1(f) \ge 0$
$\lambda_2(f) \ge 0$	$\mu_2(f) \ge 0$
$P_2(f) \ge 0$	$\mu_3(f) \ge 0$
$\lambda_2(f) P_2(f) = 0$	$\mu_4(f)$
$\int_0^{F_s} P_2(f)\, df = \mathrm{P}_2$	K_2'
$K_2 > 0$	$K_3' \ge 0$
$P_2(f) = \frac{1}{K_2 - \lambda_2(f)} - N_2(f) - \alpha_1(f) P_1(f)$	$\mu_5(f)$

Table I. Lagrangian multipliers for the problem in (13).

Taking the derivative with respect to the primal variables, we have

$$\begin{aligned}
&\frac{\partial \mathcal{L}}{\partial P_1(f)} = 0 \;\Rightarrow \frac{1}{N_1(f) + \alpha_2(f) P_2(f) + P_1(f)} = K_1' - \mu_1(f) - \alpha_1(f)\mu_5(f) \quad (a) \\
&\frac{\partial \mathcal{L}}{\partial \lambda_2(f)} = 0 \;\Rightarrow \mu_2(f) + \mu_4(f) P_2(f) - \mu_5(f)\cdot \frac{1}{\left(K_2 - \lambda_2(f)\right)^2} = 0 \quad (b) \\
&\frac{\partial \mathcal{L}}{\partial P_2(f)} = 0 \;\Rightarrow \frac{\alpha_2(f) P_1(f)}{\left(N_1(f) + \alpha_2(f) P_2(f)\right)\left(N_1(f) + \alpha_2(f) P_2(f) + P_1(f)\right)} \quad (c)\,. \\
&\qquad = \mu_3(f) + \mu_4(f)\lambda_2(f) + \mu_5(f) + K_2' \\
&\frac{\partial \mathcal{L}}{\partial K_2} = 0 \;\Rightarrow K_3' + \int_0^{F_s} \mu_5(f)\cdot \frac{1}{\left(K_2 - \lambda_2(f)\right)^2} df = 0 \quad (d)
\end{aligned} \tag{15}$$

These four equalities illustrate parts of the necessary optimality conditions. Eq. (15.a) describes the summation of the overall PSD level experienced by the user 1, which is a flat water-level in the IW algorithm. Eq. (15.c) determines foresighted user 1's signal to interference-and-noise ratio (SINR), $\frac{P_1(f)}{N_1(f) + \alpha_2(f) P_2(f)}$, at the optimum. Eq. (15.b) and

(15.d) provide additional equality constraints over the primal and dual variables.

The remaining parts of the necessary KKT conditions are given by the constraints of the primal and dual variables in Table I, and complementary slackness constraints [16]. The equalities in (15.d) can be simplified. Note that we always have $K_2 > 0$. By complementary slackness, it leads to $K_3' = 0$. Therefore, (15.d) is reduced to

$$\int_0^{F_s} \mu_5(f) \cdot \frac{1}{(K_2 - \lambda_2(f))^2} df = 0. \tag{16}$$

By careful study of all these necessary optimality conditions, some key remarks can be made as follows.

***Remark 1* :** The Nash equilibrium achieved by the IW algorithm may not satisfy the necessary KKT conditions.

It is known that at the Nash equilibrium, no user can unilaterally increase its rate by changing its transmit PSD. A Nash equilibrium is reached iff the water-filling condition is simultaneously achieved for both users, which leads to

$$\begin{aligned} &\frac{1}{N_1(f) + \alpha_2(f) P_2(f) + P_1(f)} = K_1 - \lambda_1(f) \quad (a) \\ &\frac{1}{N_2(f) + \alpha_1(f) P_1(f) + P_2(f)} = K_2 - \lambda_2(f) \quad (b) \end{aligned}, \tag{17}$$

where $K_1, K_2 > 0, \lambda_1(f), \lambda_2(f), P_1(f), P_2(f) \geq 0,$ and $\lambda_1(f) P_1(f) = \lambda_2(f) P_2(f) = 0$.

For a Nash equilibrium strategy to solve the problem in (13), it must satisfy the necessary condition. Note that (17.b) has already been included in Table I. A possible way to test whether or not (17) satisfies the necessary KKT conditions in (15) and Table I is to let $\mu_1(f) = \lambda_1(f),\ \mu_2(f) = P_2(f),\ \mu_3(f) = \lambda_2(f),\ \ \mu_4(f) = -1,\ \mu_5(f) = 0,\ \ K_1' = K_1$. It leads to $K_2' = \frac{\alpha_2(f) P_1(f)}{(N_1(f) + \alpha_2(f) P_2(f))(N_1(f) + \alpha_2(f) P_2(f) + P_1(f))}$. In general, it is hard to guarantee that the LHS of (15.c) is a constant for any $f \in [0, F_s]$. Therefore, the IW algorithm may not be the optimal solution of the problem in (13).

However, by this remark, we do not mean that the Nash-strategy is always strictly sub-optimal. There do exist some situations in which IW algorithm satisfies the necessary conditions and solves the problem in (13). For example, with $N_1(f) = C^1, N_1(f) = C^2, \alpha_1(f) = 0$ and $\alpha_2(f) = 0$ where C^1, C^2 are both constants, the IW algorithm is reduced to two separate single user water-filling solutions which provide the optimal performance and achieve the upper bound in (9). In this remark, we want to emphasize that the Nash-strategy may only provide sub-optimal solutions in many cases.

***Definition 1* :** We define the *non-Nash-equilibrium strategy* to be any power allocation strategy satisfying

$$\exists f', f'' \in [0, F_s],\ P_1(f') > 0,\ P_1(f'') = 0,\ and\ N_1(f') + \alpha_2(f') P_2(f') + P_1(f') > N_1(f'') + \alpha_2(f'') P_2(f'') \tag{18}$$

Fig. 3 illustrates such a case pictorially, where user 1 does not allocate any power in the region of $f_1 \leq f \leq f_2$ even though the noise and interference level is below the water-level in $f_0 \leq f \leq f_1$.

***Remark 2* :** Non-Nash-equilibrium strategies may satisfy the necessary conditions, because it is possible that there exist primal and dual variables together with $\mu_5(f) \neq 0, \exists f \in [0, F_s]$, which satisfy all the necessary conditions in (15) and Table I, and result in a non-Nash-equilibrium strategy. This can be intuitively explained as follows.

Obviously, for a non-Nash-equilibrium strategy in Fig. 3, we have

$$\frac{1}{N_1(f')+\alpha_2(f')P_2(f')+P_1(f')} < \frac{1}{N_1(f'')+\alpha_2(f'')P_2(f'')}, \ for \ f_0 \le f' \le f_1, f_1 \le f'' \le f_2 \tag{19}$$

For the power allocation in Fig. 3 to satisfy the necessary KKT conditions, from (15.a), it leads to

$$K_1' - \alpha_1(f')\mu_5(f') < K_1' - \mu_1(f'') - \alpha_1(f'')\mu_5(f''), \ for \ f_0 \le f' \le f_1, f_1 \le f'' \le f_2 \tag{20}$$

For the above inequality to hold, we need to find $\mu_5(f)$ with $\mu_5(f') > 0, \mu_5(f'') < 0, \ for \ f_0 \le f' \le f_1, f_1 \le f'' \le f_2$. We know from (16) that, for the optimal solution, the integration of $\frac{\mu_5(f)}{(K_2 - \lambda_2(f))^2}$ over the entire frequency band is zero. There may exist some $\mu_5(f)$ that satisfy the necessary conditions and form the non-Nash-equilibrium strategies. In other words, the dual variable $\mu_5(f)$ in Table I is used to adjust the water level such that non-Nash-equilibrium strategies may meet the necessary KKT conditions.

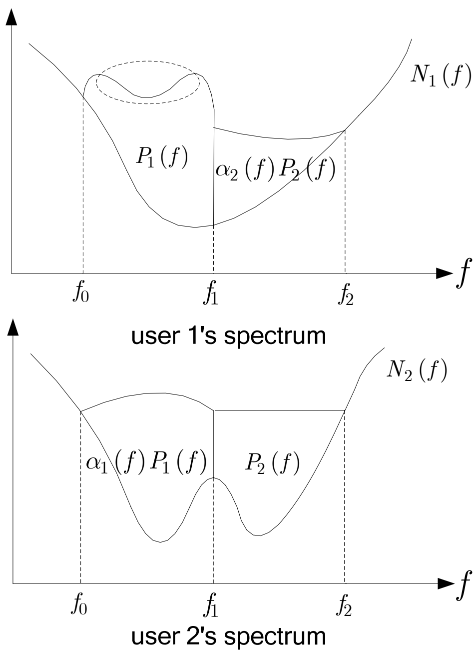

Fig. 3. An example of non-Nash-equilibrium strategy.

***Remark 3* :** If $P_1(f) > 0$ and $\lambda_2(f) > 0$, the following equality always holds:

$$\frac{1}{N_1(f)+P_1(f)} = K_1'. \tag{21}$$

The conclusion follows the complementary slackness and (15.a)-(15.b), if $P_1(f) > 0$ and $\lambda_2(f) > 0$, we have $\mu_1(f) = \mu_2(f) = P_2(f) = 0.$ By (15.b), it leads to $\mu_5(f) = 0.$ Therefore, (15.a) reduce to the form in (21).

Intuitively, $\lambda_2(f) > 0$ implies that, in this frequency band, the noise-and-interference-level is above the water-level for user 2 and user 1 enjoys an interference-free environment. Therefore, if it puts a certain amount of power such that $P_1(f) > 0$, the optimal power allocation strategy within this frequency band is to water-fill. According to this remark, user 1's power allocation in the dashed circle in Fig. 3 is not optimal.

***Remark 4*:** For a non-Nash-equilibrium strategy to satisfy the necessary conditions, it is impossible to have only these two power allocation patterns, $P_1(f) > 0, \lambda_2(f) > 0$ and $P_1(f) = 0, P_2(f) > 0$, over the whole frequency band.

We know from Remark 3 that $P_1(f) > 0, \lambda_2(f) > 0$ leads to $P_2(f) = \mu_5(f) = 0$. Similarly, for $P_1(f) = 0, P_2(f) > 0$, (15.c) can be simplified to $\mu_5(f) + K_2' = 0$. If only these two power allocation patterns exist, to assure that (16) holds, we always have $\mu_5(f) = K_2' = 0$.

By the definition of the non-Nash-equilibrium strategy, $\exists f', f'' \in [0, F_s],\ P_1(f') > 0,\ and\ P_1(f'') = 0,$ such that

$$N_1(f') + \alpha_2(f') P_2(f') + P_1(f') > N_1(f'') + \alpha_2(f'') P_2(f''). \tag{22}$$

Note that $P_1(f') > 0$ leads to $\mu_1(f') = 0$. Therefore, we have

$$K_1' - \mu_1(f') \geq K_1' - \mu_1(f''), \tag{23}$$

which is the RHS of (15.a) with $\mu_5(f) = 0$. This contradicts the inequality in (22) about the LHS of (15.a). Therefore, Remark 4 holds.

Intuitively, if $P_1(f') > 0, \lambda_2(f') > 0$ and $P_1(f'') = 0, P_2(f'') > 0$, we can reduce the power around $f = f'$ and move it to $f = f''$. Notice that for the non-Nash-equilibrium strategy, the noise and interference level around $f = f''$ is below the water-level around $f = f'$. This adjustment in the power allocation will increase user 1's achievable rate and hence the original power allocation is not optimal.

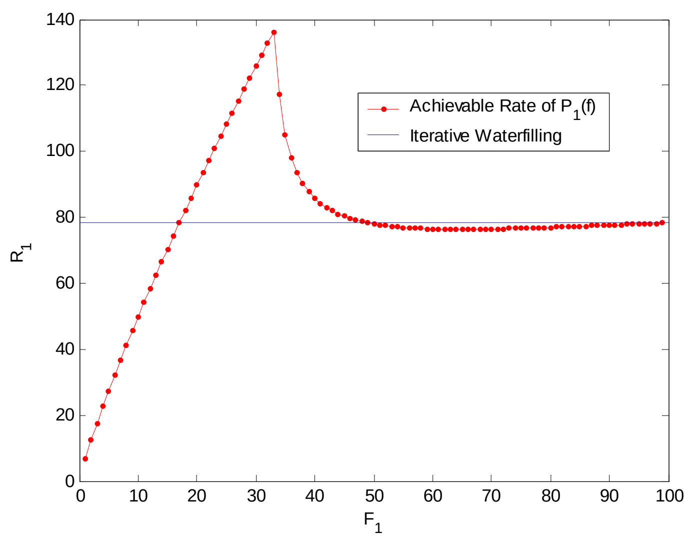

Fig. 4. Achievable rate R_1 in an interference channel as in [8].

Now we illustrate all these remarks using a simple example. We consider a two-user symmetric system similar with the case discussed in [8]. The parameter that we choose here is $f \in [0, 100], N_1(f) = N_2(f) = 0.01, \alpha_1(f) = \alpha_2(f) = 0.5$, $\mathbf{P_1} = \mathbf{P_2} = 100$. User 1's possible power allocation schemes is restrict to

$$P_1(f) = \begin{cases} \dfrac{100}{F_1}, 0 \le f \le F_1 \\ 0, \ \ F_1 \le f \le 100 \end{cases} . \tag{24}$$

User 1's achievable rate R_1 is plotted in Fig. 4. When $F_1 = 100/3$, it corresponds to the non-Nash-equilibrium strategy in Remark 2. We can see that R_1 reaches the maximum and it doubles the achievable rate of IW algorithm. Therefore, the Nash-strategy is strictly sub-optimal in this example. The cases in which $0 < F_1 < 100/3$ correspond to the situations in Remark 4 and they do provide sub-optimal performance compared with $F_1 = 100/3$.

IV. A LOW-COMPLEXITY SOLUTION

In this section, we discuss how to solve the problem in (11). We first analyze the complexity of the optimal solution. Noting that an exhaustive search for the optimal solution is computationally intractable, we develop a low-complexity sub-optimal algorithm and verify the performance of the proposed algorithm via numerical simulation. How the strategic users can obtain the required information and the extension to general multi-user cases are also discussed.

A. Optimal Solution

Since the optimization problem in (11) is non-convex, it generally can only be solved through an exhaustive search. A possible exhaustive search is to divide the whole frequency band into $N = F_s / \Delta_f$ bins. Define user k's transmit power in the ***i***-th frequency bin to be s_k^i and the granularity in the transmit PSD as Δ_P. The value of s_k^i can now be limited to the set $\{0, \Delta_P, \cdots, \mathbf{P_k}\}$. By performing an exhaustive search of all the possible combinations, the optimum can be found. Therefore, such a exhaustive search in $\left(s_k^1, \cdots, s_k^N\right)$ has a overall complexity of $\mathcal{O}((\mathbf{P_k}/\Delta_P)^N)$. Generally speaking, in order to approximate the optimal solution, we need to divide the frequency band into small bins and N will be sufficiently large. Therefore, a low-complexity solution needs to be developed. Note that the OSB and ASB algorithms [5][6] cannot be used here to reduce the computation complexity because the Stackelberg equilibrium is not necessarily Pareto efficient. Based on the necessary optimality conditions, we propose a sub-optimal approach in the following subsection.

B. A Low-Complexity Sub-optimal Approach

Note that $R_1 = \int_0^{F_s} \ln\left(1 + \dfrac{P_1(f)}{N_1(f) + \alpha_2(f) P_2(f)}\right) df$. From the necessary conditions in (15.a) and (15.c), we have

$$\frac{P_1(f)}{N_1(f) + \alpha_2(f) P_2(f)} = \frac{\mu_3(f) + \mu_4(f)\lambda_2(f) + \mu_5(f) + K_2'}{\alpha_2(f)\left[K_1' - \mu_1(f) - \alpha_1(f)\mu_5(f)\right]} . \tag{25}$$

Therefore, it follows that

$$R_1 = \int_0^{F_s} \ln\left(1 + \frac{\mu_3(f) + \mu_4(f)\lambda_2(f) + \mu_5(f) + K_2'}{\alpha_2(f)\left[K_1' - \mu_1(f) - \alpha_1(f)\mu_5(f)\right]}\right) df . \tag{26}$$

The equality above shows that R_1 can also be expressed as a function of dual variables $\mu_1(f), \mu_3(f), \mu_4(f), \mu_5(f)$, $\lambda_2(f), K_1', K_2'$, and channel conditions $\alpha_1(f), \alpha_2(f)$. Next, inspired by the derived necessary conditions, we develop a low-complexity sub-optimal algorithm to efficiently solve the problem and achieve reasonable performance.

Algorithm 1: Sub-optimal power allocation strategy[1]

---

input: $N_1(f), N_2(f), \alpha_1(f), \alpha_2(f), \mathrm{P}_1, \mathrm{P}_2$

initialization : $K^1 = \mathrm{P}_1, K^2 = 0, N = F_s/\Delta_f, \mathcal{F}_1 = \varnothing, \mathcal{F}_2 = \{1,2,\cdots,N\}, \mathcal{F} = \{1,2,\cdots,N\}, flag = 1, R_1' = 0$

procedure:

Calculate the transmit PSD $P_1^{nash}(f)$ and its achievable rate R_1^{nash} of the IW algorithm

while $flag = 1$ **do**

1) $M(f_s) = \dfrac{N_2(f_s) + \alpha_1(f_s)K^1}{N_1(f_s) + \alpha_2(f_s)K^2}, \forall f_s \in \mathcal{F}_2$

2) $f_s^{\max} = \arg\max_{f_s \in \mathcal{F}_2} M(f_s), \mathcal{F}_1' = \{f_s^{\max}\} \cup \mathcal{F}_1, \mathcal{F}_2' = \mathcal{F}_2 \setminus \{f_s^{\max}\}, f_s'^{\max} = \arg\max_{f_s \in \mathcal{F}_2'} M(f_s)$

3) $P_1(f) = waterfilling(\mathbf{P_1}, \mathcal{F}_1', N_1(f_s))$, $P_2(f) = waterfilling(\mathbf{P_2}, \mathcal{F}, N_2(f) + \alpha_1(f)P_1(f))$

4) $R_1'' = rate(P_1(f), N_1(f) + \alpha_2(f)P_2(f)), R_2' = rate(P_2(f), N_2(f) + \alpha_1(f)P_1(f))$

if $R_1'' \geq R_1'$

$\mathcal{F}_1 = \mathcal{F}_1', \mathcal{F}_2 = \mathcal{F}_2', K^1 = P_1(f_s^{\max}) \cdot |\mathcal{F}_1| / (|\mathcal{F}_1| + 1), K^2 = P_2(f_s'^{\max}) \cdot (|\mathcal{F}_2| + 1) / |\mathcal{F}_2|, R_1' = R_1''$

end if

if $R_1'' < R_1'$ or $\mathcal{F}_2' = \varnothing$

$flag = 0$

end if

end while

if $R_1^{nash} < R_1'$

return $P_1^{nash}(f)$ and R_1^{nash}

else

return $P_1(f) = waterfilling(\mathbf{P_1}, \mathcal{F}_1, N_1(f_s))$ and R_1'

end if

end procedure

Note that $\alpha_1(f)$ and $\alpha_2(f)$ are the uncontrollable channel conditions determined by the environment. For the integration over $[0, F_s]$ in (26) to be large, we expect that $\mu_5(f) > 0$ when $\alpha_2(f)$ is small and $\alpha_1(f)$ is large. Intuitively, user 1 should allocate its power such that, at the frequency band it occupies, the maximal rate is achieved

[1] $waterfilling(\mathbf{P}, \mathcal{F}, N(f))$ denotes the water-filling transmit PSD in set $\mathcal{F}$ of frequency bins which treats $N(f)$ as noise and is subject to the power constraint $\mathbf{P}$, and $rate(P(f), N(f))$ denotes the achievable rate of transmit PSD $P(f)$ with respect to the noise PSD $N(f)$.

with minimal noise and interference from the other user, i.e. $N_1(f)$ and $\alpha_2(f)$ are small. On the other hand, if user 2 wants to avoid some frequency channels, we expect that, in those channels, user 2 will experience weak channel condition and strong noise and interference, i.e. $N_2(f)$ and $\alpha_1(f)$ are large.

Based on the above arguments and the observations in Remark 3 and 4, we develop a practical sub-optimal power allocation strategy and summarize it as Algorithm 1. In this algorithm, we propose a metric $M(f_s)$ named "preference value", which is defined as a ratio between the noise and interference PSD the foresighted user and its competing user experience at frequency bin $f = f_s$. The "preference value" $M(f_s)$ reflects the incentive of the foresighted user to occupy that frequency bin. It wants to possess the frequency bin $f = f_s$ if $M(f_s)$ is large and keep away from it if $M(f_s)$ is small. The basic idea of algorithm 1 is to rank the frequency bins using this metric. Initially, user 1 owns no frequency bins and all the bins belong to user 2. According to Remark 3, user 1 water-fills the given allocated frequency bins. It continues moving the frequency bin with the largest preference value $M(f_s)$ from user 2 to user 1 until no rate improvement in R_1 can be achieved. This procedure is proposed based on the fact that for most channel realizations and sufficiently large P_1, there usually exist initially $f' \in \mathcal{F}_1 \; and \; f'' \in \mathcal{F}_2$ satisfying $P_1(f') > 0, P_2(f') = 0, P_1(f'') = 0,$ $P_2(f'') > 0,$ and

$$N_1(f') + P_1(f') > N_1(f'') + \alpha_2(f'')P_2(f''), \exists f' \in \mathcal{F}_1, f'' \in \mathcal{F}_2, \tag{27}$$

which leads to $\lambda_2(f') > 0$. Based on the argument in Remark 4, the performance might be further improved by adjusting $P_1(f)$. Note that we update K^1 and K^2 in order to calculate $M(f_s)$ in the following iteration. If the achievable rate of the above procedure is less than the IW approach, the strategic user will choose the Nash-equilibrium strategy, which guarantees that the performance of Algorithm 1 is no worse than the IW algorithm. The complexity of Algorithm 1 is only $\mathcal{O}\left(2F_s/\Delta_f\right)$, which reduces the complexity by a factor of $\mathcal{O}\left(\left(\mathbf{P_k}/\Delta_P\right)^{(F_s/\Delta_f)} \Big/ \left(2F_s/\Delta_f\right)\right)$ compared with the optimal case, which is considerably large if $\Delta_f \to 0$ and $\Delta_P \to 0$.

C. *Illustrative Results*

In this sub-section, we evaluate the performance of the proposed sub-optimal algorithm by comparing with the IW algorithm. We simulate a wireless system with 200 sub-carriers over the 10-MHz band. We assume that $\mathrm{P}_1 = \mathrm{P}_2 = 100$ and $\sigma_1(f) = \sigma_2(f) = 0.01$. To evaluate the performance, we tested 3×10^5 sets of frequency-selective fading channels where the Nash equilibrium exists, which are simulated using a four-ray Rayleigh model with the exponential power profile and 100 ns root mean square delay spread [18]. The simulated power of each ray is decreasing exponentially according to its delay. The total power of all rays of $H_{11}(f)$ and $H_{22}(f)$ is normalized as one, and that of $H_{12}(f)$ and $H_{21}(f)$ is normalized as 0.5.

Fig. 5 and 6 show the power allocations for both users using different algorithms. In IW algorithm, each user water-fills the whole frequency band by regarding its competitor's transmit PSD as background noise until the Nash equilibrium is achieved. In contrast, user 1 will not water-fill if choosing Algorithm 1. It will avoid the myopic behavior and improve its performance by considering user 2's channel state information (CSI) and power allocation strategy. For example, user 1 concentrates its power in the interval $[30,117]$ even though it can gain an immediate increase in R_1 by re-allocating some of its power in the region where the noise PSD is below its water-level, e.g. $[20,30]$ and $[117,140]$.

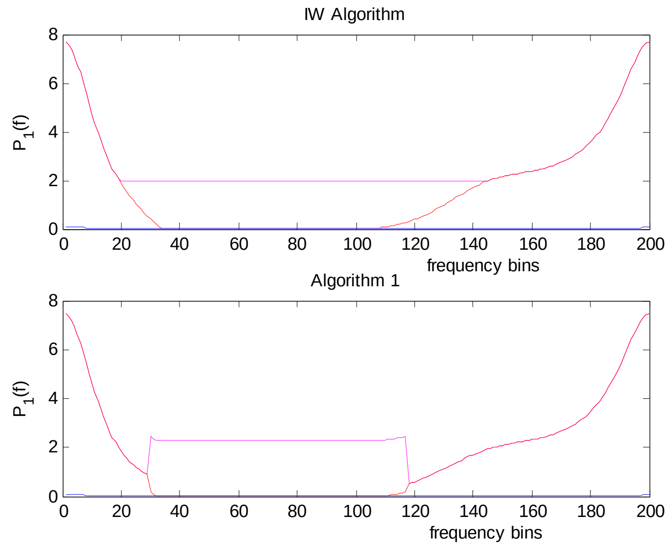

Fig. 5. User 1's power allocation using different algorithms.

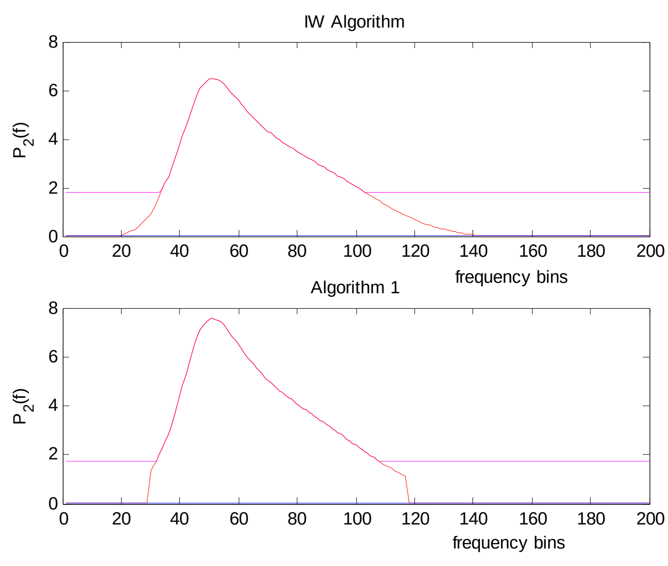

Fig. 6. User 2's power allocation using different algorithms.

Fig. 7 shows the simulated histogram of the ratio of R_1' over R_1^{nash}. If the ratio is larger than one, the proposed algorithm provides a strictly better performance than the IW algorithm. From the curve, Algorithm 1 achieves a higher rate R_1' most of the time. This is because Algorithm 1 mitigates the interference by explicitly taking the other user's CSI into account. On the other hand, there is a small probability of approximately 14% (shown as the shaded area in Fig. 7) that the rate R_1' achieved by Algorithm 1 is smaller than the rate R_1^{nash} in the IW algorithm. Note that in these cases, Algorithm 1 returns the same power allocations as the IW algorithm, which ensures a solution no worse than the IW algorithm. The average improvement of Algorithm 1 over the IW algorithm is 16.43%.

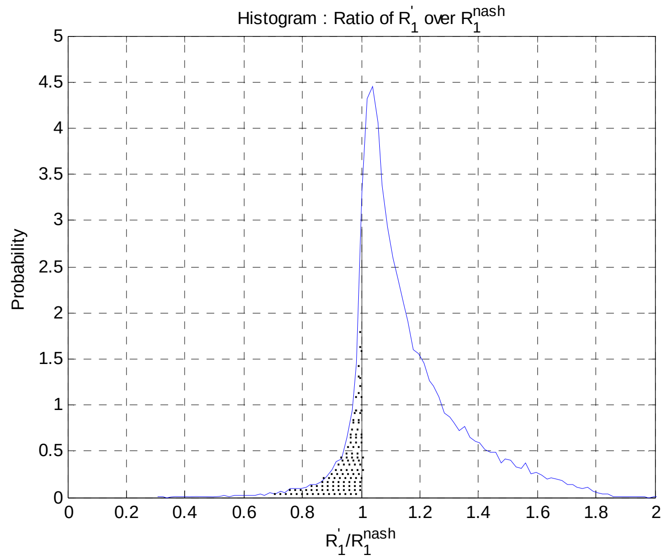

Fig. 7. Histogram for the ratio of $R_1' \,/\, R_1^{nash}$.

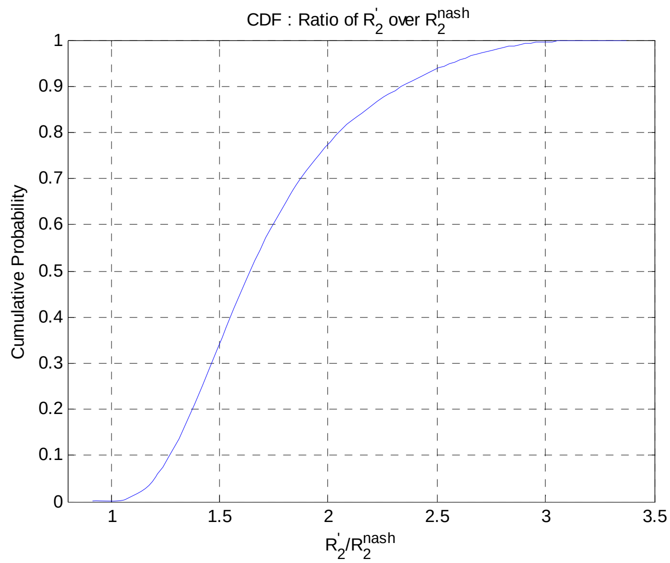

Fig. 8. cdf for the ratio of $R_2' \,/\, R_2^{nash}$.

The ratio between user 2's achievable rate R_2' in Algorithm 1 and R_2^{nash} in IW algorithm is shown in Fig. 8. It is surprising to find that, in very few cases with only a probability of 0.05%, Algorithm 1 will result in a rate R_2' smaller than R_2^{nash} in the IW algorithm. The average rate improvement for user 2 is 74%, which is significantly higher than that of user 1. This is because user 1 plays the Stackelberg equilibrium strategy that mitigates the interference caused to user 2. However, if user 1 plays the Nash strategy, user 2's achievable rate will be reduced immediately after user 1 update its transmit PSD, because its counterpart's water-filling behavior changes the noise-and-interference PSD that user 2 previously experiences.

D. Information Acquisition

It has been mentioned in the previous sections that, in order to formulate the problem in (11) and improve its performance, information about the competing user is indispensable. The information includes the other user's CSI and the strategy of allocating its power. In this section, we briefly discuss three possible ways of attaining the required information and optimizing power allocation.

The first possible way for users to gain the information is through channel state estimation and predictive modeling [10][19]. For example, user can choose the parametric approach to model the channel transfer function $\alpha_1(f)$ as

$$\alpha_1(f) = \frac{\left|\sum_{i=1}^{n_\alpha^1} A_i f^{-i}\right|^2}{\left|\sum_{i=1}^{n_\alpha^2} B_i f^{-i}\right|^2}, \tag{28}$$

in which $A_1, \cdots, A_{n_\alpha^1}, B_1, \cdots, B_{n_\alpha^2}$ are the parameters of the transfer function. Note that $N_1(f)$, $\alpha_2(f)$ and $N_2(f)$ can also be modeled in similar ways. Therefore, user 2's response to user 1's power allocation can be expressed as a function of these parameters. For example, if user 1 chooses a transmit PSD $P_1(f)$, the background noise and interference that it will experience after user 2 updates its power allocation is

$$N_1(f) + \alpha_2(f)\left[\frac{1}{K_2 - \lambda_2(f)} - N_2(f) - \alpha_1(f)P_1(f)\right]. \tag{29}$$

Therefore, if a user strategically changes the pattern of its power allocation, it can measure the resulted background noise and interference PSD and formulate a nonlinear parametric estimation problem. By solving this problem, the user can have an estimation of the required information.

An alternative of optimizing the power allocation is learning [10][20]. This technique enables users to wisely adapt their power allocations by repeatedly interacting with the environment without explicitly having the knowledge of CSI. Both no-regret learning and reinforcement learning are appropriate candidates for this approach [20].

Moreover, a myopic user has the incentive to provide the required information, because it is observed from numerical simulations that the achievable rate of the Nash-strategy player can be greatly improved by providing its own private information. Therefore, another possible choice in acquiring the necessary information is the information exchange between both users. This can also be viewed as the users' cooperative behavior to avoid mutual interference.

E. Extensions to Multi-user Games

The two-user problem formulation in Section III and IV can also be extended to the general multi-user case, although the analysis becomes much more involved. In this subsection, we briefly discuss its extensions and leave the details for future research. In the general case, we assume that the number of foresighted user is N_f and the number of myopic user is N_m. Previous sections focuses on the case in which $N_f = N_m = 1$. We discuss two more cases in the following.

1) $N_f = 1, N_m > 1$:

We can still use the bi-level programming formulation in this case. However, the lower-level problem needs to be substituted by a Nash equilibrium constraint:

$$\begin{aligned}
&\max_{P_1(f),P_i(f),\lambda_i(f),K_i} \int_0^{F_s} \ln\left(1+\frac{P_1(f)}{N_1(f)+\alpha_2(f)P_2(f)}\right)df \\
&s.t. \int_0^{F_s} P_1(f)\,df \le \mathrm{P}_1 \\
&\quad P_1(f) \ge 0 \\
&\quad \lambda_i(f) \ge 0\,, \qquad i = 2,\cdots,N_m+1,\ similarly\ hereinafter \\
&\quad P_i(f) \ge 0 \\
&\quad \lambda_i(f)P_i(f) = 0 \\
&\quad \int_0^{F_s} P_i(f)\,df \le \mathrm{P_i} \\
&\quad K_i \ge 0 \\
&\quad K_i\left(\mathrm{P_i} - \int_0^{F_s} P_i(f)\,df\right) = 0 \\
&\quad P_i(f) = \frac{1}{K_i-\lambda_i(f)} - N_i(f) - \sum_{j=1,j\ne i}^{K} \alpha_j(f)P_j(f)
\end{aligned} \tag{30}$$

As a general form of the investigated two-user case, the problem in (30) is non-convex and thus hard to handle. A possible approach is to develop sub-optimal solutions, e.g. the general multi-user version of Algorithm 1.

2) $N_f > 1, N_m \ge 1$:

If the number of foresighted user is larger than one, the single objective function in the original upper-level problem disappears and it becomes a multi-objective optimization problem. Note that using similar arguments in Theorem 1, we can easily show that the Nash equilibrium still exists in the follower's game. To these foresighted users, a reasonable outcome is to choose a Pareto-optimal operating point in the set

$$\mathcal{R}^{N_f} = \left\{\left(R_1,\cdots,R_{N_f}\right) : R_i > R_i^{nash},\ for\ all\ i = 1,\cdots,N_f\right\}, \tag{31}$$

where R_i^{nash} is user i's achievable rate if all the users are myopic. This point can be determined based on a negotiation among the multiple foresighted users. Cooperative game theory provides many solution concepts, e.g. Nash bargaining, for choosing the operating point [13]. Note that the overall game in these scenarios is neither purely cooperative nor purely non-cooperative, because the cooperation only exists among the foresighted users while non-cooperation still holds for the myopic players.

V. CONCLUSIONS

This paper considers the strategic behavior in determining the transmit power PSD for selfish users sharing a frequency-selective interference channel from the user point of view. We introduce the concept of Stackelberg equilibrium, model the two-user non-cooperative case as a bi-level programming problem, and derive the necessary optimality conditions. We show that a strategic user will avoid shortsighted Nash-strategy and improve its performance if it has the knowledge of the CSI and response strategy of the competing user. A low-complexity sub-optimal approach is proposed and numerical results show a substantial performance improvement. Possible operational methods for acquiring the necessary information are also discussed.

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