Relaying in Wireless Networks Modeled through Cooperative Game Theory

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Abstract—We analyze the capacity of wireless networks in the presence of cooperative relaying by using game theory instruments. Cooperation is approached as a cross-layer interaction between routing and medium access control; the latter is assumed to be based on memoryless time-division multiplexing, where the users have fixed probabilities to access the channel. We investigate the proper cooperation mechanism to be adopted by the users so that a gain is obtained by both those who have their transmission relayed to the final destination and also those who act as relays. Especially, we show how this gain can be directly related to a throughput improvement if the users follow specific access procedures. This incentive to cooperation works not only in an abstract information theoretic context, but also for a more direct personal advantage of the users. Numerical results are shown to confirm the validity of the proposed approach. The adopted methodology is useful both for modeling and performance analysis of communication links in relay networks, and for designing viable protocols which the users have incentives to follow.

I. INTRODUCTION

RELAY networks have been widely studied in information theory [1]. In particular, the relay channel represents one of the most common scenarios studied. Several theoretical results about the capacity of this basic network have been available in the literature since long [2], and others keep being proposed even very recently [3], [4].

At the same time, game theory [5] is being employed more and more every day by wireless telecommunication engineer. From a game theoretic perspective, the relay channel is a natural scenario to evaluate cooperation [6], which may improve the communication for users experiencing bad quality on their direct link to the destination. The concept of cooperation has a precise meaning in game theory mostly through the application of coalitional games, which we will exploit in this paper.

Yet, we do not apply these game theoretic concept to a pure information theory scenario. Instead, we focus on a precise application of the relay channel, with a real network protocol involving packet exchanges and retransmission, through cooperative Automatic Repeat reQuest (ARQ). Our proposal involves a cross-layer solution spanning on the data link (and more specifically, both channel access protocol design and ARQ) and network layers.

The contribution in this sense is two-fold. First, we give an analytical characterization of the performance of the relay channel from a game theoretic perspective. Second, we are also able to define a cooperative protocol where the overall network throughput is improved. However, differently from classic results of information theory where the capacity enhancement simply stems from spontaneous cooperation, i.e., the nodes collaborate out of goodwill, we precisely model also the node behavior so that their cooperation is not taken for granted, but rather promoted through a careful design of incentives to all the involved nodes (both those exploiting cooperative relays, and those who aid others, e.g., by forwarding their packets).

In this sense, this throughput enhancement is achievable in practical cases, and is truly beneficial for those nodes who have poor channel conditions, e.g., those at the cell edge, not only because somebody improves their throughput, but also since other nodes are actually willing to cooperate with them, since they see a concrete benefit in it.

The rest of this paper is organized as follows. In Section II we give some notions of coalitional game theory. In Section III we formulate our game theoretic resource allocation strategy for a relay channel based on coalitional games and involving Medium Access Control (MAC), routing, and throughput subdivision. We present some numerical results in Section IV, then we sketch possible extensions in Section V and finally we conclude in Section VI.

II. COALITIONAL GAME TERMINOLOGY AND NOTATION

Cooperative game theory [6] is a branch of game theory that provides analytical tools to study the behavior of rational players when they cooperate. The main area of cooperative games is represented by coalitional games [7], defined as a pair (N,v), where $N = \{1, ..., N\}$ is a discrete set of players and v is a function that quantifies the value of a coalition in a game. Each coalition $S \subseteq N$ behaves as a single player, competing against other coalitions in order to obtain a higher value of v. A coalitional game may have the following properties:

Property 1. (Characteristic form) The value of a coalition $S$ depends only on who are the members of that coalition, regardless of other coalitions

Property 2. (Transferable utility) The value of a coalition is a real number, representing the total utility achieved by the coalition, and it can be arbitrarily divided among its members

For coalitional games satisfying properties 1 and 2, the value $v : 2^N \rightarrow \mathbb{R}$ is a function that assigns to each coalition $S$ the
total utility achieved by it. The utility value can be arbitrarily divided among the coalition members and the amount of utility that a player \( i \in S \) receives, \( x_i \), is the player’s payoff. A payoff allocation is a vector \( x \in \mathbb{R}^{|S|} \) (where \( |S| \) is the cardinality of the set \( S \)) whose elements are the payoffs of players belonging to the coalition; in other words, it represents a redistribution of the total utility.

Another interesting property that a coalitional game may have is super-additivity, that for a game with properties 1 and 2 assumes the following form:

**Property 3.** (Super-additivity)

\[ v(S_1 \cup S_2) \geq v(S_1) + v(S_2) \quad \forall S_1, S_2 \subset \mathcal{N} \text{ s.t. } S_1 \cap S_2 = \emptyset \]

The super-additivity property expresses in mathematical terms that formation of a larger coalition is always beneficial. Hence, for those games where it holds, the players are encouraged to stick together, forming the grand coalition \( \mathcal{N} \).

For a game having all properties listed before, the main aspects to analyze are:

- finding a redistribution of the total utility \( v(\mathcal{N}) \) such that the grand coalition is stable, i.e., no group of players has an incentive to leave the grand coalition
- finding fairness criteria for the redistribution of the total utility
- quantifying the gain that the grand coalition can obtain with respect to non cooperative behaviors

A payoff allocation is *group rational* if \( \sum_{i=1}^{N} x_i = v(\mathcal{N}) \) and it is *individually rational* if \( x_i \geq v(\{i\}) \forall i \), i.e., if every player does not obtain a lower utility by cooperating than by acting alone. A payoff allocation having both properties is said to be an *imputation*.

The concept of *core* is also very important. It is defined as the set of imputations that guarantee that the grand coalition is stable, i.e., all payoff allocations where no group of players \( S \subset \mathcal{N} \) have an incentive to refuse the proposed payoff allocation, leaving the grand coalition and forming coalition \( S \) instead. Mathematically speaking,

\[ C = \left\{ x \text{ s.t. } \sum_{i=1}^{N} x_i = v(\mathcal{N}), \sum_{i \in S} x_i \geq v(S) \forall S \subset \mathcal{N} \right\} \quad (1) \]

Indeed, the core may be empty, in which case the grand coalition is not stable. The existence of the core ought to be checked case by case, possibly exploiting some categories of games where the existence is guaranteed [5, Ch. 13].

### III. Problem Statement

We consider the scenario of two nodes, A and B, which want to communicate with an Access Point, Z, as represented in Fig. 1. \( \gamma_A, \gamma_B, \gamma_{AB} \) and \( \gamma_{BA} \) are the signal to noise ratios (SNRs) between A and Z, B and Z, A and B and B and A respectively. We suppose that:

- \( \gamma_A, \gamma_B, \gamma_{AB} \) and \( \gamma_{BA} \) are constant over time, i.e., time invariant channels and fixed transmission powers of A and B. Actually, also slow time-varying channels can be included in this analysis. Moreover, without losing generality, we suppose \( \gamma_B \geq \gamma_A \)
- node A and B always have packets to transmit to Z
- time division multiple access (TDMA) is adopted, assuming that the access point Z manages it in a centralized manner. The process that assigns a slot to a new packet is independent identically distributed (i.i.d.) with \( P_A \) and \( P_B = 1 - P_A \) the probabilities to assign the slot to node A or node B respectively
- we consider an ARQ retransmission scheme with at most 1 retransmission (maximum total number of transmission \( F = 2 \)). In subsection III-C we will see how the analysis can be generalized for multiple retransmissions
- we focus on the uplink connection from the users to the access point, therefore we neglect the traffic from Z to nodes A and B

Once a new packet for node A is scheduled, the non cooperative transmission process of this packet can be represented by the Markov Chain in Fig. 2. Absorbing states \( R_A \) and \( N_A \) represent the events that the packet is received or not received by Z. Other states represent the actual number of packet transmissions performed by user A, so the initial state is state \( 1_A \). We define \( q(\gamma) \) as the probability that a packet is correctly received when the SNR is \( \gamma \). This function depends on the modulation scheme used and on the packet length. We define \( P_{RA}^{NC} \) as the probability to be absorbed in state \( R_A \) in the non cooperative case.

\[ P_{RA}^{NC} = q(\gamma_A) + (1 - q(\gamma_A))q(\gamma_A) \quad (2) \]

We define \( N_A^{NC} \) as the average number of transmissions of the packet in the non cooperative case.

\[ N_A^{NC} = q(\gamma_A) + 2(1 - q(\gamma_A)) = 2 - q(\gamma_A) \quad (3) \]

The transmission of a packet in the non cooperative case, from the choice of the user to packet reception (or to the maximum number of transmissions), is represented in Fig. 3. Initial state \( I \) represents the selection of the user that can transmit the packet. Users A and B are selected with probabilities \( P_A \) and \( P_B \), respectively. Once either user is selected, the
transmission process evolves analogously to the Markov Chain shown in Fig. 2. When the packet is correctly received by Z or the maximum number of transmissions is reached (i.e., an absorbing state is entered), another new packet is considered, again starting from state 1; a new user is selected, a new packet is transmitted, and so on. Renewal theory [8] allows to study this kind of situations. The beginning of each renewal cycle constitutes a regenerative epoch of the Markov process. The asymptotic metrics of the network can be obtained by studying the average behavior of the Markov process. The asymptotic bit rate of each user is calculated by considering the average number of transmitted bits and dividing it by the average time to absorption:

$$BR^NC_A = P_A P^NC_A N_{bit} / T_{pkt}$$ (4)

where $N^NC = P_A N^NC_A + P_B N^NC_B$ is the average number of transmissions for packet, $N_{bit}$ is the number of bits in a packet and $T_{pkt}$ is the time needed for a single packet transmission.

Finally, the asymptotic bit rate of the network for the non cooperative scenario is given by:

$$BR^NC = BR^NC_A + BR^NC_B$$ (5)

A. Cooperative ARQ

Now the performance of the network is evaluated for the case where cooperation is active, by means of the coalitional game framework. Nodes can cooperate, helping other nodes to retransmit a packet not correctly received by the access point.

We assume that the game satisfies properties 1 and 2. Note that in the two-user case, the former property is automatically satisfied. However, the property still holds true even if the analysis is extended to a network with more than two users, since the TDMA approach guarantees that different coalitions do not interact: each coalition tries to obtain the maximum throughput by using the slots assigned exclusively to it. For what concerns property 2, the problem of the throughput redistribution is addressed in subsection III-B.

The value $v(\cdot)$ of the coalitional game is the throughput obtained by each coalition. In a two-user case, three coalitions are possible: the two coalitions formed by the single users, $A$ and $B$, and the coalition formed by both users, i.e., the grand coalition $N = \{A, B\}$. The value of each coalition is:

$$v(\{A\}) = BR^NC_A \quad v(\{B\}) = BR^NC_B$$

$$v(N) = BR^C = BR^C_A + BR^C_B$$ (6)

where $BR^C_A$ and $BR^C_B$ are the respective asymptotic bit rates for user A and B in the cooperative scenario.

During a cooperative transmission, the packet transmitted by a node is heard by Z and the other user who actively cooperates. In our formulation, cooperation implies that, if a packet is not correctly received by Z, its retransmission is carried out by the user who has the better signal to noise ratio, provided that it received the packet correctly. Thus, the transmission process for user A can be represented by the Markov Chain in Fig. 4.

The value of each coalition is:

$$v(\{A\}) = BR^NC_A \quad v(\{B\}) = BR^NC_B$$

Fig. 4. Cooperative transmission process of a packet of user A

Therefore the game satisfies also property 3.
B. Throughput subdivision

Now we want to find a payoff allocation that belongs to the core and is fair under certain parameters. Note that, for a super-additive two player game, the core is not empty and coincides with the set of imputations.

In the considered game, the set of imputations is given by:

\[ x_A = BR^{NC}_A + (1 - w)(BR^{C}_A - BR^{NC}_A) \]
\[ x_B = BR^{NC}_B + w(BR^{C}_B - BR^{NC}_B) \] (10)

Here a cooperation weight denoted as \( w \) is introduced to determine the throughput share that each user gets. Note that \( w \) is introduced to give a proper incentive to both users to cooperate. In fact, only user A, whose channel quality to Z determines the throughput share that each user gets. Note that, for \( B \). Throughput subdivision

\[ P^{C}_{RA} = q(\gamma_A) + \sum_{i=2}^{F} P(T_{A,Z} = i) \]
\[ = q(\gamma_A) + \sum_{i=2}^{F} \left[ \sum_{k=1}^{i-1} P(T_{A,B} = k)P(T_{A,Z} = i|T_{A,B} = k) \right. \]
\[ + P(T_{A,B} > i - 1)P(T_{A,Z} = i|T_{A,B} > i - 1) \right] \]
\[ = q(\gamma_A) + \sum_{i=2}^{F} \sum_{k=1}^{i-1} q(\gamma_{AB})(1 - q(\gamma_{AB}))^{k-1}q(\gamma_B) \]
\[ \cdot (1 - q(\gamma_A))^k(1 - q(\gamma_B))^{i-k-1} \]
\[ + (1 - q(\gamma_{AB}))^{i-1}q(\gamma_A)(1 - q(\gamma_A))^{i-1} \] (13)

\[ N^{C}_A = q(\gamma_A) + \sum_{i=2}^{F-1} iP(T_{A,Z} = i) + FP(T_{A,Z} > F - 1) \]
\[ = q(\gamma_A) + \sum_{i=2}^{F-1} \left[ \sum_{k=1}^{i-1} P(T_{A,B} = k) \right. \]
\[ \cdot P(T_{A,Z} = i|T_{A,B} = k) + P(T_{A,B} > i - 1) \]
\[ + P(T_{A,Z} = i|T_{A,B} > i - 1) \right] \]
\[ + F \sum_{k=1}^{F-2} P(T_{A,B} = k) \]
\[ + P(T_{A,Z} > F - 1|T_{A,B} = k) + P(T_{A,B} > F - 2) \]
\[ + P(T_{A,Z} > F - 1|T_{A,B} > F - 2) \]
\[ = q(\gamma_A) + \sum_{i=2}^{F-1} \sum_{k=1}^{i-1} q(\gamma_{AB})(1 - q(\gamma_{AB}))^{k-1}q(\gamma_B) \]
\[ \cdot (1 - q(\gamma_A))^k(1 - q(\gamma_B))^{i-k-1} + (1 - q(\gamma_{AB}))^{i-1} \]
\[ \cdot q(\gamma_A)(1 - q(\gamma_A))^{i-1} + F \sum_{k=1}^{F-2} q(\gamma_{AB}) \]
\[ + (1 - q(\gamma_{AB}))^{F-2}(1 - q(\gamma_A))^{F-1} \] (14)

C. Multiple retransmission generalization

Previously, we have found the mathematical expressions for the asymptotic bit rates in the non-cooperative and cooperative cases for \( F = 2 \). This can be generalized to \( F > 2 \) as follows.

For the non-cooperative case we obtain:

\[ P^{NC}_{RA} = q(\gamma_A) + \sum_{i=2}^{F} q(\gamma_A)(1 - q(\gamma_A))^{i-1} \]
\[ N^{NC}_A = q(\gamma_A) + \sum_{i=2}^{F-1} iq(\gamma_A)(1 - q(\gamma_A))^{i-1} + \]
\[ + F(1 - q(\gamma_A))^{F-1} \] (12)

For the cooperative case, let \( T_{A,Z} \) and \( T_{A,B} \) be the times required by \( Z \) and \( B \), respectively, to correctly receive the packet from \( A \). Then, we have

\[ P^{C}_{RA} = q(\gamma_A) + \sum_{i=2}^{F} P(T_{A,Z} = i) \]
\[ = q(\gamma_A) + \sum_{i=2}^{F} \left[ \sum_{k=1}^{i-1} P(T_{A,B} = k)P(T_{A,Z} = i|T_{A,B} = k) \right. \]
\[ + P(T_{A,B} > i - 1)P(T_{A,Z} = i|T_{A,B} > i - 1) \right] \]
\[ = q(\gamma_A) + \sum_{i=2}^{F} \sum_{k=1}^{i-1} q(\gamma_{AB})(1 - q(\gamma_{AB}))^{k-1}q(\gamma_B) \]
\[ \cdot (1 - q(\gamma_A))^k(1 - q(\gamma_B))^{i-k-1} \]
\[ + (1 - q(\gamma_{AB}))^{i-1}q(\gamma_A)(1 - q(\gamma_A))^{i-1} \] (13)

\[ N^{C}_A = q(\gamma_A) + \sum_{i=2}^{F-1} iP(T_{A,Z} = i) + FP(T_{A,Z} > F - 1) \]
\[ = q(\gamma_A) + \sum_{i=2}^{F-1} \left[ \sum_{k=1}^{i-1} P(T_{A,B} = k) \right. \]
\[ \cdot P(T_{A,Z} = i|T_{A,B} = k) + P(T_{A,B} > i - 1) \]
\[ + P(T_{A,Z} = i|T_{A,B} > i - 1) \right] \]
\[ + F \sum_{k=1}^{F-2} P(T_{A,B} = k) \]
\[ + P(T_{A,Z} > F - 1|T_{A,B} = k) + P(T_{A,B} > F - 2) \]
\[ + P(T_{A,Z} > F - 1|T_{A,B} > F - 2) \]
\[ = q(\gamma_A) + \sum_{i=2}^{F-1} \sum_{k=1}^{i-1} q(\gamma_{AB})(1 - q(\gamma_{AB}))^{k-1}q(\gamma_B) \]
\[ \cdot (1 - q(\gamma_A))^k(1 - q(\gamma_B))^{i-k-1} + (1 - q(\gamma_{AB}))^{i-1} \]
\[ \cdot q(\gamma_A)(1 - q(\gamma_A))^{i-1} + F \sum_{k=1}^{F-2} q(\gamma_{AB}) \]
\[ + (1 - q(\gamma_{AB}))^{F-2}(1 - q(\gamma_A))^{F-1} \] (14)

It is easy to see that \( P^{C}_{RA} > P^{NC}_{RA} \) and \( N^{C}_A \leq N^{NC}_A \), therefore \( BR^{C}_A > BR^{NC}_A \). Thus, the reasonings done in subsection III-B are still valid even for \( F > 2 \).
the case where the whole throughput gain harvested from the cooperation is $= 0$ when both users get a cooperation gain, and have the proper (absolute) throughput gain. The larger its non-cooperative throughput. The larger $\gamma_A$, the higher the cooperation throughput of $A$. Cooperation from $B$ allows to save fewer slots than the previous case, but it is still beneficial to relay a packet. Nevertheless, our preliminary evaluations hinted that a cooperation gain still holds also in this scenario.

Finally, given the promising results found for a simple two-node network, it is surely worth investigating an extension to larger networks, possibly with multi-hop relaying. This development, currently under evaluation, implies both an evaluation on a larger scale and also the definition of a proper negotiation protocol to establish the cooperation roles [10].

VI. CONCLUSIONS

We presented an analysis of a relay network by coalitional game theory, where a MAC/network protocol is designed by giving to all the users benefits when they collaborate with each other. This is concretely realized by proving the theoretical condition for a stable core and properly designing a suitable throughput subdivision among the users. The resulting solution properly accounts for modeling aspects such as users’ selfishness and thus the need for proper incentives to cooperation.

REFERENCES


